D_s inclusive decays

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The availability of branching fractions for a large majority of D_s decays permits the prediction of inclusive branching fractions. This is achieved with the help of a modest amount of input from an isospin statistical model applied to nonresonant multibody D_s decays. A systematic uncertainty in these mostly small branching ratios is estimated by comparing predictions of this model with those of a model involving quark-antiquark pair production. The calculated inclusive branching fractions can be compared with data (for example, from a large sample of $D_s^+ D_s^{*-} + D_s^{*+} D_s^-$ obtained by the CLEO Collaboration) and examined for specific final states which can shed light on strong and weak decay mechanisms.

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I. INTRODUCTION

The mechanisms responsible for decays of hadrons containing heavy quarks are of interest both as probes of the strong interactions and as sources of information on the underlying weak processes. The identification of signatures of new physics in such processes often relies on a firm understanding of long-distance (nonperturbative) effects which can masquerade as such new physics.

One incompletely understood process in B meson decays is known as "weak annihilation," or (WA) [1]. One wishes to extract the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{ub}|$ from charmless semileptonic B decays. These constitute only 2% of all semileptonic B decays, as $|V_{ub}/V_{cb}|^2 \simeq 1\%$ while phase space favors $b \to u\ell\nu$ over $b \to c\ell\nu$ by a factor of 2. Strategies thus have evolved to extract the small charmless semileptonic fraction. These include, for example, the study of leptons with energies E_ℓ beyond the endpoint for $b \to c\ell\nu$. The WA process can contaminate the endpoint signal: a B^+ can turn into a soft $I = 0$ hadronic system plus a vector $\bar{b}u$ which can then annihilate freely into $\ell\nu$. (Helicity arguments greatly suppress the annihilation of a pseudoscalar $\bar{b}u$ into $\ell\nu$.)

The CLEO [2] and *BABAR* [3] Collaborations have placed upper limits for WA of a few percent of charmless semileptonic b decays, while theoretical estimates [4] lie somewhat lower. The WA process is supposed to be of order $1/m_Q^3$, where Q is the heavy quark, so it should be more visible in charm decays. A suggestion for probing this process in the semileptonic decay $D_s^+ \to \omega\ell^+\nu_\ell$ was made in Ref. [5]. It was pointed out there that by comparing the decays $D_s^+ \to \omega\ell^+\nu_\ell$ and $D_s \to \phi\ell^+\nu_\ell$ with the corresponding hadronic decays $D_s^+ \to \omega\pi^+$ and $D_s^+ \to \phi\pi^+$, one could anticipate a WA contribution to the branching fraction $\mathcal{B}(\omega\ell^+\nu_\ell)$ of order 10^{-3}, nearly an order of magnitude greater than one would expect from the process $D_s \to \phi\ell^+\nu_\ell$ taking account of $\omega - \phi$ mixing.

The hadronic decays of D_s can shed light on the WA process. The contribution of the weak process $c\bar{s} \to u\bar{d}$ to the decay $D_s \to \omega\pi^+$ is forbidden by G-parity. Charmed particle decays to VP final states ($V =$ vector, $P =$ pseudoscalar) dominated by the annihilation process $c\bar{s} \to u\bar{d}$ do not appear to have a consistent description within flavor SU(3) [6]. In particular, $D_s \to \omega\pi^+$ is expected to be suppressed, whereas it is seen with branching fraction $\mathcal{B}(D_s \to \omega\pi^+) = (2.5 \pm 0.9) \times 10^{-3}$, while the allowed mode $D_s \to \rho^0\pi^+$ is "not seen" [7].

The presence of certain D_s hadronic decay modes containing an ω thus could be regarded as evidence for different mechanisms for WA [5]. The decay $D_s \to \omega\pi^+$, forbidden by ordinary annihilation, may proceed through preradiation of the ω, whether via violation of the Okubo-Zweig-Iizuka (OZI) rule [8] or rescattering. For instance, the D_s can dissociate into two-meson states such as $D^{(*)0}K^{(*)+}$ and $D^{(*)+}K^{(*)0}$ which rescatter strongly to $(c\bar{s})\omega$ while the virtual $c\bar{s}$ state decays weakly to π^+. On the other hand, the decay $D_s \to \omega\pi^+\pi^0$ would be a possible signature for ordinary WA caused directly by $c\bar{s} \to u\bar{d}$, where the $u\bar{d}$ current couples to $\omega\pi^+\pi^0$. In this case the $c\bar{s}$ system must exchange at least two gluons with the final state in order to overcome helicity suppression. On the other hand, if $c\bar{s}$ emits a $Q = 1$ *vector* weak current, which *can* couple to $\omega\pi^+$, a state $s\bar{s}$ is left over, which can couple to η. (The phase space for η' is quite limited.) One would then expect to see $D_s \to \omega\pi^+\eta$.

The study of inclusive ω production in D_s decays is thus of interest in shedding light on mechanisms of weak decay and their interplay with long-distance (nonperturbative) physics. Additionally, the study of η and particularly η'

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inclusive production is relevant because fits based on flavor SU(3) have great difficulty in reproducing the large reported branching fractions $\mathcal{B}(D_s^+ \to \rho^+ \eta) = (13.0 \pm 2.2)\%$ and $\mathcal{B}(D_s^+ \to \rho^+ \eta') = (12.2 \pm 2.0)\%$ [6], preferring values a factor of 2 and 4 less, respectively.

The CLEO Collaboration has studied the reaction $e^+e^- \to (D_s^+ D_s^{*-} + D_s^{*+} D_s^-)$ at $\sqrt{s} = 4.17$ GeV ($\sigma \simeq 1$ nb^{-1}) accumulating a sample of about 600 nb^{-1} [9]. The dominant decay mode of $D_s^{*\pm}$ is to $\gamma D_s^\pm$. It then becomes possible to study a cleanly-identified sample of tagged D_s decays, and thus to obtain inclusive D_s branching ratios.

In the present paper we undertake a theoretical study of inclusive branching fractions in D_s^+ decays, augmenting the extensive list of known processes [7] with estimates for unobserved modes based on isospin arguments, particularly those employing a statistical model [10–12]. The statistical model is described briefly in Sec. II. Systematic uncertainties in the predictions of this model are obtained by comparison with predictions of an alternative model involving quark-aniquark pair production [13]. This model is discussed in Sec. III. We then proceed from known D_s decay modes to others in Sec. IV, showing that we thereby account for the majority of D_s decays and presenting a provisional table of all D_s branching fractions in Sec. V. These are then employed to construct a table of inclusive branching fractions in Sec. VI, while Sec. VII concludes.

II. STATISTICAL MODEL

In the absence of information on branching fractions for certain nonresonant multibody modes, we may use a statistical isospin model to relate them to other known modes. Such a model may be constructed by coupling the internal subsystems to isospin amplitudes in all possible ways and then assuming the reduced amplitudes are equal in magnitude and incoherent in phase [10–12]. We illustrate this technique with two cases of total $I = I_3 = 1$, $K\bar{K}\pi$ and 3π, using particle orders consistent with those quoted for the isospin Clebsch-Gordan coefficients in Ref. [7].

A. Example of $(K\bar{K}\pi)_{I=I_3=1}$

(1) $(K\bar{K})\pi$: We label the reduced amplitudes $A_I^{K\bar{K}}$ by $I_{K\bar{K}} = 0, 1$. Then

$$A(K^+K^-\pi^+) = \frac{1}{\sqrt{2}}A_0^{K\bar{K}} - \frac{1}{2}A_1^{K\bar{K}}, \quad (1)$$

$$A(K^0\bar{K}^0\pi^+) = -\frac{1}{\sqrt{2}}A_0^{K\bar{K}} - \frac{1}{2}A_1^{K\bar{K}}, \quad (2)$$

$$A(K^+\bar{K}^0\pi^0) = \frac{1}{\sqrt{2}}A_1^{K\bar{K}}, \quad (3)$$

so assuming incoherent and equal amplitudes $A_{0,1}^{K\bar{K}}$,

$$|A(K^+K^-\pi^+)|^2:|A(K^0\bar{K}^0\pi^+)|^2:|A(K^+\bar{K}^0\pi^0)|^2 = 3:3:2. \quad (4)$$

(2) $(\pi K)\bar{K}$: We label the reduced amplitudes $A_I^{\pi K}$ by $I_{\pi K} = 3/2, 1/2$. Then

$$A(\pi^+K^+K^-) = \frac{\sqrt{3}}{2}A_{3/2}^{\pi K}, \quad (5)$$

$$A(\pi^+K^0\bar{K}^0) = -\frac{1}{2\sqrt{3}}A_{3/2}^{\pi K} + \sqrt{\frac{2}{3}}A_{1/2}^{\pi K}, \quad (6)$$

$$A(\pi^0K^+\bar{K}^0) = -\frac{1}{\sqrt{6}}A_{3/2}^{\pi K} - \frac{1}{\sqrt{3}}A_{1/2}^{\pi K}, \quad (7)$$

leading again to the ratios (4) if $A_{1/2,3/2}^{\pi K}$ are equal and incoherent.

