

Complete Michel parameter analysis of the inclusive semileptonic $b \rightarrow c$ transitionBenjamin Dassinger,^{*} Robert Feger,[†] and Thomas Mannel[‡]*Fachbereich Physik, Universität Siegen, D-57068 Siegen, Germany*

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We perform a complete “Michel parameter” analysis of all possible helicity structures which can appear in the process $B \rightarrow X_c \ell \bar{\nu}_\ell$. We take into account the full set of operators parametrizing the effective Hamiltonian and include the complete one-loop QCD corrections as well as the nonperturbative contributions. The moments of the leptonic energy as well as the combined moments of the hadronic energy and hadronic invariant mass are calculated including the nonstandard contributions.

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I. INTRODUCTION

The experimental and theoretical developments in heavy flavor physics allow us to perform a high precision test of the flavor sector. In particular, the enormous amount of data for semileptonic B decays in combination with very reliable theoretical methods has opened the road for a precision determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements V_{cb} and V_{ub} , which are known at a relative precision of roughly 2% and 10% [1].

Aside from testing and extracting its parameters as precisely as possible, a second major goal of heavy flavor physics is to look for possible effects beyond the standard model. It is generally believed that flavor changing neutral currents are a good place to search for effects of new physics, since these decays are usually loop-induced and hence sensitive to virtual effects from high-mass states. Thus one expects here possibly an effect which is sizable compared to the standard-model contribution.

Semileptonic processes are tree level processes in the standard model and thus the relative effects from new-physics contributions are likely to be small. However, a possible right-handed admixture to the hadronic current is completely absent in the standard model and hence such an effect would be a clear signal for physics beyond the standard model.

In a recent publication [2] we considered a “Michel parameter analysis” [3] of semileptonic B decays, where we considered mainly a possible right-handed contribution to the hadronic $b \rightarrow c$ current. In the present paper we complete the analysis of [2] by extending the analysis to all possible two-quark–two-lepton operators.

There is an extensive literature on a possible non-standard model contributions to semileptonic B decays [4–8]. However, the analysis presented here is different in two respects. First of all, our analysis is completely model-independent; however, we neglect the lepton

masses and hence our analysis would need to be extended straightforwardly to include e.g. a discussion of a charged Higgs contribution as in [5,6,8]. Secondly, we consider different observables (i.e. the moments of spectra) which have become available only recently through the precise data of the B factories; in this way a much better sensitivity to a nonstandard contribution is expected.

Experimentally, there are limits on nonstandard contributions from various sources. In the leptonic sector the measurement of the μ and τ decay parameters yields already stringent limits [9]; likewise the neutrino data yield limits on right-handed admixtures. In total, the data in the leptonic sector indicate that a right-handed admixture for charged leptons is well below 1%.

Limits on a right-handed contribution can also be inferred from high-energy processes [9]. The global analysis of the LEP data does not give any indication for a right-handed admixture, which would show up in inconsistencies in the determination of the weak mixing angle. From the electroweak fit one obtains a mass limit for a right-handed W boson of $M_{W_R} > 715$ GeV, which yields a limit for a right-handed admixture in the charged currents of $c_R < 1.3\%$.

Finally one may also use the data on exclusive B decays to obtain limits on a right-handed contribution. In [4] the data on $B \rightarrow D^{(*)} \ell \bar{\nu}_\ell$ have been used to extract a limit on a right-handed admixture. This limit of $c_R = (14 \pm 18)\%$ is not as stringent as the ones mentioned above, but refers to the specific transition $b \rightarrow c$.

In the next section we perform an effective-field-theory analysis of possible new-physics contribution, which is kept completely model-independent. It turns out that only very few operators contribute to the semileptonic $b \rightarrow c$ transition. Compared to the usual Michel-parameter analysis, well known from muon decay, this effective theory analysis also yields order-of-magnitude estimates of the various contributions.

Based on these effective interactions we recompute the spectra of inclusive semileptonic $b \rightarrow c$ transitions including the new interactions. We make use of the standard heavy quark expansion (HQE) and include QCD radiative corrections as well as nonperturbative contributions.

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In Sec. III we shall perform the HQE including the new-physics operators. In subsection III A we compute the QCD radiative corrections for the various helicity combinations of the hadronic current. We adopt the kinetic scheme as it has been used for the calculation of semileptonic moments in [10] and perform the complete one-loop calculation for the new-physics terms. We note that the standard-model calculations have been performed already to order α_s^2 in [11,12].

In subsection III C we calculate the nonperturbative contributions of the new-physics operators to order $1/m_b^2$. Finally, in Sec. IV we quote our results for the various moments which are frequently used in the analysis of semileptonic decays and conclude.

II. EFFECTIVE-FIELD-THEORY ANALYSIS OF

$$b \rightarrow c\ell\bar{\nu}_\ell$$

It is well known that any contribution to the effective Lagrangian of some yet unknown physics at a high scale Λ can be written as contributions of operators with dimensions larger than 4. These operators are $SU(3) \times SU(2) \times U(1)$ invariant and suppressed by an appropriate power of $1/\Lambda$. We note that for such an analysis we have to make an assumption of the yet unestablished Higgs sector: We shall stick with our analysis to the single Higgs doublet case; an extension to a type-II two-Higgs doublet as e.g. needed for supersymmetry is straightforward.

The lowest dimension relevant for our analysis is six; the list of relevant operators has been given in [13] and we shall use the notations of our previous paper [2]. The quark and lepton fields are grouped into

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \quad \begin{pmatrix} t_L \\ b_L \end{pmatrix} \quad \text{left-handed quarks} \quad (1)$$

$$Q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix}, \quad \begin{pmatrix} c_R \\ s_R \end{pmatrix}, \quad \begin{pmatrix} t_R \\ b_R \end{pmatrix} \quad \text{right-handed quarks} \quad (2)$$

$$L_L = \begin{pmatrix} \nu_{e,L} \\ e_L \end{pmatrix}, \quad \begin{pmatrix} \nu_{\mu,L} \\ \mu_L \end{pmatrix}, \quad \begin{pmatrix} \nu_{\tau,L} \\ \tau_L \end{pmatrix} \quad \text{left-handed leptons} \quad (3)$$

$$L_R = \begin{pmatrix} \nu_{e,R} \\ e_R \end{pmatrix}, \quad \begin{pmatrix} \nu_{\mu,R} \\ \mu_R \end{pmatrix}, \quad \begin{pmatrix} \nu_{\tau,R} \\ \tau_R \end{pmatrix} \quad \text{right-handed leptons,} \quad (4)$$

where Q_L and L_L are doublets under $SU(2)_L$ and Q_R and L_R are doublets under an (explicitly broken) $SU(2)_R$. Note that we also introduced a right-handed neutrino in order to complete the right-handed lepton doublets.

The Higgs field and its charge conjugate are written as a 2×2 matrix

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_0 + i\chi_0 & \sqrt{2}\phi_+ \\ \sqrt{2}\phi_- & \phi_0 - i\chi_0 \end{pmatrix} \quad (5)$$

transforming under $SU(2)_L \times SU(2)_R$. The potential of the Higgs fields leads to a vacuum expectation value (VEV) for the field ϕ_0 .

The dimension-6 operators fall into two classes, the two-quark operators with gauge and Higgs fields and the two-quark–two-lepton operators. In our previous analysis [2] we considered only the first class, and the first step towards a full analysis is to also take into account the second class.

