Double-lepton polarization asymmetries in $B_s \to \phi \ell^+ \ell^-$ decay in the fourth-generation standard model

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In this paper, we investigate the effects of the fourth generation of quarks on the double-lepton polarization asymmetries in the $B_s \to \phi \ell^+ \ell^-$ decay. It is shown that most of these asymmetries in $B_s \to \phi \ell^+ \ell^-$ are quite sensitive to the fourth-generation parameters. We also compare these asymmetries with those of $B \to K \ell^+ \ell^-$ decay and show that $\langle P_{LT} \rangle$, $\langle P_{TL} \rangle$, $\langle P_{NN} \rangle$, and $\langle P_{TT} \rangle$ in $B_s \to \phi \tau^+ \tau^-$ decay are more sensitive to the fourth-generation parameters in comparison with those of $B \to K \tau^+ \tau^-$ decay. We conclude that an efficient way to establish the existence of the fourth generation of quarks could be the study of these asymmetries in the $B_s \to \phi \ell^+ \ell^-$ decay.

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I. INTRODUCTION

Although the standard model (SM) is a successful theory, there is no clear theoretical argument within this model to restrict the number of generations to three, and therefore the possibility of a new generation should not be ruled out. Based on this possibility, a number of theoretical and experimental investigations have been performed. The measurement of the Z decay widths restricts the number of light neutrino for $m_\nu < m_Z/2$ to three [1]. However, if a heavy neutrino exits, the possibility of extra generations of heavy quarks is not excluded from the experiment. Moreover the electroweak data [2] supports an extra generation of heavy quarks, if the mass difference between the new up- and down-type quarks is not too large.

Many authors who support the existence of a fourth generation studied those effects in various areas, for instance, Higgs and neutrino physics, cosmology, and dark matter [3–8]. For example, in [8] it is argued that the fourth generation of quarks and leptons can be generated in the Higgs boson production at the Tevatron and the LHC, before actually being detected. By the detailed study of this process at the Tevatron and LHC, the number of generations in the SM can be determined. Moreover, the flavor democracy (democratic mass matrix approach) [9] favors the existence of the nearly degenerate fourth SM family, while the fifth SM family is disfavored both by the mass phenomenology and precision tests of the SM [10]. The main restrictions on the new SM families come from the experimental data on the ρ and S parameters [10]. However, the common mass of the fourth quark ($m_{t'}$) lies between 320 GeV and 730 GeV considering the experimental value of $\rho = 1.0002^{+0.0007}_{-0.0004}$ [11]. The last value is close to the upper limit on heavy quark masses $m_q \leq 700$ GeV $\approx 4m_t$, which follows from partial-wave unitarity at high energies [12]. It should be noted that with the preferable value $a \approx g_w$ flavor democracy predicts $m_{t'} \approx 8m_w \approx 640$ GeV.

One of the promising areas in the experimental search for the fourth generation, via its indirect loop effects, is the rare B meson decays. Based on this idea, serious attempts to probe the effects of the fourth generation on the rare B meson were made by many researchers. The fourth generation can affect physical observables, i.e., branching ratio, CP asymmetry, polarization asymmetries, and forward-backward asymmetries. The study of these physical observables is a good tool to use to look for the fourth generation of up-type quarks [13–29].

Recently, the sensitivity of the double-lepton polarization asymmetries to the fourth generation in the transition of B to a pseudoscalar meson ($B \to K \ell^+ \ell^-$) has been investigated, and it is found out that this observable is sensitive to the fourth-generation parameters ($m_{t'}$, $V_{t'b}V^*_{t's}$) [24]. In this work, we investigate the effects of the fourth generation of quarks (b', t') on the double-lepton polarizations in the transition of B to a vector meson ($B_s \to \phi \ell^+ \ell^-$) and compare our results with those of $B \to K \ell^+ \ell^-$ decay presented in Ref. [24]. It should be mentioned that both decays occur through the $b \to s$ transition in which the sequential fourth generation of up quarks (t'), like u, c, t quarks, contributes at the loop level. Hence, this new generation will change only the values of the Wilson coefficients via the virtual exchange of the fourth-generation up quark t', and the full operator set is exactly the same as in SM.

The paper is organized as follows: In Sec. II, the expressions for the matrix element and double-lepton polarizations of $B_s \to \phi \ell^+ \ell^-$ in the SM have been presented. The effect of the fourth generation of quarks on the effective Hamiltonian and the double-lepton polarization asymmetries have been discussed in Sec. III. The sensitivity of these polarizations to the fourth-generation parameters ($m_{t'}$, $r_{\rm sb}$, $\phi_{\rm sb}$) have been numerically analyzed in the final section.

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II. THE MATRIX ELEMENT AND DOUBLE-LEPTON POLARIZATIONS OF $B_s \to \phi \ell^+ \ell^-$ IN THE SM

In the SM, the relevant effective Hamiltonian for $B_s \to \phi \ell^+ \ell^-$ decay, which is described by the $b \to s\ell^+\ell^-$ transition at quark level, can be written as

$$\mathcal{H}_{\rm eff} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) \mathcal{O}_i(\mu), \tag{1}$$

where the complete set of the operators $\mathcal{O}_i(\mu)$ and the corresponding expressions for the Wilson coefficients $C_i(\mu)$ are given in [30]. Using the above effective Hamiltonian, the one-loop matrix elements of $b \to s\ell^+\ell^-$ can be written in terms of the tree-level matrix elements of the effective operators as

$$\begin{aligned}\mathcal{M}(b \to s\ell^+\ell^-) &= \langle s\ell^+\ell^- | \mathcal{H}_{\rm eff} | b\rangle \\ &= -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i C_i^{\rm eff}(\mu) \langle s\ell^+\ell^- | \mathcal{O}_i | b\rangle^{\rm tree}. \\ &= -\frac{G_F \alpha}{2\pi\sqrt{2}} V_{tb} V_{ts}^* \Big[\tilde{C}_9^{\rm eff} \bar{s}\gamma_\mu (1-\gamma_5) b \bar{\ell}\gamma_\mu \ell \\ &\quad + \tilde{C}_{10}^{\rm eff} \bar{s}\gamma_\mu (1-\gamma_5) b \bar{\ell}\gamma_\mu \gamma_5 \ell \\ &\quad - 2C_7^{\rm eff} \frac{m_b}{q^2} \bar{s} i\sigma_{\mu\nu} q^\nu (1+\gamma_5) b \bar{\ell}\gamma_\mu \ell \Big],\end{aligned} \tag{2}$$

where $q^2 = (p_1 + p_2)^2$ and p_1 and p_2 are the final leptons four-momenta and the effective Wilson coefficients at μ scale, are given as [30,31]

$$\begin{aligned} C_7^{\rm eff} &= C_7 - \frac{1}{3} C_5 - C_6 \qquad C_{10}^{\rm eff} = \frac{\alpha}{2\pi} \tilde{C}_{10}^{\rm eff} = C_{10} \\ C_9^{\rm eff} &= \frac{\alpha}{2\pi} \tilde{C}_9^{\rm eff} = C_9 + \frac{\alpha}{2\pi} Y(s). \end{aligned} \tag{3}$$

In Eq. (3), $s = q^2/m_b^2$ and the function $Y(s)$ contains the short-distance contributions due to the one-loop matrix element of the four quark operators $Y_{\rm per}(s)$, as well as the long-distance contributions coming from the real $c\bar{c}$ intermediate states, i.e., J/ψ, ψ', $\cdots$. The latter contributions are taken into account by introducing Breit-Wigner form of the resonance propagator, which leads to the second term in the following formula [see Eq. (4)] [32–34]. As a result, the function $Y(s)$ can be written as

$$\begin{aligned} Y(s) &= Y_{\rm per}(s) + \frac{3\pi}{\alpha^2}(3C_1 + C_2 + 3C_3 + C_4 + 3C_5 \\ &\quad + C_6) \sum_{V_i = \psi_i} \kappa_i \frac{m_{V_i} \Gamma(V_i \to \ell^+\ell^-)}{m_{V_i}^2 - s m_b^2 - i m_{V_i} \Gamma_{V_i}}, \end{aligned} \tag{4}$$

where

$$\begin{aligned} Y_{\rm per}(s) &= g\left(\frac{m_c}{m_b}, s\right)(3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) \\ &\quad - \frac{1}{2} g(1, s)(4C_3 + 4C_4 + 3C_5 + C_6) - \frac{1}{2} g(0, s) \\ &\quad \times (C_3 + 3C_4) + \frac{2}{9}(3C_3 + C_4 + 3C_5 + C_6). \end{aligned} \tag{5}$$

The explicit expressions for the g functions can be found in [30], and the phenomenological parameters κ_i in Eq. (4) can be determined from

$$\mathcal{B}(B \to K^* V_i \to K^* \ell^+\ell^-) = \mathcal{B}(B \to K^* V_i) \mathcal{B}(V_i \to \ell^+\ell^-), \tag{6}$$

where the data for the right-hand side is given in [35]. For the lowest resonances J/ψ and ψ', one can use $\kappa = 1.65$ and $\kappa = 2.36$, respectively, (see [36]). In this study, we neglect the long-distance contributions for simplicity and like Ref. [30], to have a scheme independent matrix element, we use the leading order as well as the next-to-leading order QCD corrections to C_9 and the leading order QCD corrections to the other Wilson coefficients.

