Non-Gaussianity in island cosmology

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In this paper we fully calculate the non-Gaussianity of primordial curvature perturbation of the island universe by using the second order perturbation equation. We find that for the spectral index $n_s \simeq 0.96$, which is favored by current observations, the non-Gaussianity level $f_{\rm NL}$ seen in an island will generally lie between 30 and 60, which may be tested by the coming observations. In the landscape, the island universe is one of anthropically acceptable cosmological histories. Thus the results obtained in some sense mean the coming observations, especially the measurement of non-Gaussianity, will be significant to clarify how our position in the landscape is populated.

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The vacua in the landscape will be populated during eternal inflation (see e.g. Refs. [1–3] for recent reviews). From an anthropical viewpoint, how a vacuum like ours is populated may be more crucial, since the history of populating determines our observations. Recently, it has been argued that the island cosmology in the landscape can be consistent with our real world [4] (see earlier Refs. [5,6] for discussions based on the background with the cosmological constant observed). The large fluctuations with the null energy condition violation can stride over the barrier between vacua, and directly create some regions full with radiation, i.e. islands, in new or baby vacua. These islands will evolve with the standard cosmology, some of which under certain conditions may correspond to our observable universe (see Ref. [7] for details). From the usual viewpoint, in order to have a universe like ours in the landscape, the slow roll inflation with adequate period is generally required [8]. This can be implemented only by a potential with a long plain above the corresponding minimum, which obviously means a fine-tuning, since the regions with such potentials are generally expected to be quite rare in a random landscape. Meanwhile the island can actually emerge for any potential, independent of whether the potential has a long plain. Thus in principle as long as we can wait, the islands of observable universes will be able to appear in any corner of the landscape.

The island universe model brings a distinct anthropically acceptable cosmological history. Thus it is quite interesting to ask how we can determine whether we live in an island or in a reheating region after slow roll inflation, which might be significant to understand why and how our vacuum in the landscape is selected. In principle, this can be judged by the observations of primordial perturbations. However, in the level of first order scalar perturbation, the island universe is actually degenerated with the slow roll inflation, which in some sense is a reflection of the duality between their background evolutions, i.e. between the slow expansion [9] and the nearly exponent expansion (see Refs. [7,10] for details). Thus in principle it is hardly possible to distinguish them by the spectrum index and amplitude of curvature perturbation. However, recently it has been found that the non-Gaussianity of perturbation in island cosmology is generally large [7], while that predicted by the simple slow roll inflation model is quite small. Thus in this sense the non-Gaussianity might be a powerful discriminator.

The current bound placed by the Five-Year Wilkinson Microwave Anisotropy Probe (WMAP5) is $-9 < f_{\rm NL} < 111$ [11], which seems to slightly prefer a net positive $f_{\rm NL}$, though $f_{\rm NL} = 0$ is still at 95% confidence. The analysis of a large-scale structure combined with the WMAP5 gave the further limit $-1 < f_{\rm NL} < 70$ [12]. Further, the future Planck satellite will be expected to give $\Delta f_{\rm NL} \sim 5$ [13]. These valuable observations are placing the island universe in an interesting and tested regime. In Ref. [7], the non-Gaussianity is roughly estimated in terms of three point function, which is determined only by the cubic interaction term of field. However, this neglects other sources for non-Gaussianity. Here the curvature perturbation is actually induced by the entropy perturbation, and thus the nonlinear relation between the curvature perturbation and the entropy perturbation can also contribute to the non-Gaussianity. This is reflected in the second order perturbation equation correlating both. It seems that the coming observations, especially the measurement of non-Gaussianity, have had the ability to identify the cosmological history in which we live, and thus show how our position in the landscape is populated. Thus in order to have a definite prediction tested by coming and precise observations, a full study for the non-Gaussianity of the island universe is obviously urgently required. This will be done in this paper by applying the second order perturbation equation.

