Tachyon field in intermediate inflation

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The tachyonic inflationary universe model in the context of intermediate inflation is studied. General conditions for this model to be realizable are discussed. In the slow-roll approximation, we describe in great detail the characteristics of this model.

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I. INTRODUCTION

Nowadays cosmology presents explosive activity, which is principally due to theoretical developments and accurate astronomical data. In this context, cosmology allows to use astrophysics to perform tests of fundamental theories, otherwise inaccessible to terrestrial accelerators. In order to do this task it is necessary to perform a study about how the Universe evolves during its different periods. In fact, this study leads to considering at some stage in the early Universe an inflationary phase, which is to date the most compelling solution to many long-standing problems of the big bang model (horizon, flatness, monopoles, etc.) [1,2].

The source of inflation is a scalar field (the inflaton field), which plays an important role in providing a causal interpretation of the origin of the observed anisotropy of the cosmic microwave background radiation, and also the distribution of large scale structures [3,4]. The identity of this scalar field may be found by considering one of the extensions of the standard model of particle physics based on grand unified theories, supergravity, or string theory.

In what concerns the scalar inflaton field, its dynamics usually is determined by the Klein-Gordon action. However, more recently, and motivated by string theory, it is extremely natural to consider other nonstandard scalar field action. In this context, the deep interplay between small-scale nonperturbative string theory and large-scale braneworld scenarios has aroused interest in a tachyon field as an inflationary mechanism, especially in the Dirac-Born-Infeld action formulation as a description of the D-brane action [5–10]. Here, rolling tachyon matter is associated with unstable D branes. The decay of these D branes produces a pressureless gas with finite energy density that resembles classical dust. Cosmological implications of this rolling tachyon were first studied by Gibbons [11], and in this context it is quite natural to consider scenarios in which inflation is driven by the rolling tachyon field. In recent years, the possibility of an inflationary phase described by the potential of a tachyon field has been considered in a quite large diversity of topics [12–28].

On the other hand, string/M theory suggests that in order to have a ghost-free action high order curvature, invariant corrections to the Einstein-Hilbert action must be proportional to the Gauss-Bonnet (GB) term [29]. GB terms arise naturally as the leading order of the α expansion to the low-energy string effective action, where α is the inverse string tension [30]. This kind of theory has been applied to possible resolution of the initial singularity problem [31], to the study of black-hole solutions [32], and accelerated cosmological solutions [33]. In particular, very recently, it has been found that for a dark energy model the GB interaction in four dimensions with a dynamical dilatonic scalar field coupling leads to a solution of the form $a(t) = a_0 \exp(At^f)$ [34]. Here, the constant A is given by $A = \frac{2}{\kappa n}$ and $f = 1/2$, with $\kappa^2 = 8\pi G$, and n is a constant. Therefore, we may argument that intermediate inflation comes from an effective theory at a low dimension of a more fundamental string theory.

In general, in the context of inflation we have the particular scenario of ''intermediate inflation,'' in which the scale factor evolves as $a(t) = \exp(At^f)$. Therefore, the expansion of the Universe is slower than standard de Sitter inflation ($a(t) = \exp(Ht)$), but faster than power law inflation ($a(t) = t^p;\ p > 1$). The intermediate inflationary model was introduced as an exact solution for a particular scalar field potential of the type $V(\phi) \propto \phi^{-4(f^{-1}-1)}$, where f is a free parameter [35]. With this sort of potential, and with $1 > f > 0$, it is possible in the slow-roll approximation to have a spectrum of density perturbations, which presents a scale-invariant spectral index $n_s = 1$, i.e., the so-called Harrizon-Zel'dovich spectrum of density perturbations, provided f takes the value of $2/3$ [36]. Even though this kind of spectrum is disfavored by the current Wilkinson Microwave Anisotropy Probe (WMAP) data [3,4], the inclusion of tensor perturbations, which could be present at some point by inflation and parametrized by the tensor-to-scalar ratio r, the conclusion that $n_s \geq 1$ is allowed providing that the value of r is significantly nonzero [37]. In fact, in Ref. [38] it was shown that the combination $n_s = 1$, and $r > 0$ is given by a version of the intermediate inflation in which the scale factor varies as $a(t) \propto e^{t^{2/3}}$ within the slow-roll approximation.

