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Baryon number violation and a new electroweak interaction

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We introduce a new supercurrent in the electroweak sector of the standard model. Its interaction with the hypergauge field influences the mass of the Z boson but has no effect on the $W^{\pm}$ boson masses. In the leptonic sector it affects the numerical value of the vector and axial coupling constants between neutral currents and the Z boson, and a comparison with experimental values yields an upper bound to the strength of the coupling between the supercurrent and the hypergauge field. In the baryonic sector the supercurrent gives a new contribution to the anomaly equation for baryon number current. As a consequence it may have an effect on baryogenesis.

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In the conventional superconductor the supercurrent $\vec{J}$ is the gauge-invariant vector field [1],

$$\vec{J} = e\psi^{\star}\Big(i\frac{\hbar}{2m}\overleftrightarrow{\nabla} - \frac{2e}{mc}\vec{A}\Big)\psi \equiv e|\psi|^2\vec{C}, \qquad (1)$$

where $\vec{A}$ is the Maxwellian $U(1)$ gauge field and ψ is the wave function of Cooper pairs. When we use this relation to eliminate $\vec{A}$ in favor of $\vec{J}$ in the Landau-Ginzburg Hamiltonian that describes the conventional superconductivity

$$\mathcal{H} = \tfrac{1}{2}\vec{B}^2 + |(\vec{\nabla} - 2ie\vec{A})\psi|^2 + \lambda(|\psi|^2 - v^2)^2,$$

where $\vec{B}$ is the magnetic field, we obtain a Hamiltonian that involves only the vector field $\vec{C}$, the Cooper pair density $\rho = |\psi|$, and the phase of the Cooper pair $\theta = \arg\psi$ as independent and manifestly gauge-invariant field variables,

$$\mathcal{H} = \frac{1}{4}\Big(C_{ij} + \frac{\pi}{e}\tilde{\sigma}_{ij}\Big)^2 + (\vec{\nabla}\rho)^2 + \rho^2\vec{C}^2 + \lambda(\rho^2 - v^2)^2 \qquad (2)$$

with

$$C_{ij} = \nabla_i C_j - \nabla_j C_j$$

and

$$\tilde{\sigma}_{ij} = \epsilon_{ijk}\sigma_k = \frac{1}{2\pi}[\nabla_i, \nabla_j]\theta \qquad (3)$$

is the string current with support that coincides with the world sheet of the core of the Abrikosov vortex. When (2) describes such a vortex, (3) subtracts a similar singular contribution that appears in C_{ij}. In the third term this singularity becomes suppressed since the density ρ vanishes at the core of the vortex. Furthermore, whenever the ground state value of ρ is nonvanishing, the vector field $\vec{C}$ is massive and becomes subject to the Meißner effect.

In the present paper we are interested in the physical consequences when a non-Abelian generalization of (1) and (2) is implemented in the Weinberg-Salam model of electroweak interactions. Indeed, the structure of the Weinberg-Salam Lagrangian closely resembles that of the conventional superconductor: Now the gauge field and the Higgs field both transform under the non-Abelian gauge group $G_{\rm EW} = SU_L(2) \times U_Y(1)$, and the Higgs field supports three independent angular (phase) variables that can be combined with three of the gauge fields into the neutral Z boson and the charged $W^{\pm}$ bosons. In the low temperature phase these vector fields become massive, in parallel with the Meißner effect of superconductivity.

In the sequel our motivation lies in the following: In the case of an ordinary superconductor, the supercurrent can be used to entirely remove the gauge field. But in the case of the electroweak theory there are *four* independent gauge fields. As a consequence there may be as many as four independent supercurrents [2]. We shall be mainly interested in the properties of a scrupulously chosen supercurrent that appears to lead to a new electroweak interactions with topological ramifications. In particular, we propose that our supercurrent affects the anomaly equation for the baryon number current in a manner that enhances baryon production. This effect is catalyzed by the presence of the (embedded) topological defects, the Nambu monopoles, in the electroweak model [3].

Presently, there are many approaches for explaining baryon asymmetry of the Universe [4,5]. According to the hot baryogenesis scenario [6], the cooling of the early Universe produced baryons by sphaleron driven thermal activation [7]. But the standard model predicts that instead of a finite temperature phase transition, the cooling of the early Universe proceeded through a crossover transition that retained a thermodynamic equilibrium and thus the hot baryogenesis contrasts the third Sakharov condition [8].