When the island emerges, the change of local background may be depicted by $\epsilon \ll -1$, which is determined by the evolution of local Hubble parameter "h," where "local" means that the quantities, such as the scale factor "a" and "h," denote the values of the null energy condition-violating region only, and $\epsilon \ll -1$ means the energy density of the local emerging island is rapidly increased. In order to phenomenologically describe and simulate this behavior, we appeal the field same with the normal scalar fields but with the minus sign in their kinetic terms, which is usually called a ghost field. The evolution of such a ghost field is climbing up along its potential and the steeper its potential is, the faster it climbs, which is determined by the property of this kind of field, e.g. Ref. [14]. Thus in Ref. [6], it has been argued that such a field can be suitable for depicting the emergence of an island. In the scenario of an island universe, as detailed in Refs. [4,7], initially the background is de Sitter (dS)'s, and then in some regions the islands emerge, in which the local background experiences a jump. There actually are not ghost fields presented in the entire scenario, since this phenomenon is quantum. We introduce the ghost field artificially, since we found that in the classical sense it can describe the evolution of the emerging island well, which enables us to semiclassically explore the island universe model and its possible predictions. In this sense, the ghost field introduced serves only the evolution of background, by which we can do some analytical and numerical calculations for primordial perturbations. Further, for this purpose, this introduced field should be required to satisfy some conditions which ensures the scenario of the island universe is not changed; for example, it is not expected to participate in other quantum processes.

We assume that ϵ is constant during the emergence of an island for simplicity. Thus we can have the scale factor

$$a \sim \frac{1}{(-t)^{1/|\epsilon|}} \sim h^{1/|\epsilon|}, \tag{1}$$

which is nearly unchanged since $|\epsilon| \gg 1$, which in some sense is also why we call such a fluctuation an emergent island (see Fig. 1 in Ref. [15]). Thus the efolding number of mode with some scale $\sim 1/k$ leaving the horizon before the thermalization can be written as $\mathcal{N} \simeq \ln(\frac{h_e}{h_i})$ [6], where the subscript "i" and "e" denote the initial and end values of relevant quantities, respectively. The observable cosmology requires $\mathcal{N} \sim 50$. Thus in order to have an adequate efolding number, an adequately low scale of the parent vacuum should be selected.

The emergence of an island in the landscape will generally involve the upward fluctuations of a number of fields, or moduli. Thus it is inevitable that there are entropy perturbations, which can source the curvature perturbation. The method that we use to calculate the curvature perturbation is similar to that applied in ekpyrotic models [16,17] (see also [18] and earlier Refs. [19,20]). The calculation of the non-Gaussianity is similar to that implemented in Refs. [21–24]. The difference lies in the character of the fields used. Here, as has been mentioned, what we have used are the normal scalar fields but with the minus sign in their kinetic terms. Thus compared to the corresponding equations for perturbations of normal scalar fields, there will be some slight discrepancies in relevant perturbation equations; i.e., a difference of sign before some terms, however, will lead to distinct results.

In principle, for both such fields, the rotation in field space can be made, which decomposes fields into the field φ along the motion direction in field space, and the field s orthogonal to the motion direction [25]. In this case the evolution of background will be determined only by φ, whose potential is relevant only with the background parameter $|\epsilon|$, while s will contribute only to the entropy perturbation (see Ref. [7] for details). Here $v_k = a\delta s_k$ is set for our convenience, and thus $v_k^{(i)} = a\delta s_k^{(i)}$, where the superscript denotes the ith order perturbation. Hereafter, we will study the equations of perturbations with this replacement. The equation of first order entropy perturbation and the detailed analysis of solutions have been presented in Refs. [7,10], which thus will be neglected here. In terms of Ref. [7], the spectrum index of δs field is

$$n_{\delta s} - 1 \simeq \frac{2}{\epsilon}, \tag{2}$$

which means that the spectrum of entropy perturbation is nearly scale invariant with a slightly red tilt, since $\epsilon \ll -1$. Here we have assumed that usual the quantum field theory can be applied even for such ghost fields.[1] The amplitude of perturbation spectrum is

$$\mathcal{P}_{\delta s}^{1/2} = k^{3/2}\left|\frac{v_k^{(1)}(\eta_e)}{a}\right| \simeq \frac{1}{\sqrt{2}a(-\eta_e)}, \tag{3}$$

which is calculated at the end time η_e of null energy violating evolution, i.e. the emergence of an island, since the amplitude of perturbation on the super horizon scale is increased all along up to the end [7], where η is conformal time. Noting a is nearly unchanged, which can be given from Eq. (1) since $|\epsilon| \gg 1$ and is actually a reflection that the island is emerging very quickly, we have $a\eta \simeq t$; thus the amplitude of spectrum can be rewritten as $\mathcal{P}_{\delta s} \simeq \frac{1}{2(-t_e)^2}$. We can see that these results are determined only by the evolution of background during the emergence of the island but not dependent on other details.