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In this paper we would like to study intermediate inflationary Universe model in which a tachyon field theory is taken into account. We will solve the Friedmann and tachyon field equations for an intermediate expansion of the scale factor and results will be compared with those obtained in the same situation, but where a standard scalar field is considered. We should note that this sort of problem has been studied in the literature [39]. Here, in this paper we would like to go further and thus constrain the parameters of our model by taking into account the WMAP 3 and 5 yr data.

The outline of the paper is as follows: The next section presents a short review of the tachyonic-intermediate inflationary phase. Section III deals with the calculations of cosmological perturbations in general term. Finally, in Sec. IV we conclude with our finding.

II. TACHYON-INTERMEDIATE INFLATION MODEL

We begin by writing the Friedmann equation for a flat universe and the conservation equation

$$H^2 = \frac{\kappa^2}{3}\rho, \tag{1}$$

and

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{2}$$

where $H = \dot{a}/a$ denotes the Hubble parameter, $\kappa^2 = 8\pi G = 8\pi/m_p^2$ (m_p represents the Planck mass), and the dots mean derivatives with respect to the cosmological time t. For convenience we will use units in which $c = \hbar = 1$. For a tachyonic field the energy density and the pressure are given by

$$\rho = \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}}, \quad \text{and} \quad p = -V(\phi)\sqrt{1-\dot{\phi}^2},$$

respectively. Here, ϕ is the tachyonic scalar field, and $V(\phi)$ its scalar potential, which satisfies $dV/d\phi < 0$, and $V(\phi \to \infty) \to 0$, characteristic of any tachyon field potential [5].

From Eqs. (1) and (2) we get for the velocity of the tachyonic scalar field, and its evolution equation becomes

$$\dot{\phi} = \sqrt{-\frac{2\dot{H}}{3H^2}} \tag{3}$$

and

$$\frac{\ddot{\phi}}{1-\dot{\phi}^2} + 3H\dot{\phi} = -\frac{V'}{V}, \tag{4}$$

respectively. Here, $V' = \partial V(\phi)/\partial\phi$.

On the other hand, in intermediate inflation it is assumed that the scale factor follows the law

$$a(t) = a_0 \exp(At^f); \qquad 0 < f < 1, \tag{5}$$

where $A > 0$ has the dimension of m_p^f. Note that this assumption immediately determines the behavior of $\dot{\phi}$ and V'/V, as we can see from Eqs. (3) and (4). Note also that $\dot{H} < 0$, since $0 < f < 1$. From Eqs. (3) and (4) we get for the scalar field ϕ and the scalar potential $V(\phi)$

$$\phi = \phi_0 + \left[\frac{8(1-f)}{3Af(2-f)^2}\right]^{1/2} t^{(2-f)/2}, \tag{6}$$

and

$$V(\phi) = \alpha(\phi - \phi_0)^{-4(1-f)/(2-f)} \times \sqrt{1 - B(\phi-\phi_0)^{-2f/(2-f)}}, \tag{7}$$

with

$$\alpha = \frac{3}{\kappa^2}\left[Af\left(\frac{3(2-f)^2}{8(1-f)}\right)^{(f-1)}\right]^{2/(2-f)},$$

and

$$B = 2\left[\frac{(1-f)}{3Af}\right]^{2/(2-f)}\left[\frac{(2-f)^2}{8}\right]^{-f/(2-f)},$$

respectively.

The Hubble parameter as a function of ϕ becomes

$$H(\phi) = \sqrt{\frac{\alpha\kappa^2}{3}}(\phi-\phi_0)^{2(f-1)/(2-f)}. \tag{8}$$

Without loss of generality ϕ_0 can be taken to be vanished.

During the inflationary epoch the energy density associated to the tachyon field is of the order of the potential, i.e., $\rho \sim V$. Assuming the set of slow-roll conditions, i.e., $\dot{\phi}^2 \ll 1$ and $\ddot{\phi} \ll 3H\dot{\phi}$ [11,12], Eqs. (1)–(4) become

$$H^2 \approx \frac{\kappa^2}{3}V, \tag{9}$$

and

$$\frac{V'}{V} \approx -3H\dot{\phi}. \tag{10}$$

In this approximation the scalar field potential, $V(\phi)$ becomes

$$V(\phi) \approx \alpha\phi^{-2\beta},$$

where

$$\beta \equiv \frac{2(1-f)}{2-f}.$$

Note that this result is also obtained from Eq. (7) by taking $1 \gg B\phi^{-2f/(2-f)}$.