The alternative, cold baryogenesis scenario [9,10] allows for fluctuations that may have caused the early Universe to abandon a state of thermal equilibrium. This may have occurred during the inflationary epoch of the early Universe, at a TeV scale and in combination with the tachyonic transition [11]. Indeed, if the inflaton couples to the Higgs field in a manner that forces the effective Higgs mass to change its sign, a sudden and rapid production of a large amount of particles could take place. Tachyonic preheating [12] may thus lead to a net baryon number production due to a change in the Chern-Simons (CS) contribution to the anomaly equation for the baryon number current.

In both of these scenarios a central role is played by various embedded defects or their bound states. For example, the electroweak sphaleron that drives hot baryogenesis can be interpreted as a pair of a Nambu monopole [3] and antimonopole, bound to each other by a Z vortex [13]. It is also presumed that both Nambu monopoles and electroweak vortices were copious during the hot phase [14]. Additional defects such as textures [9], half-knots [10], knotted hypermagnetic fields [15], linked and twisted vortices [16], center vortices [17], etc., may also have relevance.

We start by recalling how the electroweak gauge group $G_{\rm EW}$ acts on the non-Abelian gauge field $\hat{A}_\mu \equiv \vec{A}_\mu \cdot \vec{\tau}$, the hypergauge field Y_μ, and the Higgs field Φ,

$$G_{\rm EW}: \begin{cases} \hat{A}_\mu \to \Omega \hat{A}_\mu \Omega^\dagger - \frac{2i}{g}\Omega \partial_\mu \Omega^\dagger, \\ Y_\mu \to Y_\mu - \frac{2}{g'}\partial_\mu \omega_Y, \\ \Phi \to \exp\{i\omega_Y\}\Omega\Phi. \end{cases} \tag{4}$$

Here Ω is the $SU_L(2)$ gauge matrix, ω_Y parametrizes a noncompact $U_Y(1)$ hypergauge transformation, and g and g' are the two couplings of the Weinberg-Salam model with the Weinberg angle given by $\tan\theta_W = g'/g$. We now employ these transformation laws to introduce an electroweak supercurrent: The hypercharge transformation has no effect on the $SU_L(2)$ gauge field $\hat{A}_\mu$, while the Higgs field has a nontrivial hypercharge. Thus we can use these fields to construct a supercurrent in a direct non-Abelian generalization of (1) [18],

$$\mathcal{J}_\mu = \frac{2}{g'}\Phi^\dagger\left(-\frac{g}{2}\vec{A}_\mu \cdot \vec{\tau} + i\partial_\mu\right)\Phi = \Phi^\dagger\Phi \cdot \mathcal{Y}_\mu. \tag{5}$$

We also introduce the unit vector

$$\vec{n} = -\frac{\Phi^\dagger \vec{\tau}\Phi}{\Phi^\dagger\Phi} \tag{6}$$

that determines the direction of the isospin polarization in the $SU_L(2)$ gauge group.

It is important to notice that the non-Abelian supercurrent $\mathcal{Y}_\mu$ in (5) is $SU_L(2)$ gauge invariant but under $U_Y(1)$ it transforms like the hypergauge field Y_μ so that

$$U_Y(1)\colon\ \mathcal{Y}_\mu \to \mathcal{Y}_\mu - \frac{2}{g'}\partial_\mu \omega_Y. \tag{7}$$

Thus we can modify the Lagrangian of the standard electroweak model by executing the following *shift* of Y_μ

$$Y_\mu \to (1-\kappa)Y_\mu + \kappa\mathcal{Y}_\mu, \tag{8}$$

in the Lagrangian. In Eq. (8) κ is an *ab initio* free parameter. This leads to a modification of the Lagrangian by a gauge-invariant operator. Indeed, since the gauge transformation properties of the original and shifted Y_μ are identical, the shift (8) preserves all local gauge symmetries of the original electroweak Lagrangian.

Furthermore, this shift (8) can be introduced independently in the Higgs sector and in the fermionic sector of the Lagrangian ($\kappa_\Phi \neq \kappa_f$), and the parameter κ may also be different for different fermionic flavors ($\kappa_f \neq \kappa_{f'}$).