The entropy perturbation can source the curvature perturbation by $\dot{\mathcal{R}}^{(1)} \simeq \frac{2h\dot{\theta}}{\dot{\varphi}}\delta s^{(1)}$ [25]. Thus if $\dot{\theta} = 0$, i.e. the motion in field space is a straight line, the entropy pertur-

[1]Here we need a normal quantization condition, such as the usual field theory, to set initial conditions for primordial perturbation, which seems to contradict with that of a ghost field. However, this might be justified as follows. Initially the background is dS's, in which there are not ghost fields; thus in principle the normal quantization condition of usual field theory can be applied. Then the island emerges, and the local background enters into a null energy violating evolution, which the ghost field is introduced to describe. Thus the primordial perturbation induced by such fields must have a normal quantization condition as its initial condition, or it cannot be matched to that of the initial dS background.

bation will not couple to the curvature perturbation. However, when there is a sharp change of direction of field motion, $\dot{\theta}$ must not be equal to 0; in this case $\dot{\mathcal{R}}^{(1)}$ will inevitably obtain a corresponding change induced by δs. We take the rapid transition approximation,[2] which means that all relevant quantities at a split second before the thermalization are nearly unchanged, except θ changes from its initial fixed value $\theta = \theta_*$ to $\theta \simeq 0$. Thus we have

$$\mathcal{R}^{(1)} \simeq \frac{2h_e\theta_*}{\dot{\varphi}}\delta s^{(1)}, \qquad (4)$$

which means that $\mathcal{R}^{(1)}$ acquires a jump induced by the entropy perturbation $\delta s^{(1)}$ and thus inherits the nearly scale invariant spectrum of $\delta s^{(1)}$ given by Eq. (2). We can substitute Eq. (3) and $\frac{h^2}{\dot{\varphi}^2} = \frac{4\pi}{|\epsilon|}$ into Eq. (4), and obtain the resulting amplitude of curvature perturbation as $\mathcal{P}_{(\delta s \to \mathcal{R})} \simeq 16\theta_*^2 \cdot |\epsilon| \frac{h_e^2}{\pi}$, which is approximately $|\epsilon| h_e^2$. We can see that it and Eq. (2) can be related to those of the usual slow roll inflation by replacing ϵ as $-\frac{1}{\epsilon}$, which actually exactly gives the spectral index and amplitude of slow roll inflation to the first order of slow roll parameters, noting that this duality is valid not only for constant $|\epsilon|$ [10] but also when $|\epsilon|$ is changed [7].

The intrinsic non-Gaussianity in entropy perturbation can be generated during the emergence of an island. This can be obtained by considering the motion equation of second order entropy perturbation, which, when $\dot{\theta} = 0$, is

$$v_k^{(2)\prime\prime} + (k^2 - f(\eta))v_k^{(2)} + g(\eta)(v_k^{(1)})^2 = 0, \qquad (5)$$

where $f(\eta) = \frac{a''}{a} + a^2\mathcal{V}_{(2)}$ and $g(\eta) = -\frac{a\mathcal{V}_{(3)}}{2}$. The sign between both terms of $f(\eta)$ is plus while the sign in $g(\eta)$ is minus, which is just the reverse of that of the normal scalar field [26]. Here $\mathcal{V}_{(i)}$ denotes the i times derivative for s, and $\mathcal{V}_{(2)} \simeq \frac{2}{a^2\eta^2}$ and $\mathcal{V}_{(3)} \simeq \frac{8\alpha\sqrt{\pi}}{a^2\eta^2}\sqrt{|\epsilon|}$ for $|\epsilon| \gg 1$, which can be obtained by Eq. (5) in Ref. [7], where the constant $\alpha \equiv \sqrt{1/x} - \sqrt{x})$, and $\theta = \mathrm{arctg}(x)$ is determined by the cubic interaction of potential on s field. We care only about the solution at long wavelength. Thus taking $k \to 0$, we can obtain

$$v_k^{(2)} \simeq \frac{\alpha\sqrt{\pi|\epsilon|}(v_k^{(1)})^2}{a}. \qquad (6)$$

Thus we have $\delta s^{(2)} \simeq \alpha\sqrt{\pi|\epsilon|}(\delta s^{(1)})^2$, since $v_k^{(i)} = a\delta s^{(i)}$.