(3) $(\pi\bar{K})K$: We label the reduced amplitudes $A_I^{\pi\bar{K}}$ by $I_{\pi\bar{K}} = 3/2, 1/2$. Then

$$A(\pi^+K^-K^+) = -\frac{1}{2\sqrt{3}}A_{3/2}^{\pi\bar{K}} + \sqrt{\frac{2}{3}}A_{1/2}^{\pi\bar{K}}, \quad (8)$$

$$A(\pi^+\bar{K}^0K^0) = \frac{\sqrt{3}}{2}A_{3/2}^{\pi\bar{K}}, \quad (9)$$

$$A(\pi^0\bar{K}^0K^+) = -\frac{1}{\sqrt{6}}A_{3/2}^{\pi\bar{K}} - \frac{1}{\sqrt{3}}A_{1/2}^{\pi\bar{K}}, \quad (10)$$

leading again to (4) if $A_{1/2,3/2}^{\pi\bar{K}}$ are equal and incoherent.

B. Example of $(3\pi)_{I=I_3=1}$

The only couplings to consider are $(\pi\pi)\pi$. For example, choosing a particular order,

$$A(\pi^+\pi^+\pi^-) = \sqrt{\frac{3}{5}}A_2, \quad (11)$$

$$A(\pi^+\pi^0\pi^0) = -\sqrt{\frac{3}{20}}A_2 + \frac{1}{2}A_1, \quad (12)$$

so if A_2 and A_1 are equal and incoherent,

$$|A(\pi^+\pi^+\pi^-)|^2:|A(\pi^+\pi^0\pi^0)|^2 = 3:2. \quad (13)$$

Coupling the pions in a different order one can encounter also an amplitude with $I_{\pi\pi} = 0$, but the same result is obtained.

C. Tables of relative branching fractions

Results for Cabibbo-favored D_s decays are quoted from Ref. [11] for $(K\bar{K}n\pi)_{I=I_3=1}$ final states in Table I and for

TABLE I. Statistical model predictions for charge states in $(K\bar{K}n\pi)_{I=I_3=1}$.

$n(\pi^+) + n(\pi^-)$	0	1	2		3
$Q(\bar{K})$	0	− or 0	−	0	− or 0
$n = 0$	1	-	-	-	-
1	1/4	3/8	-	-	-
2	1/10	9/40	3/20	3/10	-
3	1/30	7/60	2/15	4/15	1/6

TABLE II. Statistical model predictions for charge states in $(n\pi)_{I=I_3=1}$.

$n(\pi^+) + n(\pi^-)$	1	3	5
$n = 1$	1	-	-
2	1	-	-
3	2/5	3/5	-
4	1/5	4/5	-
5	3/35	22/35	10/35
6	1/28	12/28	15/28

TABLE III. Statistical model predictions for charge states in $K + (n\pi)$ arising from singly-Cabibbo-suppressed D_s decays. A statistical average of contributions from $I = 1/2$ and $I = 3/2$ final states has been taken as in Table I of Ref. [12].

$n(\pi^+) + n(\pi^-)$	0	1	2	3
$n = 1$	1/2	1/2	-	-
2	3/20	8/20	9/20	-
3	3/45	9/45	21/45	12/45

$(3\pi)_{I=I_3=1}$ in Table II. Higher-multiplicity D_s decays appear to account for a very small fraction of the total [7]. Results for singly-Cabibbo suppressed D_s decays to $K + (n\pi)$ can be transcribed from Table I of Ref. [12], which applies to a statistical average of $I = 1/2$ and $I = 3/2$ states for $\bar{D}^0 \to K + (n\pi)$ arising from $\bar{s}\,\bar{u}\,du$. Here the $K + (n\pi)$ states arise from $\bar{s}\,\bar{d}\,ud$, which is related to $\bar{s}\,\bar{u}\,du$ by isospin reflection. The results are shown in Table III.

III. QUARK-ANTIQUARK PAIR PRODUCTION MODEL

In order to obtain estimates for systematic theoretical uncertainties in predictions of the statistical model we present in this section predictions of an alternative model involving quark-antiquark ($q\bar{q}$) pair production.

A. Description of the model

In this model final nonresonant states involving several pseudoscalar mesons are formed by the production of $u\bar{u}$ and $d\bar{d}$ quark pairs in every possible way, in association with a corresponding four-quark process in hadronic D_s decays, and in association with the $u\bar{d}$ weak current in semileptonic τ decays. One assumes that amplitudes for a given number of quark pairs are equal in magnitudes and incoherent in phase. For completeness, we describe below quark diagrams contributing to amplitudes for the three distinct classes studied in Sec. II:

(i) Cabibbo-favored $D_s \to n\pi$ and $\tau^+ \to (n\pi)^+\bar{\nu}_\tau$. Here one fills in $q\bar{q}$ pairs ($q = u, d$) between the $u\bar{d}$ pair produced by the weak current.

(ii) Cabibbo-favored $D_s \to K\bar{K}n\pi$. These decay processes obtain contributions from two tree amplitudes which are assumed to add incoherently, a "color-favored" amplitude (T) and a "color-suppressed" amplitude (C). In the first amplitude (T), involving the weak subprocess $c \to (u\bar{d})s$ with a spectator $\bar{s}$, one fills in with equal probabilities $q\bar{q}$ pairs between the color-singlet $u\bar{d}$ pair produced by the weak current and between the final s and spectator $\bar{s}$. In the second amplitude (C), involving the subprocess $c \to (s\bar{d})u$ with a spectator $\bar{s}$, we fill in with equal probabilities $q\bar{q}$ pairs between the color-singlet $s\bar{d}$ pair in the weak subprocess and between the $u\bar{s}$ pair.

(iii) Cabibbo-suppressed $D_s \to Kn\pi$. These decays obtain contributions from both color-favored and color-suppressed amplitudes. In the first, based on the weak subprocess $c \to (u\bar{d})d$ with a spectator $\bar{s}$, we fill in with equal probabilities $q\bar{q}$ pairs between the color-singlet $u\bar{d}$ pair in the weak subprocess and between the final d and spectator $\bar{s}$. In the color-suppressed amplitude, involving the weak subprocess $c \to (d\bar{d})u$ with a spectator $\bar{s}$, one fills in with equal probabilities $q\bar{q}$ pairs between the color-singlet $d\bar{d}$ pair in the weak subprocess and between the final u and spectator $\bar{s}$.

Color-favored (T) and color-suppressed (C) tree amplitudes, in processes involving pions and a $K\bar{K}$ pair or pions and a kaon, are independent quantities assumed to add incoherently in partial decay rates, $\Gamma = |T|^2 + |C|^2$. Consider two D_s decay processes $D_s \to f_1$ and $D_s \to f_2$ within the same class, $f = K\bar{K}n\pi$ or $f = Kn\pi$ for a given n. Denoting corresponding pairs of amplitudes by (T_1, C_1) and (T_2, C_2) and assuming for illustration $|C_1|/|C_2| < |T_1|/|T_2|$, one obtains the following lower and upper bounds on ratios of corresponding partial decay rates or ratios of branching ratios:

$$\frac{|C_1|^2}{|C_2|^2} < \frac{\mathcal{B}_1}{\mathcal{B}_2} = \frac{\Gamma_1}{\Gamma_2} < \frac{|T_1|^2}{|T_2|^2}. \tag{14}$$

Ratios $|T_1|^2/|T_2|^2$ and $|C_1|^2/|C_2|^2$ are calculated in the next subsection. They provide lower and upper bounds on ratios of branching ratios. These bounds, and ratios of branching fractions calculated in this model for $D_s \to n\pi$

TABLE IV. Predictions of the $q\bar{q}$ pair production model for charge states in $(n\pi)_{I=I_3=1}$.

$n(\pi^+) + n(\pi^-)$	1	3	5
$n = 1$	1	-	-
2	1	-	-
3	3/7	4/7	-
4	1/5	4/5	-
5	5/61	40/61	16/61

TABLE V. Predictions of the $q\bar{q}$ pair production model for charge states in $(K\bar{K}n\pi)_{I=I_3=1}$, for color-favored (T) and color-suppressed (C) amplitudes.