Note that this kind of potential does not present a minimum. This characteristic of the potential makes the study of reheating of the Universe in a nonstandard way [40].

At this stage, it is convenient to introduce the slow-roll parameters ε and η, such that

$$\varepsilon = -\frac{\dot{H}}{H^2} \approx \frac{V'^2}{\kappa^2 V^3} \approx \frac{4\beta^2}{\kappa^2\alpha}\phi^{2(\beta-1)}, \tag{11}$$

and

$$\eta = -\frac{\ddot{\phi}}{H\dot{\phi}} \approx \frac{V'^2}{\kappa^2 V^3} - \frac{V''}{\kappa^2 V^2} \approx -\frac{2\beta}{\kappa^2\alpha}\phi^{2(\beta-1)}, \tag{12}$$

which will be useful in the study of perturbations of the model.

On the hand, the number of e-folds between two different values $\phi(t=t_1)=\phi_1$ and $\phi(t=t_2)=\phi_2>\phi_1$ is given by

$$N = \int_{t_1}^{t_2} H dt = \frac{\kappa^2\alpha}{4\beta(1-\beta)}[\phi_2^{-2(\beta-1)} - \phi_1^{-2(\beta-1)}]. \tag{13}$$

Here, we have used Eq. (6). This expression allows us to determine the value of ϕ_2 in terms of N, A, and f.

Following Refs. [35,36], ϕ_1 it is obtained from the condition $\varepsilon = 1$ (at the beginning of inflation), that is, at $\phi_1^{2(\beta-1)} = \frac{\kappa^2\alpha}{4\beta^2}$.

III. PERTURBATION

In this section we will study the scalar and tensor perturbations for our model. The general expression for the perturbed metric of the flat Friedmann-Robertson-Walker is

$$ds^2 = -(1+2B)dt^2 + 2a(t)D_{,i}dx^i dt + a^2(t)[(1-2\psi)\delta_{ij} + 2E_{,i,j} + 2h_{ij}]dx^i dx^j,$$

where B, D, ψ, and E are the scalar-type metric perturbations, and h_{ij} characterizes the transverse-traceless tensor perturbation. For a tachyon field in the slow-roll approximation the power spectrum of the curvature perturbation becomes [41]

$$\mathcal{P}_{\mathcal{R}} = \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2 \frac{1}{Z_S} \approx \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2 \frac{1}{V} \approx \frac{\kappa^6}{12\pi^2}\frac{V^4}{V'^2}, \tag{14}$$

where $Z_S = V(1-\dot{\phi}^2)^{-3/2} \approx V$ [42]. From this equation we can derive the spectral index given as $n_s - 1 = \frac{d\ln\mathcal{P}_{\mathcal{R}}}{d\ln k}$, where the interval of wave number k is related to the number of e-folds by $d\ln k \simeq dN$. In terms of the slow-roll parameters it is given in first-order approximation by [12]

$$n_s \approx 1 - 2(\varepsilon + \eta), \tag{15}$$

and from Eqs. (11) and (12) we get

$$n_s \approx 1 - \frac{4}{\alpha\kappa^2}\beta(2\beta-1)\phi^{2(\beta-1)}.$$

Since $1>f>0$, we clearly see that the Harrison-Zel'dovich model, i.e., $n_s=1$ occurs for $\beta=1/2$ or equivalently $f=2/3$. For $n_s>1$ we have $\beta<1/2$, and $n_s<1$ is for $\beta>1/2$ (recall that $\beta = 2(1-f)/(2-f)$).

One of the interesting features of the 5 yr data set from WMAP is that it hints at a significant running in the scalar spectral index $dn_s/d\ln k = n_{run}$ [3,4]. From Eq. (15) we get that the running of the scalar spectral index becomes

$$n_{\rm run} = \left(\frac{4V}{V'}\right)[\varepsilon_{,\phi} + \eta_{,\phi}]\varepsilon. \tag{16}$$

In models with only scalar fluctuations the marginalized value for the derivative of the spectral index is approximately -0.03 from WMAP-5 yr data only [3].

