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Exotic hadrons with hidden charm and strangeness

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We investigate on exotic tetraquark hadrons of the kind $[cs][\bar{c}\,\bar{s}]$ by computing their spectrum and decay modes within a constituent diquark-antidiquark model. We also compare these predictions with the present experimental knowledge.

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I. INTRODUCTION

Motivated by our former study on the interpretation of the $Y(4260)$ resonance [1] (see also [2,3]), we analyze the possibility of a spectroscopy of particles with hidden strangeness and charm embodied in diquark-antidiquark structures of the kind $[cs][\bar{c}\,\bar{s}]$, where $\mathbf{q} = [cs]$ is a $\bar{\mathbf{3}}_{\mathbf{c}}$ diquark. We predict the mass spectrum for these states and discuss which might be their prominent decay modes on the basis of quark rearrangements in the $\mathbf{q}\bar{\mathbf{q}}$ system. The mass spectrum is computed using the nonrelativistic spin-spin interactions Hamiltonian, supposed to remove the degeneracy among the various $[cs][\bar{c}\,\bar{s}]$ states with assigned spins and angular momenta

$$H_{SS} = \sum_{\text{pairs}} \frac{\kappa_{ij}}{m_i m_j} (\vec{S}_i \cdot \vec{S}_j)\delta^3(\vec{r}_{ij}). \tag{1}$$

The couplings are inversely proportional to quark masses: we will incorporate this dependency in the color-magnetic moments κ.

II. THE MODEL

Adopting the approach discussed at length in [4,5] we will determine the mass spectrum of $[cs][\bar{c}\,\bar{s}]$ hadrons by diagonalization of the following nonrelativistic effective Hamiltonian for a $[q_1q_2][\bar{q}_1\bar{q}_2]$ diquark-antidiquark hadron

$$H = 2m_{\mathbf{q}} + H_{SS}^{(qq)} + H_{SS}^{(q\bar{q})} + H_{SL} + H_L, \tag{2}$$

where

$$\begin{aligned} H_{SS}^{(qq)} &= 2\kappa_{\mathbf{q}}(\vec{S}_{q_1} \cdot \vec{S}_{q_2} + \vec{S}_{\bar{q}_1} \cdot \vec{S}_{\bar{q}_2}), \\ H_{SS}^{(q\bar{q})} &= 2\kappa_{q_1\bar{q}_2}(\vec{S}_{q_1} \cdot \vec{S}_{\bar{q}_2} + \vec{S}_{\bar{q}_1} \cdot \vec{S}_{q_2}) + 2\kappa_{q_1\bar{q}_1}\vec{S}_{q_1} \cdot \vec{S}_{\bar{q}_1} \\ &\quad + 2\kappa_{q_2\bar{q}_2}\vec{S}_{q_2} \cdot \vec{S}_{\bar{q}_2} \end{aligned} \tag{3}$$

represent the chromomagnetic interactions between quarks in the tetraquark system whereas the spin-orbit and orbital contributions are given by

$$H_{SL} = 2A(\vec{S}_{\mathbf{q}} \cdot \vec{L} + \vec{S}_{\bar{\mathbf{q}}} \cdot \vec{L}), \qquad H_L = B\frac{L(L+1)}{2}, \tag{4}$$

respectively. The symbol $\vec{S}_{\mathbf{q}}$ represents the *total* spin of the diquark $\mathbf{q} = [q_1q_2]$. A and B are coefficients to be determined by data. To diagonalize H we need to estimate the diquark mass $m_{\mathbf{q}}$ and the coupling constants κ; then we have to specify the states having assigned J^{PC} quantum numbers and find their masses. The $[cs]$ diquark mass is simply estimated by $m_{[cs]} = m_{[qs]} + m_c - m_q = 1955$ MeV, where $m_{c,q}$ are constituent masses and $m_{[qs]}$ is obtained by comparison with light scalar mesons data [6]. We have used $m_{[qs]} = 590$ MeV, $m_c = 1670$ MeV, and $m_q = 305$ MeV.

As shown in [4], quark-antiquark spin-spin couplings are estimated by comparison with the mass spectra of ordinary $q\bar{q}$ mesons. A calculation made along these lines provides us with the chromomagnetic couplings for $q\bar{q}$ color singlets $(\kappa_{q\bar{q}})_{\mathbf{1}}$. We obtain $\kappa_{cs} = 25$ MeV, $(\kappa_{c\bar{s}})_{\mathbf{1}} = 72$ MeV, $(\kappa_{s\bar{s}})_{\mathbf{1}} = 121$ MeV, $(\kappa_{c\bar{c}})_{\mathbf{1}} = 59$ MeV, $A = 22$ MeV, and $B = 495$ MeV.