The list of two-quark–two-lepton operators with $SU(2)_L \times SU(2)_R$ consists of¹

$$\mathcal{O}_{LL,LL}^{(i)} = (\bar{Q}_L \Gamma_i Q_L)(L_L \Gamma_i L_L) \quad (6)$$

$$\mathcal{P}_{LL,LL}^{(i)} = (\bar{Q}_L \tau^a \Gamma_i Q_L)(L_L \tau^a \Gamma_i L_L) \quad (7)$$

$$\mathcal{O}_{LL,RR}^{(i)} = (\bar{Q}_L \Gamma_i Q_L)(L_R \Gamma_i L_R) \quad (8)$$

$$\mathcal{O}_{RR,LL}^{(i)} = (\bar{Q}_R \Gamma_i Q_R)(L_L \Gamma_i L_L) \quad (9)$$

$$\mathcal{O}_{RR,RR}^{(i)} = (\bar{Q}_R \Gamma_i Q_R)(L_R \Gamma_i L_R) \quad (10)$$

$$\mathcal{P}_{RR,RR}^{(i)} = (\bar{Q}_R \tau^a \Gamma_i Q_R)(L_R \tau^a \Gamma_i L_R) \quad (11)$$

while the operators with explicitly broken $SU(2)_R$ read

$$\mathcal{R}_{LL,RR}^{(i)} = (\bar{Q}_L \Gamma_i Q_L)(L_R \Gamma_i \tau^3 L_R) \quad (12)$$

$$\mathcal{R}_{RR,LL}^{(i)} = (\bar{Q}_R \Gamma_i \tau^3 Q_R)(L_L \Gamma_i L_L) \quad (13)$$

$$\mathcal{R}_{RR,RR}^{(i)} = (\bar{Q}_R \Gamma_i Q_R)(L_R \Gamma_i \tau^3 L_R) \quad (14)$$

$$\mathcal{S}_{RR,RR}^{(i)} = (\bar{Q}_R \tau^a \Gamma_i Q_R)(L_R \tau^a \tau^3 \Gamma_i L_R) \quad (15)$$

$$\mathcal{T}_{RR,RR}^{(i)} = (\bar{Q}_R \tau^a \tau^3 \Gamma_i Q_R)(L_R \tau^a \tau^3 \Gamma_i L_R). \quad (16)$$

Here we have defined

$$\begin{aligned} \Gamma_i \otimes \Gamma_i &= 1 \otimes 1, & \gamma_\mu \otimes \gamma^\mu, \\ \gamma_\mu \gamma_5 \otimes \gamma_5 \gamma^\mu, & & \sigma_{\mu\nu} \otimes \sigma^{\mu\nu}. \end{aligned} \quad (17)$$

Note that these operators are not all independent. Furthermore, note that operators with the helicity combinations such as $(LR)(LR)$ cannot appear at the level of $SU(2)_W \times U(1)_Y$ -invariant dimension-6 operators; they will appear as dimension-8 operators and will be of the

¹In order to have a streamlined notation we suppress all flavor indices in the following.

general form

$$(\bar{Q}_L \Gamma_i H Q_R)(\bar{L}_L \Gamma_i H L_R)$$

involving additional Higgs fields.

We shall assume that the right-handed neutrino acquires a large Majorana mass in which case it can be integrated out at some high scale, which we assume to lie well above Λ . In this case $SU(2)_R$ is “maximally broken” for the leptons, which means that the possible operators always have a projection $P_- = (1 - \tau^3)/2$ and thus only right-handed interactions involving the right-handed charged leptons remain.

For the case at hand we are interested in the charged current interactions containing a $b \rightarrow c$ transition. Since we eliminated the right-handed neutrino and helicities are conserved for both currents we end up with the conclusion that the charged leptonic current has to be left-handed. Thus we have only the operators

$$O_1 = (\bar{b}_L \gamma_\mu c_L)(\bar{\nu}_{\ell,L} \gamma^\mu \ell_L) \quad (18)$$

$$O_2 = (\bar{b}_R \gamma_\mu c_R)(\bar{\nu}_{\ell,L} \gamma^\mu \ell_L), \quad (19)$$

where $\ell = e, \mu, \text{ or } \tau$, since any helicity changing combination has to originate from dimension-8 operators yielding an additional suppression of a factor v^2/Λ^2 relative to the dimension-6 contributions.

However, as has been discussed in our previous paper, helicity violating combinations such as $(LR)(LL)$ operators can appear from the two-quark operators with gauge and Higgs fields. These operators induce anomalous gauge-boson couplings which are suppressed by a factor v^2/Λ^2 . They originate from two-quark operators, which are (at the scale of the weak bosons)

$$O_{LL}^{(1)} = \bar{Q}_L \not{L} Q_L \quad (20)$$

$$O_{LL}^{(2)} = \bar{Q}_L \not{L}_3 Q_L \quad (21)$$

with

$$L^\mu = H(iD^\mu H)^\dagger + (iD^\mu H)H^\dagger \quad (22)$$

$$L_3^\mu = H\tau_3(iD^\mu H)^\dagger + (iD^\mu H)\tau_3 H^\dagger. \quad (23)$$

The terms proportional to τ_3 have once again been included to break the custodial symmetry explicitly.

In the same spirit we define RR operators

$$O_{RR}^{(1)} = \bar{Q}_R \not{R} Q_R \quad (24)$$

$$O_{RR}^{(2)} = \bar{Q}_R \{\tau_3, \not{R}\} Q_R \quad (25)$$

$$O_{RR}^{(3)} = i\bar{Q}_R [\tau_3, \not{R}] Q_R \quad (26)$$

$$O_{RR}^{(4)} = \bar{Q}_R \tau_3 \not{R} \tau_3 Q_R \quad (27)$$

with

$$R^\mu = H^\dagger(iD^\mu H) + (iD^\mu H)^\dagger H. \quad (28)$$

Using an odd number of Higgs fields we can construct invariant LR operators. For our analysis the relevant operators are

$$O_{LR}^{(1)} = \bar{Q}_L(\sigma_{\mu\nu} B^{\mu\nu})H Q_R + \text{H.c.} \quad (29)$$

$$O_{LR}^{(2)} = \bar{Q}_L(\sigma_{\mu\nu} W^{\mu\nu})H Q_R + \text{H.c.} \quad (30)$$

$$O_{LR}^{(3)} = \bar{Q}_L(iD_\mu H)iD^\mu Q_R + \text{H.c.} \quad (31)$$

After spontaneous symmetry breaking the LL and RR operators contain anomalous quark-boson couplings of the order of magnitude v^2/Λ^2 . For the LR operators the field strengths of the weak bosons appear, inducing an additional factor of a quark momentum p , and hence the order of magnitude is $pv/\Lambda^2 \sim m_q v/\Lambda^2$.

At the scale of the bottom quark these anomalous coupling terms have the same power counting as the two-quark–two-lepton operators: Integrating out the weak bosons, their propagator together with the gauge couplings reduce to a pointlike interaction proportional to $g^2/M_W^2 = 1/v^2$. Combining this with the order of magnitude of the anomalous coupling of LL and RR v^2/Λ^2 we find that at the scale of the bottom mass these contributions scale in the same way as the two-quark–two-lepton operators directly induced at the high-scale Λ . For the case of LR the additional momentum p of the quark is of the order of its mass, and hence is as well of the order v^2/Λ^2 , possibly further suppressed by a small quark Yukawa coupling.

This conclusion may be altered in a two-Higgs doublet model in the case of large $\tan\beta$, i.e. of a large ratio of Higgs vacuum expectation values. After integrating out the heavy degrees of freedom $\tan\beta$ will play the role of a coupling constant which then may be enhanced by a large value. In e.g. [8] such a scenario has been considered, where a sizable value of $\tan\beta$ overcomes the suppression of the factor $(m_\ell m_b)/m_{H^\pm}^2$ in the amplitude; in this case also scalar contributions to the leptonic current have to be taken into account.

To this end, the parametrization introduced in [2] remains valid also in the general case, if only dimension-6 operators are included. Thus the effective Hamiltonian reads

$$\mathcal{H}_{\text{eff}} = \frac{4G_F V_{cb}}{\sqrt{2}} J_{q,\mu} J_l^\mu, \quad (32)$$

where $J_l^\mu = \bar{e} \gamma^\mu P_- \nu_e$ is the usual leptonic current and $J_{h,\mu}$ is the generalized hadronic $b \rightarrow c$ current which is given by

$$\begin{aligned}
 J_{h,\mu} = & c_L \bar{c} \gamma_\mu P_- b + c_R \bar{c} \gamma_\mu P_+ b + g_L \bar{c} i \overleftrightarrow{D}_\mu P_- b \\
 & + g_R \bar{c} i \overleftrightarrow{D}_\mu P_+ b + d_L i \partial^\nu (\bar{c} i \sigma_{\mu\nu} P_- b) \\
 & + d_R i \partial^\nu (\bar{c} i \sigma_{\mu\nu} P_+ b),
 \end{aligned} \quad (33)$$

where P_\pm denotes the projector on positive/negative chirality and D_μ is the QCD covariant derivative. Note that the term proportional to c_L contains the standard-model contribution as well as a possible new-physics contribution and c_R may now also contain a contribution from a two-quark–two-lepton operator induced at the high scale Λ . The gauge part $i g_3 A_\mu^a \lambda_a / 2$ of the QCD covariant derivative D_μ gives rise to a new quark-quark-gluon-boson vertex.