In order to compute the decay width and other physical observables of $B_s \to \phi \ell^+\ell^-$ decay, we need to sandwich the matrix elements in Eq. (2) between the final and initial meson states. Therefore, the hadronic matrix elements for the $B_s \to \phi \ell^+\ell^-$ can be parameterized in terms of form factors. For the vector meson ϕ with polarization vector ε_μ the semileptonic form factors of the V–A current is defined as

$$\begin{aligned} &\langle \phi(p_\phi, \epsilon) \mid \bar{s}\gamma_\mu(1-\gamma_5) b \mid B(p_{B_s})\rangle \\ &\quad = -\frac{2V(q^2)}{m_{B_s} + m_\phi} \epsilon_{\mu\nu\rho\sigma} p_\phi^\rho q^\sigma \epsilon^{*\nu} \\ &\qquad - i\Big[\epsilon_\mu^* (m_{B_s} + m_\phi) A_1(q^2) - (\epsilon^* q)(p_{B_s} + p_\phi)_\mu \\ &\qquad \times \frac{A_2(q^2)}{m_{B_s} + m_\phi} - q_\mu (\epsilon^* q) \frac{2m_\phi}{q^2}(A_3(q^2) - A_0(q^2))\Big], \end{aligned} \tag{7}$$

where $q = p_{B_s} - p_\phi$, and $A_3(q^2 = 0) = A_0(q^2 = 0)$ (this condition ensures that there is no kinematical singularity in the matrix element at $q^2 = 0$). Also, the form factor $A_3(q^2)$ can be written as a linear combination of the form factors A_1 and A_2

$$A_3(q^2) = \frac{1}{2m_\phi}[(m_{B_s} + m_\phi) A_1(q^2) - (m_{B_s} - m_\phi) A_2(q^2)]. \tag{8}$$

The other semileptonic form factors coming from the dipole operator $\sigma_{\mu\nu} q^\nu (1+\gamma_5) b$ can be defined as

$$
\begin{aligned}
&\langle\phi(p_\phi,\varepsilon)|\bar{s}i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|B(p_{B_s})\rangle \\
&\quad = 4\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}p^\rho q^\sigma T_1(q^2) + 2i[\varepsilon^*_\mu(m^2_{B_s}-m^2_\phi) \\
&\qquad - (p_{B_s}+p_\phi)_\mu(\varepsilon^* q)]T_2(q^2) \\
&\qquad + 2i(\varepsilon^* q)\left[q_\mu - (p_{B_s}+p_\phi)_\mu\frac{q^2}{m^2_{B_s}-m^2_\phi}\right]T_3(q^2).
\end{aligned} \tag{9}
$$

As seen from Eqs. (7)–(9), we have to compute the form factors to obtain the physical observables at hadronic level. The form factors are related to the nonperturbative sector of QCD and can be evaluated only by using nonperturbative methods. In the present work, we use the light cone QCD sum rule predictions for the form factors in which one-loop radiative corrections to twist-2 and twist-3 contributions are taken into account. The form factors

$$
\begin{aligned}
F(q^2) \in \{&V(q^2), A_0(q^2), A_1(q^2), A_2(q^2), A_3(q^2), \\
&\times T_1(q^2), T_2(q^2), T_3(q^2)\}
\end{aligned}
$$

are fitted to the following functions [37,38]:

TABLE I. The form factors for $B_s \to \phi\ell^+\ell^-$ in a three-parameter fit [37].

	$F(0)$	a_F	b_F
$A_0^{B_s\to\phi}$	0.382	1.77	0.856
$A_1^{B_s\to\phi}$	0.296	0.87	−0.061
$A_2^{B_s\to\phi}$	0.255	1.55	0.513
$V^{B_s\to\phi}$	0.433	1.75	0.736
$T_1^{B_s\to\phi}$	0.174	1.82	0.825
$T_2^{B_s\to\phi}$	0.174	0.70	−0.315
$T_3^{B_s\to\phi}$	0.125	1.52	0.377

$$
F(q^2) = \frac{F(0)}{1 - a_F\frac{q^2}{m^2_{B_s}} + b_F(\frac{q^2}{m^2_{B_s}})^2}, \tag{10}
$$

where the parameters $F(0)$, a_F and b_F are listed in the Table I.

Using Eqs. (7)–(9), the matrix element of the $B_s \to \phi\ell^+\ell^-$ decay can be written as follows:

$$
\begin{aligned}
\mathcal{M}(B_s\to\phi\ell^+\ell^-) = \frac{G\alpha}{4\sqrt{2}\pi}V_{tb}V^*_{ts}\{&\bar{\ell}\gamma^\mu(1-\gamma_5)\ell[-2B_0\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu}p^\lambda_\phi q^\sigma - iB_1\varepsilon^*_\mu + iB_2(\varepsilon^* q)(p_{B_s}+p_\phi)_\mu + iB_3(\varepsilon^* q)q_\mu] \\
&+ \bar{\ell}\gamma^\mu(1+\gamma_5)\ell[-2C_1\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu}p^\lambda_\phi q^\sigma - iD_1\varepsilon^*_\mu + iD_2(\varepsilon^* q)(p_{B_s}+p_\phi)_\mu + iD_3(\varepsilon^* q)q_\mu]\},
\end{aligned} \tag{11}
$$

where

$$
\begin{aligned}
B_0 &= (\tilde{C}_9^{\rm eff} - \tilde{C}_{10}^{\rm eff})\frac{V}{m_{B_s}+m_\phi} + 4(m_{B_s}+m_s)C_7^{\rm eff}\frac{T_1}{q^2}, \\
B_1 &= (\tilde{C}_9^{\rm eff} - \tilde{C}_{10}^{\rm eff})(m_{B_s}+m_\phi)A_1 + 4(m_{B_s}-m_s)C_7^{\rm eff}(m^2_{B_s}-m^2_\phi)\frac{T_2}{q^2}, \\
B_2 &= \frac{\tilde{C}_9^{\rm eff} - \tilde{C}_{10}^{\rm eff}}{m_{B_s}+m_\phi}A_2 + 4(m_{B_s}-m_s)C_7^{\rm eff}\frac{1}{q^2}\left[T_2 + \frac{q^2}{m^2_{B_s}-m^2_\phi}T_3\right], \\
B_3 &= 2(\tilde{C}_9^{\rm eff} - \tilde{C}_{10}^{\rm eff})m_\phi\frac{A_3-A_0}{q^2} - 4(m_{B_s}-m_s)C_7^{\rm eff}\frac{T_3}{q^2}, \\
C_1 &= B_0(\tilde{C}_{10}^{\rm eff}\to -\tilde{C}_{10}^{\rm eff}), \qquad D_i = B_i(\tilde{C}_{10}^{\rm eff}\to -\tilde{C}_{10}^{\rm eff}), \quad (i=1,2,3).
\end{aligned}
$$

From the above equations for the differential decay width, we get the following result:

$$
\frac{d\Gamma}{d\hat{s}}(B_s\to\phi\ell^+\ell^-) = \frac{G^2\alpha^2 m_{B_s}}{2^{14}\pi^5}|V_{tb}V^*_{ts}|^2\lambda^{1/2}(1,\hat{r},\hat{s})v\Delta(\hat{s}), \tag{12}
$$

with

$$
\begin{aligned}
\Delta = \frac{2}{3\hat{r}_\phi\hat{s}}m^2_{B_s}{\rm Re}[&-12m^2_{B_s}\hat{m}^2_l\lambda\hat{s}\{(B_3-D_2-D_3)B^*_1 - (B_3+B_2-D_3)D^*_1\} + 12m^4_{B_s}\hat{m}^2_l\lambda\hat{s}(1-\hat{r}_\phi)(B_2-D_2)(B^*_3-D^*_3) \\
&+ 48\hat{m}^2_l\hat{r}_\phi\hat{s}(3B_1D^*_1 + 2m^4_{B_s}\lambda B_0C^*_1) - 16m^4_{B_s}\hat{r}_\phi\hat{s}\lambda(\hat{m}^2_l - \hat{s})\{|B_0|^2 + |C_1|^2\} - 6m^4_{B_s}\hat{m}^2_l\lambda\hat{s}\{2(2+2\hat{r}_\phi-\hat{s})B_2D^*_2 \\
&- \hat{s}|(B_3-D_3)|^2\} - 4m^2_{B_s}\lambda\{\hat{m}^2_l(2-2\hat{r}_\phi+\hat{s}) + \hat{s}(1-\hat{r}_\phi-\hat{s})\}(B_1B^*_2 + D_1D^*_2) + \hat{s}\{6\hat{r}_\phi\hat{s}(3+v^2) + \lambda(3-v^2)\} \\
&\times\{|B_1|^2 + |D_1|^2\} - 2m^4_{B_s}\lambda\{\hat{m}^2_l[\lambda - 3(1-\hat{r}_\phi)^2] - \lambda\hat{s}\}\{|B_2|^2 + |D_2|^2\}],
\end{aligned}
$$

where $\hat{s} = q^2/m^2_{B_s}$, $\hat{r}_\phi = m^2_\phi/m^2_{B_s}$ and $\lambda(a,b,c) = a^2+b^2+c^2-2ab-2ac-2bc$, $\hat{m}_\ell = m_\ell/m_{B_s}$, $v = \sqrt{1-4\hat{m}^2_\ell/\hat{s}}$ are the final lepton velocity.

Having obtained the matrix element for the $B_s \to \phi\ell^+\ell^-$, we can now calculate the double-polarization asymmetries. For this purpose, we define the orthogonal unit vectors $s_i^{\pm\mu}$ in the rest frame of leptons, where $i = L$, N, or T refer to the longitudinal, normal, and transversal polarization directions, respectively,

$$\begin{aligned}
s_L^{-\mu} &= (0, \vec{e}_L^{\,-}) = \left(0, \frac{\vec{p}_-}{|\vec{p}_-|}\right),\\
s_L^{+\mu} &= (0, \vec{e}_L^{\,+}) = \left(0, \frac{\vec{p}_+}{|\vec{p}_+|}\right),\\
s_N^{-\mu} &= (0, \vec{e}_N^{\,-}) = \left(0, \frac{\vec{p}_\phi \times \vec{p}_-}{|\vec{p}_\phi \times \vec{p}_-|}\right),\\
s_N^{+\mu} &= (0, \vec{e}_N^{\,+}) = \left(0, \frac{\vec{p}_\phi \times \vec{p}_+}{|\vec{p}_\phi \times \vec{p}_+|}\right),\\
s_T^{-\mu} &= (0, \vec{e}_T^{\,-}) = (0, \vec{e}_N^{\,-} \times \vec{e}_L^{\,-}),\\
s_T^{+\mu} &= (0, \vec{e}_T^{\,+}) = (0, \vec{e}_N^{\,+} \times \vec{e}_L^{\,+}).
\end{aligned} \tag{13}$$

In the above equations $\vec{p}_\mp$ and $\vec{p}_\phi$ are the three-momenta of the leptons $\ell^\mp$ and ϕ meson, respectively. Then by Lorentz transformation these unit vectors are boosted from the rest frame of leptons to the center of mass (CM) frame of leptons. Under this transformation only the longitudinal unit vectors $s_L^{\pm\mu}$ change, but the other two vectors remain unchanged. $s_L^{\pm\mu}$ in the CM frame of leptons are obtained as

$$\begin{aligned}
(s_L^{-\mu})_{\rm CM} &= \left(\frac{|\vec{p}_-|}{m_\ell}, \frac{E\vec{p}_-}{m_\ell|\vec{p}_-|}\right),\\
(s_L^{+\mu})_{\rm CM} &= \left(\frac{|\vec{p}_-|}{m_\ell}, -\frac{E\vec{p}_-}{m_\ell|\vec{p}_-|}\right).
\end{aligned} \tag{14}$$

The polarization asymmetries can now be calculated using the spin projector $1/2(1 + \gamma_5 \not{s}_i^-)$ for ℓ^- and the spin projector $1/2(1 + \gamma_5 \not{s}_i^+)$ for ℓ^+.