On the other hand, the couplings $\kappa_{q\bar{q}}$ in Eq. (3) are not necessarily in the singlet channel as those estimated since octet couplings $(\kappa_{\mathbf{8}})$ are also possible between quarks and antiquarks in a $\mathbf{q}\bar{\mathbf{q}}$ system [7]. The octet couplings are estimated with the aid of the one-gluon exchange model as follows. For the diquark attraction in the $\bar{\mathbf{3}}$-color channel, we can write $\mathbf{q}^i = [cs]^i := \epsilon^{ijk}c_js_k$, neglecting spin. i, j, and k are color indices in the fundamental representation of $SU(3)$. Then the color neutral hadron is

$$[cs][\bar{c}\,\bar{s}] = (c_j\bar{c}^j)(s_k\bar{s}^k) - (c_j\bar{s}^j)(s_k\bar{c}^k). \tag{5}$$

We then use the following $SU(N)$ identity for the Lie algebra generators $\sum_{a=1}^{N^2-1}\lambda_{ij}^a\lambda_{kl}^a = 2(\delta_{il}\delta_{jk} - \frac{1}{N}\delta_{ij}\delta_{kl})$ where N is the number of colors. A color octet ($N = 3$) $q\bar{q}$ state can be written as $\bar{q}^i\lambda_{ij}^a q^j$, and consequently

$$(\bar{c}^i\lambda_{ij}^a c^j)(\bar{s}^k\lambda_{kl}^a s^l) = 2\left[(c_i\bar{s}^i)(s_k\bar{c}^k) - \frac{1}{N}(c_i\bar{c}^i)(s_k\bar{s}^k)\right]. \tag{6}$$

This allows one to extract from (5) the octet term as follows:

$$[cs][\bar{c}\,\bar{s}] = \tfrac{2}{3}(c_j\bar{c}^j)(s_k\bar{s}^k) - \tfrac{1}{2}(\bar{c}^i\lambda_{ij}^a c^j)(\bar{s}^k\lambda_{kl}^a s^l). \tag{7}$$

This formula gives information about the relative weights of a singlet and an octet color state in a diquark-antidiquark tetraquark. We have three colors running in the sum $c_i\bar{c}^i$, whereas $a = 1, \ldots, 8$ in $\bar{c}^i \lambda^a_{ij} c^j$. Therefore the probability of finding (projecting onto) a particular $q\bar{q}$ pair in color singlet, for example, to find $c\bar{c}$ in the color singlet state $c_j \bar{c}^j$, is half the probability of finding the same pair in color octet $\bar{c}\lambda^a c$ as $3 \times 2/3 = 1/2(8 \times 1/2)$. We write then

$$\kappa_{c\bar{c}}([cs][\bar{c}\,\bar{s}]) = \tfrac{1}{3}(\kappa_{c\bar{c}})_{\mathbf{1}} + \tfrac{2}{3}(\kappa_{c\bar{c}})_{\mathbf{8}}, \tag{8}$$

where $(\kappa_{c\bar{c}})_{\mathbf{1}}$ have been reported above. For the determination of the quantity $(\kappa_{c\bar{c}})_{\mathbf{8}}$ we have to resort to the one-gluon exchange model.

Then, if $\kappa_{c\bar{c}}(\mathbf{R})$ is the weight of the quark-antiquark interaction, we have $\kappa_{c\bar{c}}(\mathbf{R}) \sim (C^{(2)}(\mathbf{R}) - C^{(2)}(\mathbf{3}) - C^{(2)}(\bar{\mathbf{3}}))$. We recall that $C^{(2)}(\mathbf{R}) = 0$, $4/3$, $4/3$, and 3 as $\mathbf{R} = \mathbf{1}$, $\mathbf{3}$, $\bar{\mathbf{3}}$, and $\mathbf{8}$. Then it is immediately found that $(\kappa_{c\bar{c}})_{\mathbf{1}} \sim -\frac{8}{3}$, $(\kappa_{c\bar{c}})_{\mathbf{8}} \sim \frac{1}{3} = -\frac{1}{8}(\kappa_{c\bar{c}})_{\mathbf{1}}$. Finally, from Eq. (8), we have

$$\kappa_{c\bar{c}}([cs][\bar{c}\,\bar{s}]) = \tfrac{1}{4}(\kappa_{c\bar{c}})_{\mathbf{1}}. \tag{9}$$