III. OPERATOR PRODUCT EXPANSION

The operator product expansion (OPE) for inclusive decays has become textbook material [14]. For the case of inclusive semileptonic decays the OPE is formulated for the T product of the two hadronic currents

$$\begin{aligned}
 T_{\mu\nu} = & \int d^4x e^{-ix(m_b v - q)} \\
 & \cdot \langle B(p) | \bar{b}_\nu(x) \Gamma_\mu c(x) \bar{c}(0) \Gamma_\nu^\dagger b_\nu(0) | B(p) \rangle,
 \end{aligned} \quad (34)$$

where Γ is the combination of Dirac matrices and derivatives given in (33), $v = p/M_B$ is the four velocity of the decaying B meson and q is the momentum transferred to the leptons. The quantity $T_{\mu\nu}$ is expanded in inverse powers of the scale of the order m_b , where m_b is the heavy quark mass. Technically this procedure is an OPE for the product of the two currents.

The standard-model calculation has been performed at tree level up to order $1/m_b^4$ and it turns out that the nonperturbative corrections are small. The radiative corrections have been computed to order α_s , $\beta_0 \alpha_s^2$, and recently also to order α_s^2 for the leading (i.e. the parton model) term [11] and to order α_s for the term of order $1/m_b^2$ involving μ_π^2 .

In the following we shall consider the perturbative and nonperturbative contributions to the OPE, performed with the modified current (33). We shall compute the complete one-loop contributions as well as the leading nonperturbative corrections proportional to μ_π^2 and μ_G^2 .

A. QCD corrections and renormalization group analysis

The calculation of the QCD radiative corrections has been performed in [15] and the results in the kinetic scheme have been given in [10] for the semileptonic moments in the standard model. In order to perform an analysis of possible nonstandard contributions we have to calculate the QCD radiative corrections for the current (33) to order α_s . Thus we have to evaluate the Feynman diagrams shown in Figs. 1 and 2 for the real and virtual corrections, respectively. Note that the scalar current (i.e.

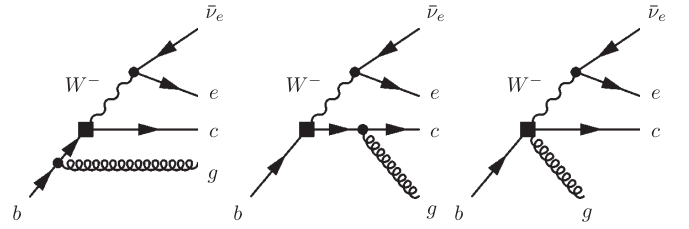


FIG. 1. Real corrections.

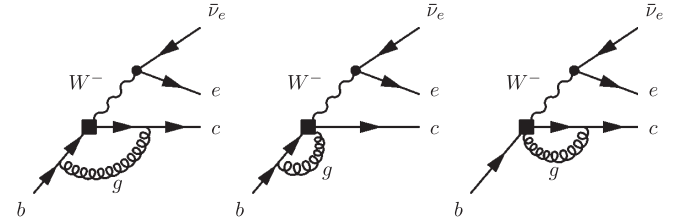


FIG. 2. Virtual corrections.

the terms proportional to $g_{L/R}$) induces new vertices shown in the Feynman diagrams at right. The real and virtual corrections are individually IR-divergent. We regulated the IR divergence by introducing a gluon mass which drops out upon summation of the real and virtual correction being IR-convergent. In the calculation of the virtual corrections the wave function renormalizations of the b and c quark field also have to be included.

The total amplitude consists of the sum of the standard-model contribution and the one from the new-physics operators. Since the new-physics piece is of order $1/\Lambda^2$, we shall include only the interference term of the standard model with this contribution. The square of the new-physics term is already of order $1/\Lambda^4$ and has to be neglected, since we compute only up to this order. Thus we compute

$$\begin{aligned}
 d\Gamma = & \frac{1}{2m_b} \langle \langle cl\nu | c_L \mathcal{H}_{\text{eff}}^{\text{SM}} | b \rangle \langle cl\nu | \mathcal{H}_{\text{eff}} | b \rangle^* \\
 & + \langle \langle cl\nu | \mathcal{H}_{\text{eff}} | b \rangle \langle cl\nu | c_L \mathcal{H}_{\text{eff}}^{\text{SM}} | b \rangle^* \rangle d\phi_{\text{PS}},
 \end{aligned} \quad (35)$$

where $d\phi_{\text{PS}}$ is the corresponding phase space (PS) element and

$$\mathcal{H}_{\text{eff}}^{\text{SM}} = \frac{4G_F V_{cb}}{\sqrt{2}} (\bar{c} \gamma_\mu P_- b) (\bar{e} \gamma^\mu P_- \nu_e)$$

is the standard-model effective Hamiltonian, which has the same helicity structure as the new-physics contribution proportional to c_L .

The relevant Feynman rules for the new-physics operators at tree level can be read off from (33); note that the terms involving g_L and g_R yield a boson-gluon-quark-antiquark vertex in order to maintain QCD gauge invariance.

It is well known that the left- and right-handed currents do not have anomalous dimensions and hence the parts of

(33) with c_L and c_R are not renormalized. However, the scalar and tensor contributions have anomalous dimensions and hence we need to normalize these operators at some scale and run them down to the scale of the bottom quark.

To this end, we have to calculate the anomalous-dimension matrix of these currents to set up the renormalization group equation. It can be obtained from the requirement that the physical matrix elements must not depend on the renormalization scale μ :

$$0 = \frac{d}{d \ln \mu} \langle c \ell \nu_e | \mathcal{H}_{\text{eff}} | b \rangle. \quad (36)$$

Inserting the OPE for the Hamiltonian we get:

$$\begin{aligned} \langle c \ell \nu_e | \mathcal{H}_{\text{eff}} | b \rangle &= \frac{4G_F V_{cb}}{\sqrt{2}} \cdot \langle c \ell \nu_e | [c_L (\bar{c} \gamma_\mu P_- b) (\bar{e} \gamma^\mu P_- \nu_e) \\ &\quad + c_R (\bar{c} \gamma_\mu P_+ b) (\bar{e} \gamma^\mu P_- \nu_e)] | b \rangle \\ &\quad + \frac{4G_F V_{cb}}{\sqrt{2}} \vec{C} \cdot \langle c \ell \nu_e | \vec{\mathcal{O}} | b \rangle, \end{aligned} \quad (37)$$

with

$$\begin{aligned} \vec{C} &= \begin{pmatrix} g_L \\ g_R \\ d_L \\ d_R \\ c_{L/R}^{m_b} \\ c_{L/R}^{m_c} \end{pmatrix} \\ \vec{\mathcal{O}} &= \begin{pmatrix} (\bar{c} i \vec{D}_\mu P_- b) (\bar{e} \gamma^\mu P_- \nu_e) \\ (\bar{c} i \vec{D}_\mu P_+ b) (\bar{e} \gamma^\mu P_- \nu_e) \\ (i \partial^\nu (\bar{c} i \sigma_{\mu\nu} P_- b)) (\bar{e} \gamma^\mu P_- \nu_e) \\ (i \partial^\nu (\bar{c} i \sigma_{\mu\nu} P_+ b)) (\bar{e} \gamma^\mu P_- \nu_e) \\ (m_b \bar{c} \gamma_\mu P_- b) (\bar{e} \gamma^\mu P_- \nu_e) \\ (m_b \bar{c} \gamma_\mu P_+ b) (\bar{e} \gamma^\mu P_- \nu_e) \\ (m_c \bar{c} \gamma_\mu P_- b) (\bar{e} \gamma^\mu P_- \nu_e) \\ (m_c \bar{c} \gamma_\mu P_+ b) (\bar{e} \gamma^\mu P_- \nu_e) \end{pmatrix}. \end{aligned} \quad (38)$$

where the operators $\vec{\mathcal{O}}$ are of dimension-7.