Considering the above explanations, we can define the double-lepton polarization asymmetries as in [39]:

$$P_{ij}(\hat{s}) = \frac{(\frac{d\Gamma}{d\hat{s}}(\vec{s}_i^{\,-}, \vec{s}_j^{\,+}) - \frac{d\Gamma}{d\hat{s}}(-\vec{s}_i^{\,-}, \vec{s}_j^{\,+})) - (\frac{d\Gamma}{d\hat{s}}(\vec{s}_i^{\,-}, -\vec{s}_j^{\,+}) - \frac{d\Gamma}{d\hat{s}}(-\vec{s}_i^{\,-}, -\vec{s}_j^{\,+}))}{(\frac{d\Gamma}{d\hat{s}}(\vec{s}_i^{\,-}, \vec{s}_j^{\,+}) + \frac{d\Gamma}{d\hat{s}}(-\vec{s}_i^{\,-}, \vec{s}_j^{\,+})) + (\frac{d\Gamma}{d\hat{s}}(\vec{s}_i^{\,-}, -\vec{s}_j^{\,+}) + \frac{d\Gamma}{d\hat{s}}(-\vec{s}_i^{\,-}, -\vec{s}_j^{\,+}))}, \tag{15}$$

where $i, j = L, N, T$, and the first index i corresponds to lepton, while the second index j corresponds to antilepton, respectively. After doing the straightforward calculation we obtain the following expressions for $P_{ij}(\hat{s})$:

$$\begin{aligned}
P_{LL} = \frac{m_{B_s}^2}{3\hat{r}_\phi\hat{s}\Delta} \mathrm{Re}\{&-24m_{B_s}^2\hat{m}_\ell^2\hat{s}\lambda[(B_1 - D_1)(B_3^* - D_3^*)] + 12m_{B_s}^3\hat{m}_\ell\hat{s}\lambda(1 - \hat{r}_\phi)[2m_{B_s}\hat{m}_\ell(B_2 - D_2)(B_3^* - D_3^*)]\\
&- 8m_{B_s}^4\hat{r}_\phi\hat{s}^2\lambda(1 + 3v^2)(|B_0|^2 + |C_1|^2) + 12m_{B_s}^4\hat{m}_\ell^2\hat{s}^2\lambda|B_3 - D_3|^2 + 8m_{B_s}^2\hat{m}_\ell^2\lambda(4 - 4\hat{r}_\phi - \hat{s})(B_1D_2^* + B_2D_1^*)\\
&- 32\hat{m}_\ell^2(\lambda + 3\hat{r}_\phi\hat{s})B_1D_1^* - 8m_{B_s}^4\hat{m}_\ell^2\lambda[\lambda + 3(1 - \hat{r}_\phi)^2]B_2D_2^* - 64m_{B_s}^4\hat{m}_\ell^2\hat{r}_\phi\hat{s}\lambda B_0C_1^* + 8m_{B_s}^2\lambda[\hat{s} - \hat{s}(\hat{r}_\phi + \hat{s})\\
&- 3\hat{m}_\ell^2(2 - 2\hat{r}_\phi - \hat{s})](B_1B_2^* + D_1D_2^*) - m_{B_s}^4\hat{s}\lambda[\lambda(1 + 3v^2) - 3(1 - \hat{r}_\phi)^2(1 - v^2)](|B_2|^2 + |D_2|^2)\\
&+ 4[6\hat{m}_\ell^2(\lambda + 6\hat{r}_\phi\hat{s}) - \hat{s}(\lambda + 12\hat{r}_\phi\hat{s})](|B_1|^2 + |D_1|^2)\},
\end{aligned} \tag{16}$$

$$\begin{aligned}
P_{LN} = \frac{\pi m_{B_s}^2}{2\hat{r}_\phi\Delta}\sqrt{\frac{\lambda}{\hat{s}}}\mathrm{Im}\{&-4m_{B_s}^4\hat{m}_\ell\lambda(1 - \hat{r}_\phi)B_2D_2^* + 2m_{B_s}^4\hat{m}_\ell\hat{s}\lambda B_2B_3^* - 2m_{B_s}^4\hat{m}_\ell\hat{s}\lambda[B_3D_2^* + (B_2 + D_2)D_3^*]\\
&- 2m_{B_s}^2\hat{m}_\ell\hat{s}(1 + 3\hat{r}_\phi - \hat{s})(B_1B_2^* - D_1D_2^*) - 4\hat{m}_\ell(1 - \hat{r}_\phi - \hat{s})B_1D_1^* - 2m_{B_s}^2\hat{m}_\ell\hat{s}(1 - \hat{r}_\phi - \hat{s})(B_1 + D_1)(B_3^* - D_3^*)\\
&+ 2m_{B_s}^2\hat{m}_\ell[\lambda + (1 - \hat{r}_\phi)(1 - \hat{r}_\phi - \hat{s})](B_2D_1^* + B_1D_2^*)\},
\end{aligned} \tag{17}$$

$$P_{NL} = -P_{LN}, \tag{18}$$

$$\begin{aligned}
P_{LT} = \frac{\pi m_{B_s}^2 v}{\hat{r}_\phi\Delta}\sqrt{\frac{\lambda}{\hat{s}}}\mathrm{Re}\{&m_{B_s}^4\hat{m}_\ell\lambda(1 - \hat{r}_\phi)|B_2 - D_2|^2 - 8m_{B_s}^2\hat{m}_\ell\hat{r}_\phi\hat{s}(B_0B_1^* - C_1D_1^*) + m_{B_s}^4\hat{s}\lambda\hat{m}_\ell B_2B_3^*\\
&- m_{B_s}^4\hat{m}_\ell\hat{s}\lambda(B_2D_3^* + B_3D_2^* - D_2D_3^*) + \hat{m}_\ell(1 - \hat{r}_\phi - \hat{s})|B_1 - D_1|^2 + m_{B_s}\hat{s}(1 - \hat{r}_\phi - \hat{s})\\
&\times[-m_{B_s}\hat{m}_\ell(B_1 - D_1)(B_3^* - D_3^*)] - m_{B_s}^2\hat{m}_\ell[\lambda + (1 - \hat{r}_\phi)(1 - \hat{r}_\phi - \hat{s})](B_1 - D_1)(B_2^* - D_2^*)\},
\end{aligned} \tag{19}$$

$$\begin{aligned}
P_{TL} = \frac{\pi m_{B_s}^2 v}{\hat{r}_\phi\Delta}\sqrt{\frac{\lambda}{\hat{s}}}\mathrm{Re}\{&m_{B_s}^4\hat{m}_\ell\lambda(1 - \hat{r}_\phi)|B_2 - D_2|^2 + 8m_{B_s}^2\hat{m}_\ell\hat{r}_\phi\hat{s}(B_0B_1^* - C_1D_1^*) + m_{B_s}^4\hat{s}\lambda\hat{m}_\ell B_2B_3^*\\
&- m_{B_s}^4\hat{m}_\ell\hat{s}\lambda(B_2D_3^* + B_3D_2^* - D_2D_3^*) + \hat{m}_\ell(1 - \hat{r}_\phi - \hat{s})|B_1 - D_1|^2 - m_{B_s}\hat{s}(1 - \hat{r}_\phi - \hat{s})\\
&\times[m_{B_s}\hat{m}_\ell(B_1 - D_1)(B_3^* - D_3^*)] - m_{B_s}^2\hat{m}_\ell[\lambda + (1 - \hat{r}_\phi)(1 - \hat{r}_\phi - \hat{s})](B_1 - D_1)(B_2^* - D_2^*)\},
\end{aligned} \tag{20}$$

$$P_{NT} = \frac{2m_{B_s}^2 v}{3\hat{r}_\phi \Delta} \,\mathrm{Im}\{4\lambda(B_1 D_1^* + m_{B_s}^4 \lambda B_2 D_2^*) - 16 m_{B_s}^4 \hat{s}\lambda\hat{r}_\phi B_0 C_1^* - 4m_{B_s}^2 \lambda(1-\hat{r}_\phi - \hat{s})(B_1 D_2^* + B_2 D_1^*)\}, \tag{21}$$

$$P_{TN} = -P_{NT}, \tag{22}$$

$$\begin{aligned} P_{NN} = {}& \frac{2m_{B_s}^2}{3\hat{r}_\phi \Delta} \,\mathrm{Re}\{-24\hat{m}_\ell^2 \hat{r}_\phi(|B_1|^2 + |D_1|^2) + 16 m_{B_s}^4 \hat{s}\lambda\hat{r}_\phi v^2 B_0 C_1^* + 6m_{B_s}^2 \hat{m}_\ell^2 \lambda[-2B_1(B_2^* + B_3^* - D_3^*) \\ & + 2D_1(B_3^* - D_2^* - D_3^*)] + 6m_{B_s}^3 \hat{m}_\ell \lambda(1-\hat{r}_\phi)[2m_{B_s}\hat{m}_\ell(B_2 - D_2)(B_3^* - D_3^*)] + 6m_{B_s}^4 \hat{m}_\ell^2 \lambda(2 + 2\hat{r}_\phi - \hat{s}) \\ & \times (|B_2|^2 + |D_2|^2) + 6m_{B_s}^4 \hat{m}_\ell^2 \hat{s}\lambda|B_3 - D_3|^2 + m_{B_s}^2 \lambda[3(2 - 2\hat{r}_\phi - \hat{s}) - v^2(2 - 2\hat{r}_\phi + \hat{s})](B_1 D_2^* + B_2 D_1^*) \\ & - m_{B_s}^4 \lambda[(3+v^2)\lambda + 3(1-v^2)(1-\hat{r}_\phi)^2]B_2 D_2^* - 2[6\hat{r}_\phi \hat{s}(1-v^2) + \lambda(3-v^2)]B_1 D_1^*\}, \end{aligned} \tag{23}$$