	$n(\pi^+) + n(\pi^-)$	0	1	2		3
	$Q(\bar{K})$	0	$-$ or 0	$-$	0	$-$ or 0
T	$n = 0$	1	-	-	-	-
	1	0	1/2	-	-	-
	2	0	3/10	1/5	1/5	-
	3	0	3/22	2/11	2/11	2/11
C	$n = 0$	1	-	-	-	-
	1	1/3	1/3	-	-	-
	2	1/9	2/9	4/27	8/27	-
	3	1/27	1/9	4/27	8/27	4/27

TABLE VI. Predictions of the $q\bar{q}$ pair production model for charge states in $K + (n\pi)$, for color-favored (T) and color-suppressed (C) amplitudes.

	$n(\pi^+) + n(\pi^-)$	0	1	2	3
T	$n = 1$	0	1	-	-
	2	0	3/5	2/5	-
	3	0	6/22	4/11	4/11
C	$n = 1$	1	0	-	-
	2	1/4	1/4	1/2	-
	3	3/37	6/37	20/37	8/37

and $\tau^+ \to (n\pi)^+ \bar{\nu}_\tau$, will be compared in Secs. IV and V with predictions of the statistical isospin model in order to estimate systematic theoretical errors in these predictions.

B. Tables of relative branching fractions

Results for relative branching fractions obtained in the $q\bar{q}$ pair production model are shown in Tables IV, V, and VI. The upper and lower parts of the last two tables, denoted T and C respectively, correspond to color-favored and color-suppressed amplitudes.

IV. FROM KNOWN D_s DECAYS TO OTHER MODES

A. Hadronic modes from $D_s^+ \to \tau^+\nu_\tau$

As the branching ratio $\mathcal{B}(D_s \to \tau\nu_\tau) = (6.6 \pm 0.6)\%$ [7,14], and τ has important decays to hadrons, we must take account of such decays. The known τ^+ branching ratios to hadronic final states are summarized in Table VII, with values in brackets inferred by applying the statistical isospin model to $\mathcal{B}(\tau^+ \to (5\pi)^+\bar{\nu}_\tau) = (7.6 \pm 0.5) \times 10^{-3}$ [7]. The second error in each of the three values of $\mathcal{B}_f$ corresponding to $f^+ = (5\pi)^+$ is a systematic error obtained by comparing predictions of this model with those of the $q\bar{q}$ pair production model. Also shown are inclusive branching ratios of τ^+ decays to π^+, π^0, π^-, K^+, and K^0. These are multiplied by $\mathcal{B}(D_s \to \tau\nu_\tau)$ to obtain contributions to inclusive D_s decays. One sees that the contributions, particularly to secondary π^+ and π^0, are significant.

B. Secondary particles from η, η', ϕ, and ω

The decays of the η contribute to secondary pions and photons. In Table VIII we note these contributions for pions. Secondary photons will not be considered here.

The decays of the η' contribute to a number of inclusive final states. These are summarized in Table IX for the main η' decay modes (those with branching fractions greater than 1%).

The ϕ is produced copiously in D_s decays. Its decays to K^+K^- and $K^0\bar{K}^0$ occur with different branching fractions because of differences in the limited phase space. Moreover, it has non-negligible branching fraction into $\pi^+\pi^-\pi^0$ and $\eta\gamma$. In Table X we collect these branching fractions and their implications for inclusive signals.

Although few D_s modes involve ω, we consider the main modes of ω for completeness in Table XI.

C. Treatment of neutral kaons

Often branching fractions are quoted for decay of a zero-strangeness system into XK_S, where X has a known strangeness. Thus if $S(X) = 1$, we infer $\mathcal{B}(X\bar{K}^0) = 2\mathcal{B}(XK_S)$, $\mathcal{B}(XK^0) = 0$, while if $S(X) = -1$, we infer $\mathcal{B}(XK^0) = 2\mathcal{B}(XK_S)$, $\mathcal{B}(X\bar{K}^0) = 0$. Similarly, if a state of known strangeness contains a single K_S we double the branching fraction and quote it for the appropriate K^0 or $\bar{K}^0$. Bracketed states in the table of branching fractions will indicate modes for which this has been done.

D. $K\bar{K}n\pi$ final states

We use the fact that $D_s \to K\bar{K}\pi$ is dominated by quasi-two-body $\phi\pi^+$, $\bar{K}^{*0}K^+$, and $\bar{K}^0K^{*+}$ final states to calculate the relative abundances of three charge states, using a statistical model for the left over (presumably nonresonant or broad S-wave) component. We start with branching ratios quoted in Ref. [7]: $\mathcal{B}(D_s \to \phi\pi^+) = (4.38 \pm 0.35)\%$, $\mathcal{B}(D_s \to \bar{K}^{*0}K^+) = (3.9 \pm 0.6)\%$ (inferred from the quoted value of $(2.6 \pm 0.4)\%$ with K^{*0} decaying to $K^+\pi^-$), and $\mathcal{B}(D_s \to \bar{K}^0K^{*+}) = (5.3 \pm 1.2)\%$. We summarize in Table XII the contributions of each subsystem to the three $K\bar{K}\pi$ charge states, as well as to $2\pi^+\pi^0\pi^-$

TABLE VII. Branching fractions $\mathcal{B}_f \equiv \mathcal{B}(\tau^+ \to f^+ \bar{\nu}_\tau)$ and contributions to inclusive pion and kaon production. Modes with branching fractions less than 10^{-4} are omitted. Brackets denote modes with values obtained from $\mathcal{B}(\tau^+ \to (5\pi)^+ \bar{\nu}_\tau) = (7.6 \pm 0.5) \times 10^{-3}$ [7] and the statistical model coefficients of Table II. The last line is calculated with $\mathcal{B}(D_s) \equiv \mathcal{B}(D_s \to \tau^+ \nu_\tau) = (6.6 \pm 0.6)\%$ [7]. Contributions to inclusive $\bar{K}^0$ and K^- production are found to be $\mathcal{B}(\tau^+ \to \bar{K}^0 X \bar{\nu}_\tau) = (0.52 \pm 0.05)\%$, $\mathcal{B}(\tau^+ \to K^- X \bar{\nu}_\tau) = (0.140 \pm 0.005)\%$, or 0.03 and 0.01 when multiplied by $\mathcal{B}(D_s \to \tau \nu_\tau)$. Columns 3–7 have been rounded off for convenience.

f^+	$\mathcal{B}_f$ (%)	$\mathcal{B}(\pi^+)$ (%)	$\mathcal{B}(\pi^0)$ (%)	$\mathcal{B}(\pi^-)$ (%)	$\mathcal{B}(K^+)$ (%)	$\mathcal{B}(K^0)$ (%)
π^+	10.91 ± 0.07	10.91 ± 0.07	0	0	0	0
$\pi^+\pi^0$	25.52 ± 0.10	25.52 ± 0.10	25.52 ± 0.10	0	0	0
$\pi^+ 2\pi^0$	9.27 ± 0.12	9.27 ± 0.12	18.54 ± 0.24	0	0	0
$2\pi^+\pi^-$	9.03 ± 0.06	18.06 ± 0.12	0	9.03 ± 0.06	0	0
$2\pi^+\pi^-\pi^0$	4.48 ± 0.06	8.96 ± 0.12	4.48 ± 0.06	4.48 ± 0.06	0	0
$\pi^+ 3\pi^0$	1.04 ± 0.07	1.04 ± 0.07	3.12 ± 0.21	0	0	0
K^+	0.695 ± 0.023	0	0	0	0.70 ± 0.02	0
$K^0\pi^+$	0.84 ± 0.04	0.84 ± 0.04	0	0	0	0.84 ± 0.04
$K^+\pi^0$	0.428 ± 0.015	0	0.43 ± 0.02	0	0.43 ± 0.02	0
$K^+ 2\pi^0$	0.063 ± 0.023	0	0.13 ± 0.05	0	0.06 ± 0.02	0
$K^+\pi^+\pi^-$	0.287 ± 0.016	0.29 ± 0.02	0	0.29 ± 0.02	0.29 ± 0.02	0
$K^0\pi^+\pi^0$	0.39 ± 0.04	0.39 ± 0.04	0.39 ± 0.04	0	0	0.39 ± 0.04
$K^+\pi^+\pi^-\pi^0$	0.081 ± 0.012	0.08 ± 0.01	0.08 ± 0.01	0.08 ± 0.01	0.08 ± 0.01	0
$K^+ 3\pi^0$	0.047 ± 0.021	0	0.14 ± 0.06	0	0.05 ± 0.02	0
$K^0\pi^+ 2\pi^0$	0.026 ± 0.024	0.03 ± 0.02	0.05 ± 0.05	0	0	0.03 ± 0.02
$K^+\bar{K}^0$	0.158 ± 0.016	0	0	0	0.16 ± 0.02	0
$K^+\bar{K}^0\pi^0$	0.158 ± 0.020	0	0.16 ± 0.02	0	0.16 ± 0.02	0
$K^0\bar{K}^0\pi^+$	0.17 ± 0.04	0.17 ± 0.04	0	0	0	0.17 ± 0.04
$K^+K^-\pi^+$	0.140 ± 0.005	0.14 ± 0.01	0	0	0.14 ± 0.01	0
$K^0\bar{K}^0\pi^+\pi^0$	0.031 ± 0.023	0.03 ± 0.02	0.03 ± 0.02	0	0	0.03 ± 0.02
$[3\pi^+ 2\pi^-]$	$0.217 \pm 0.014 \pm 0.018$	0.65 ± 0.07	0	0.43 ± 0.05	0	0
$[2\pi^+\pi^- 2\pi^0]$	$0.478 \pm 0.031 \pm 0.020$	0.96 ± 0.07	0.96 ± 0.07	0.48 ± 0.04	0	0
$[\pi^+ 4\pi^0]$	$0.065 \pm 0.004 \pm 0.003$	0.07	0.26 ± 0.02	0	0	0
Totals:		77.40 ± 0.28	54.28 ± 0.36	14.79 ± 0.11	2.06 ± 0.05	1.46 ± 0.08
$\times\mathcal{B}(D_s)$		5.11 ± 0.46	3.58 ± 0.33	0.98 ± 0.09	0.14 ± 0.01	0.10 ± 0.01