Now that we know its input parameters, we are ready to diagonalize the Hamiltonian (2). We label the particle states by using the notation $|S_{\mathbf{q}}, S_{\bar{\mathbf{q}}}; S_{\mathbf{q}\bar{\mathbf{q}}}, J\rangle$, where $S_{\mathbf{q}\bar{\mathbf{q}}}$ is the total spin of the diquark-antidiquark system. States are organized in order to have definite J^{PC} quantum numbers. For negative parity ones a unit of relative angular momentum between the diquark and the antidiquark is required ($L_{\mathbf{q}\bar{\mathbf{q}}} = 1$). Altogether we have

(a) Two positive parity states with $J^{PC} = 0^{++}$

$$\begin{aligned} |0^{++}\rangle_1 &= |0_{cs}, 0_{\bar{c}\,\bar{s}}; 0_{\mathbf{q}\bar{\mathbf{q}}}, J = 0\rangle, \\ |0^{++}\rangle_2 &= |1_{cs}, 1_{\bar{c}\,\bar{s}}; 0_{\mathbf{q}\bar{\mathbf{q}}}, J = 0\rangle. \end{aligned} \tag{10}$$

(b) Three states with $J = 0$ and negative parity ($L_{\mathbf{q}\bar{\mathbf{q}}} = 1$ required)

$$\begin{aligned} |A\rangle &= |1_{cs}, 0_{\bar{c}\,\bar{s}}; 1_{\mathbf{q}\bar{\mathbf{q}}}, J = 0\rangle, \\ |B\rangle &= |0_{cs}, 1_{\bar{c}\,\bar{s}}; 1_{\mathbf{q}\bar{\mathbf{q}}}, J = 0\rangle, \\ |C\rangle &= |1_{cs}, 1_{\bar{c}\,\bar{s}}; 1_{\mathbf{q}\bar{\mathbf{q}}}, J = 0\rangle. \end{aligned} \tag{11}$$

State $|C\rangle$ is even under charge conjugation. Taking symmetric and antisymmetric combinations of states $|A\rangle$ and $|B\rangle$, we obtain a C-odd and a C-even state, respectively; therefore we have two states with $J^{PC} = 0^{-+}$

$$|0^{-+}\rangle_1 = \frac{1}{\sqrt{2}}(|A\rangle - |B\rangle), \qquad |0^{-+}\rangle_2 = |C\rangle, \tag{12}$$

and one state with $J^{PC} = 0^{--}$

$$|0^{--}\rangle = \frac{1}{\sqrt{2}}(|A\rangle + |B\rangle). \tag{13}$$

(c) Three states with $J = 1$ and positive parity

$$\begin{aligned} |D\rangle &= |1_{cs}, 0_{\bar{c}\,\bar{s}}; 1_{\mathbf{q}\bar{\mathbf{q}}}, J = 1\rangle, \\ |E\rangle &= |0_{cs}, 1_{\bar{c}\,\bar{s}}; 1_{\mathbf{q}\bar{\mathbf{q}}}, J = 1\rangle, \\ |F\rangle &= |1_{cs}, 1_{\bar{c}\,\bar{s}}; 1_{\mathbf{q}\bar{\mathbf{q}}}, J = 1\rangle. \end{aligned} \tag{14}$$

$|F\rangle$ is an eigenvector under charge conjugation, with negative eigenvalue. Operating on $|D\rangle$ and $|E\rangle$ in the same way as for states $|A\rangle$ and $|B\rangle$ we obtain the $J^{PC} = 1^{++}$ state

$$|1^{++}\rangle = \frac{1}{\sqrt{2}}(|D\rangle + |E\rangle) \tag{15}$$

and the $J^{PC} = 1^{+-}$ ones

$$|1^{+-}\rangle_1 = \frac{1}{\sqrt{2}}(|D\rangle - |E\rangle), \qquad |1^{+-}\rangle_2 = |F\rangle. \tag{16}$$