We shall now consider the consequences of (8) in the ensuing *low energy effective* Weinberg-Salam model. In particular, we shall argue that in the fermionic sector of the electroweak Lagrangian, the shift (8) may have a profound effect on the anomaly equation that governs the baryon number nonconservation in early universe baryogenesis. But we first inspect whether (8) leads to other observable effects, and, in particular, whether we can obtain an estimate for the numerical value of the parameter κ.

The kinetic term of the hypergauge field $\mathcal{Y}_\mu$ is a gauge-invariant quantity and thus it may be independently added to the Lagrangian. But here we restrict our attention solely to the effects of the shift (8) to the interactions between the Higgs, vector boson, and fermion fields.

We first observe that if we introduce the shift (8) (with parameter κ_Φ) in the Higgs kinetic term, this leads to a correction of the Z-boson mass,

$$M_Z^2 = \left[\frac{g\upsilon(1-\kappa_\Phi)}{2\cos\theta_W}\right]^2, \tag{9}$$

and has no effect to the $W^\pm$-boson mass.

We proceed to the fermionic sector of the electroweak Lagrangian. There are N_f flavors of the left-handed and right-handed fermions, and in the following we shall not always differ between leptons and quarks. If we implement the shift (8) (with $\kappa_f \equiv \kappa$), besides rescaling the familiar hypergauge interaction by a factor $(1-\kappa)$ it leads to a new interaction between the Higgs field, the $SU_L(2)$ gauge vectors and the fermions which is of the form

$$\frac{i}{2}g'\kappa \mathsf{Y}_\ell \bar{\Psi}_{f,\ell}\gamma^\mu \mathcal{Y}_\mu \Psi_{f,\ell}. \tag{10}$$

Here $\ell = L, R$, and the lepton hypercharges are $\mathsf{Y}_L = -1$ and $\mathsf{Y}_R = -2$. The essential feature of (10) is that it leaves the interactions between the charged weak currents and the $W^\pm$ bosons unchanged. But the neutral sector of the electroweak Lagrangian

$$\mathcal{L}_{\psi}^{(0)} = -gJ_3^{\mu}W_{\mu}^3 - (g'/2)J_Y^{\mu}Y_{\mu} \tag{11}$$

becomes modified by the shifts in the neutral currents

$$J_3^{\mu} \to J_3^{\mu} + \kappa J_Y^{\mu}/2 \quad \text{and} \quad J_Y^{\mu} \to (1-\kappa)J_Y^{\mu}. \tag{12}$$

When we specify $f = e, \nu$, then

$$J_3^{\mu} = (\bar{\psi}_{\nu,L}\gamma^{\mu}\psi_{\nu,L} - \bar{\psi}_{e,L}\gamma^{\mu}\psi_{e,L})/2,$$
$$J_Y^{\mu} = -(\bar{\psi}_{\nu,L}\gamma^{\mu}\psi_{\nu,L} + \bar{\psi}_{e,L}\gamma^{\mu}\psi_{e,L} + 2\bar{\psi}_{e,R}\gamma^{\mu}\psi_{e,R}),$$

and we find that the coupling between the electric current and the electromagnetic gauge field still leads to the standard definition of the Weinberg angle θ_W, with electric charge given by $e = g\sin\theta_W$. Defining the field combinations in the usual manner,

$$\begin{pmatrix} A_{\mu} \\ Z_{\mu} \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Y_{\mu} \\ A_{\mu}^3 \end{pmatrix}, \tag{13}$$

we then get a familiar form for the Lagrangian that describes the neutral current sector,

$$\mathcal{L}_L^{(0)} = j_e^{\mu}A_{\mu} - \frac{g}{2\cos\theta_W}\sum_{f=e,\nu}\bar{\Psi}_f\gamma^{\mu}(g_V^{(f)} - g_A^{(f)}\gamma^5)\Psi_f Z_{\mu}.$$

But now the vector g_V and axial g_A couplings of the neutral currents to the Z-boson field have been modified,

$$g_V^{(f)} = (1-\kappa)T_3^{(f)} - 2Q^{(f)}(\sin^2\theta_W - \kappa_f),$$
$$g_A^{(f)} = (1-\kappa)T_3^{(f)}. \tag{14}$$