arising from $\phi \to \pi^+\pi^-\pi^0$. Here we have used the branching fractions for ϕ in Table X, and have assumed K^* decays are governed by isospin Clebsch-Gordan coefficients.

The subtotals represent the sums of quasi-two-body contributions to different charge states. As $\mathcal{B}(D_s \to K^+K^-\pi^+) = (5.50 \pm 0.28)\%$ is measured, we add a nonresonant ("NR") or broad S-wave background of $(0.74 \pm 0.51)\%$ to reach this total. The inferred branching ratios $\mathcal{B}(D_s \to K^0\bar{K}^0\pi^+) = (5.76 \pm 0.96)\%$ and $\mathcal{B}(D_s \to K^+\bar{K}^0\pi^0) = (3.56 \pm 0.67)\%$ (where the error includes a small contribution from a systematic model uncertainty) are not very different from the respective predictions of $(5.50 \pm 0.28)\%$ and $(3.67 \pm 0.19)\%$ of a pure statistical model.

Observed quasi-two-body decays of D_s contributing to $K\bar{K}2\pi$ final states are to $\phi(\to K^+K^-)\rho^+$, with branching fraction $(4.0^{+1.1}_{-1.2})\%$, and $K^{*+}\bar{K}^{*0}$, with branching fraction $(7.0 \pm 2.5)\%$. As seen in Table XIII, these two contributions almost completely saturate the final state $K^+K^-\pi^+\pi^0$, leaving a very small NR amount to be treated model-dependently.

The only subsystem for which one has reliable information in $D_s \to K\bar{K}3\pi$ is $\phi 2\pi^+\pi^-$, with quoted branching fraction $\mathcal{B}(D_s \to \phi(\to K^+K^-)2\pi^+\pi^-) = (0.59 \pm 0.11)\%$. Using the ϕ branching fractions in Table X, one

TABLE VIII. Branching fractions in η decays [7] and contributions to inclusive pions. Branching fractions less than 1% are omitted.

η mode	$\mathcal{B}$ (%)	$\mathcal{B}(\pi^+)$ (%)	$\mathcal{B}(\pi^0)$ (%)	$\mathcal{B}(\pi^-)$ (%)
$\gamma\gamma$	39.31 ± 0.20	0	0	0
$3\pi^0$	32.56 ± 0.20	0	97.68 ± 0.60	0
$\pi^+\pi^-\pi^0$	22.73 ± 0.28	22.73 ± 0.28	22.73 ± 0.28	22.73 ± 0.28
$\pi^+\pi^-\gamma$	4.60 ± 0.16	4.60 ± 0.16	0	4.60 ± 0.16
Total		27.33 ± 0.32	120.41 ± 0.66	27.33 ± 0.32

TABLE IX. Branching fractions in η' decays [7] and contributions to inclusive pions, η, and ω. In calculating pion production we use the branching fractions of Table VIII for η and note that $\mathcal{B}(\omega \to \pi^+\pi^-\pi^0) = (89.2 \pm 0.7)\%$, $\mathcal{B}(\omega \to \pi^0\gamma) = (8.92 \pm 0.24)\%$, $\mathcal{B}(\omega \to \pi^+\pi^-) = (1.53^{+0.13}_{-0.11})\%$ (see Table XI).

η' mode	$\mathcal{B}$ (%)	$\mathcal{B}(\pi^+)$ (%)	$\mathcal{B}(\pi^0)$ (%)	$\mathcal{B}(\pi^-)$ (%)	$\mathcal{B}(\eta)$ (%)	$\mathcal{B}(\omega)$ (%)
$\pi^+\pi^-\eta$	44.6 ± 1.4	56.8 ± 1.8	53.7 ± 1.7	56.8 ± 1.8	44.6 ± 1.4	0
$\pi^+\pi^-\gamma$	29.4 ± 0.9	29.4 ± 0.9	0	29.4 ± 0.9	0	0
$\pi^0\pi^0\eta$	20.7 ± 1.2	5.66 ± 0.33	66.3 ± 3.8	5.66 ± 0.33	20.7 ± 1.2	0
$\omega\gamma$	3.02 ± 0.31	2.74 ± 0.28	2.96 ± 0.30	2.74 ± 0.28	0	3.02 ± 0.31
Total		94.6 ± 2.1	123.0 ± 4.2	94.6 ± 2.1	65.3 ± 1.8	3.02 ± 0.31

TABLE X. Branching fractions of ϕ [7] and their implications for inclusive pion production. In addition the branching fractions for kaon and η production in ϕ decay are $\mathcal{B}(K^+) = \mathcal{B}(K^-) = (49.2 \pm 0.6)\%$, $\mathcal{B}(K^0) = \mathcal{B}(\bar{K}^0) = (34.2 \pm 0.5)\%$, and $\mathcal{B}(\eta) = (1.30 \pm 0.03)\%$.

ϕ mode	$\mathcal{B}$ (%)	$\mathcal{B}(\pi^+)$ (%)	$\mathcal{B}(\pi^0)$ (%)	$\mathcal{B}(\pi^-)$ (%)
K^+K^-	49.2 ± 0.6	0	0	0
$K^0\bar{K}^0$	34.0 ± 0.5	0	0	0
$\pi^+\pi^-\pi^0$	15.25 ± 0.35	15.25 ± 0.35	15.25 ± 0.35	15.25 ± 0.35
$\eta\gamma$	1.304 ± 0.025	0.36 ± 0.01	1.57 ± 0.03	0.36 ± 0.01
Total		15.61 ± 0.35	16.82 ± 0.35	15.61 ± 0.35

TABLE XI. Branching fractions in ω decays [7] and contributions to inclusive pions.

ω mode	$\mathcal{B}$ (%)	$\mathcal{B}(\pi^+)$ (%)	$\mathcal{B}(\pi^0)$ (%)	$\mathcal{B}(\pi^-)$ (%)
$\pi^+\pi^-\pi^0$	89.2 ± 0.7	89.2 ± 0.7	89.2 ± 0.7	89.2 ± 0.7
$\pi^0\gamma$	8.92 ± 0.24	0	8.92 ± 0.24	0
$\pi^+\pi^-$	$1.53^{+0.11}_{-0.13}$	$1.53^{+0.11}_{-0.13}$	0	$1.53^{+0.11}_{-0.13}$
Total		90.7 ± 0.7	98.1 ± 0.7	90.7 ± 0.7

TABLE XII. Contributions of quasi-two-body D_s decays to $K\bar{K}\pi$ and $2\pi^+\pi^-\pi^0$ charge states, in percent branching ratio from D_s.