The curvature perturbation induced by the second order of entropy perturbation can be given as

$$\dot{\mathcal{R}}^{(2)} \simeq \frac{2h\dot{\theta}}{\dot{\varphi}}\delta s^{(2)} - \frac{h(4\dot{\theta}^2 - \mathcal{V}_{(2)})}{\dot{\varphi}^2}(\delta s^{(1)})^2 \qquad (7)$$

on large scale. The only difference here from Ref. [26] is that there is a minus sign before $\mathcal{V}_{(2)}$. The non-Gaussianity is generated when modes are outside the horizon, and thus here the non-Gaussianity is expected to be local. The level of non-Gaussianity is usually expressed in terms of parameter f_{NL} as defined in Refs. [27,28]:

$$\begin{aligned} f_{\mathrm{NL}} &= -\frac{5\mathcal{R}^{(2)}}{3(\mathcal{R}^{(1)})^2} \\ &\simeq -\frac{5}{3(\mathcal{R}^{(1)})^2}\int\left(\frac{2h\dot{\theta}}{\dot{\varphi}}\delta s^{(2)} - \frac{h(4\dot{\theta}^2 - \mathcal{V}_{(2)})}{\dot{\varphi}^2}(\delta s^{(1)})^2\right)dt, \end{aligned} \qquad (8)$$

where Eq. (7) has been applied.

The terms in Eq. (8), proportional to $\dot{\theta}$, are not 0 only at a split second before the thermalization. Thus the rapid transition approximation can be applied in the calculations. The first term corresponds to the intrinsic non-Gaussianity of δs. This can be inherited by the curvature perturbation, which is

$$-\frac{5}{3(\mathcal{R}^{(1)})^2}\int\frac{2h\dot{\theta}}{\dot{\varphi}}\delta s^{(2)}dt \simeq -\frac{5\alpha}{12\theta_*}|\epsilon|, \qquad (9)$$

where Eqs. (4) and (6) have been used. This result in fact equals that calculated by using the three point function [7]. The second term in Eq. (8) corresponds to the nonlinear correction for the linear relation between $\mathcal{R}$ and δs. It will also contribute to the non-Gaussianity of the curvature perturbation, which is

$$\frac{5}{3(\mathcal{R}^{(1)})^2}\int\frac{h(4\dot{\theta}^2 - \mathcal{V}_{(2)})}{\dot{\varphi}^2}(\delta s^{(1)})^2dt \simeq \frac{5}{6\theta_*}|\epsilon|, \qquad (10)$$

where $\mathcal{V}_{(2)} \simeq \frac{2}{a^2\eta^2} \simeq \frac{2}{|t|^2}$ for $|\epsilon| \gg 1$, and also we set $\dot{\theta} \simeq \frac{1}{|t|}$ for calculation. The latter means the period Δt_* of change of θ can be deduced from $\int\dot{\theta}dt \simeq 1$. Thus we have $\Delta t_* \simeq |t_e| \simeq \frac{1}{|\epsilon|h_e}$, noting that t is negative. While the total time that the emergence of an island lasts is $T \simeq \frac{1}{|\epsilon|h_i}$ [6,7], it is far shorter than one Hubble time since $|\epsilon| \gg 1$ and thus is consistent with the claim that the emergence of the island is a quantum fluctuation in the corresponding dS background. The adequate efolding number requires $h_e/h_i \gtrsim e^{50}$, and thus we have $T \simeq \Delta t_* e^{50}$; i.e. the period of change of θ is far less than the time the emergence of the island lasts. This is consistent with the rapid transition approximation used.

[2]Here, during the null energy violating evolution, i.e. the emergence of an island, there is $\dot{\theta} = 0$ until the end time; however, around the end time $\dot{\theta}$ must deviate from 0, and thus in this sense this corresponds to a rapid transition for θ. In general, the period that $\dot{\theta}$ deviates from 0 is far shorter than that of $\dot{\theta} = 0$, which is the meaning of rapid transition approximation. Noting the approximation used here is similar to that used in Refs. [16–18,23], in e.g. [17], this approximation is called the rapid transition approximation, and thus here we follow this term. The null energy violating transition means the total period of the null energy violating evolution, i.e. the emergence of an island, in which $\dot{\theta} = 0$ while $\dot{\theta} \neq 0$ occurs only around its end time.