In the following we consider the renormalization group mixing of these dimension-4 operators. The calculation of the one-loop anomalous dimension is standard. We define the anomalous-dimension matrix γ by

$$\frac{d\vec{C}}{d \ln \mu} = \gamma^T(\mu) \vec{C} \quad (39)$$

and compute γ from the divergencies of the renormalization constants in the usual way. We obtain

$$\gamma^T(\mu) = \frac{2\alpha_s(\mu)}{3\pi} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mathbf{1} & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{1} & 0 & \mathbf{1} & 0 & 0 & 0 & 0 \\ 0 & \mathbf{3} & 0 & 0 & \mathbf{3} & 0 & 0 & 0 \\ \mathbf{3} & 0 & 0 & 0 & 0 & \mathbf{3} & 0 & 0 \\ \mathbf{3} & 0 & 0 & 0 & 0 & 0 & \mathbf{3} & 0 \\ 0 & \mathbf{3} & 0 & 0 & 0 & 0 & 0 & \mathbf{3} \end{pmatrix}. \quad (40)$$

The renormalization group (RG) equation for the Wilson coefficient is

$$\left(\frac{\partial}{\partial \ln \mu} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} \right) \vec{C} = \gamma^T(\alpha_s(\mu)) \vec{C}. \quad (41)$$

We seek a solution of this equation with the initial conditions

$$c_{L/R}^{m_b}(\Lambda) = 0 = c_{L/R}^{m_c}(\Lambda), \quad (42)$$

since the matching of the left- and right-handed currents is performed by fixing the coefficients c_L and c_R and all additional contributions are only due to renormalization group running. Inserting the one-loop expressions we obtain

$$\begin{aligned} c_{L/R}(\mu) &= c_{L/R}(\Lambda) & g_{L/R}(\mu) &= g_{L/R}(\Lambda) \\ d_{L/R}(\mu) &= (g_{L/R}(\Lambda) + d_{L/R}(\Lambda)) \left(\frac{\alpha_s(\Lambda)}{\alpha_s(\mu)} \right)^{4/(3\beta_0)} \\ &\quad - g_{L/R}(\Lambda) \\ c_{L/R}^{m_b}(\mu) &= g_{R/L}(\Lambda) \left(\left(\frac{\alpha_s(\Lambda)}{\alpha_s(\mu)} \right)^{4/(\beta_0)} - 1 \right) \\ c_{L/R}^{m_c}(\mu) &= g_{L/R}(\Lambda) \left(\left(\frac{\alpha_s(\Lambda)}{\alpha_s(\mu)} \right)^{4/\beta_0} - 1 \right). \end{aligned} \quad (43)$$

One may reexpand (43) using the one-loop expression for the strong coupling constant and obtain the logarithmic terms of the one-loop calculation. However, we have performed the complete one-loop calculation which yields also nonlogarithmic terms. Except for the vector and axial vector currents these nonlogarithmic contributions depend on the renormalization scheme, for which we have used the usual \overline{MS} scheme.

It is well known that in order to fix this dependence on the renormalization scheme, one would need to include the RG running at two loops, which, however, goes beyond the scope of the present paper. In the present paper, we use \overline{MS} and fix the scale to be $\mu = m_b$, which is the relevant scale of the decay processes.

The advantage of keeping the nonlogarithmic terms is that they lead to kinematic effects as e.g. a bremsstrahlung spectrum for the hadronic invariant mass. Thus they lead to a distortion of the spectra and hence have an impact on the moments. In particular the partonic mass moments $\langle (M_x^2 - m_c^2)^n \rangle$ with $n > 0$ will start at order α_s .

B. Mass scheme

The calculation of the process is usually set up with pole masses of the particles; the pole mass is known not to be a well-defined mass. The problems manifest themselves by abnormally large radiative corrections when the pole mass scheme is used. It has been discussed extensively in the literature that an appropriately defined short-distance mass is better suited for the OPE calculation of an inclusive semileptonic rate.

In the present analysis we will use the kinetic mass scheme, where the mass is defined by a nonrelativistic sum rule for the kinetic energy [16]. At one-loop level the kinetic mass is related to the pole mass by

$$m_q^{\text{kin}}(\mu_f) = m_q^{\text{pole}} \left[1 - \frac{4}{3} \frac{\alpha_s}{\pi} \left(\frac{4}{3} \frac{\mu_f}{m_b} + \frac{\mu_f^2}{2m_b^2} \right) \right], \quad (44)$$

where μ_f is a factorization scale for removing contributions below from the mass definition. The factorization scale is set to 1 GeV since this is the typical energy release in the process. This low renormalization scale is in fact the reason why the \overline{MS} scheme is inappropriate.

The ratio $\rho = m_c^2/m_b^2$ is rather stable under the choice of schemes (provided that the same scheme is chosen for both m_b and m_c) and thus the choice of the mass scheme enters only through the m_b^5 dependence of the rates. It is well known from the calculation in the standard model that the $\mathcal{O}(\alpha_s)$ corrections from the relation of the kinetic mass with the pole mass

$$\left(\frac{m_q^{\text{pole}}}{m_q^{\text{kin}}(1 \text{ GeV})} \right)^5 \approx 1 + 2.0899 \frac{\alpha_s}{\pi}. \quad (45)$$

compensate to a large extent the radiative corrections to the rates computed in the pole scheme, leaving only small QCD radiative corrections. It turns out that this also is the case in our calculation including the anomalous couplings.

C. Nonperturbative corrections

The nonperturbative corrections at tree level, including the modified current (33), have been studied in [17], however, these results have not yet been published, and hence we shall quote these results in the following.

The calculation of the nonperturbative corrections at tree level requires to compute the Feynman diagram shown in Fig. 3, where the one-gluon graph is needed to obtain the

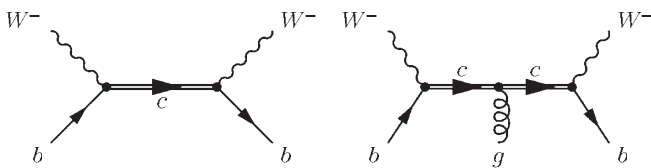


FIG. 3. Nonperturbative corrections at tree level.

matching coefficient of the chromomagnetic moment operator.

The calculation is standard and yields somewhat lengthy results, thus we defer the presentation of these expressions to Appendix A.

To the order we calculate, the nonperturbative effects are parametrized by the kinetic energy μ_π^2 and the chromomagnetic moment μ_G^2 , which are small quantities compared to the b -quark mass. Inserting the values extracted from $b \rightarrow c$ semileptonic decays we find

$$\frac{\mu_\pi^2}{m_b^2} \sim \frac{\mu_g^2}{3m_b^2} \sim 0.02. \quad (46)$$

Hence the nonperturbative corrections are tiny compared to the leading terms, as long as there are no abnormally large coefficients or the leading term vanishes. As it has been investigated in [17] this as well holds true for the new-physics contributions parametrized by (33).