Mode	$K^+K^-\pi^+$	$K^0\bar{K}^0\pi^+$	$K^+\bar{K}^0\pi^0$	$2\pi^+\pi^-\pi^0$
$\phi\pi^+$	2.15 ± 0.17	1.49 ± 0.12	0	0.67 ± 0.05
$\bar{K}^{*0}K^+$	2.6 ± 0.4	0	1.3 ± 0.2	0
$\bar{K}^0K^{*+}$	0	3.53 ± 0.80	1.77 ± 0.40	0
Subtotal	4.76 ± 0.43	5.02 ± 0.81	3.07 ± 0.45	0.67 ± 0.05
NR	0.74 ± 0.51	0.74 ± 0.51	$0.49 \pm 0.34^{+0.25}_{-0.49}$	0
Total	5.50 ± 0.28	5.76 ± 0.96	3.56 ± 0.67	0.67 ± 0.05

TABLE XIII. Contributions of quasi-two-body D_s decays to $K\bar{K}2\pi$ charge states, in percent branching ratio from D_s. Not shown: A contribution of $D_s \to \phi\rho^+$ to the $2\pi^+\pi^-2\pi^0$ final state with branching fraction $(4.0^{+1.1}_{-1.2})\% \cdot (15.25 \pm 0.35)\%/(49.2 \pm 0.6)\% = (1.24 \pm 0.36)\%$. Totals with smaller errors are observed branching fractions [7] and are used instead of model estimates. (Branching fractions to $K^+\bar{K}^0\pi^+\pi^-$ and $K^0K^-2\pi^+$ are inferred by doubling reported branching fractions involving K_S.)

Mode	$K^+K^-\pi^+\pi^0$	$K^0\bar{K}^0\pi^+\pi^0$	$K^+\bar{K}^0\pi^+\pi^-$	$K^0K^-\pi^+\pi^+$	$K^+\bar{K}^0\pi^0\pi^0$
$\phi\rho^+$	$4.0^{+1.1}_{-1.2}$	2.76 ± 0.79	0	0	0
$K^{*+}\bar{K}^{*0}$	1.56 ± 0.56	1.56 ± 0.56	0	3.11 ± 1.11	0.78 ± 0.28
Subtotal	5.56 ± 1.28	4.32 ± 0.97	0	3.11 ± 1.11	0.78 ± 0.28
NR	0.04 ± 1.37	0.04 ± 1.37	$0.05 \pm 1.83 \pm 0.02$	0.03 ± 0.91	0.02 ± 0.61
Total	5.6 ± 0.5	4.36 ± 1.68	1.92 ± 0.26	3.28 ± 0.24	0.80 ± 0.67

infers

$$\mathcal{B}(D_s \to \phi 2\pi^+\pi^-) = (1.20 \pm 0.22)\%,$$
$$\mathcal{B}(D_s \to \phi(\to K^+K^-)2\pi^+\pi^-) = (0.59 \pm 0.11)\%,$$
$$\mathcal{B}(D_s \to \phi(\to K^0\bar{K}^0)2\pi^+\pi^-) = (0.41 \pm 0.08)\%,$$
$$\mathcal{B}(D_s \to \phi(\to \pi^+\pi^-\pi^0)2\pi^+\pi^-) = (0.18 \pm 0.03)\%. \quad (15)$$

The $\pi^+\pi^0\pi^0$ system accompanies the ϕ 2/3 and 3/4 as frequently as $2\pi^+\pi^-$ in the statistical model and the $q\bar{q}$ production model, respectively, (see Tables II and IV). Thus one gets the corresponding predictions

$$\mathcal{B}(D_s \to \phi\pi^+2\pi^0) = (0.80 \pm 0.15 \pm 0.10)\%,$$
$$\mathcal{B}(D_s \to \phi(\to K^+K^-)\pi^+2\pi^0) = (0.39 \pm 0.07 \pm 0.05)\%,$$
$$\mathcal{B}(D_s \to \phi(\to K^0\bar{K}^0)\pi^+2\pi^0) = (0.27 \pm 0.05 \pm 0.04)\%,$$
$$\mathcal{B}(D_s \to \phi(\to \pi^+\pi^-\pi^0)\pi^+2\pi^0) = (0.12 \pm 0.02 \pm 0.02)\%, \quad (16)$$

where the second error represents the difference between predictions of the two models. The only $D_s \to K\bar{K}3\pi$ modes for which branching fractions have been measured are $D_s^+ \to K^+K^-2\pi^+\pi^-$, with $\mathcal{B} = (0.88 \pm 0.16)\%$, and $D_s \to K_SK_S2\pi^+\pi^-$, with $\mathcal{B} = (8.4 \pm 3.5) \times 10^{-4}$. It may be risky to use the latter to extrapolate to the $K^0\bar{K}^02\pi^+\pi^-$ mode if the kaon pair favors one CP eigenstate over another. Consequently, we shall use the difference between $\mathcal{B}(D_s \to K^+K^-2\pi^+\pi^-)$ and its contribution from the ϕ to estimate the amount of the $K\bar{K}3\pi$ final state to be treated model-dependently. The results are shown in Table XIV, where symmetric systematic errors are quoted using the maximum of positive and negative errors.

E. Multiple pions from $\phi(n\pi)$ final states

We have already calculated branching fractions for D_s to decay to multipion states via the mode $\phi \to \pi^+\pi^-\pi^0$, but summarize the results here.

(1) The state $2\pi^+\pi^-\pi^0$ receives a contribution from $D_s \to \phi\pi^+$, resulting in $\mathcal{B}(D_s^+ \to 2\pi^+\pi^-\pi^0) = (4.38 \pm 0.35)\% \cdot (15.25 \pm 0.35)\% = (0.67 \pm 0.06)\%$.
(2) The state $2\pi^+\pi^-2\pi^0$ receives a contribution from $D_s \to \phi\rho^+$, resulting in $\mathcal{B}(D_s^+ \to 2\pi^+\pi^-2\pi^0) = (4.0^{+1.1}_{-1.2})\% \cdot (15.25 \pm 0.35)\%/(49.2 \pm 0.6)\%$, resulting in $\mathcal{B}(D_s^+ \to 2\pi^+\pi^-2\pi^0) = (1.24 \pm 0.36)\%$.
(3) The state $3\pi^+2\pi^-\pi^0$ receives a contribution from $D_s \to \phi 2\pi^+\pi^-$, resulting in $\mathcal{B}(D_s^+ \to 3\pi^+2\pi^-\pi^0) = (0.18 \pm 0.03)\%$.
(4) The state $2\pi^+\pi^-3\pi^0$ receives a contribution from $D_s \to \phi\pi^+2\pi^0$, resulting in $\mathcal{B}(D_s^+ \to 2\pi^+\pi^-3\pi^0) = (0.12 \pm 0.03)\%$.

F. Avoiding double counting from $D_s \to (\eta, \eta') + (\pi^+, \rho^+)$

The decays $D_s \to (\eta\pi^+, \eta'\pi^+, \eta\rho^+, \eta'\rho^+)$ can populate numerous multi-pion final states. Of these, the only inclusive branching fraction explicitly quoted in Ref. [7] is $\mathcal{B}(D_s \to 3\pi^+2\pi^-\pi^0) = (4.9 \pm 3.2)\%$. We estimate using quoted branching fractions that $\mathcal{B}(D_s \to \eta'\pi^+)\mathcal{B}(\eta' \to \eta\pi^+\pi^-)\mathcal{B}(\eta \to \pi^+\pi^-\pi^0) = (0.39 \pm 0.04)\%$, leaving $(4.5 \pm 3.2)\%$ to be accounted for elsewhere. It is this figure that we include in the table of D_s branching fractions.

V. PROVISIONAL TABLES OF BRANCHING FRACTIONS

We summarize D_s branching fractions to leptonic and semileptonic final states in Table XV and to hadronic final states in Table XVI. Averages in the literature [7] have been supplemented where necessary with statistical model estimates, but an attempt has been made to use as much input from data as possible. We treat ηX, $\eta' X$, and ωX final states explicitly, but quote only final states for ϕX. The sum of the entries in Tables XV and XVI is $(102.7 \pm 5.3)\%$. The largest source of error is a rather early measurement [16], $\mathcal{B}(D_s \to 3\pi^+2\pi^-\pi^0) = (4.9 \pm 3.2)\%$, which includes a small contribution from $D_s \to \eta'\pi^+$. Replacement of this number and many estimated numbers with observed ones will be possible given the excellent particle identification and electromagnetic calorimetry of the CLEO-c detector. We do not extrapolate to other 6π

TABLE XIV. Contributions in percent of $\phi(3\pi)$ and model contributions to charge states in $D_s \to K\bar{K}3\pi$. The latter contributions, based on the statistical model, contain a statistical error and systematic error obtained by comparison with the $q\bar{q}$ production model.