The term in Eq. (7), proportional to $\mathcal{V}_{(2)}$, is not relevant with $\dot{\theta}$. Thus there exists a nonlinear dependence of $\mathcal{R}$ to δs during the entire evolutive period of fluctuation. In this case this term will contribute an integrated non-Gaussianity. When $\dot{\theta}=0$, Eq. (7) becomes $\dot{\mathcal{R}}^{(2)}=\frac{h\mathcal{V}_{(2)}(\delta s^{(1)})^2}{\dot{\varphi}^2}$. Then we make the integral for this equation, and can obtain the relation of $(\delta s^{(1)})^2\sim -\mathcal{R}^{(2)}$, noting that here Eq. (3) needs to be used. Thus the contribution of this integral effect for non-Gaussianity can be written as

$$-\frac{5\mathcal{R}^{(2)}}{3(\mathcal{R}^{(1)})^2}\simeq \frac{5}{12\theta_*^2}|\epsilon|, \qquad (11)$$

which is inverse to θ_*^2, not like Eqs. (9) and (10). When $\theta_*\ll 1$, this term will make $f_{\rm NL}$ very large.

Thus the total non-Gaussianity of the curvature perturbation is

$$f_{\rm NL}\cong \frac{5(-\alpha\theta_*+2\theta_*+1)}{12\theta_*^2}|\epsilon|, \qquad (12)$$

which is the sum of the results given in Eqs. (9)–(11). We can see that in general the non-Gaussianity in island cosmology is large, since $|\epsilon|$ is large. However, since here α is also the function of θ_*, where θ_* takes its value between 0 and $\pi/2$, thus for a fixed $|\epsilon|$, the value of $f_{\rm NL}$ may be a larger or smaller dependent of θ_*. In general without any fine tuning, θ_* should be about 1. For $\theta_*\simeq 1$, and $n_s\simeq 0.96$ meaning $|\epsilon|\simeq 50$ from Eq. (2), we can have $f_{\rm NL}\simeq 43$, which is a preferred positive value by the current observations. A smaller θ_* means a larger fine tuning, and also a larger $f_{\rm NL}$, which is not favored. In addition, in principle there can be an accident cancellation for all θ_*-dependent terms in Eq. (12) for some value of θ_*, in this case $f_{\rm NL}\simeq 0$. This value is about 1.26, beyond which $f_{\rm NL}<0$.

We can obtain $f_{\rm NL}\sim 1/|n_s-1|$ by combining Eqs. (2) and (12), which means that $f_{\rm NL}\sim \mathcal{O}(10)$ since the red shift $|n_s-1|>0.01$, and the redder the spectrum is, the smaller $f_{\rm NL}$ is. The reason is that a redder spectrum corresponds to a smaller $|\epsilon|$, thus $f_{\rm NL}$. This result is different from that in a simple slow roll inflation model, in which $f_{\rm NL}$ is not inversely proportional to $|n_s-1|$ like in an island, but proportional to it, e.g. Ref. [29]. This predestines that the non-Gaussianity in simple slow roll inflation is quite small. We plot a $f_{\rm NL}-n_s$ plane in Fig. 1 for further illustration. This figure can be distinguished from that in ekpyrotic and cyclic models [21,22], in which, in principle, the redder the spectrum is, the larger the non-Gaussianity is (see also Fig. 5 in Ref. [12]). Though it seems that there requires $|\epsilon|\gg 1$ both in our model and in the cyclic model, and the only difference is that ϵ is negative in our model and positive in the latter, it is this difference that means that their behavior is distinctly contrary in the $f_{\rm NL}-n_s$ plane. In the cyclic model, the spectrum index obtained is the same with that of the island universe model. However, since $\epsilon\gg 1$, when ϵ is constant, the spectrum will be blue, which can be seen in Eq. (2). Thus to have a red spectrum favored by the observations, the change of ϵ must be considered. In this case, the spectrum index is $n_s-1\simeq \frac{2}{\epsilon}-\frac{d\ln|\epsilon|}{d\mathcal{N}}$. The red spectrum requires $\frac{d\ln|\epsilon|}{d\mathcal{N}}>\frac{2}{\epsilon}$. This may be implemented only by introducing a larger ϵ, since this can lead to a smaller $\frac{2}{\epsilon}$. Thus in this case a redder spectrum corresponds to a larger $f_{\rm NL}$. In order to have an adequate red spectrum, for example $n_s\simeq 0.97$, ϵ must be large and change with $\mathcal{N}$ more rapidly than $\epsilon\sim\mathcal{N}$. However, in an island universe, this is not necessary, since $\epsilon\ll -1$, which ensures that its spectrum is naturally red. Including the change of ϵ does not alter our result qualitatively.