IV. RESULTS AND DISCUSSION

We have evaluated the new-physics contributions to the various moments of the leptonic and hadronic energy and the hadronic invariant mass spectra. We have included tree-level partonic and $1/m_b^2$ corrections as well as the QCD radiative corrections at one-loop with a renormalization group treatment as described in the last section. The hadronic energy and the hadronic invariant mass of the decay products can be written as

$$\begin{aligned} E_{\text{Had}} &= \mathbf{v} \cdot (\mathbf{p}_B - \mathbf{q}) = m_B - \mathbf{v} \cdot \mathbf{q} \\ s_{\text{Had}} &= (\mathbf{p}_B - \mathbf{q})^2 = m_B^2 - 2m_B \mathbf{v} \cdot \mathbf{q} + q^2, \end{aligned} \quad (47)$$

where m_B and $\mathbf{p}_B = m_B \mathbf{v}$ are the mass and the momentum of the B meson and \mathbf{q} is the momentum of the leptonic system. The B -meson mass can be expanded as

$$m_B = m_b + \bar{\Lambda} + \frac{\mu_\pi^2 + \mu_g^2}{2m_b} + \dots. \quad (48)$$

Thus it is possible to relate the hadronic variables in (47) to the partonic ones

$$\begin{aligned} \hat{E}_0 &= \frac{E_0}{m_b} = \frac{\mathbf{v} \cdot (\mathbf{p}_b - \mathbf{q})}{m_b} = 1 - \mathbf{v} \cdot \hat{\mathbf{q}} \\ \hat{s}_0 &= \frac{s_0}{m_b^2} = \frac{(\mathbf{p}_b - \mathbf{q})^2}{m_b^2} = 1 - 2\mathbf{v} \cdot \hat{\mathbf{q}} + \hat{q}^2, \end{aligned} \quad (49)$$

where \mathbf{p}_b is the b -quark momentum. In the following we shall quote the results in terms of the partonic variables (49).

We have also included a cut on the charged lepton energy since such a cut has to be used in the experimental analysis. The results may be obtained as FORTRAN code from the authors. In order to have a qualitative discussion of the results, we give the results in Tables I, II, III, IV, and V for various moments without an energy cut for the

charged lepton energy E_l and with a cut of 1 GeV for this quantity. In Table I we list the results for the moments

$$L_n = \frac{1}{\Gamma_0} \int_{E_{\text{cut}}} d\hat{E}_l \hat{E}_l^n \frac{d\Gamma}{d\hat{E}_l}, \quad (50)$$

in Table II we consider the scale dependence of the L_n , and in the Tables III and V we quote

$$H_{ij} = \frac{1}{\Gamma_0} \int_{E_{\text{cut}}} d\hat{E}_l \int d\hat{s}_0 d\hat{E}_0 (\hat{s}_0 - \rho)^i E_l^j \frac{d^3\Gamma}{d\hat{E}_0 d\hat{s}_0 d\hat{E}_l} \quad (51)$$

with $\rho = m_b^2/m_c^2$, where the normalization

$$\Gamma_0 = \frac{G_F^2 |V_{cb}|^2 m_b^5}{192\pi^3} (1 - 8\rho - 12\rho^2 \ln\rho + 8\rho^3 - \rho^4) \quad (52)$$

is given in terms of the partonic rate. Note that we perform the calculations in the kinetic scheme, and we also insert the value of the kinetic mass in the normalization Γ_0 .

The entries in the tables contain the coefficients corresponding to the expansion of the various moments:

$$L_n = c_L^2 L_n^{(c_L c_L)} + c_L c_R L_n^{(c_L c_R)} + c_L d_L L_n^{(c_L d_L)} + c_L d_R L_n^{(c_L d_R)} + c_L g_L L_n^{(c_L g_L)} + c_L g_R L_n^{(c_L g_R)} \quad (53)$$

$$H_{ij} = c_L^2 H_{ij}^{(c_L c_L)} + c_L c_R H_{ij}^{(c_L c_R)} + c_L d_L H_{ij}^{(c_L d_L)} + c_L d_R H_{ij}^{(c_L d_R)} + c_L g_L H_{ij}^{(c_L g_L)} + c_L g_R H_{ij}^{(c_L g_R)}, \quad (54)$$

where all the coefficients have an expansion in α_s and in $1/m_b$

$$\begin{aligned} L_n^{(c_1 c_2)} &= L_n^{(c_1 c_2; m_b^0, \alpha_s^0)} + \frac{\mu_\pi^2}{m_b^2} L_n^{(c_1 c_2; m_b^2, \alpha_s^0)} + \frac{\mu_g^2}{3m_b^2} L_n^{(c_1 c_2; m_b^2, \alpha_s^0)} \\ &+ \dots + \frac{\alpha_s}{\pi} L_n^{(c_1 c_2; m_b^0, \alpha_s^1)} + \dots \\ H_{ij}^{(c_1 c_2)} &= H_{ij}^{(c_1 c_2; m_b^0, \alpha_s^0)} + \frac{\mu_\pi^2}{m_b^2} H_{ij}^{(c_1 c_2; m_b^2, \alpha_s^0)} \\ &+ \frac{\mu_g^2}{3m_b^2} H_{ij}^{(c_1 c_2; m_b^2, \alpha_s^0)} + \dots + \frac{\alpha_s}{\pi} H_{ij}^{(c_1 c_2; m_b^0, \alpha_s^1)} \\ &+ \dots, \end{aligned}$$

where we have only shown the terms which we have calculated. Note that the one-loop calculation of the moments yields results of the (schematic) form

$$\begin{aligned} L_n &= L_n^{\text{Born}} \left[1 + a_n \frac{\alpha_s}{\pi} \ln\left(\frac{\Lambda^2}{m_b^2}\right) + b_n^{\overline{MS}} \frac{\alpha_s}{\pi} \right] \\ &= L_n(\mu) \left[1 + a_n \frac{\alpha_s}{\pi} \ln\left(\frac{\mu^2}{m_b^2}\right) + b_n^{\overline{MS}} \frac{\alpha_s}{\pi} \right] \quad (55) \end{aligned}$$

and similar expression for the hadronic moments.

As discussed above, the nonlogarithmic contribution b_n depends on the chosen scheme, and

$$L_n(\mu) = L_n^{\text{Born}} \left[1 + a_n \frac{\alpha_s}{\pi} \ln\left(\frac{\Lambda^2}{\mu^2}\right) \right]$$

is replaced by the RG-improved results given in (43).

The results of the calculations are displayed in Tables I, II, III, IV, and V in Appendix B. Within those tables the scales have been chosen to be $\Lambda = M_W$ and $\mu = m_b$. Table I contains the results for the leptonic moments normalized to the total leptonic rate at tree level (52) for all lepton energies and for a cut of 1 GeV on the lepton energy. It turns out that the radiative corrections to the scalar and tensor admixtures are sizable, i.e. the α_s/π coefficients are large. In addition, these coefficients have the opposite sign as the tree-level piece, and hence a substantial reduction of the tree result is expected.

Table II contains the sum of the tree level and the α_s contributions using the one-loop expression for the running coupling α_s . As discussed above, the full next-to-leading order (NLO) expressions for the scalar and tensor couplings are not available yet and hence a residual scale dependence remains. We expect the scale μ to be of the order of m_b and hence we evaluate the expressions for $\mu = m_b/2$, m_b , and $2m_b$. For c_L^2 as well as for $c_L c_R$ the scale dependence is weak and originates from yet unknown NNLO effects. Because of the large α_s/π coefficients the scale dependence for the tensor couplings is sizable, while it is huge for the scalar couplings, since the tree contribution is almost canceled by the radiative correction. A full NLO calculation will very likely not improve this situation and hence we have to conclude that we will not have a good sensitivity to the tensor couplings and practically no sensitivity to the scalar couplings, at least for the lepton-energy moments.

The coefficients of the nonperturbative contributions at tree level are in general of similar size as the ones of the α_s corrections. Since $\alpha_s/\pi \sim \mu_\pi/m_b^2$, the nonperturbative corrections are of similar importance as the radiative ones. However, the leptonic moments are all dominated by the tree-level contribution and hence the radiative as well as the nonperturbative corrections to the moments are small.

Tables III and V contain the various hadronic moments computed without and with a cut on the lepton energy. For the $i = 0$ moments we have to draw the same conclusion as for the leptonic moments: The scalar and tensor couplings have large and opposite-sign coefficients compared to the tree-level piece; this leads in the same way to a sizable

reduction of the tree-level result as well as to a large scale dependence, which is shown in Table IV, where the result up to order α_s is shown.