(d) Six states with $J = 1$ and negative parity. To start with consider the following two

$$\begin{aligned} |G\rangle &= |1_{cs}, 0_{\bar{c}\,\bar{s}}; 1_{\mathbf{q}\bar{\mathbf{q}}}, J = 1\rangle, \\ |H\rangle &= |0_{cs}, 1_{\bar{c}\,\bar{s}}; 1_{\mathbf{q}\bar{\mathbf{q}}}, J = 1\rangle, \end{aligned} \tag{17}$$

differing from $|D\rangle$ and $|E\rangle$ as we have $L_{\mathbf{q}\bar{\mathbf{q}}} = 1$ here. When symmetrized and antisymmetrized these give the combinations

$$\begin{aligned} |1^{-+}\rangle_1 &= \frac{1}{\sqrt{2}}(|G\rangle - |H\rangle), \\ |1^{--}\rangle_1 &= \frac{1}{\sqrt{2}}(|G\rangle + |H\rangle). \end{aligned} \tag{18}$$

Moreover we have the following four charge conjugation eigenstates:

$$\begin{aligned} |1^{-+}\rangle_2 &= |1_{cs}, 1_{\bar{c}\,\bar{s}}; 1_{\mathbf{q}\bar{\mathbf{q}}}, J = 1\rangle, \\ |1^{--}\rangle_2 &= |0_{cs}, 0_{\bar{c}\,\bar{s}}; 0_{\mathbf{q}\bar{\mathbf{q}}}, J = 1\rangle, \\ |1^{--}\rangle_3 &= |1_{cs}, 1_{\bar{c}\,\bar{s}}; 0_{\mathbf{q}\bar{\mathbf{q}}}, J = 1\rangle, \\ |1^{--}\rangle_4 &= |1_{cs}, 1_{\bar{c}\,\bar{s}}; 2_{\mathbf{q}\bar{\mathbf{q}}}, J = 1\rangle. \end{aligned} \tag{19}$$

The action of the spin operators in Eq. (3) on the states here listed is independent of the specific $L_{\mathbf{q}\bar{\mathbf{q}}}$ value. Let us write

$$|S_{\mathbf{q}}, S_{\bar{\mathbf{q}}}; S_{\mathbf{q}\bar{\mathbf{q}}}, J\rangle = |c^T \Gamma s, \bar{c}^T \Gamma \bar{s}; S_{\mathbf{q}\bar{\mathbf{q}}}, J\rangle, \tag{20}$$

where the Γ can be $\Gamma^0 = 1/\sqrt{2}\sigma_2$ and $\Gamma^i = 1/\sqrt{2}\sigma_2\sigma^i$ for spin 0 and spin 1, respectively. The numerical factors are chosen in such a way to preserve the normalization $\mathrm{Tr}[(\Gamma^\alpha)^\dagger \Gamma^\beta] = \delta^{\alpha\beta}$. Then the action of a spin-spin interaction operator, e.g., $\vec{S}_c \cdot \vec{S}_s$, on the generic state in Eq. (20) is

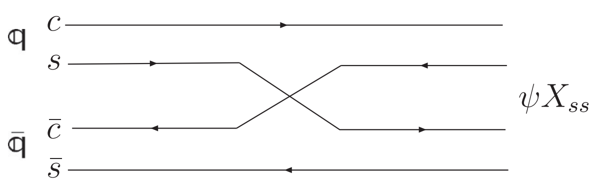
$$(\vec{S}_c \cdot \vec{S}_s)|c^T\Gamma s, \bar{c}^T\Gamma\bar{s}; S_{\mathbf{q}\bar{\mathbf{q}}}, J\rangle = \frac{1}{4}\sum_j |c^T\sigma_j^T\Gamma\sigma_j s, \bar{c}^T\Gamma\bar{s}; S_{\mathbf{q}\bar{\mathbf{q}}}, J\rangle \quad (21)$$

and similarly for the other operators.

III. RESULTS AND DISCUSSION

The final results on the mass spectrum determination are summarized in Table I together with the spin, orbital quantum numbers, and decay modes and widths, when calculable. We include in parentheses the mass shifts in MeV due to turning off the spin-spin interactions whence orbital angular momentum effectively increases the diquark-antidiquark distance.

Suppose that the tetraquark system could be described energetically by a double-well potential with the two light quarks lying in the two wells induced by the charm quarks, which can be considered as static color sources. The potential barrier separating the two wells prevents a quark in the diquark to bind with an antiquark in the antidiquark (and vice versa). This process occurs anyway at the rate of the barrier penetration. We shall assume that this is the case for the quark passing through the barrier to bind with the antiquark in the other well, as represented in the following diagram:



where $X_{ss} = \phi$, ω, η, and η'. The charm quarks have no other choice than to neutralize color in a charmonium meson. An alternative process is the formation of open charm mesons by rearranging the strange quarks. We shall suppose that the latter two alternatives occur at almost the same rate (both of them pay the same energetic price of breaking the diquark bindings).

