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Model calculations of the Sivers function satisfying the Burkardt sum rule

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It is shown that, at variance with previous analyses, the MIT bag model can explain the available data of the Sivers function and satisfies the Burkardt sum rule to a few percent accuracy. The agreement is similar to the one recently found in the constituent quark model. Therefore, these two model calculations of the Sivers function are in agreement with the present experimental and theoretical wisdom.

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The study of the partonic properties of transversely polarized hadrons will answer crucial questions on their structure, such as their relativistic nature and their angular momentum content. Experiments are progressing fast, motivating a strong theoretical activity [1]. One of the quantities under scrutiny is the Sivers function, the object of this study. Semi-inclusive deep inelastic scattering (SIDIS), i.e. the process $A(e, e'h)X$, with the detection in the final state of a hadron h with the scattered electron e', is one of the processes to access the transversity parton distributions (PDs). For several years it has been known that SIDIS off a transversely polarized target shows the so called ''single spin asymmetries'' (SSAs). It can be shown (see, i.e., Ref. [2] and references therein), that one of the mechanisms generating the SSAs is governed by the Sivers function [3]. The latter describes the amplitude of modulation of the number density of unpolarized quarks in a transversely polarized target due to the correlation between the transverse spin of the target and the intrinsic transverse parton momentum. The Sivers function is a transverse momentum dependent (TMD) PD, denoted $f_{1T}^{\perp\mathcal{Q}}(x, k_T)$, where x is the Bjorken variable and k_T is the transverse momentum of the parton $\mathcal{Q}$. It is a time reversal odd object [1] and for this reason, for several years, it was believed to vanish. However, this argument was invalidated by a calculation in a spectator model, following the discovery of final state interactions (FSI) at leading-twist, i.e., not kinematically suppressed in DIS [4]. The current wisdom is that a nonvanishing Sivers function is generated by FSI, technically represented by the gauge link in the definition of TMD parton distributions [5]. Recently, the first data of SIDIS off transversely polarized targets have shown a strong flavor dependence of the Sivers mechanism [6]. Complementary experiments on transversely polarized ^{3}He target, addressed in [7], are being performed at JLab [8]. Parametrizations of $f_{1T}^{\perp\mathcal{Q}}(x, k_T)$ are available [9–11], and new data are expected soon. From the theoretical point of view, a model independent constraint on calculations of $f_{1T}^{\perp\mathcal{Q}}(x, k_T)$ is the Burkardt sum rule (SR) [12]. It states that the average transverse momentum of all the partons in a hadron, $\langle\vec{k}_T\rangle$, which can be defined through $f_{1T}^{\perp\mathcal{Q}}(x, k_T)$, has to vanish. If the proton is polarized in the positive x direction, the Burkardt SR reads:

$$\sum_{\mathcal{Q}=u,d,s,g..}\langle k_y^{\mathcal{Q}}\rangle = -\int_0^1 dx \int d\vec{k}_T \frac{k_y^2}{M} f_{1T}^{\perp\mathcal{Q}}(x, k_T) = 0. \quad (1)$$

Given the present situation of increasing experimental activity, estimates of $f_{1T}^{\perp\mathcal{Q}}(x, k_T)$, subject to solid theoretical constraints, can be very useful. Since a direct calculation in QCD is not yet feasible, this quantity has been calculated in several models: a quark-diquark model [4,13]; the MIT bag model, in its simplest version [14] and introducing an instanton contribution [15]; the constituent quark model (CQM) [16]. To distinguish between the model estimates, data and model independent relations, such as the Burkardt SR, can be used. In all the models used so far the total momentum of the proton is carried by the quarks of flavor u and d. According to Eq. (1), this implies that the magnitude of $f_{1T}^{\perp\mathcal{Q}}$ for $\mathcal{Q} = u$ and d has to be similar and the sign has to be opposite. This is also the trend of the parametrizations of the data [9,11]. In different versions of the diquark model the magnitude of the d contribution is much smaller than that of the u. In the MIT bag model [14] u and d-quark contributions of opposite sign are found to be proportional by a factor of -4. Even in the modified version of the MIT bag model of Ref. [15], the Burkardt SR is not fulfilled. On the contrary, in the CQM, we found a satisfactory description of the data and therefore the calculation fulfills the Burkardt SR at the 2% level [16]. This puzzling situation deserves to be investigated. To this end, we next analyze the MIT bag model calculation to understand the origin of the discrepancy with the CQM calculation. One should realize that, in the CQM, even if a pure S-wave description of the proton is used, i.e., a pure $SU(6)$ wave function, we are able to reproduce the gross features of the data. The same $SU(6)$ spin-flavor structure is used in the bag calculation of