Here $T_3^{(f)}$ is the weak isospin ($T_3^{(\nu)} = +1/2$ for $f = \nu$ and $T_3^{(\ell)} = -1/2$ for $f = \ell \equiv e, \mu, \tau$), and $Q^{(f)}$ is the electric charge ($Q^{(\nu)} = 0$ and $Q^{(\ell)} = -1$). When we use the uncertainties in constraints on the $g_{V,A}$ couplings published by the Particle Data Group [19] we get an estimate for the experimentally allowed value of κ,

$$|\kappa^{(\ell)}| \lesssim 3\times 10^{-4}, \qquad |\kappa^{\nu_e}| \lesssim 10^{-1}, \qquad |\kappa^{\nu_\mu}| \lesssim 2\times 10^{-2}. \tag{15}$$

In the leptonic sector the shift (8) has no effect on the electromagnetic interaction involving the massless gauge field A_{μ}. There are also no changes in the charged weak interactions that involve the charged currents $W_{\nu}^{\pm}$. In particular, there is no effect on the fermion masses: The only effect of the shift (8) appears in the neutral weak sector involving the exchange of the Z boson.

However, when we proceed to the baryonic sector we find that the shift (8) does have an impact on the processes that violate $B + L$ conservation. Thus it may well have an influence on baryogenesis. For this we recall the standard nonconservation (anomaly) equation for the baryon number current which in terms of our shifted field becomes

$$\partial_{\mu}j_B^{\mu} = \frac{N_f}{32\pi^2}(-g^2\vec{G}_{\mu\nu}\tilde{\vec{G}}_{\mu\nu} + g'^2F_{\mu\nu}^{(\kappa)}\tilde{F}_{\mu\nu}^{(\kappa)}). \tag{16}$$

Here

$$F_{\mu\nu}^{(\kappa)} = (1-\kappa)(\partial_{\mu}Y_{\nu} - \partial_{\nu}Y_{\mu}) + \kappa\mathcal{F}_{\mu\nu},$$
$$\mathcal{F}_{\mu\nu} = \partial_{\mu}\mathcal{Y}_{\nu} - \partial_{\nu}\mathcal{Y}_{\mu} = \frac{g}{g'}\mathcal{G}_{\mu\nu}(\vec{W},\vec{n}) + \frac{4\pi}{g'}\tilde{\Sigma}_{\mu\nu}^{\mathsf{S}_0}, \tag{17}$$

and we have introduced the gauge-invariant 't Hooft tensor [20],

$$\mathcal{G}_{\mu\nu}(\vec{A},\vec{n}) = \vec{n}\cdot\vec{G}_{\mu\nu} - \frac{1}{g}\vec{n}\cdot \mathrm{D}_{\mu}\vec{n}\times\mathrm{D}_{\nu}\vec{n}, \tag{18}$$

where $\vec{n}$ is defined in (6),

$$\vec{G}_{\mu\nu} = \partial_{\mu}\vec{A}_{\nu} - \partial_{\nu}\vec{A}_{\mu} + g\vec{A}_{\mu}\times\vec{A}_{\nu}$$

and $\mathrm{D}_{\mu}^{ab} = \delta^{ab}\partial_{\mu} + g\epsilon^{acb}A_{\mu}^c$. The tensor $\Sigma_{\mu\nu}^{\mathsf{S}_0}$ denotes a (Dirac-like) string contribution that describes the two-dimensional world sheet of a closed Z vortex [21,22]. In the unitary gauge where $\vec{n}$ becomes aligned with the (positive) z axis in the isospin space, the 't Hooft tensor is

$$\mathcal{G}_{\mu\nu} = \partial_{\mu}W_{\nu}^3 - \partial_{\nu}W_{\mu}^3 + \frac{4\pi}{g}\tilde{\Sigma}_{\mu\nu}^{\mathsf{S}'_\mathsf{C}}, \tag{19}$$

where S'_C denotes the conventional Dirac string of the Nambu monopole. As a consequence the total world surface determined by the string structures of the Nambu monopoles is $\mathsf{S}_\mathsf{C} = \mathsf{S}_0 + \mathsf{S}'_\mathsf{C}$. In this gauge we can write

$$F_{\mu\nu}^{(\kappa)} = \partial_{\mu}\mathcal{B}_{\nu} - \partial_{\nu}\mathcal{B}_{\mu} + \frac{4\pi\kappa}{g'}\tilde{\Sigma}_{\mu\nu}^{\mathsf{S}_\mathsf{C}}, \tag{20}$$

$$\mathcal{B}_{\mu} = \cos\theta_W A_{\mu} - \left(\sin\theta_W - \frac{\kappa}{\sin\theta_W}\right)Z_{\mu}. \tag{21}$$