$$\begin{aligned} P_{TT} = {}& \frac{2m_{B_s}^2}{3\hat{r}_\phi \hat{s}\Delta} \,\mathrm{Re}\{8m_{B_s}^4 \hat{r}_\phi \hat{s}\lambda[4\hat{m}_\ell^2(|B_0|^2 + |C_1|^2) + 2\hat{s}B_0 C_1^*] - 6m_{B_s}^2 \hat{m}_\ell^2 \hat{s}\lambda[-2(B_1 - D_1)(B_3^* - D_3^*)] \\ & - 6m_{B_s}^3 \hat{m}_\ell \hat{s}\lambda(1-\hat{r}_\phi)[2m_{B_s}\hat{m}_\ell(B_2 - D_2)(B_3^* - D_3^*)] - 6m_{B_s}^4 \hat{m}_\ell^2 \hat{s}^2 \lambda|B_3 - D_3|^2 + 4m_{B_s}^2 \hat{m}_\ell^2 \lambda(4 - 4\hat{r}_\phi - \hat{s}) \\ & \times (B_1 B_2^* + D_1 D_2^*) + 2\hat{s}[6\hat{r}_\phi \hat{s}(1-v^2) + \lambda(1-3v^2)]B_1 D_1^* - 2m_{B_s}^4 \hat{m}_\ell^2 \lambda[\lambda + 3(1-\hat{r}_\phi)^2](|B_2|^2 + |D_2|^2) \\ & - m_{B_s}^2 \hat{s}\lambda[2 - 2\hat{r}_\phi + \hat{s} - 3v^2(2 - 2\hat{r}_\phi - \hat{s})](B_1 D_2^* + B_2 D_1^*) - 8\hat{m}_\ell^2(\lambda - 3\hat{r}_\phi \hat{s})(|B_1|^2 + |D_1|^2) \\ & - m_{B_s}^4 \hat{s}\lambda[(1+3v^2)\lambda - 3(1-v^2)(1-\hat{r}_\phi)^2]B_2 D_2^*\}. \end{aligned} \tag{24}$$

The analytical dependence of the double-lepton polarizations on the fourth quark mass ($m_{t'}$) and the product of quark mixing matrix elements ($V_{t'b}^* V_{t's} = r_{sb} e^{i\phi_{sb}}$) are studied in the next section.

III. EFFECTS OF THE FOURTH-GENERATION

As we mentioned in the introduction, the inclusion of the fourth-generation in the standard model (SM4) does not lead to new operators in the $\mathcal{H}_{\rm eff}$, and all Wilson coefficients receive additional terms as $\frac{\lambda_{t'}}{\lambda_t} C_i^{\rm SM4}$ either via virtual exchange of the fourth-generation up-type quark t' ($C_3, \ldots, C_{10}$) or via using the unitarity of the Cabibbo-Kobayashi-Maskawa matrix (C_1, C_2). Consequently, one can write the new effective Hamiltonian as

$$\mathcal{H}_{\rm eff} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i^{\rm new}(\mu)\mathcal{O}_i(\mu), \tag{25}$$

where $C_i^{\rm new}$ are

$$C_i^{\rm new}(\mu) = C_i(\mu) + \frac{\lambda_{t'}}{\lambda_t} C_i^{\rm SM4}(\mu), \quad i = 1\ldots 10. \tag{26}$$

In the above equation, $\lambda_f = V_{fb}^* V_{fs}$ and $\lambda_{t'}$ can be parameterized as

$$\lambda_{t'} = V_{t'b} V_{t's}^* = r_{\rm sb} e^{i\phi_{sb}}. \tag{27}$$

Now by using the above effective Hamiltonian, we can reobtain the one-loop matrix elements of $b \to s\ell^+\ell^-$ by replacing $C_i^{\rm eff}(\tilde{C}_i^{\rm eff})$ with $C_i^{\rm eff\,new}(\tilde{C}_i^{\rm eff\,new})$ in Eq. (2), where $C_i^{\rm eff\,new}$ and $\tilde{C}_i^{\rm eff\,new}$ are given as

$$\begin{aligned} C_i^{\rm eff\,new}(\mu) &= C_i^{\rm eff}(\mu) + \frac{\lambda_{t'}}{\lambda_t} C_i^{\rm eff\,SM4}(\mu), \qquad i = 7, \\ \tilde{C}_i^{\rm eff\,new}(\mu) &= \tilde{C}_i^{\rm eff}(\mu) + \frac{\lambda_{t'}}{\lambda_t} \tilde{C}_i^{\rm eff\,SM4}(\mu), \qquad i = 9, 10. \end{aligned} \tag{28}$$

Here the effective Wilson coefficients $C_i^{\rm eff\,SM4}$ and $\tilde{C}_i^{\rm eff\,SM4}$ are defined in the same way as Eqs. (3) by substituting C_i with $C_i^{\rm SM4}$. It is worth noting that the explicit forms of $C_i^{\rm eff\,SM4}$ and $\tilde{C}_i^{\rm eff\,SM4}$ can also be found from the corresponding Wilson coefficients in SM by replacing $m_t \to m_{t'}$ [30]. Based on the preceding explanations, in order to obtain the matrix element and the double-lepton polarization asymmetries for $B_s \to \phi\ell^+\ell^-$ decay in the presence of the fourth generation, one should replace $C_i^{\rm eff}(\tilde{C}_i^{\rm eff})$ with $C_i^{\rm eff\,new}(\tilde{C}_i^{\rm eff\,new})$ in all equations of the previous section.

The unitary quark mixing matrix is now 4×4, which can be written in terms of 6 mixing angles and 3 *CP* violating phases. The relevant elements of this matrix for $b \to s$ transition satisfy the relation

$$\lambda_u + \lambda_c + \lambda_t + \lambda_{t'} = 0. \tag{29}$$

Consequently, as required by the Glashow-Iliopoulos-Maiani mechanism, the factor $\lambda_t C_i^{\rm new}$ should be modified to $\lambda_t C_i$ when $m_{t'} \to m_t$ or $\lambda_{t'} \to 0$. We can easily check the validity of this condition by using Eq. (29)

$$\begin{aligned} \lambda_t C_i^{\rm new} &= \lambda_t C_i + \lambda_{t'} C_i^{\rm SM4} \\ &= -(\lambda_u + \lambda_c)C_i + \lambda_{t'}(C_i^{\rm SM4} - C_i) \\ &= -(\lambda_u + \lambda_c)C_i = \lambda_t C_i. \end{aligned} \tag{30}$$

The numerical analysis of the dependence of the double-lepton polarizations on the fourth quark mass ($m_{t'}$) and the product of quark mixing matrix elements ($V_{t'b}^* V_{t's} = r_{sb} e^{i\phi_{sb}}$) are presented in the next section.

IV. RESULTS AND DISCUSSIONS

The main input parameters in the calculations are the form factors for which we have chosen the predictions of light cone QCD sum rule method [37,38], as pointed out in Sec. II. Besides the form factors, we use the other input parameters as follows:

$$m_{B_s} = 5.37\ \mathrm{GeV}, \quad m_b = 4.8\ \mathrm{GeV}, \quad m_c = 1.5\ \mathrm{GeV},$$
$$m_\tau = 1.77\ \mathrm{GeV}, \quad m_\mu = 0.105\ \mathrm{GeV}, \quad m_\phi = 1.020\ \mathrm{GeV},$$
$$|V_{tb}V_{ts}^*| = 0.0385, \quad \alpha^{-1} = 129,$$
$$G_f = 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}, \quad \tau_{B_s} = 1.46 \times 10^{-12} s. \quad (31)$$

In order to present a quantitative analysis of the double-lepton polarization asymmetries, the values of the fourth-generation parameters are needed. Considering the experimental values of $B \to X_s \gamma$ and $B \to X_s \ell^+ \ell^-$ decays the value of the r_{sb} parameter is restricted to the range {.01–.03} for $\phi_{\mathrm{sb}} \sim \{0°–360°\}$ and $m_{t'} \sim \{200–600\}$ GeV [17,27]. Using the B_s mixing parameter Δm_{B_s}, a sharp restriction on ϕ_{sb} has been obtained ($\phi_{\mathrm{sb}} \sim 90°$)[13]. Therefore, in our following numerical analysis, the corresponding values of above ranges are $r_{\mathrm{sb}} = \{.01, .02, .03\}$, $\phi_{\mathrm{sb}} = \{60°, 90°, 120°\}$, $m_{t'} = 175 \le m_{t'} \le 600$.