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Refs. [14,15] and no agreement with the data is found. This situation is in contradiction with previous calculations of other PDs in the bag model [17] and in the CQM [18] which both have been able to reproduce the gross features of the data.

If the proton is polarized in the positive x direction, the Sivers function can be written, in a helicity basis for the proton, as [16,19]

$$f_{1T}^{\perp\mathcal{Q}}(x,k_T) = 2\Re\left\{\frac{M}{4k_y}\int\frac{d\xi^- d^2\vec{\xi}_T}{(2\pi)^3} \times e^{-i(x\xi^- P^+ - \vec{\xi}_T\cdot\vec{k}_T)}\langle PS_z=1|\hat{O}_{\mathcal{Q}}|PS_z=-1\rangle\right\}, \tag{2}$$

where $\hat{O}_{\mathcal{Q}} = \bar{\psi}_{\mathcal{Q}}(0,\xi^-,\vec{\xi}_T)\mathcal{L}^\dagger_{\vec{\xi}_T}(\infty,\xi^-)\gamma^+\mathcal{L}_0(\infty,0)\psi_{\mathcal{Q}}(0)$, $\psi_{\mathcal{Q}}(\xi)$ is the quark field, $\mathcal{L}_{\vec{\xi}_T}(\infty,\xi^-)$ is the gauge link [11]. In the following, the framework and the notation of Ref. [14] are used to calculate Eq. (2) in the MIT bag model [17,20]. By expanding the gauge link to next-to-leading order, inserting in Eq. (2) the bag model wave function in momentum space, $\varphi(k)$ [20], and using for the definition of the quark helicity and momentum labels the ones in Fig. 1, $f_{1T}^{\perp\mathcal{Q}}(x,k_T)$ can be written as

$$f_{1T}^{\perp\mathcal{Q}}(x,k_\perp) = -g^2\frac{ME_P}{k^y}2\Re\left\{\int\frac{d^2q_\perp}{(2\pi)^5}\frac{i}{q^2} \times \sum_{\{m\},\beta}C^{\mathcal{Q},\beta}_{\{m\}}\varphi^\dagger_{m_1}(\vec{k}-\vec{q}_\perp)\gamma^0\gamma^+\varphi_{m_2}(\vec{k}) \times \int\frac{d^3k_3}{(2\pi)^3}\varphi^\dagger_{m_3}(\vec{k}_3)\gamma^0\gamma^+\varphi_{m_4}(\vec{k}_3-\vec{q}_\perp)\right\}. \tag{3}$$

This equation corresponds to Eq. (17) in Ref. [14] modified to follow the Trento convention [19] which implies an additional factor of 1/2. Here g is the strong coupling constant, $C^{\mathcal{Q},\beta}_{\{m\}} = T^a_{ij}T^a_{kl}\langle PS_z = 1|b^{i\dagger}_{\mathcal{Q}m_1}b^j_{\mathcal{Q}m_2}b^{k\dagger}_{\beta m_3}b^l_{\beta m_4}|PS_z=-1\rangle$ with $\{m\}=m_1$, m_2, m_3, m_4; M is the proton mass, E_p its energy, $b^i_{\mathcal{Q},m}$ is the annihilation operator for a quark with flavor $\mathcal{Q}$, helicity m, and color index i, and T^a_{ij} is a Gell-Mann matrix. In turn, the k_3 integral can be written as

$$\int\frac{d^3k_3}{(2\pi)^3}\varphi^\dagger_{m_3}(\vec{k}_3)\gamma^0\gamma^+\varphi_{m_4}(\vec{k}_3-\vec{q}_\perp) \equiv F_{m_3}(\vec{q}_\perp)\delta_{m_3m_4} + H_{m_3}(\vec{q}_\perp)\delta_{m_3,-m_4}, \tag{4}$$