State	$\phi(3\pi)$	Model	Total
$K^+K^-2\pi^+\pi^-$	0.59 ± 0.11	0.29 ± 0.19	0.88 ± 0.16
$K^0\bar{K}^02\pi^+\pi^-$	0.41 ± 0.08	0.29 ± 0.19	0.70 ± 0.21
$K^+\bar{K}^0\pi^+\pi^-\pi^0$	0	$0.46 \pm 0.30 \pm 0.17$	0.46 ± 0.34
$K^0K^-2\pi^+\pi^0$	0	$0.23 \pm 0.15 \pm 0.06$	0.23 ± 0.16
$K^+K^-\pi^+2\pi^0$	0.39 ± 0.09	$0.20 \pm 0.13 \pm 0.02$	0.59 ± 0.16
$K^0\bar{K}^0\pi^+2\pi^0$	0.27 ± 0.06	$0.20 \pm 0.13 \pm 0.02$	0.47 ± 0.15
$K^+\bar{K}^03\pi^0$	0	$0.06 \pm 0.04 \pm 0.06$	0.06 ± 0.06

TABLE XV. D_s branching fractions to leptonic and semileptonic modes [7,14,15]. Only modes contributing to inclusive hadron production are shown. Modes with $\mathcal{B} < 10^{-3}$ are omitted. Each $D_s \to X\ell^+\nu_\ell$ represents the assumed sum of $Xe^+\nu$ and $X\mu^+\nu$ modes and is doubled from the value quoted in Ref. [7].

Mode f	$\mathcal{B}(D_s \to f)$ (%)	Remarks
$\tau^+\nu_\tau$	6.6 ± 0.6	See Table VII for τ^+ hadronic decays
$\eta\ell^+\nu_\ell$	5.8 ± 1.2	See Table VIII for η hadronic decays
$\eta'\ell^+\nu_\ell$	2.04 ± 0.66	See Table IX for η' hadronic decays
$\phi\ell^+\nu_\ell$	4.72 ± 0.52	See Table X for ϕ hadronic decays
Total	19.16 ± 1.58	

TABLE XVI. D_s branching fractions to hadronic modes [7]. Values for bracketed modes are inferred from arguments in Secs. II and III. Second errors are systematic uncertainties obtained by differences between predictions of the statistical isospin model and the model based on $q\bar{q}$ production. Modes with $\mathcal{B} < 10^{-3}$ are omitted.

Mode f	$\mathcal{B}(D_s \to f)$ (%)	Remarks
$[K^+\bar{K}^0]$	2.98 ± 0.18	Doubled from quoted K^+K_S value
$K^+K^-\pi^+$	5.50 ± 0.28	
$[K^0\bar{K}^0\pi^+]$	5.76 ± 0.96	Quasi-2-body + model for NR
$[K^+\bar{K}^0\pi^0]$	3.56 ± 0.67	" "
$K^+K^-\pi^+\pi^0$	5.6 ± 0.5	
$[K^0\bar{K}^0\pi^+\pi^0]$	4.36 ± 1.68	Quasi-2-body + model for NR
$[K^+\bar{K}^0\pi^+\pi^-]$	1.92 ± 0.26	Doubled from quoted value with K_S
$[K^0K^-\pi^+\pi^+]$	3.28 ± 0.24	" "
$[K^+\bar{K}^0\pi^0\pi^0]$	0.80 ± 0.67	Quasi-2-body + model for NR
$K^+K^-\pi^+\pi^+\pi^-$	0.88 ± 0.16	
$[K^0\bar{K}^0\pi^+\pi^+\pi^-]$	0.70 ± 0.21	$\phi(3\pi)$ + model
$[K^+\bar{K}^0\pi^+\pi^-\pi^0]$	0.46 ± 0.34	" "
$[K^0K^-\pi^+\pi^+\pi^0]$	0.23 ± 0.16	" "
$[K^+K^-\pi^+\pi^0\pi^0]$	0.59 ± 0.16	" "
$[K^0\bar{K}^0\pi^+\pi^0\pi^0]$	0.47 ± 0.15	" "
$[K^+\bar{K}^0\pi^0\pi^0\pi^0]$	0.06 ± 0.06	" "
$\pi^+\pi^+\pi^-$	1.11 ± 0.08	
$[\pi^+\pi^0\pi^0]$	$0.74 \pm 0.05 \pm 0.09$	Model from $\pi^+\pi^+\pi^-$
$[\pi^+\pi^+\pi^-\pi^0]$	0.67 ± 0.06	From $\phi\pi^+$
$\eta\pi^+$	1.58 ± 0.21	
$\omega\pi^+$	0.25 ± 0.09	
$3\pi^+2\pi^-$	0.80 ± 0.09	
$[2\pi^+\pi^-2\pi^0]$	$3.00 \pm 0.41 \pm 0.24$	From $\phi\rho^+$ and model from $3\pi^+2\pi^-$
$[\pi^+4\pi^0]$	$0.24 \pm 0.03 \pm 0.01$	Model from $3\pi^+2\pi^-$
$\eta\rho^+$	13.0 ± 2.2	
$[3\pi^+2\pi^-\pi^0]$	4.5 ± 3.2	Subtracting 0.39% for $\eta'\pi^+$
$2\pi^+\pi^-3\pi^0$	0.12 ± 0.03	From $D_s \to \phi\pi^+\pi^0\pi^0$
$\eta'\pi^+$	3.8 ± 0.4	
$\eta'\rho^+$	12.2 ± 2.0	
$[K^0\pi^+]$	0.25 ± 0.03	Doubled from $K_S\pi^+$
$K^+\pi^0$	0.08 ± 0.02	(Statistical: 0.25 ± 0.03, $q\bar{q}$: unrestricted)
$K^+\eta$	0.141 ± 0.031	
$K^+\eta'$	0.16 ± 0.05	
$K^+\pi^+\pi^-$	0.69 ± 0.05	
$[K^0\pi^+\pi^0]$	$0.61 \pm 0.04^{+0.43}_{-0.26}$	Model from $K^+\pi^+\pi^-$
$[K^+\pi^0\pi^0]$	$0.23 \pm 0.02^{+0.12}_{-0.23}$	" "
$[K^0\pi^+\pi^+\pi^-]$	0.60 ± 0.22	Doubled from $K_S\pi^+\pi^+\pi^-$
$[K^+\pi^+\pi^-\pi^0]$	$1.05 \pm 0.39 \pm 0.45$	Model from $K^0\pi^+\pi^+\pi^-$
$[K^0\pi^+\pi^0\pi^0]$	0.45 ± 0.17	" "
$[K^+3\pi^0]$	$0.15 \pm 0.06^{+0.08}_{-0.15}$	" "
Total	83.57 ± 5.05	

modes from $3\pi^+2\pi^-\pi^0$, preferring to wait until it is better-measured.

The last 11 entries in Table XVI sum to a branching fraction of $(4.41 \pm 0.78)\%$ for hadronic Cabibbo-suppressed decays of D_s, consistent with a fraction $|V_{cd}/V_{cs}|^2$ of the Cabibbo-favored hadronic decays which amounts to $(4.24 \pm 0.27)\%$.

VI. PREDICTED INCLUSIVE BRANCHING FRACTIONS

With the branching fractions in Tables XV and XVI, it now becomes possible to calculate inclusive particle yields. These are summarized for pions in Table XVII, for kaons in Table XVIII, and for η, η', ϕ, and ω in Table XIX. Errors in π^+ and π^- inclusive branching ratios

TABLE XVII. Inclusive yields of pions from various final states in D_s decays.