It is clear from the expressions of all nine double-lepton polarization asymmetries that they depend on the momentum transfer q^2 and the new parameters ($m_{t'}$, r_{sb}, ϕ_{sb}). Consequently, it may be experimentally difficult to investigate these dependencies at the same time. One way to deal with this problem is to integrate over q^2 and study the averaged double-lepton polarization asymmetries. The average of P_{ij} over q^2 is defined as

$$\langle P_{ij} \rangle = \frac{\int_{4\hat{m}_\ell^2}^{(1-\sqrt{\hat{r}_\phi})^2} P_{ij} \frac{d\mathcal{B}}{d\hat{s}} d\hat{s}}{\int_{4\hat{m}_\ell^2}^{(1-\sqrt{\hat{r}_\phi})^2} \frac{d\mathcal{B}}{d\hat{s}} d\hat{s}}. \quad (32)$$

We have used the above formula and depicted the dependency of $\langle P_{ij} \rangle$ on the fourth-generation parameters in Figs. 1–7. In the following, we compare our results for $B_s \to \phi \ell^+ \ell^-$ decay with the results of Ref. [24] for $B \to K \ell^+ \ell^-$ decay. Since the overall behavior of $\langle P_{ij} \rangle$ versus $m_{t'}$, r_{sb}, and ϕ_{sb} are almost the same as that of $B \to K \ell^+ \ell^-$ decay, we discuss the differences of these two decays and some aspects which have not been discussed in Ref. [24]:

(i) *Figure* 1: Similar to the $B \to K \mu^+ \mu^-$ decay, $\langle P_{LL} \rangle$ is not sensitive to the fourth-generation quark parameters; therefore, the $\langle P_{LL} \rangle$ plots for μ channel have been omitted. However, for the τ channel, the maximum deviation from SM is about 50%, which can be seen at $m_{t'} \sim 600$ GeV. In comparison with the results of Ref. [24], it is understood that the deviation from SM for $B_s \to \phi \tau^+ \tau^-$ is twice that of $B \to K \tau^+ \tau^-$ decay. Therefore, the magnitude of $\langle P_{LL} \rangle$ in $B_s \to \phi \tau^+ \tau^-$ compared with that in $B \to K \tau^+ \tau^-$ decay has more chance to show the existence of the fourth generation.

(ii) *Figure* 2: The value of $\langle P_{LN} \rangle_{\max}$ for μ channel is about 0.04, which is almost 2 times smaller than that for $B \to K$ decay. However, for τ channel such value is at most around 0.3, which is approximately equal to the maximum value of $\langle P_{LN} \rangle$ for $B \to K$ decay. Furthermore, in μ and τ channels by increasing r_{sb} and keeping the values of ϕ_{sb} fixed, the maximum deviation from SM occurs at smaller values of $m_{t'}$. This result can be interesting since the maximum deviation from SM happens for $r_{\mathrm{sb}} \sim \{0.02–0.03\}$ and $m_{t'} \sim \{300–400\}$ GeV. Therefore, similar to $B \to K$ decay, the new generation has a chance to be observed around $m_{t'} \sim \{300–400\}$ GeV. Our analysis shows that to measure the effect of the fourth generation in $\langle P_{LN} \rangle$, the τ channel of $B_s \to \phi$ and $B \to K$ are more important than the μ channel of these decays, knowing that in the μ channel the $B \to K$ decay is more significant than the $B_s \to \phi$ decay.

(iii) *Figure* 3: For the μ channel, the magnitude of $\langle P_{LT} \rangle$ in $B_s \to \phi$ decay changes at most about 80% compared with the SM prediction, while the maximum change in $B \to K$ decay reaches up to 60%. For the τ case, unlike $B \to K$ decay, the magnitude of $\langle P_{LT} \rangle$ in $B_s \to \phi$ transition exhibits strong dependence on the fourth quark mass ($m_{t'}$) and the product of quark mixing matrix elements ($|V_{t'b}V_{t's}^*| = r_{sb}$). As seen from Fig. (3) the maximum deviation from SM in the τ channel is much more than that in the μ channel. Therefore, for establishing the fourth-generation of quarks the measurement of $\langle P_{LT} \rangle$ for $B_s \to \phi \tau^+ \tau^-$ decay is more suitable than such measurement for $B_s \to \phi \mu^+ \mu^-$ and $B \to K \mu^+ \mu^-$ decays.

(iv) *Figure* 4: It is seen from Eqs. (19) and (20) that contrary to $B \to K$ decay, P_{TL} is neither symmetric nor antisymmetric under the exchange of subscripts L and T, which leads to different values for P_{TL} and P_{LT}. For the μ channel, the magnitude of $\langle P_{TL} \rangle$ in $B_s \to \phi$ decay changes at most about 40% compared with the SM prediction, while the maximum change in the case of $B \to K$ decay reaches up to 60%. For the τ case, unlike $B \to K$ decay, the magnitude of $\langle P_{TL} \rangle$ in $B_s \to \phi$ transition changes at most about 60% compared with the SM prediction. Therefore, in the measurement of $\langle P_{TL} \rangle$, the decays $B_s \to \phi \ell^+ \ell^- (\ell = \mu,\ \tau)$ and $B \to K \mu^+ \mu^-$ have the same significance for finding the new generation of quarks.

(v) *Figure* 5: By comparing this figure with Fig. 2, one can find that the overall behavior of $\langle P_{TN} \rangle$ and $\langle P_{LN} \rangle$ are the same. Furthermore, the magnitude of $\langle P_{TN} \rangle_{\max}$ for the μ channel is about 0.22, which is 4 times smaller than that for $B \to K$ decay and for the τ channel; such value is at most around 0.0075, which is approximately 10 times smaller than $\langle P_{TN} \rangle_{\max}$ for $B \to K$ decay. Although the measurement of $\langle P_{TN} \rangle$ in $B \to K \tau^+ \tau^-$ decay for finding the new generation is useful, such measurement in the decays $B_s \to \phi \mu^+ \mu^-$ and $B \to K \mu^+ \mu^-$ are more significant.

(vi) *Figure 6*: For the τ channel in $B_s \to \phi$ decay, the value of $\langle P_{NN} \rangle$ shows stronger dependence on the fourth-generation parameters ($m_{t'}$, r_{sb}, ϕ_{sb}) in comparison with that in $B \to K$ decay. However, for the μ channel, both $B_s \to \phi$ and $B \to K$ decays show similar strong dependence on the fourth-generation parameters. Furthermore, the situation for $B_s \to \phi\tau^+\tau^-$ decay is even more interesting than $B_s \to \phi\mu^+\mu^-$ decay, since for fixed values of ϕ_{sb} and r_{sb}, an increase in $m_{t'}$ changes the sign of $\langle P_{NN} \rangle$. So, the study of the magnitude and the sign of $\langle P_{NN} \rangle$ for $B_s \to \phi\tau^+\tau^-$ decay as well as the magnitude of this asymmetry in $B_s \to \phi\mu^+\mu^-$ and $B_s \to K\mu^+\mu^-$ decays can serve as good tests for discovering the new physics beyond the SM (see Ref. [24]).

(vii) *Figure 7*: A comparison between the τ channel of this figure and an analogous figure for $B \to K\tau^+\tau^-$ shows that the values of $\langle P_{TT} \rangle$ for $B_s \to \phi\tau^+\tau^-$ decay has considerable dependency on the fourth-generation parameters ($m_{t'}$, r_{sb}, ϕ_{sb}). Furthermore, for the μ channel, both $B_s \to \phi$ and $B \to K$ decays show strong dependence on the fourth-generation parameters, comparable to each other. Therefore, compared with the $B \to K\tau^+\tau^-$ decay in Ref. [24], the study of the magnitude of $\langle P_{TT} \rangle$ in $B_s \to \phi\tau^+\tau^-$ provides a better opportunity to see the effect of the new physics beyond the SM. On the other hand, the decays $B_s \to \phi\mu^+\mu^-$ and $B \to K\mu^+\mu^-$ have the same significance for discovering this new physics.

In the above discussion, we did not consider the theoretical and experimental uncertainties. These issues will be discussed in the following: The theoretical uncertainties come from the higher order calculation of Wilson coefficients as well as the hadronic uncertainties due to the form factors.

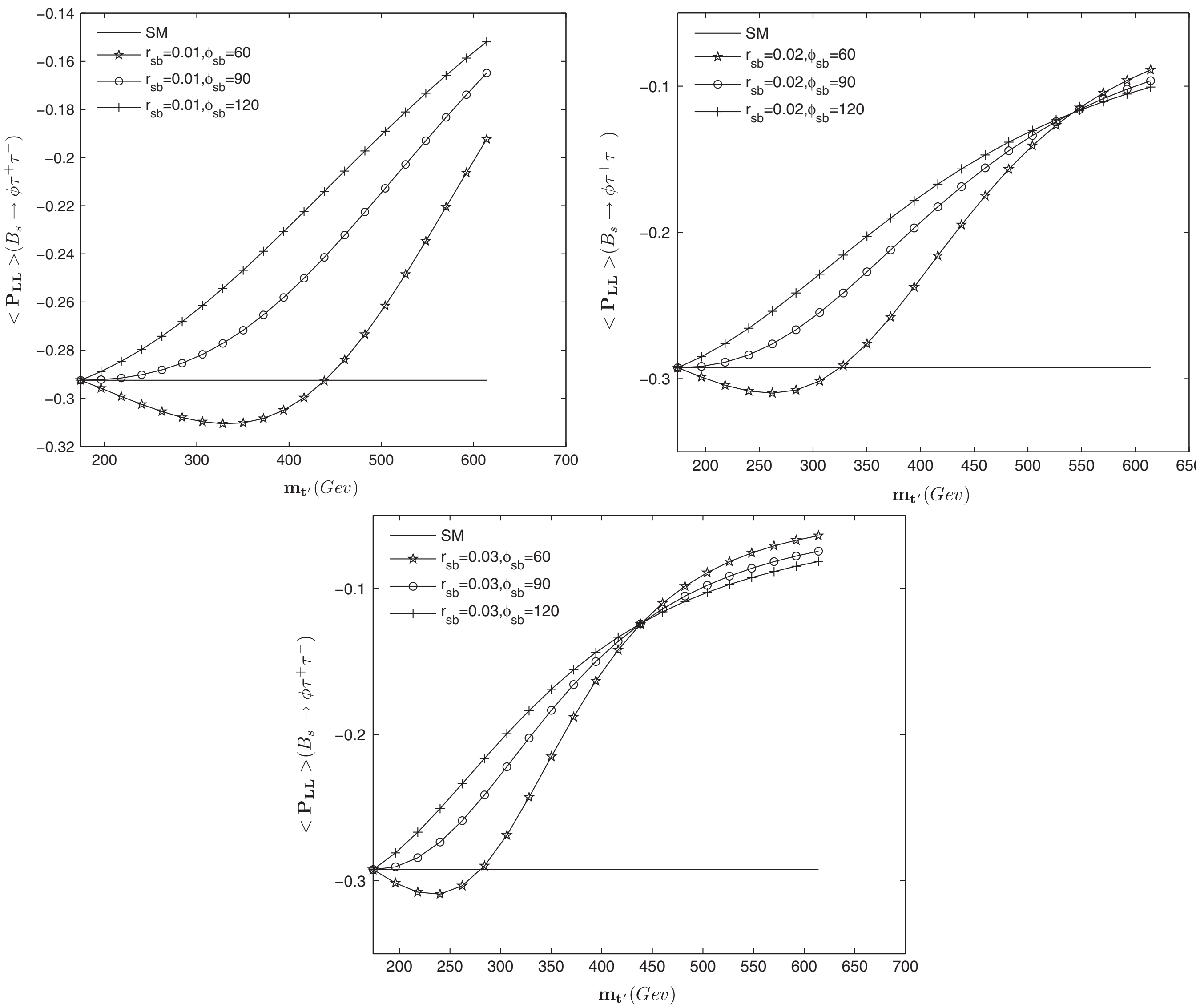

FIG. 1. The dependence of the $\langle P_{LL} \rangle$ on the fourth-generation quark mass $m_{t'}$ for three different values of $\phi_{sb} = \{60°, 90°, 120°\}$ and $r_{sb} = \{0.01, 0.02, 0.03\}$ for the τ channel.