Clearly the moments with $i > 0$ do not have a tree-level contribution at the partonic level since the tree-level partonic rate is proportional to the mass shell delta function $\delta(\hat{s}_0 - \rho)$. For these moments the leading contributions are at order α_s or $1/m_b^2$. Hence their dependence on the scale is given by the dependence of α_s . However, here the radiative corrections are small compared to the nonperturbative ones. The nonperturbative corrections at tree level contain also derivatives of the mass shell delta function $\delta(\hat{s}_0 - \rho)$, where at leading order $1/m_b$ the maximum number of derivatives is two. Because of this, the first and second i moments are of order $1/m_b^2$; higher moments with $i > 2$ will only have contributions of order $1/m_b^3$ or higher.

The sensitivity to a possible new-physics contribution is mainly limited by the precision of the standard-model calculation. Current analyses use up to the second moments in both the leptonic energy and the invariant mass squared. The highest moments included in the standard-model analyses are (roughly) sensitive to terms of the order $1/m_b^3$ which is the highest order in the $1/m_b$ expansion included in the fit. The size of these terms together with the size of the α_s^2 corrections may serve as a conservative estimate of the uncertainties of the standard-model calculation, which at the end determines the sensitivity to a possible new-physics contribution. Furthermore, an inclusion of higher moments in the fit, including the new contributions (in particular with $i > 2$), needs the calculation of the $1/m_b^3$ terms for the new-physics contributions. As the impact of such hadronic mass moments to the fit is small we did not include a table of them in this paper, but the results of the calculation can be obtained from the authors in a FORTRAN or MATHEMATICA file.

V. SUMMARY AND CONCLUSIONS

This work completes the analysis of possible new-physics effects in inclusive semileptonic B decays. Starting from a general ansatz for anomalous couplings in semileptonic decays we compute the effects on leptonic and hadronic moments which are used in the analysis of inclusive semileptonic decays.

As far as the leptonic moments are concerned, the QCD radiative corrections turn out to be as important as the nonperturbative ones. We have presented the complete expressions to order α_s and to order $1/m_b^2$ including the new-physics pieces.

This holds also true for the hadronic energy moments. However, the hadronic mass moments (taken with respect to m_c^2) do not have a tree-level contribution. Hence the nonperturbative corrections of order $1/m_b^2$ as well as the terms of order α_s are the leading contributions in the heavy quark expansion. It turns out that, numerically, the nonperturbative contributions are in general dominant.

For the leptonic moments and for the $i = 0$ hadronic moments the radiative correction for the scalar and tensor couplings turns out to be sizable. This leads to a substantial reduction of the moments and in the case of the scalar coupling to a near cancellation between tree level and the radiative corrections, which induces a large scale dependence. Hence the sensitivity to scalar and tensor couplings of the moments is limited. However, the moments with $i \neq 0$ appear first at order α_s and have a reasonable sensitivity to scalar and tensor couplings.

The standard analysis in semileptonic decays is to perform a combined fit of V_{cb} , the quark masses and the HQE parameters, usually up to order $1/m_b^3$. We propose to use the results given here to include the anomalous couplings induced by possible new physics into such a fit.

The effective-theory analysis indicates that a right-handed admixture could be the largest effect. Since the radiative corrections to the right-handed currents are completely known to NLO and the size of the coefficients indicates a good sensitivity to the anomalous coupling c_R this coefficient should be the first to be searched for in a moment analysis.

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APPENDIX A: NONPERTURBATIVE CORRECTIONS TO THE NONSTANDARD CURRENTS

In this appendix we show the results for the new-physics contributions based on (33). We have calculated the interference term of the standard-model contribution with the current (33) and list the terms proportional to the coupling constants. We obtain

$$\begin{aligned} \left(\frac{d\Gamma}{dy}\right)^{c_L c_L} &= \left(\frac{2(y-3)y^2\rho^3}{(y-1)^3} - \frac{6y^2\rho^2}{(y-1)^2} - 6y^2\rho + 2(3-2y)y^2\right) + \left(\frac{4(y^2-5y+10)\rho^3y^3}{3(y-1)^5} + \frac{2(5-2y)\rho^2y^3}{(y-1)^4} + \frac{10y^3}{3}\right)\frac{\mu_\pi^2}{m_b^2} \\ &+ \left(\frac{10y^2(y^2-4y+6)\rho^3}{(y-1)^4} - \frac{18(y-2)y^2\rho^2}{(y-1)^3} + \frac{12y^2(2y-3)\rho}{(y-1)^2} + 2y^2(5y+6)\right)\frac{\mu_g^2}{3m_b^2} \end{aligned}$$

$$\begin{aligned} \left(\frac{d\Gamma}{dy}\right)^{c_L c_R} &= \sqrt{\rho} \left(-\frac{12\rho^2 y^2}{(y-1)^2} - \frac{24\rho y^2}{y-1} - 12y^2 \right) + \sqrt{\rho} \left(\frac{4(5-2y)\rho^2 y^3}{(y-1)^4} + \frac{4(5-3y)\rho y^3}{(y-1)^3} \right) \frac{\mu_\pi^2}{m_b^2} \\ &+ \sqrt{\rho} \left(\frac{12\rho y^3}{(y-1)^2} - \frac{36(y-2)\rho^2 y^2}{(y-1)^3} + \frac{24(2y-3)y^2}{y-1} \right) \frac{\mu_g^2}{3m_b^2} \end{aligned}$$

$$\begin{aligned} \left(\frac{d\Gamma}{dy}\right)^{c_L g_L} &= \left(-\frac{12\rho^2 y^2}{y-1} - 12(y-1)y^2 - 24\rho y^2 \right) m_b + \left(-\frac{2(4y^2-9y+3)\rho^2 y^2}{(y-1)^3} - 12\rho y^2 + 6y^2 \right) \frac{\mu_\pi^2}{m_b^2} \\ &+ \left(\frac{6(3-2y)\rho^2 y^2}{(y-1)^2} - \frac{12(y-3)\rho y^2}{y-1} + 18y^2 \right) \frac{\mu_g^2}{3m_b^2} \end{aligned}$$

$$\begin{aligned} \left(\frac{d\Gamma}{dy}\right)^{c_L g_R} &= \sqrt{\rho} \left(-\frac{12\rho^2 y^2}{y-1} - 12(y-1)y^2 - 24\rho y^2 \right) m_b + \sqrt{\rho} \left(-\frac{2(4y^2-9y+3)\rho^2 y^2}{(y-1)^3} - 12\rho y^2 + 6y^2 \right) \frac{\mu_\pi^2}{m_b^2} \\ &+ \sqrt{\rho} \left(\frac{30(3-2y)\rho^2 y^2}{(y-1)^2} - \frac{60(y-3)\rho y^2}{y-1} + 90y^2 \right) \frac{\mu_g^2}{3m_b^2} \end{aligned}$$

$$\begin{aligned} \left(\frac{d\Gamma}{dy}\right)^{c_L d_L} &= \left(-\frac{8\rho^3 y^3}{(y-1)^3} - \frac{12\rho^2 y^3}{(y-1)^2} + 4y^3 \right) m_b + \left(\frac{4(-4y^2+11y+5)\rho^3 y^3}{3(y-1)^5} + \frac{2(-3y^2+4y+5)\rho^2 y^3}{(y-1)^4} - \frac{10y^3}{3} \right) \frac{\mu_\pi^2}{m_b^2} \\ &+ \left(\frac{12(5-2y)\rho^3 y^3}{(y-1)^4} - \frac{6(y-3)\rho^2 y^3}{(y-1)^3} + \frac{24(y-2)\rho y^3}{(y-1)^2} + 6y^3 \right) \frac{\mu_g^2}{3m_b^2} \end{aligned}$$