Here $\mathcal{B}_{\mu}$ is an Abelian gauge field without Dirac singularities. Then we introduce the conserved ($\partial^{\mu}k_{\mu}^{\mathsf{C}} = 0$) current of the Nambu monopole,

$$k_{\mu}^{\mathsf{C}} = \kappa\cdot\frac{2\pi}{g'}\epsilon_{\mu}{}^{\nu\rho\sigma}\partial_{\nu}\tilde{\Sigma}_{\rho\sigma}^{\mathsf{S}_\mathsf{C}}.$$

When we integrate both sides of (16) over the four-dimensional volume between two distant, time separated three-dimensional spatial surfaces, we obtain an equation for the net production of baryon number,

$$\Delta Q_B = N_f\left(\Delta N_{\mathrm{CS}} - \gamma_{\kappa}\Delta n_{\mathrm{CS}} + \frac{e\kappa}{4\pi}\Phi^{\mathsf{C}} + \frac{\kappa^2}{2}N_{\mathsf{S}_\mathsf{C}}\right). \tag{22}$$

The first two terms are the conventional results [modulo the $\gamma_{\kappa} \equiv (1-\kappa)^2$ factor], reflecting the change in the non-Abelian Chern-Simons number N_{CS} and the Abelian Chern-Simons number n_{CS}. These terms also contain contributions from linked and twisted Z vortices [16].

The third and fourth terms in Eq. (22) are new and entirely due to our supercurrent shift. The third term has its origin in the presence of Nambu monopoles. Using the definition of the monopole current and assuming that C is a large enough trajectory so that the contribution from the massive Z-boson field can be ignored we get

$$\Phi^{\mathsf{C}} = \frac{1}{\cos\theta_W}\int d^4x k^{\mathsf{C}}_\mu \mathcal{B}^\mu = \oint_{\mathsf{C}} dx_\mu A^\mu + \cdots. \quad (23)$$

Thus Φ^{C} coincides with the standard electromagnetic flux that emanates from the string structure of the Nambu monopoles. The third contribution to the baryon number production (22) can formally be interpreted as the Witten effect [23], as it makes the Nambu monopoles to serve as dyons with electric charge $q_e \propto e\kappa$.

The fourth term in (22) counts the number of (self-) intersections of the string world sheets [24]:

$$N_I[\mathsf{S_C}] = \frac{1}{2}\int d^4x \Sigma^{\mathsf{S_C}}_{\mu\nu}\tilde{\Sigma}^{\mathsf{S_C}}_{\mu\nu}.$$

This quantity receives contributions both from transversal intersection points and from the twisting points, and it can be entirely presented in terms of the evolution of the writhing number of the surface [24]. Note that while the ensuing contribution to the change in the baryon number depends on κ, it is entirely independent of the couplings g and g' of the electroweak Lagrangian.

We conclude with a few remarks: A shift of the form (8) introduces a modification of the electroweak theory by a gauge-invariant operator that appears to be fully consistent with the local symmetries of the Weinberg-Salam Lagrangian. The energy scale of the new transition may be different from the electroweak scale in which case the Z boson might either remain massive above the electroweak transition, or acquire the additional mass correction (9) at some lower energy scale.

Note that in the leptonic sector the sole effect of the shift appears to be in a correction to the value of the vector and axial couplings of neutral currents—this is a tree-level effect.

Finally, in the baryonic sector of the electroweak theory the shift (8) appears to affect the anomaly in the baryon current conservation. In particular, if the early universe went through a period with copious production and subsequent annihilation of Nambu monopoles, the shift could have played a central role in baryogenesis. Being of a nonperturbative nature, this aspect deserves to be studied in greater detail numerically within an adopted model of electroweak history of the Universe.

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