As for the decay widths, consider, for example, the decay into $J/\psi\phi$ of the 0^{-+} particle with mass $M = 4277$ MeV in Table I. The S-matrix element is $\langle J/\psi(\eta, p')\phi(\epsilon, q)|Y_{4277}(p)\rangle = \mathcal{G}\epsilon^{\mu\nu\rho\sigma}\eta_\mu\epsilon_\nu p_\rho q_\sigma$, where $\mathcal{G}$ must have dimensions of the inverse of a mass to let the width have dimensions of energy. The decay at hand is a P-wave decay, therefore

$$\Gamma(Y_{4277} \to J/\psi\phi) = \frac{1}{3}\frac{A^2}{M_Y^2}\left(\frac{1}{8\pi M_Y^2}p^{*3}\right), \quad (22)$$

where p^* is the decay momentum in the reaction and we assumed $\mathcal{G} = g/M_Y$, the dimensionless g being written as $g = A/(\sqrt{2}M_Y)$. Assuming that the exchange diagrams in $[c(u, d)][\bar{c}(\bar{u}, \bar{d})]$ states have the same amplitudes as the ones in $[cs][\bar{c}\bar{s}]$ [4], i.e., assuming $A = 2.6$ GeV, we obtain the decay modes reported in Table I. Similarly, S-wave modes, like the decays into $J/\psi\eta$ of the 1^{+-} states in Table I, would be associated to a matrix element of the form $\langle J/\psi(\eta, p')\eta(q)|Y(p, \epsilon)\rangle = \mathcal{F}$, where $\mathcal{F}$ must have dimensions of mass (one can set $\mathcal{F} = gM_Y$). Matrix elements of the form $\langle J/\psi(\eta, p')\eta(q)|Y(p, \epsilon)\rangle = \mathcal{N}(p \cdot \eta)(q \cdot \epsilon)$ would instead give the D-wave contribution. As for P-wave decays like those of the 0^{--} states we use the parametrization $\langle J/\psi(\epsilon, p')\eta(q)|0^{--}\rangle = \mathcal{H}(p \cdot \epsilon)$, where the dimensionless $\mathcal{H}$ is $\mathcal{H} = A/\sqrt{2}$. The standard $\eta\eta'$, $\omega\phi$ mixing schemes are used in the calculation.

TABLE I. Quantum numbers and masses for $[cs][\bar{c}\bar{s}]$ states. We include in parentheses the mass shifts in MeV due to turning off the spin-spin interactions whence orbital angular momentum effectively increases the diquark-antidiquark distance. The dominant two-body decay channels are also indicated together with the relative angular momentum of the two produced particles and the partial width ($\Gamma_{\rm part}$) of the exchange diagram dominated decays. As for the estimated masses, we put in parentheses the corrections in MeV due to turning off the spin-spin interactions whence orbital angular momentum effectively increases the diquark-antidiquark distance. By the notation $D^{(*)}$ we mean D or D^*.

$S_{\mathbf{q}}$	$S_{\bar{\mathbf{q}}}$	$S_{\mathbf{q}\bar{\mathbf{q}}}$	$L_{\mathbf{q}\bar{\mathbf{q}}}$	J^{PC}	M (MeV)	Decay channel [$\Gamma_{\rm part}$ (KeV)]	Relative wave
0	0	0	0	0^{++}	3834	...	
1	1	0	0	0^{++}	3927	Multihadron	...
1	0	1	1	0^{-+}	4277(+15)	$J/\psi\phi$ [35], $D_s^{*+}D_s^{*-}$ [10]	P
1	1	1	1	0^{-+}	4312(+30)	$J/\psi\phi$ [46], $D_s^{*+}D_s^{*-}$ [24]	P
1	0	1	1	0^{--}	4297(−5)	$\psi\eta(\eta')$ [245(110)], $D_s^{+}D_s^{*-}$ [500]	P
1	0	1	0	1^{++}	3890	Multihadron	...
1	0	1	0	1^{+-}	3870	$J/\psi\eta$ [610]	S
1	1	1	0	1^{+-}	3905	$J/\psi\eta$ [650]	S
1	0	1	1	1^{-+}	4321(+15)	$J/\psi\phi$ [52]	P
1	1	1	1	1^{-+}	4356(+30)	$J/\psi\phi$ [64]	P
0	0	0	1	1^{--}	4330	$\psi\eta(\eta')$ [90(45)], $D_s^{(*)+}D_s^{(*)-}$ [27]; $J/\psi f_0(980)$	P; S
1	0	1	1	1^{--}	4341(−5)	$\psi\eta(\eta')$[92(48)], $D_s^{(*)+}D_s^{(*)-}$ [31]; $J/\psi f_0(980)$	P; S
1	1	0	1	1^{--}	4390(+40)	$\psi\eta(\eta')$ [100(58)], $D_s^{(*)+}D_s^{(*)-}$ [51]; $J/\psi f_0(980)$	P; S
1	1	2	1	1^{--}	4289(−41)	$\psi\eta(\eta')$ [83(36)], $D_s^{(*)+}D_s^{(*)-}$ [13]; $J/\psi f_0(980)$	P; S