$$\begin{aligned} \left(\frac{d\Gamma}{dy}\right)^{c_L d_R} &= \sqrt{\rho} \left(\frac{12(y-3)\rho^2 y^2}{(y-1)^2} + 4(y-3)y^2 + \frac{12(y-3)\rho y^2}{y-1} \right) m_b \\ &+ \sqrt{\rho} \left(\frac{8(y^2-5y+10)\rho^3 y^3}{3(y-1)^5} + \frac{2(3y^2-16y+25)\rho^2 y^3}{(y-1)^4} + \frac{4(5-3y)\rho y^3}{(y-1)^3} - \frac{10y^3}{3} \right) \frac{\mu_\pi^2}{m_b^2} \\ &+ \sqrt{\rho} \left(\frac{20y^2(y^2-4y+6)\rho^3}{(y-1)^4} + \frac{6y^2(5y^2-25y+36)\rho^2}{(y-1)^3} + \frac{12(6-5y)y^2\rho}{(y-1)^2} - \frac{2y^2(5y^2-5y+12)}{y-1} \right) \frac{\mu_g^2}{3m_b^2} \end{aligned}$$

$$\frac{d\Gamma}{dy} = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}| \left(\left(\frac{d\Gamma}{dy}\right)^{c_L c_L} c_L^2 + \left(\frac{d\Gamma}{dy}\right)^{c_L c_R} c_L c_R + \left(\frac{d\Gamma}{dy}\right)^{c_L g_L} c_L g_L + \left(\frac{d\Gamma}{dy}\right)^{c_L g_R} c_L g_R + \left(\frac{d\Gamma}{dy}\right)^{c_L d_L} c_L d_L + \left(\frac{d\Gamma}{dy}\right)^{c_L d_R} c_L d_R \right).$$

APPENDIX B: TABLES

In this appendix we present the results from the calculation introduced in Sec. IV. We show tables for the various moments of the leptonic and hadronic invariant mass spectra for the new-physics contributions. These tables include tree-level partonic and $1/m_b^2$ corrections as well as radiative corrections at one-loop, computed in the \overline{MS} scheme. For the numerical analysis we use $m_b^{\text{kin}}(1 \text{ GeV}) = 4.6 \text{ GeV}$ and $\rho = m_c^2/m_b^2 = 0.0625$.

TABLE I. Tree level and α_s/π coefficients of the leptonic moments without E_l cuts and with a cut $E_l > 1$ GeV. Note that we have redefined $\mathbf{d}_{L/R} = m_B d_{L/R}$ and $\mathbf{g}_{L/R} = m_B g_{L/R}$ with $m_B = 5.279$ GeV in order to tabulate dimensionless quantities. The scale-dependent couplings have to be taken at the scale $\Lambda = M_W$ and μ is taken to be $\mu = m_b$.

			n	c_L^2	c_{LCR}	c_{LGL}	c_{LGR}	$c_L d_L$	$c_L d_R$
No E_l cuts	Tree-level	Parton	0	1.0000	-0.6685	0.2212	0.5400	0.3315	-0.6597
			1	0.3072	-0.2092	0.0613	0.1372	0.0977	-0.2307
			2	0.1030	-0.0708	0.0188	0.0388	0.0314	-0.0845
			3	0.0365	-0.0252	0.0062	0.0118	0.0107	-0.0319
	μ_π^2/m_b^2 coeff.	0	-0.5000	0.3342	-0.0017	0.1703	-0.1652	0.3288	
		1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
		2	0.0858	-0.0590	0.0365	0.1146	0.0261	-0.0702	
		3	0.0730	-0.0503	0.0210	0.0575	0.0214	-0.0637	
	μ_g^2/m_b^2 coeff.	0	-1.9449	4.9934	1.0232	1.5624	-2.1536	3.7106	
		1	-0.9625	1.8578	0.3253	0.6011	-0.7986	1.5873	
		2	-0.4495	0.7237	0.1124	0.2427	-0.3081	0.6840	
		3	-0.2052	0.2902	0.0410	0.1008	-0.1220	0.2966	
	α_s/π coeff.	0	0.3125	0.8009	-2.6592	-8.8212	-2.1497	4.3637	
		1	0.0908	0.2284	-0.7171	-2.3141	-0.5594	1.4880	
		2	0.0276	0.0739	-0.2174	-0.6843	-0.1660	0.5394	
		3	0.0085	0.0260	-0.0711	-0.2189	-0.0538	0.2039	
$E_l > 1$ GeV cut	Tree-level	Parton	0	0.8148	-0.5617	0.1621	0.3586	0.2631	-0.6161
			1	0.2776	-0.1919	0.0520	0.1089	0.0867	-0.2232
			2	0.0979	-0.0678	0.0172	0.0340	0.0296	-0.0831
			3	0.0356	-0.0246	0.0059	0.0109	0.0104	-0.0317
	μ_π^2/m_b^2 coeff.	0	-0.4504	0.3225	0.0433	0.3440	-0.1479	0.3631	
		1	0.0087	-0.0021	0.0564	0.2247	0.0031	0.0059	
		2	0.0874	-0.0594	0.0377	0.1194	0.0267	-0.0691	
		3	0.0733	-0.0504	0.0213	0.0583	0.0215	-0.0635	
	μ_g^2/m_b^2 coeff.	0	-2.1029	4.6903	0.8592	1.4595	-2.0451	3.7102	
		1	-0.9883	1.8078	0.2989	0.5845	-0.7805	1.5871	
		2	-0.4540	0.7149	0.1078	0.2398	-0.3049	0.6840	
		3	-0.2060	0.2886	0.0401	0.1003	-0.1214	0.2966	
	α_s/π coeff.	0	0.2640	0.5740	-1.8506	-5.9374	-1.3992	3.9213	
		1	0.0828	0.1930	-0.5920	-1.8692	-0.4440	1.4126	
		2	0.0262	0.0679	-0.1964	-0.6098	-0.1467	0.5260	
		3	0.0083	0.0249	-0.0674	-0.2058	-0.0504	0.2014	

 TABLE II. Summed up tree-level and α_s/π coefficients of the leptonic moments without E_l cuts for $\mu = 2.3, 4.6,$ and 9.2 GeV.

μ	n	c_L^2	c_{LCR}	c_{LGL}	c_{LGR}	$c_L d_L$	$c_L d_R$
2.3 GeV	0	1.0253	-0.6037	0.0042	-0.1916	0.1533	-0.2983
	1	0.3145	-0.1907	0.0028	-0.0552	0.0512	-0.1074
	2	0.1052	-0.0648	0.0011	-0.0182	0.0176	-0.0397
	3	0.0372	-0.0231	0.0004	-0.0065	0.0062	-0.0150
4.6 GeV	0	1.0208	-0.6151	0.0441	-0.0474	0.1883	-0.3692
	1	0.3132	-0.1940	0.0135	-0.0169	0.0604	-0.1317
	2	0.1048	-0.0658	0.0043	-0.0068	0.0204	-0.0485
	3	0.0371	-0.0234	0.0015	-0.0028	0.0071	-0.0184
9.2 GeV	0	1.0177	-0.6231	0.0715	0.0752	0.2146	-0.4223
	1	0.3123	-0.1963	0.0208	0.0164	0.0674	-0.1499
	2	0.1046	-0.0666	0.0065	0.0034	0.0225	-0.0552
	3	0.0370	-0.0237	0.0022	0.0005	0.0078	-0.0209

TABLE III. Tree-level and α_s/π coefficients of the hadronic moments without E_l cuts. The partonic tree-level moments for $i > 1$ are all zero. Note that we have redefined $\mathbf{d}_{L/R} = m_B d_{L/R}$ and $\mathbf{g}_{L/R} = m_B g_{L/R}$ with $m_B = 5.279$ GeV in order to tabulate dimensionless quantities. The scale-dependent couplings have to be taken at the scale $\Lambda = M_W$ and μ is taken to be $\mu = m_b$.