On the other hand, the generation of tensor perturbations during inflation would produce gravitational waves, and its amplitudes are given by [43]

$$\mathcal{P}_{\mathcal{T}} = 8\kappa^2\left(\frac{H}{2\pi}\right)^2 \simeq \frac{2\kappa^4}{3\pi^2}V, \tag{17}$$

where the spectral index n_g is given by $n_g = \frac{d\mathcal{P}_{\mathcal{T}}}{d\ln k} = -2\varepsilon$.

From Eqs. (14) and (17) we write the tensor-scalar ratio as

$$r = \left(\frac{\mathcal{P}_{\mathcal{T}}}{\mathcal{P}_{\mathcal{R}}}\right) \approx \frac{8V'^2}{\kappa^2 V^3}. \tag{18}$$

From expressions (15) and (18) we write the relation between n_s and r as

$$n_s \approx 1 - \frac{2-3f}{16(1-f)}r, \tag{19}$$

i.e., n_s depends linearly with respect to r.

Note that Eq. (19) exactly coincides with the expression obtained in Ref. [38], where a standard scalar field was considered. Therefore, it may come as a surprise that, on the basic of the intermediate inflation, the trajectories in the $n_s - r$ plane between standard field and tachyon field cannot be distinguished at lowest order. Actually, this coincidence has already been noted in Ref. [44]. However, tachyon inflation leads to a deviation at second order in the consistency relations, i.e., $n_s = n_s(r)$. From the same reference, the scalar spectral index n_s up to second order in the slow-roll parameter becomes

$$n_s \approx 1 - 2(\varepsilon+\eta) - [(2\varepsilon^2 + 2(2C+3-2\tilde{\alpha})\varepsilon\eta + 2C\eta\gamma], \tag{20}$$

where $C \simeq -0.72$ is a numerical constant and $\eta\gamma = (9m_p^4/2)(2V''V'/V^4 - 10V''V'^2/V^5 + 9V'^4/V^6)$. In the standard case we have $\tilde{\alpha}=0$, and $\tilde{\alpha}=1/6$ for tachyon inflation. Also, at second order, the expression for the ratio r is given by $r = 16\varepsilon(1+2C\eta - 2\tilde{\alpha}\varepsilon)$. These calculations show that the difference at second order of the consistency relations become $n_s^T - n_s^S \simeq \varepsilon\eta/3$, where n_s^T is the spectral index n_s associated to the tachyon field, meanwhile n_s^S is the same parameter for the standard scalar field. At this point, we should notice that the relation between the r and the n_g parameters becomes given by $r = -8c_s n_g$ for a tachyonic field, where the speed of sound c_s results to be $c_s^2 = 1 - (\dot\phi)^2$ [45]. However, at first order it becomes $r \simeq -8n_g$ [44]. From now on, we will consider first-order approximation only, so that we will work with this latter consistency relation.

In the following we will study the case in which $\varepsilon \ll \eta$ [12,44]. In this case, this condition gives us a constraint for the values of f. To see this we write down the ratio between η and ε, and we find for the absolute value

$$|\frac{\eta}{\varepsilon}| \approx 1 + \frac{3f-2}{4(1-f)},$$

so, for $\eta > \varepsilon$ we need to have $f > 2/3$.

The scalar spectral index n_s, for $\varepsilon \ll \eta$, is given by

$$n_s \approx 1 - 2\eta. \tag{21}$$

From Eq. (12) this expression is equivalent to

$$n_s \approx 1 + \frac{4\beta}{\kappa^2\alpha}\phi^{2(\beta-1)}. \tag{22}$$

Using that ϕ_1 it is obtained from the condition $|\ \eta\ |= 1$ (at the beginning of inflation), then Eq. (22) can be reexpressed in terms of the number of e-folding N, resulting in

$$n_s = 1 + \frac{2}{1+2(1-\beta)N} = 1 - \frac{2(1-f/2)}{(1/2-N)f - 1}.$$

Note that a value does not exist for f in which $n_s = 1$, in contrast with the standard case [38] (which occurs for $f = 2/3$). This means that it is not possible to have a Harrison-Zel'dovich spectrum in this case.