Mode	$\mathcal{B}$ (%)	$\mathcal{B}(\pi^+)$ (%)	$\mathcal{B}(\pi^0)$ (%)	$\mathcal{B}(\pi^-)$ (%)
$\tau^+\nu_\tau$	6.6 ± 0.6	5.11 ± 0.46	3.58 ± 0.33	0.98 ± 0.09
$\eta\ell^+\nu_\ell$	5.8 ± 1.2	1.59 ± 0.33	6.98 ± 1.45	1.59 ± 0.33
$\eta'\ell^+\nu_\ell$	2.04 ± 0.66	1.93 ± 0.63	2.51 ± 0.82	1.93 ± 0.63
$\phi\ell^+\nu_\ell$	4.72 ± 0.52	0.74 ± 0.08	0.79 ± 0.09	0.74 ± 0.08
$K^+K^-\pi^+$	5.50 ± 0.28	5.50 ± 0.28	0	0
$K^0\bar{K}^0\pi^+$	5.76 ± 0.96	5.76 ± 0.96	0	0
$K^+\bar{K}^0\pi^0$	3.56 ± 0.67	0	3.56 ± 0.67	0
$K^+K^-\pi^+\pi^0$	5.6 ± 0.5	5.6 ± 0.5	5.6 ± 0.5	0
$K^0\bar{K}^0\pi^+\pi^0$	4.36 ± 1.68	4.36 ± 1.68	4.36 ± 1.68	0
$K^+\bar{K}^0\pi^+\pi^-$	1.92 ± 0.26	1.92 ± 0.26	0	1.92 ± 0.26
$K^0K^-\pi^+\pi^+$	3.28 ± 0.24	6.56 ± 0.48	0	0
$K^+\bar{K}^0\pi^0\pi^0$	0.80 ± 0.67	0	1.60 ± 1.34	0
$K^+K^-\pi^+\pi^+\pi^-$	0.88 ± 0.16	1.76 ± 0.32	0	0.88 ± 0.16
$K^0\bar{K}^0\pi^+\pi^+\pi^-$	0.70 ± 0.21	1.40 ± 0.42	0	0.70 ± 0.21
$K^+\bar{K}^0\pi^+\pi^-\pi^0$	0.46 ± 0.34	0.46 ± 0.34	0.46 ± 0.34	0.46 ± 0.34
$K^0K^-\pi^+\pi^+\pi^0$	0.23 ± 0.16	0.46 ± 0.32	0.23 ± 0.16	0
$K^+K^-\pi^+\pi^0\pi^0$	0.59 ± 0.16	0.59 ± 0.16	1.18 ± 0.32	0
$K^0\bar{K}^0\pi^+\pi^0\pi^0$	0.47 ± 0.15	0.47 ± 0.15	0.94 ± 0.30	0
$K^+\bar{K}^0\pi^0\pi^0\pi^0$	0.06 ± 0.06	0	0.18 ± 0.18	0
$\pi^+\pi^+\pi^-$	1.11 ± 0.08	2.22 ± 0.16	0	1.11 ± 0.08
$\pi^+\pi^0\pi^0$	0.74 ± 0.10	0.74 ± 0.10	1.48 ± 0.20	0
$\pi^+\pi^+\pi^-\pi^0$	0.67 ± 0.06	1.34 ± 0.12	0.67 ± 0.06	0.67 ± 0.06
$\eta\pi^+$	1.58 ± 0.21	2.01 ± 0.27	1.90 ± 0.25	0.43 ± 0.06
$\omega\pi^+$	0.25 ± 0.09	0.48 ± 0.17	0.25 ± 0.09	0.22 ± 0.08
$3\pi^+2\pi^-$	0.80 ± 0.09	2.40 ± 0.27	0	1.60 ± 0.18
$2\pi^+\pi^-2\pi^0$	3.00 ± 0.48	6.00 ± 0.96	6.00 ± 0.96	3.00 ± 0.48
$\pi^+4\pi^0$	0.24 ± 0.03	0.24 ± 0.03	0.96 ± 0.12	0
$\eta\rho^+$	13.0 ± 2.2	16.55 ± 2.80	28.65 ± 4.85	3.55 ± 0.60
$3\pi^+2\pi^-\pi^0$	4.5 ± 3.2	13.5 ± 9.6	4.5 ± 3.2	9.0 ± 6.4
$2\pi^+\pi^-3\pi^0$	0.12 ± 0.03	0.24 ± 0.06	0.36 ± 0.09	0.12 ± 0.03
$\eta'\pi^+$	3.8 ± 0.4	7.39 ± 0.79	4.67 ± 0.49	3.59 ± 0.38
$\eta'\rho^+$	12.2 ± 2.0	23.74 ± 3.89	27.21 ± 4.46	11.54 ± 1.89
$K^0\pi^+$	0.25 ± 0.03	0.25 ± 0.03	0	0
$K^+\pi^0$	0.08 ± 0.02	0	0.08 ± 0.02	0
$K^+\eta$	0.141 ± 0.031	0.04 ± 0.01	0.17 ± 0.04	0.04 ± 0.01
$K^+\eta'$	0.16 ± 0.05	0.15 ± 0.05	0.20 ± 0.06	0.15 ± 0.05
$K^+\pi^+\pi^-$	0.69 ± 0.05	0.69 ± 0.05	0	0.69 ± 0.05
$K^0\pi^+\pi^0$	0.61 ± 0.35	0.61 ± 0.35	0.61 ± 0.35	0
$K^+\pi^0\pi^0$	0.23 ± 0.18	0	0.46 ± 0.36	0
$K^0\pi^+\pi^+\pi^-$	0.60 ± 0.22	1.20 ± 0.44	0	0.60 ± 0.22
$K^+\pi^+\pi^-\pi^0$	1.05 ± 0.60	1.05 ± 0.60	1.05 ± 0.60	1.05 ± 0.60
$K^0\pi^+\pi^0\pi^0$	0.45 ± 0.17	0.45 ± 0.17	0.90 ± 0.34	0
$K^+3\pi^0$	0.15 ± 0.13	0	0.45 ± 0.39	0
Total		125.5 ± 11.1	112.5 ± 8.0	46.6 ± 6.8

TABLE XVIII. Inclusive yields of kaons from various final states in D_s decays.

Mode	$\mathcal{B}$ (%)	$\mathcal{B}(K^+)$ (%)	$\mathcal{B}(K^0)$ (%)	$\mathcal{B}(K^-)$ (%)	$\mathcal{B}(\bar{K}^0)$ (%)
$\tau^+\nu_\tau$	6.6 ± 0.6	0.14 ± 0.01	0.10 ± 0.01	0.03	0.01
$\phi\ell^+\nu_\ell$	4.72 ± 0.52	2.32 ± 0.26	1.60 ± 0.18	2.32 ± 0.26	1.60 ± 0.18
$K^+\bar{K}^0$	2.98 ± 0.18	2.98 ± 0.18	0	0	2.98 ± 0.18
$K^+K^-\pi^+$	5.50 ± 0.28	5.50 ± 0.28	0	5.50 ± 0.28	0
$K^0\bar{K}^0\pi^+$	5.76 ± 0.96	0	5.76 ± 0.96	0	5.76 ± 0.96
$K^+\bar{K}^0\pi^0$	3.56 ± 0.67	3.56 ± 0.67	0	0	3.56 ± 0.67
$K^+K^-\pi^+\pi^0$	5.6 ± 0.5	5.6 ± 0.5	0	5.6 ± 0.5	0
$K^0\bar{K}^0\pi^+\pi^0$	4.36 ± 1.68	0	4.36 ± 1.68	0	4.36 ± 1.68
$K^+\bar{K}^0\pi^+\pi^-$	1.92 ± 0.26	1.92 ± 0.26	0	0	1.92 ± 0.26
$K^0K^-\pi^+\pi^+$	3.28 ± 0.24	0	3.28 ± 0.24	3.28 ± 0.24	0
$K^+\bar{K}^0\pi^0\pi^0$	0.80 ± 0.67	0.80 ± 0.67	0	0	0.80 ± 0.67
$K^+K^-\pi^+\pi^+\pi^-$	0.88 ± 0.16	0.88 ± 0.16	0	0.88 ± 0.16	0
$K^0\bar{K}^0\pi^+\pi^+\pi^-$	0.70 ± 0.21	0	0.70 ± 0.21	0	0.70 ± 0.21
$K^+\bar{K}^0\pi^+\pi^-\pi^0$	0.46 ± 0.34	0.46 ± 0.34	0	0	0.46 ± 0.34
$K^0K^-\pi^+\pi^+\pi^0$	0.23 ± 0.16	0	0.23 ± 0.16	0.23 ± 0.16	0
$K^+K^-\pi^+\pi^0\pi^0$	0.59 ± 0.16	0.59 ± 0.16	0	0.59 ± 0.16	0
$K^0\bar{K}^0\pi^+\pi^0\pi^0$	0.47 ± 0.15	0	0.47 ± 0.15	0	0.47 ± 0.15
$K^+\bar{K}^0\pi^0\pi^0\pi^0$	0.06 ± 0.06	0.06 ± 0.06	0	0	0.06 ± 0.06
$K^0\pi^+$	0.25 ± 0.03	0	0.25 ± 0.03	0	0
$K^+\pi^0$	0.08 ± 0.02	0.08 ± 0.02	0	0	0
$K^+\eta$	0.141 ± 0.031	0.14 ± 0.03	0	0	0
$K^+\eta'$	0.16 ± 0.05	0.16 ± 0.05	0	0	0
$K^+\pi^+\pi^-$	0.69 ± 0.05	0.69 ± 0.05	0	0	0
$K^0\pi^+\pi^0$	0.61 ± 0.35	0	0.61 ± 0.35	0	0
$K^+\pi^0\pi^0$	0.23 ± 0.18	0.23 ± 0.18	0	0	0
$K^0\pi^+\pi^+\pi^-$	0.60 ± 0.22	0	0.60 ± 0.22	0	0
$K^+\pi^+\pi^-\pi^0$	1.05 ± 0.60	1.05 ± 0.60	0	0	0
$K^0\pi^+\pi^0\pi^0$	0.45 ± 0.17	0	0.45 ± 0.17	0	0
$K^+3\pi^0$	0.15 ± 0.13	0.15 ± 0.13	0	0	0
Total		27.3 ± 1.4	18.4 ± 2.0	18.4 ± 0.7	22.7 ± 2.2

TABLE XIX. Inclusive yields of η, η', ϕ, and ω from various final states in D_s decays.