		i	j	c_L^2	$c_L c_R$	$c_L g_L$	$c_L g_R$	$c_L d_L$	$c_L d_R$	
Tree-level	Parton	0	0	1.0000	-0.6685	0.2212	0.5400	0.3315	-0.6597	
		0	1	0.4220	-0.2500	0.0961	0.2556	0.1217	-0.2559	
		0	2	0.1832	-0.0964	0.0429	0.1219	0.0461	-0.1021	
		0	3	0.0815	-0.0383	0.0196	0.0586	0.0180	-0.0418	
			$i > 0$	j	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		μ_π^2/m_b^2 coeff.	0	0	-0.5000	0.3342	-0.0017	0.1703	-0.1652	0.3288
			0	1	-0.5000	0.3342	-0.100	-0.2229	-0.1652	0.3288
			0	2	-0.2902	0.1836	-0.0773	-0.2119	-0.0899	0.1840
			0	3	-0.1382	0.0837	-0.0448	-0.1348	-0.0406	0.0853
			1	0	-0.5780	0.4185	-0.2038	-0.5937	-0.2091	0.4025
			1	1	-0.1584	0.1172	-0.0695	-0.2158	-0.0585	0.1129
			1	2	-0.0283	0.0280	-0.0217	-0.0718	-0.0143	0.0258
			2	0	0.1609	-0.0728	0.0386	0.1159	0.0337	-0.0809
			2	1	0.0735	-0.0302	0.0180	0.0561	0.0138	-0.0343
			3	0	0.0000	0.0000	0.0000	0.0000	0.0000	-0.0000
		μ_g^2/m_b^2 coeff.	0	0	-1.9449	4.9934	1.0232	1.5624	-2.1536	3.7106
			0	1	-0.3850	1.2777	0.4097	0.4782	-0.5223	0.9700
			0	2	-0.0302	0.2833	0.1576	0.1391	-0.1109	0.2254
			0	3	0.0298	0.0342	0.0578	0.0350	-0.0146	0.0347
			1	0	0.3143	-0.6395	-0.1100	-0.2167	0.2027	-0.4360
		1	1	0.1195	-0.2561	-0.0529	-0.0925	0.0744	-0.1709	
		1	2	0.0466	-0.1059	-0.0254	-0.0405	0.0282	-0.0689	
		$i > 1$	j	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
α_s/π coeff.		0	0	0.3128	0.8007	-2.6592	-8.8212	-2.1497	4.3637	
		0	1	0.1631	0.3441	-1.2391	-4.1901	-0.8839	1.8575	
		0	2	0.0910	0.1477	-0.5850	-2.0067	-0.3694	0.8017	
		0	3	0.0526	0.0632	-0.2793	-0.9681	-0.1568	0.3505	
		1	0	0.0901	-0.0363	0.0028	0.0176	0.0032	-0.0095	
		1	1	0.0470	-0.0178	0.0014	0.0093	0.0015	-0.0046	
		1	2	0.0251	-0.0090	0.0007	0.0050	0.0007	-0.0023	
		2	0	0.0091	-0.0033	0.0001	0.0015	0.0002	-0.0008	
		2	1	0.0053	-0.0019	0.0000	0.0009	0.0001	-0.0004	
		3	0	0.0018	-0.0006	0.0000	0.0003	0.0000	-0.0001	

TABLE IV. Summed up tree-level and α_s/π coefficients of the nonzero tree-level hadronic moments without E_l cuts for $\mu = 2.3, 4.6, \text{ and } 9.2$ GeV.

μ	i	j	c_L^2	$c_L c_R$	$c_L g_L$	$c_L g_R$	$c_L d_L$	$c_L d_R$
2.3 GeV	0	0	1.0253	-0.6037	0.0042	-0.1916	0.1533	-0.2983
	0	1	0.4352	-0.2222	-0.0051	-0.0914	0.0486	-0.1024
	0	2	0.1906	-0.0845	-0.0049	-0.0440	0.0156	-0.0360
	0	3	0.0857	-0.0331	-0.0033	-0.0214	0.0050	-0.0129
4.6 GeV	0	0	1.0208	-0.6151	0.0441	-0.0474	0.1883	-0.3692
	0	1	0.4329	-0.2271	0.0136	-0.0234	0.0628	-0.1322
	0	2	0.1892	-0.0866	0.0040	-0.0117	0.0215	-0.0487
	0	3	0.0850	-0.0340	0.0010	-0.0059	0.0075	-0.0185
9.2 GeV	0	0	1.0177	-0.6231	0.0715	0.0752	0.2146	-0.4223
	0	1	0.4312	-0.2305	0.0269	0.0335	0.0734	-0.1545
	0	2	0.1883	-0.0880	0.0104	0.0151	0.0258	-0.0582
	0	3	0.0845	-0.0347	0.0041	0.0069	0.0093	-0.0226

TABLE V. Tree-level and α_s/π coefficients of the hadronic moments with a cut $E_l > 1$ GeV. The partonic tree-level moments for $i > 1$ are all zero. Note that we have redefined $\mathbf{d}_{L/R} = m_B d_{L/R}$ and $\mathbf{g}_{L/R} = m_B g_{L/R}$ with $m_B = 5.279$ GeV in order to tabulate dimensionless quantities. The scale-dependent couplings have to be taken at the scale $\Lambda = M_W$ and μ is taken to be $\mu = m_b$.

		i	j	c_L^2	$c_L c_R$	$c_L g_L$	$c_L g_R$	$c_L d_L$	$c_L d_R$	
Tree-level	Parton	0	0	0.8148	-0.5617	0.1621	0.3586	0.2631	-0.6161	
		0	1	0.3341	-0.2037	0.0682	0.1676	0.0922	-0.2365	
		0	2	0.1411	-0.0761	0.0295	0.0789	0.0332	-0.0933	
		0	3	0.0612	-0.0293	0.0131	0.0375	0.0123	-0.0378	
		$i > 0$	j	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
	μ_π^2/m_b^2 coeff.	0	0	-0.4504	0.3225	0.0433	0.3440	-0.1479	0.3631	
		0	1	-0.4505	0.2921	-0.0597	-0.0843	-0.1329	0.3332	
		0	2	-0.2673	0.1561	-0.0532	-0.1300	-0.0695	0.1841	
		0	3	-0.1337	0.0706	-0.0327	-0.0935	-0.0308	0.0859	
		1	0	-0.5424	0.3590	-0.1687	-0.4845	-0.1685	0.3887	
		1	1	-0.1639	0.1022	-0.0598	-0.1852	-0.0478	0.1115	
		1	2	-0.0417	0.0262	-0.0204	-0.0678	-0.0126	0.0273	
		2	0	0.1203	-0.0547	0.0258	0.0742	0.0223	-0.0729	
		2	1	0.0538	-0.0221	0.0118	0.0355	0.0087	-0.0306	
		3	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
		μ_g^2/m_b^2 coeff.	0	0	-2.1029	4.6903	0.8592	1.4595	-2.0451	3.7102
			0	1	-0.4609	1.2205	0.3461	0.4476	-0.5005	0.9855
			0	2	-0.0660	0.2921	0.1348	0.1332	-0.1119	0.2391
			0	3	0.0131	0.0538	0.0507	0.0363	-0.0194	0.0439
			1	0	0.3074	-0.5095	-0.0803	-0.1804	0.1654	-0.4093
1	1		0.1171	-0.1971	-0.0381	-0.0751	0.0583	-0.1590		
1	2		0.0458	-0.0789	-0.0180	-0.0321	0.0211	-0.0635		
$i > 1$	j		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000		
α_s/π coeff.	0		0	0.2642	0.5739	-1.8506	-5.9373	-1.3992	3.9213	
	0		1	0.1216	0.2462	-0.8449	-2.7806	-0.5529	1.6572	
	0	2	0.0608	0.1057	-0.3919	-1.3149	-0.2221	0.7103		
	0	3	0.0323	0.0455	-0.1842	-0.6272	-0.0907	0.3086		
	1	0	0.0576	-0.0231	0.0018	0.0101	0.0018	-0.0079		
	1	1	0.0288	-0.0108	0.0009	0.0052	0.0008	-0.0038		
	1	2	0.0147	-0.0052	0.0004	0.0027	0.0004	-0.0018		
	2	0	0.0046	-0.0016	0.0001	0.0007	0.0001	-0.0006		
	2	1	0.0026	-0.0009	0.0000	0.0004	0.0000	-0.0003		
	3	0	0.0007	-0.0002	0.0000	0.0001	0.0000	-0.0001		

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