From Eq. (21) we can obtain for the running scalar spectral index

$$n_{\rm run} \approx \left(\frac{4\beta}{\kappa^2\alpha}\right)^2(\beta-1)\phi^{4(\beta-1)}. \tag{23}$$

From Eq. (22) we get a relation between n_s and $n_{\rm run}$, which becomes

$$n_s \simeq 1 + \sqrt{\frac{2-f}{f}}\sqrt{-n_{\rm run}}.$$

On the other hand, from Eq. (18) we write the tensor-scalar ratio as

$$r \approx \frac{32}{\kappa^2\alpha}\beta^2\phi^{2(\beta-1)}, \tag{24}$$

and in terms of the e-folding parameter N, we write

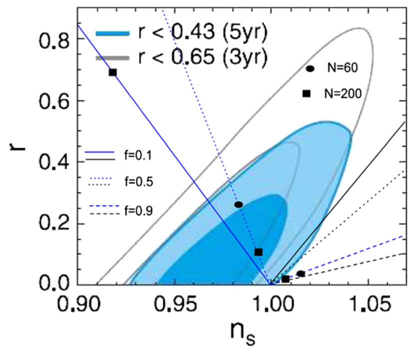

FIG. 1 (color online). The plot shows n_s versus r for our models, and they are compared with the WMAP data (3 yr and 5 yr). The curves in black represent the case $\varepsilon \ll \eta$ and the blue ones specify the tachyon lowest order case for different values of f. The two contours correspond to the 68% and 95% levels of confidence [3]. The small black dots and squares represent the number of e-folds for the values $N = 60$ and $N = 200$, respectively.

$$r = \frac{16\beta}{1+2(1-\beta)N}. \tag{25}$$

Also, from Eqs. (22) and (24) we obtain a relation between n_s and r, which is

$$n_s \approx 1 + \frac{2-f}{16(1-f)}r. \tag{26}$$

In Fig. 1 we show the dependence of the tensor-scalar ratio on the spectral index for different values of the parameter f for the tachyon lowest order (shown by the black line) and the regimen where $\varepsilon \ll \eta$ (shown by the blue line). In this plot we have used Eqs. (19) and (26).

The two contours in the plot show the 68% and 95% levels of confidence for the $r - n_s$ plane, which are defined at $k_0 = 0.002\ \mathrm{Mpc}^{-1}$ [3]. The 5 yr WMAP data places stronger limits on r (blue) than the 3 yr data (grey)[46]. For tachyon lowest order case any value the parameter f, (restricted to the range $1 > f > 0$), is well supported by the data, as can be seen from Fig. 1.

On the other hand, when we considered the regime where $\varepsilon \ll \eta$ (given in Ref. [12]), we see that for $f = 0.1$ the curve $r = r(n_s)$ barely enters the 95% confidence region for $r = 0.2$, which corresponds to $N = 54$. From Fig. 1 the best values of f occur for the range $0.5 > f > 0$.

IV. CONCLUSIONS

In this paper we have studied the tachyon-intermediate inflationary model. We have found in this model an exact solution of the Friedmann equation for a flat Universe containing a scalar field $\phi(t)$, with tachyonic scalar potential $V(\phi)$. In the slow-roll approximation we have found a general relation among the scalar potential and its derivatives. We have also obtained explicit expressions for the corresponding power spectrum of the curvature perturbations $P_{\mathcal{R}}$, tensor-scalar ratio r, scalar spectrum index n_s, and its running $n_{\rm run}$. Here, we noted that Eq. (19) exactly coincides at lowest order with the expression obtained in Ref. [38], where a standard scalar field was studied.

In order to bring some explicit results we have taken the constraint in the $r - n_s$ plane to first order in the tachyon lowest order case and the regime in which $\varepsilon \ll \eta$. In the tachyon lowest order case, we noted that the parameter f, which lies in the range $1 > f > 0$, the model is well supported by the data as could be seen from Fig. 1 for any value of f. But in the other case, i.e., when $\varepsilon \ll \eta$, the parameter f lies within the range $0.5 > f > 0$, in order to be in agreement with current WMAP astronomical data.

We should mention that we have not addressed the phenomena of reheating and the possible transition to the standard cosmology (see e.g., Refs. [40,47,48]). A calculation for the reheating temperature in the high-energy scenario would give new constraints on the parameters of the models. We hope to return to this point in the near future.

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