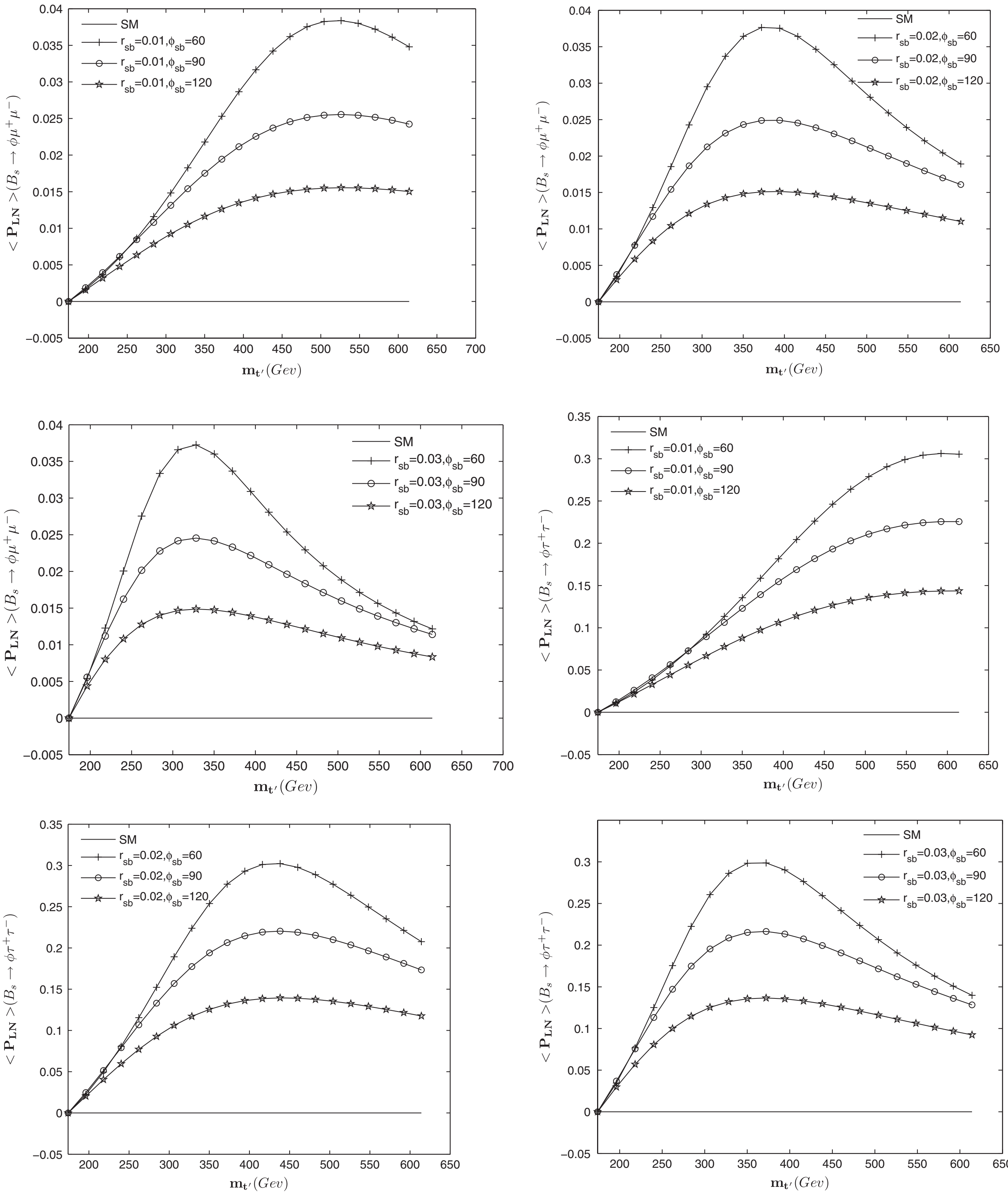

FIG. 2. The dependence of the $\langle P_{LN} \rangle$ on the fourth-generation quark mass $m_{t'}$ for three different values of $\phi_{\rm sb} = \{60°, 90°, 120°\}$ and $r_{\rm sb} = \{0.01, 0.02, 0.03\}$ for the μ and τ channels.

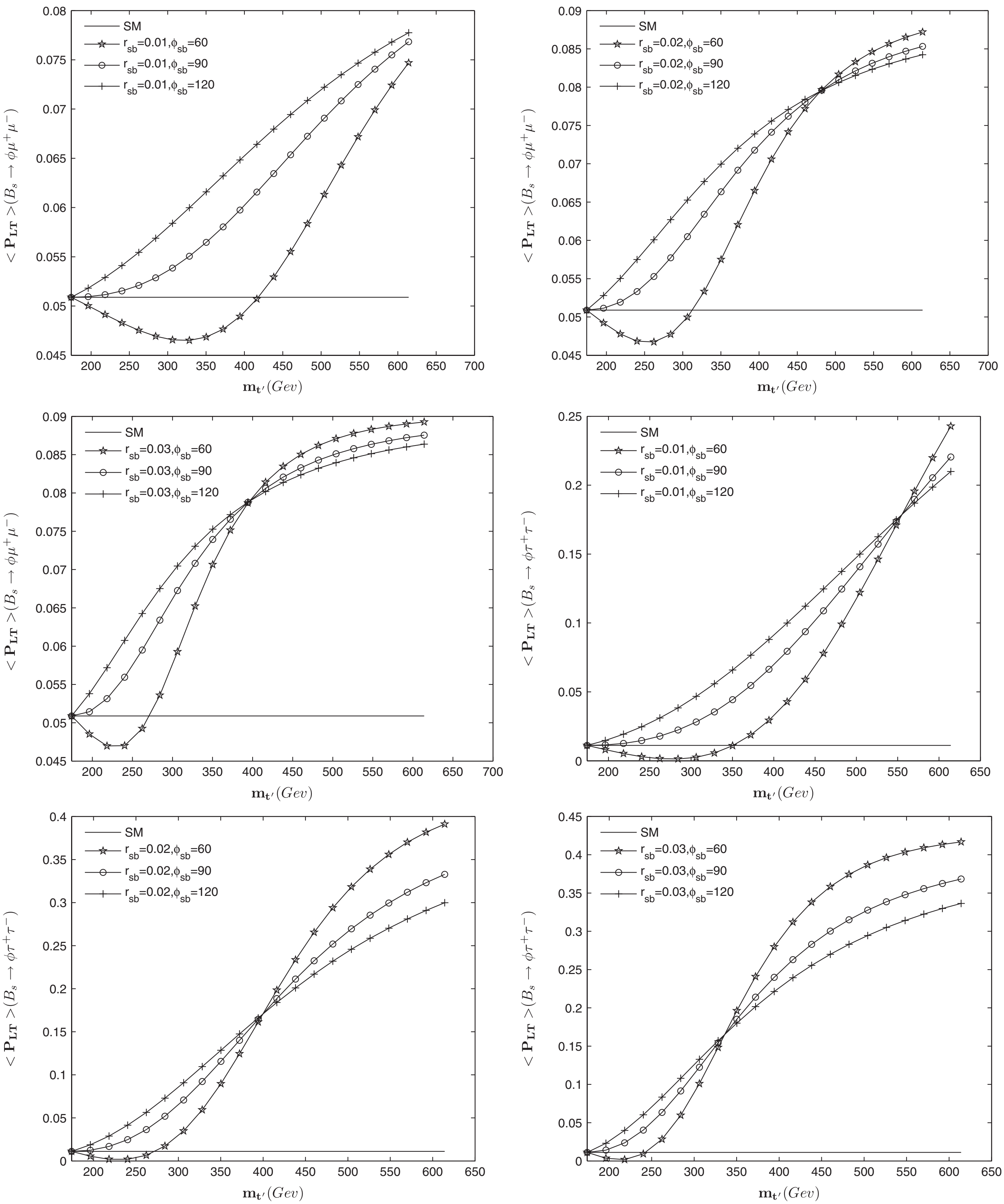

FIG. 3. The dependence of the $\langle P_{LT} \rangle$ on the fourth-generation quark mass $m_{t'}$ for three different values of $\phi_{sb} = \{60°, 90°, 120°\}$ and $r_{sb} = \{0.01, 0.02, 0.03\}$ for the μ and τ channels.