Mode	$\mathcal{B}$ (%)	$\mathcal{B}(\eta)$ (%)	$\mathcal{B}(\eta')$ (%)	$\mathcal{B}(\phi)$ (%)	$\mathcal{B}(\omega)$ (%)
$\eta\ell^+\nu_\ell$	5.8 ± 1.2	5.8 ± 1.2	0	0	0
$\eta'\ell^+\nu_\ell$	2.04 ± 0.66	1.33 ± 0.43	2.04 ± 0.66	0	0.06 ± 0.02
$\phi\ell^+\nu_\ell$	4.72 ± 0.52	0.06 ± 0.01	0	4.72 ± 0.52	0
$\phi\pi^+$	4.38 ± 0.35	0.06 ± 0.01	0	4.38 ± 0.35	0
$\phi\rho^+$	8.13 ± 2.34	0.11 ± 0.03	0	8.13 ± 2.34	0
$\phi 2\pi^+\pi^-$	1.20 ± 0.22	0.02	0	1.20 ± 0.22	0
$\phi\pi^+ 2\pi^0$	0.80 ± 0.15	0.01	0	0.80 ± 0.15	0
$\eta\pi^+$	1.58 ± 0.21	1.58 ± 0.21	0	0	0
$\omega\pi^+$	0.25 ± 0.09	0	0	0	0.25 ± 0.09
$\eta\rho^+$	13.0 ± 2.2	13.0 ± 2.2	0	0	0
$\eta'\pi^+$	3.8 ± 0.4	2.48 ± 0.26	3.8 ± 0.4	0	0.11 ± 0.01
$\eta'\rho^+$	12.2 ± 2.0	7.97 ± 1.32	12.2 ± 2.0	0	0.37 ± 0.07
$K^+\eta$	0.141 ± 0.031	0.14 ± 0.03	0	0	0
$K^+\eta'$	0.16 ± 0.05	0.10 ± 0.03	0.16 ± 0.05	0	0
Total		32.7 ± 2.9	18.2 ± 2.1	19.2 ± 2.4	0.8 ± 0.1

can be reduced considerably by improving the branching fraction measurement for $D_s \to 3\pi^+ 2\pi^- \pi^0$.

VII. CONCLUSIONS

We have calculated the inclusive branching fractions of D_s mesons to several species, using the fact that the observed branching fractions, together with modest assumptions about unseen charge states, account for all the D_s decays to an accuracy of about 5%. Calculations of branching ratios for unseen modes, based mostly on a statistical isospin model, involve small systematic theoretical uncertainties estimated by comparison with a model using quark-antiquark pair production. While many aspects of this analysis bear some resemblance to an itemized tax return, several notable features have emerged.

(i) The greatest errors on extracting inclusive branching fractions from exclusive modes are due to a few final states, notably $3\pi^+ 2\pi^- \pi^0$, $\eta\rho^+$, and $\eta'\rho^+$. Improvement of information on these modes would be very helpful.

(ii) The large predicted values for the inclusive branching ratios $\mathcal{B}(\eta)$ and $\mathcal{B}(\eta')$ may be helpful in determining whether, as suspected in SU(3) fits (see, e.g., Ref. [6]) and in factorization calculations or modifications [17,18] based on the observed semileptonic decays $D_s \to \eta^{(\prime)}\ell\nu$, the branching fractions for $D_s \to \eta\rho^+$ and particularly $\eta'\rho^+$ are too high. It could be possible that part of the large "signals" for $D_s \to \eta^{(\prime)}\rho^+$ come from misidentified kinematic reflections from other final states.
For example, both $\eta'(\to \pi^+\pi^-\eta)\rho^+(\to \pi^+\pi^0)$ and $\eta\pi^+\omega(\to \pi^+\pi^-\pi^0)$ contain the same final-state particles $\eta 2\pi^+\pi^-\pi^0$. In the CLEO paper reporting large branching fractions for $D_s \to \eta^{(\prime)}\rho^+$ (where η was identified by its two-photon decay mode) [17], no distributions in $M(\eta)$ or $M(\eta')$ were shown. If a sizable fraction of the measured value $\mathcal{B}(D_s \to \eta'\rho^+) = (12.2 \pm 2.0)\%$ is, for instance, due to underlying $D_s \to \eta\pi^+\omega$ events, then our prediction for inclusive $\mathcal{B}(\eta')$ becomes smaller while $\mathcal{B}(\omega)$ becomes correspondingly larger. The effect of such events on $\mathcal{B}(\eta)$ combines a decrease from a smaller value of $\mathcal{B}(D_s \to \eta'\rho^+)$, where η' decays dominantly to states involving η (see Table VI), and a positive contribution from $\mathcal{B}(D_s \to \eta\pi^+\omega)$.
Similarly, the decay mode $D_s \to \omega(\to \pi^0\gamma)\pi^+\pi^0$, where one photon from π^0 may be missing in some events, can mimic $D_s \to \eta(\to \gamma\gamma)\rho^+(\to \pi^+\pi^0)$. If a small fraction of $(D_s \to \eta\rho^+) = (13.0 \pm 2.2)\%$ stems from underlying $D_s \to \omega\pi^+\pi^0$ events, then our predictions for inclusive (η) and (ω) decrease and increase, respectively, roughly (i.e. neglecting differences in efficiencies) by a ratio $(\omega \to \pi^0\gamma)$:$(\eta \to \gamma\gamma) = 1$:4.4.

(iii) The measurement of $\mathcal{B}(\eta)$, $\mathcal{B}(\eta')$, and $\mathcal{B}(\omega)$ may help to shed light on specific decay mechanisms. For example, the decay $D_s \to \omega\pi^+\pi^0$, represented by the quark annihilation process $c\bar{s} \to u\bar{d}$, could have a sizable branching ratio. While $D_s \to \omega\pi^+$ with $\mathcal{B} = (0.25 \pm 0.09)\%$ is forbidden by G-parity and requires preradiation of ω or rescattering [5], $D_s \to \omega\pi^+\pi^0$ can occur through ordinary quark annihilation (as long as some mechanism overcomes helicity suppression), and is expected to have a considerably larger decay rate. As a second example, the decay $D_s \to \omega\pi^+\eta$ could arise either from WA or from the transition $c\bar{s} \to s\bar{s}+$ (charged weak vector current), where the charged weak vector current produces $\omega\pi^+$.

We look forward to experimental tests of these predictions.

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Note added in proof.— Inclusive D_s decays have now been reported by the CLEO Collaboration [19]. They are largely in agreement with the predictions of Tables XVII, XVIII, and XIX, with the notable exceptions of $\mathcal{B}(\eta') = (11.7 \pm 1.7 \pm 0.7)\%$ and $\mathcal{B}(\omega) = (6.1 \pm 1.4 \pm 0.3)\%$. Thus, current world averages [7] apparently overestimate the sum of D_s branching fractions involving η' and greatly underestimate those involving ω.

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[15] New semileptonic branching fractions not included in these averages are $\mathcal{B}(D_s \to \eta e^+ \nu_e) = (2.48 \pm 0.29 \pm 0.13)\%$, $\mathcal{B}(D_s \to \eta' e^+ \nu_e) = (0.91 \pm 0.33 \pm 0.05)\%$, $\mathcal{B}(D_s \to \phi e^+ \nu_e) = (2.29 \pm 0.37 \pm 0.01)\%$, $\mathcal{B}(D_s \to K^0 e^+ \nu_e) = (0.37 \pm 0.10 \pm 0.02)\%$, $\mathcal{B}(D_s \to K^{*0} e^+ \nu_e) = (0.18 \pm 0.07 \pm 0.01)\%$, $\mathcal{B}(D_s \to f_0 e^+ \nu_e) \times \mathcal{B}(f_0 \to \pi^+ \pi^-) = (0.13 \pm 0.04 \pm 0.01)\%$. See J. Yelton *et al.* (CLEO Collaboration), arXiv:0903.0601.
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