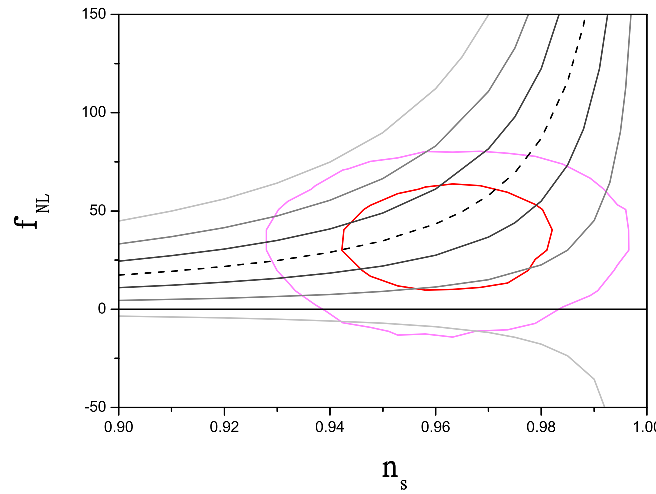

FIG. 1 (color online). The $f_{\rm NL}-n_s$ plane, in which the solid lines from top to down correspond to $\theta_*=0.7$, 0.8, 0.9, 1.1, 1.2, and 1.3, respectively. The dashed line is $\theta_*=1.0$. The 1σ and 2σ contours on $f_{\rm NL}-n_s$ is plotted by using the data in Ref. [12]. We can see that for $\theta_*\simeq 1.0$, $f_{\rm NL}\simeq 30\sim 60$ is definitely predicted by the current observations for n_s.

In Eq. (12), $f_{\rm NL}\sim|\epsilon|$ should be general, since $|\epsilon|$ is only determined by the evolution of background, which is independent of modeling. While the details of modeling only change the factor between $|\epsilon|$, it is inevitable that this factor is dependent of the parameters of model. However, this dependence is actually not important for the natural values of parameters of the model—here it is obvious that the resulting $f_{\rm NL}$ is mainly determined by $|\epsilon|$. The generalization of $f_{\rm NL}\sim|\epsilon|$ can also be seen for simple slow roll inflation, in which $f_{\rm NL}\sim\epsilon$, e.g. [29]. It can be noted that in ekpyrotic and cyclic models[21,22], $f_{\rm NL}\sim\sqrt{\epsilon}$. This is because they required that the entropy perturbation induces the curvature perturbation, which occurs during the kinetic energy domination after ekpyrotic phase. When it is required to occur during ekpyrotic phase, the result will be same with $f_{\rm NL}\sim|\epsilon|$. However, in this case, as has been mentioned, in order to have a red tilt spectrum, a larger ϵ must be introduced, which will conflict with the bound for non-Gaussianity from current observations. Thus in there this case is not adopted.

In summary, the non-Gaussianity of the island universe model is calculated fully by using the second order perturbation equation. We found that for the best fit value $n_s \simeq 0.96$ given by the current observations, without any fine tuning of relevant parameter, $f_{\rm NL} \simeq 43$, which is about between 30 and 60 when the uncertainty for n_s from the WMAP5 is included. In the simple slow roll inflation model, the non-Gaussianity is generally quite small. Thus in order to obtain a large positive value, some special operations for perturbations or models must be appealed, which means that its prediction has certain randomicity. Thus compared with the inflation, the distinct prediction of the island universe for the non-Gaussianity makes it able to be falsified definitely by coming observations. In this sense if the cosmological dynamics are actually controlled by a landscape of vacua, the results of coming observations, especially the measurement of non-Gaussianity, will be significant to clarify whether we live in an island or in a reheating region after slow roll inflation, which will be significant to understanding why and how our position in the landscape is populated.

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