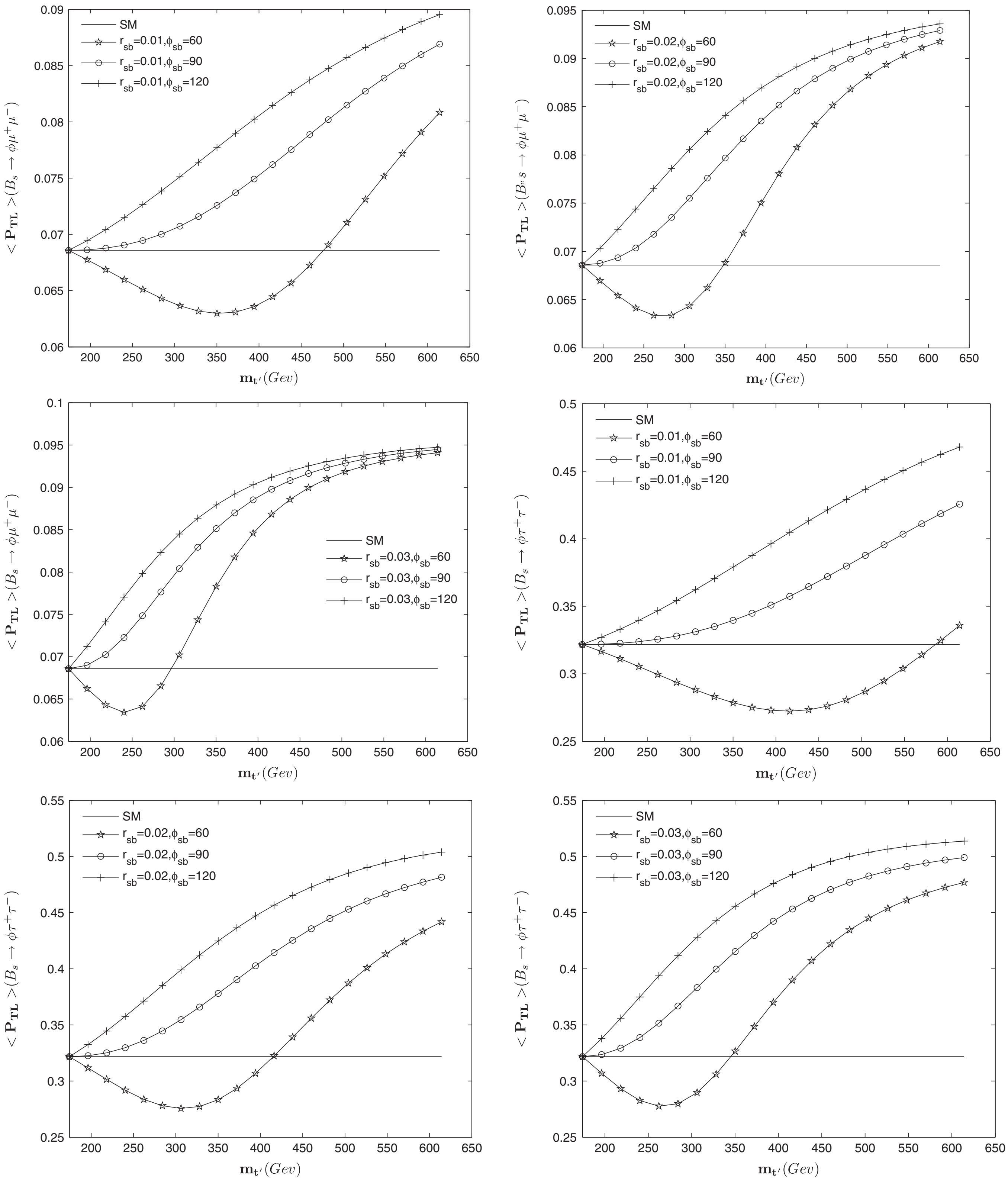

FIG. 4. The dependence of the $\langle P_{TL} \rangle$ on the fourth-generation quark mass $m_{t'}$ for three different values of $\phi_{\rm sb} = \{60°, 90°, 120°\}$ and $r_{\rm sb} = \{0.01, 0.02, 0.03\}$ for the μ and τ channels.

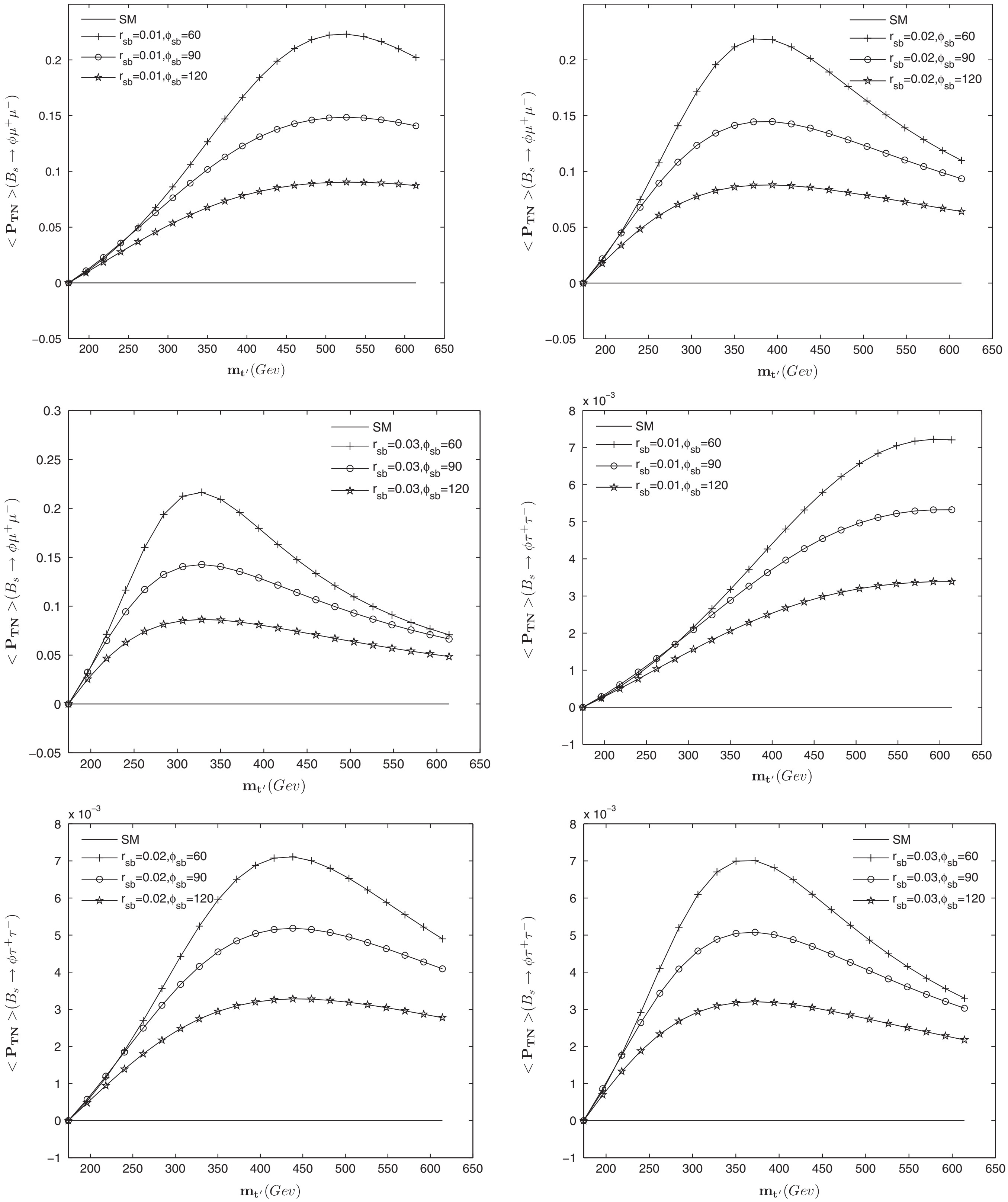

FIG. 5. The dependence of the $\langle P_{TN} \rangle$ on the fourth-generation quark mass $m_{t'}$ for three different values of $\phi_{\rm sb} = \{60°, 90°, 120°\}$ and $r_{\rm sb} = \{0.01, 0.02, 0.03\}$ for the μ and τ channels.