with

$$F_{m_3}(\vec{q}_\perp) = \frac{C}{\sqrt{2}}\int d^3k_3[t^3_0t'^3_0 + k^z_3t^3_1t'^3_0/k_3 + k'^z_3t'^3_1t^3_0/k'^3 + (\vec{k}_3\cdot\vec{k}'_3 + iv^zd_{m_3})t^3_1t'^3_1/(k'_3k_3)], \tag{5}$$

$$H_{m_3}(\vec{q}_\perp) = \frac{C}{\sqrt{2}}\int d^3k_3[(ik^y_3 - k^x_3d_{m_3})t'^3_0t^3_1/k_3 - (ik'^y_3 - k'^x_3d_{m_3})t^3_0t'^3_1/k'_3 + (v^yd_{m_3} + iv^x)t^3_1t'^3_1/(k_3k'_3)], \tag{6}$$

where $k_3 = |\vec{k}_3|$, $k'_3 = |\vec{k}'_3|$, $d_{m_3} \equiv (\delta_{m_3(1/2)} - \delta_{m_3,-(1/2)})$, $\vec{v} = \vec{q}_\perp\times\vec{k}_3$, $\vec{k}'_3 = \vec{k}_3 - \vec{q}$, $C = 16\omega^4/(\pi^2j_0^2(\omega)\times(\omega-1)M_P^3)$, with ω being the bag model mode [20] and the function $t^3_i = t_i(k_3)$, $t'^3_i = t_i(k'_3)$, $i=0,1$ are defined in [14].

In Ref. [14], only the first term in the right-hand side of Eq. (4) is calculated. This term corresponds to the helicity conserving contribution ($m_3 = m_4$) associated with the quark in the lower part of Fig. 1. This result only applies if the integral is performed taking $\vec{q}_\perp$ along the z direction. However, this is incorrect in the present case. As in any DIS process, the direction of the virtual photon determines the operator structure, i.e. γ^+ in here. The fixing of the photon direction leads to a γ_3 matrix in this operator. Therefore we do not have anymore the freedom to choose z as the direction of the exchanged gluon, which must lie in the (x,y) plane. Besides, one can check that the integral Eq. (4) does depend on the direction of $\vec{q}_\perp$. Moreover, if the findings of Ref. [14] were correct, it would mean that a helicity flip could occur only for the quark which interacts with the photon and not for the quark in the lower part of Fig. 1, a restriction which does not have any physical motivation. Thus the present calculation differs from the previous one in that we take into consideration both terms of Eq. (4). By the same argument, the expression $\varphi^\dagger_{m_1}(\vec{k}-\vec{q}_\perp)\gamma^0\gamma^+\varphi_{m_2}(\vec{k})$ in Eq. (3) also contains both helicity-flip and nonflip terms. One should notice that, in Ref. [15], a helicity-flip term (with $m_3 = -m_4$) has been found to contribute to the Sivers function. In that paper, instanton effects have been added to the pure MIT bag model calculation of Ref. [14] and the presence of this helicity-flip term is due solely to the instanton contribution. However, in the calculation of Ref. [15], the term depending on $\delta_{m_3-m_4}$ in Eq. (4), associated to the perturbative one-gluon exchange, should appear and has not been considered [21]. It is interesting to realize that, in a completely different sce-

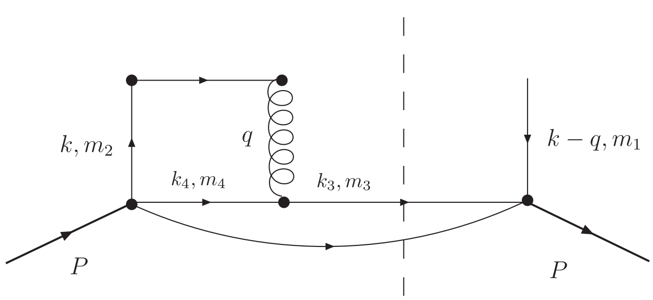

FIG. 1. The contributions to the Sivers function in the present approach. The graph has been drawn using JaxoDraw [24].

nario, the CQM calculation satisfying the Burkardt SR of Ref. [16], a contribution is found for helicity conserving and helicity flip of the quark in the lower part of Fig. 1, as it happens in our MIT bag model calculation.