Annihilation diagrams, also expected to be rather suppressed at the charm quark scale, could produce final states as $DD\pi$.

IV. EXPERIMENTAL EVIDENCES

The most interesting match between predicted and observed states is in the $J/\psi\omega$ invariant mass spectrum, studied both by Belle [8] and by *BABAR* [9] in $B \to J/\psi\omega K$ decays. A state with mass $m_Y = 3913 \pm 4$ MeV, according to the more accurate *BABAR* measurement, is observed to decay predominantly in this final state. This paper shows how the $J^{PC} = 0^{++}$ state, which decays predominantly into $J/\psi\omega$ and can be produced in B decays in pair with kaons if $L = 0$, has a predicted mass of 3927 MeV.

The $J/\psi\eta$ invariant mass was studied by *BABAR* [10] in $B \to J/\psi\eta K$ decays, and the resulting background subtracted distribution is reported in Fig. 1. Vertical lines refer to predicted mass value for the $J^{PC} = 1^{--}$ states. Even if states with different J^{PC} quantum number decay in the $J/\psi\eta$ final state, selection rules forbid $J^{PC} = 0^{--}$ or $J^{PC} = 1^{+-}$ states.

The $\psi f_0(980)$ decay mode was instead studied when $f_0(980)$ decays into two pions, i.e., in the $\psi\pi\pi$ final state, where ψ can be either J/ψ or $\psi(2S)$. Exotic mesons are searched in initial state radiation (i.e., $e^+e^- \to Y\gamma$ processes) and can therefore only be $J^{PC} = 1^{--}$. The published spectra [11,12] show several structures at $m = 4260$, 4350, and 4660 MeV. Although only the latter one shows $\pi\pi$ invariant masses clearly consistent with an $f_0(980)$ production, it is interesting to notice that the invariant masses predicted here for $J^{PC} = 1^{--}$ states are in the same mass range.

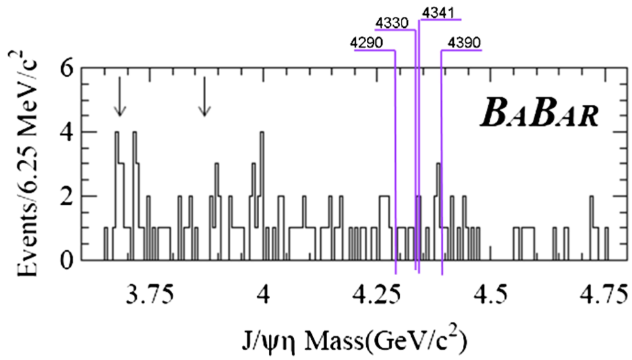


FIG. 1 (color online). $J/\psi\eta$ events at *BABAR*.

Finally, possible exotic states decaying into $J/\psi\phi$ and $J/\psi\eta'$ have been searched in Refs. [13,14], respectively, but no significant signal has been observed even integrating over the mass spectrum.

V. CONCLUSIONS

In this paper we have studied the consequences of allowing $[cs][\bar{c}\,\bar{s}]$ diquark-antidiquark particles with different J^{PC} quantum numbers. We present their spectrum and main decay modes in Table I.

After our paper appeared, the CDF Collaboration presented a 3.8σ evidence of a narrow structure in $J/\psi\phi$ at about 4143 MeV [15]. As it is clear from Fig. 1, we do not have anything close in our spectrum. Other interpretations of the CDF structure can be found in [16]. In the same CDF paper, another peak in $J/\psi\phi$ is found with a significance $\lesssim 3\sigma$ at about 4277 MeV. In this case we predict a 0^{-+} state at 4277 MeV decaying into $J/\psi\phi$ in the P wave.

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