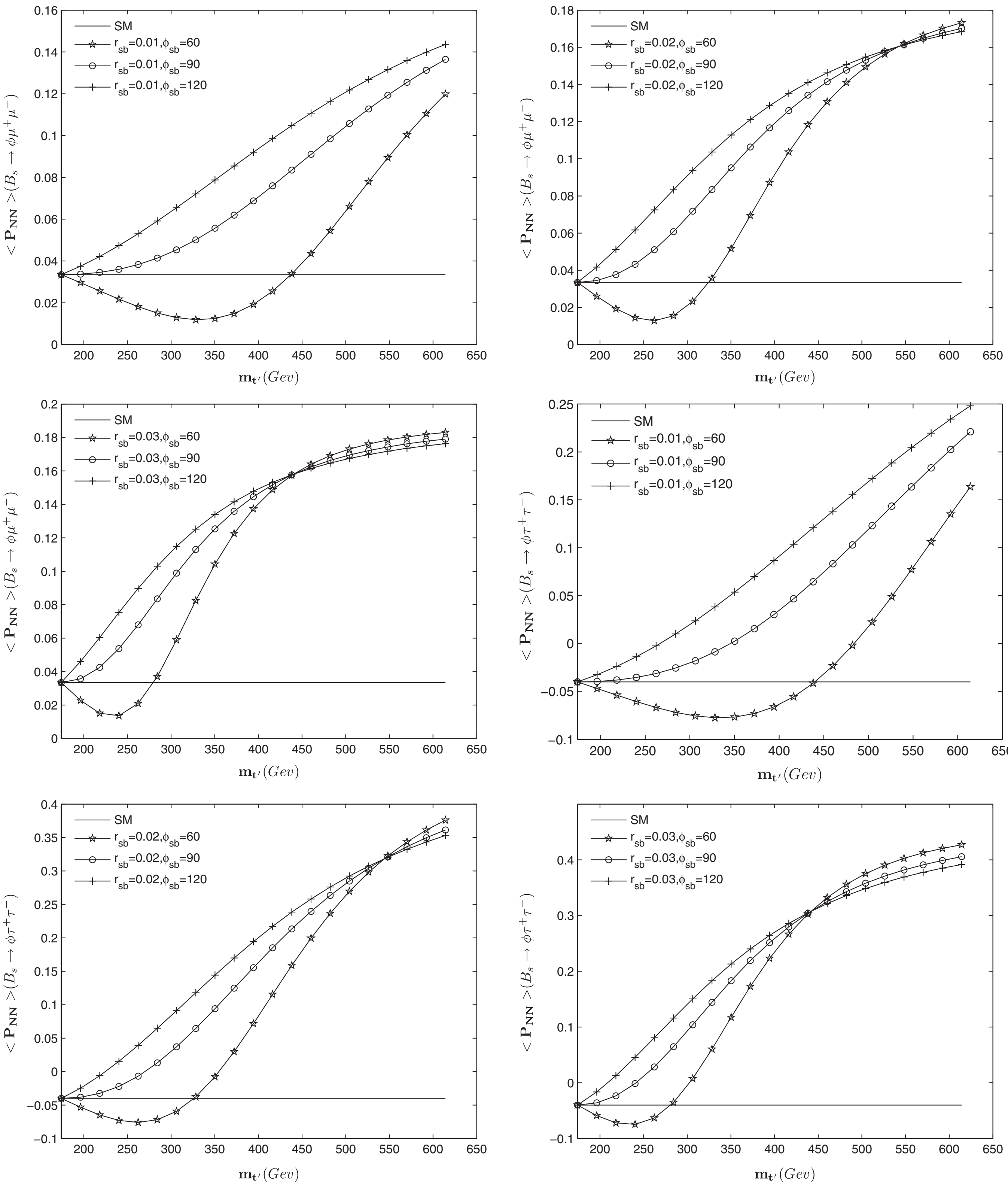

FIG. 6. The dependence of the $\langle P_{NN} \rangle$ on the fourth-generation quark mass $m_{t'}$ for three different values of $\phi_{\rm sb} = \{60°, 90°, 120°\}$ and $r_{\rm sb} = \{0.01, 0.02, 0.03\}$ for the μ and τ channels.

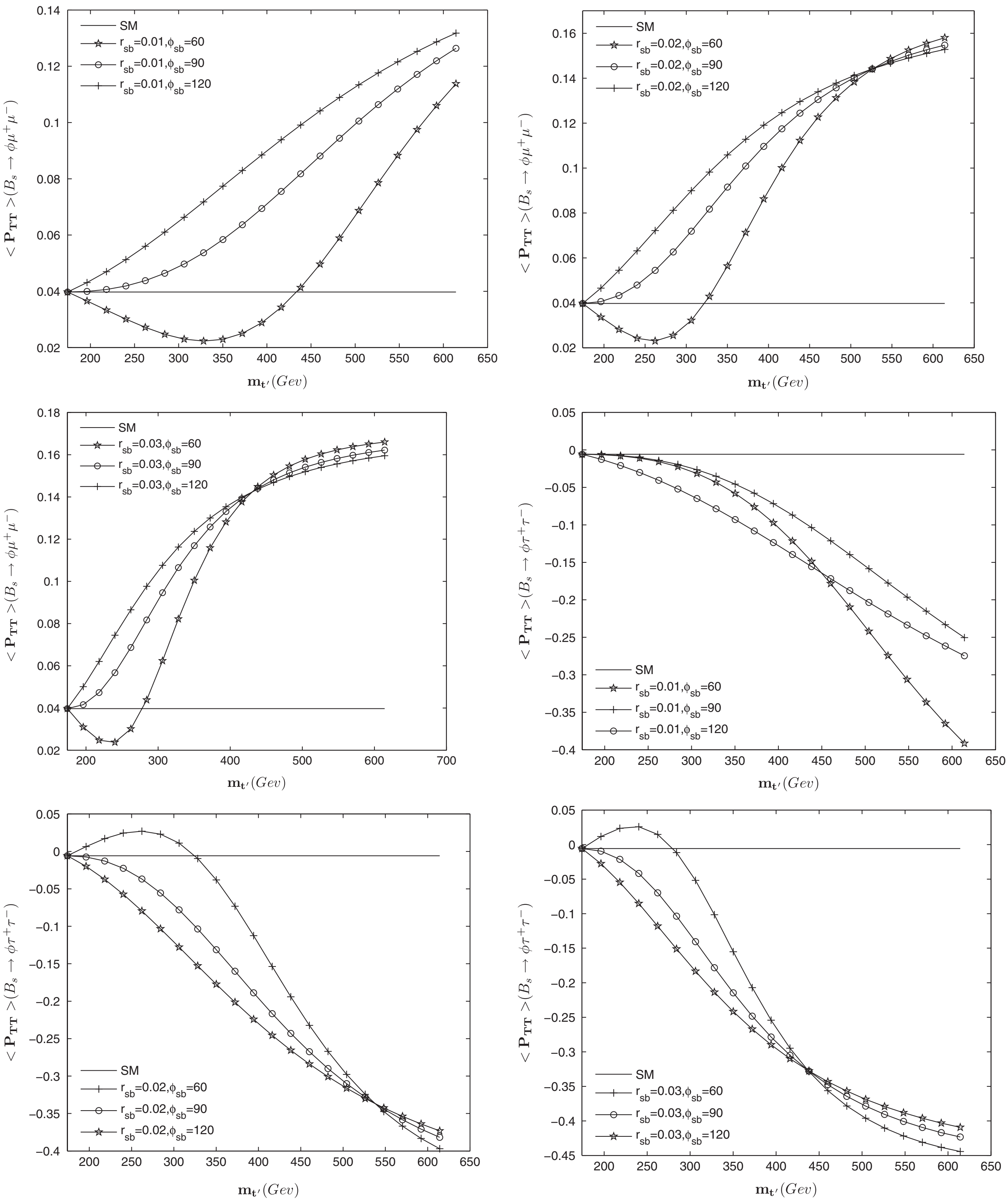

FIG. 7. The dependence of the $\langle P_{TT} \rangle$ on the fourth-generation quark mass $m_{t'}$ for three different values of $\phi_{sb} = \{60°, 90°, 120°\}$ and $r_{sb} = \{0.01, 0.02, 0.03\}$ for the μ and τ channels.

The uncertainties for the Wilson coefficients should be less than 10%, which can be ignored. Therefore, we have only calculated the uncertainty due to the form factors. For both channels of B to K decays, the uncertainty is always small compared to the maximum deviation from SM due to the fourth generation. The maximum uncertainties of SM happen in the value of $\langle P_{NN} \rangle$, which are 32% for $B \to K\mu^+\mu^-$ decay and 21% for $B \to K\tau^+\tau^-$ decay. On the other hand, for the case of B_s to ϕ decays, in general the uncertainty of these asymmetries are greater than those of B to K decays. However, these uncertainties are always small compared to the maximum deviations from SM due to the fourth generation except for the case of $\langle P_{LL} \rangle$ in $B_s \to \phi\tau^+\tau^-$ decay. For this case, the hadronic uncertainty is 50%, which is almost the same as the maximum deviation from SM coming from fourth generation. Therefore, this channel is not suitable for searching new physics. We have also computed the SM uncertainties due to the experimental parameters such as m_t, m_b, m_c and Wolfestein parameters in double-lepton polarization asymmetries. We found that these uncertainties for B_s to ϕ decays are always greater than those for B to K decays. However, for all channels such uncertainties are small compared to maximum effects of fourth generation on SM values.

Finally, let us briefly discuss whether it is possible to measure the lepton polarization asymmetries in experiments or not. Experimentally, to measure an asymmetry $\langle P_{ij} \rangle$ of the decay with branching ratio $\mathcal{B}$ at $n\sigma$ level, the required number of events (i.e., the number of $B\bar{B}$) is given by the formula

$$N = \frac{n^2}{\mathcal{B} s_1 s_2 \langle P_{ij} \rangle^2},$$

where s_1 and s_2 are the efficiencies of the leptons. Typical values of the efficiencies of the τ leptons vary from 50% to 90% for their different decay modes [40], and the error in τ-lepton polarization is estimated to be about (10–15)% [41]. So, the error in measurement of the τ-lepton asymmetries is approximately (20–30)%, and the error in obtaining the number of events is about 50%.

Looking at the expression of N, it can be understood that in order to detect the lepton polarization asymmetries in the μ and τ channels at 3σ level, the minimum number of required events are (for the efficiency of τ lepton we take 0.5):

(i) for $B_s \to \phi\mu^+\mu^-$ decay

$$N \sim \begin{cases} 10^6 & (\text{for } \langle P_{LL} \rangle), \\ 10^7 & (\text{for } \langle P_{NT} \rangle, \langle P_{TN} \rangle), \\ 10^8 & (\text{for } \langle P_{LT} \rangle, \langle P_{TL} \rangle, \langle P_{NN} \rangle, \langle P_{TT} \rangle), \\ 10^9 & (\text{for } \langle P_{LN} \rangle, \langle P_{NL} \rangle), \end{cases}$$

(ii) for $B_s \to \phi\tau^+\tau^-$ decay

$$N \sim \begin{cases} 10^8 & (\text{for } \langle P_{LT} \rangle, \langle P_{TL} \rangle, \langle P_{NN} \rangle, \langle P_{TT} \rangle), \\ 10^9 & (\text{for } \langle P_{LL} \rangle, \langle P_{LN} \rangle, \langle P_{NL} \rangle), \\ 10^{12} & (\text{for } \langle P_{NT} \rangle, \langle P_{TN} \rangle). \end{cases}$$

Considering the above values for N and the number of $B\bar{B}$ pairs ($\sim 10^{12}$ per year), which will be produced at LHC experiments (ATLAS, CMS, LHCb), one can conclude that except $\langle P_{NT} \rangle$ and $\langle P_{TN} \rangle$ for τ channel, all double-lepton polarizations can be detected at the LHC.

In summary, in this paper we have presented the analyses of the double-lepton polarization asymmetries in $B_s \to \phi\ell^+\ell^-$ decay using the SM with the fourth generation of quarks. We found that most of these asymmetries have strong dependency on the fourth-generation parameters, which can be detected at the LHC. We also compared $B_s \to \phi\ell^+\ell^-$ decay with $B \to K\ell^+\ell^-$ decay and showed that some of these asymmetries in $B_s \to \phi\ell^+\ell^-$ decay are more sensitive to the fourth-generation parameters. Therefore, by looking at $B_s \to \phi\ell^+\ell^-$ decay, one has a very good chance to investigate the correctness of the fourth generation of quarks hypothesis in the near future.

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