Evaluating the matrix elements for the valence quarks and assuming an $SU(6)$ proton state in Eq. (3), one gets

$$f_{1T}^{\perp \mathcal{Q}}(x, k_\perp) = -\frac{g^2}{2}\frac{ME_P}{k^y}C^2 \int \frac{d^2 q_\perp}{(2\pi)^2}\frac{1}{q^2}[C_{\mathcal{Q}}^{-+} Y(\vec{q}_\perp, k_T) + C_{\mathcal{Q}}^{+-} U(\vec{q}_\perp, k_T)], \tag{7}$$

with

$$Y(\vec{q}_\perp, k_T) = [k'^y t'_1 t_0/k' - k_y t_1 t'_0/k - v^y t_1 t'_1/(kk')] \times \int d^3k_3 [t_0^3 t_0'^3 + k_3^z t_1^3 t_0'^3/k_3 + k_3'^z t_1'^3 t_0^3/k'_3 + (k_3^2 - \vec{k}_3 \cdot \vec{q}_\perp) t_1^3 t_1'^3/(k_3 k'_3)], \tag{8}$$

$$U(\vec{q}_\perp, k_T) = [t'_0 t_0 + k^z (t'_1 t_0/k' + t'_0 t_1/k) + (k^2 - \vec{q}_\perp \cdot \vec{k}) t_1 t'_1/(kk')] \int d^3k_3 [k_3^y t_0'^3 t_1^3/k_3 - k'^y t_0^3 t_1'^3/k'_3 + v^x t_1^3 t_1'^3/(k'_3 k_3)], \tag{9}$$

where $k = |\vec{k}|$, $k' = |\vec{k}'|$, $t_i = t_i(k)$, $t'_i = t_i(k')$, $i = 0, 1$ and $C_u^{-+} = -16/9$ $(C_d^{-+} = 4/9)$, $C_u^{+-} = -4/9$ $(C_d^{-+} = -8/9)$ for $\mathcal{Q} = u(d)$. Let us recall that in Ref. [14] only the first term of Eq. (7), proportional to $C_{\mathcal{Q}}^{-+}$, contributes to $f_{1T}^{\perp \mathcal{Q}}$. It is therefore found that $f_{1T}^{\perp u} = -4 f_{1T}^{\perp d}$. Notice that, in order to calculate $f_{1T}^{\perp \mathcal{Q}}$, one is using a two-body operator associated with FSI and therefore one should not expect a proportionality between the u and d results. On the contrary, in the calculation of conventional PDs, in any SU(6) model calculation, the used operators are of one-body type and therefore the results turn out to be proportional [17].

Numerical results are shown in Figs. 2 and 3 for the first moment of $f_{1T}^{\perp \mathcal{Q}}$, i.e.

$$f_{1T}^{\perp(1)\mathcal{Q}}(x) = \int d^2\vec{k}_T \frac{k_T^2}{2M^2} f_{1T}^{\perp \mathcal{Q}}(x, k_T). \tag{10}$$

In Fig. 2 the dashed curves are the ones obtained in Ref. [14] (cf. Fig. 4 in Ref. [14] adapted to the Trento convention [19], i.e., reduced by a factor of 2). The obtained value for the Burkardt SR Eq. (1) turns out to be 8.73 MeV. To have an estimate of the quality of the agreement of this result with the SR, we consider the ratio $r = (\langle k_x^d \rangle + \langle k_x^u \rangle)/(\langle k_x^d \rangle - \langle k_x^u \rangle)$, obtaining $r \simeq 0.60$, i.e., the Burkardt SR seems to be violated by 60%. The full curve in Fig. 2 is the result of the present calculation. Clearly, the d contribution becomes comparable in magnitude to the u one. The obtained value for the Burkardt SR is -0.78 MeV and $r \simeq 0.05$, i.e., it is only violated by 5%. These results are comparable in quality to those obtained

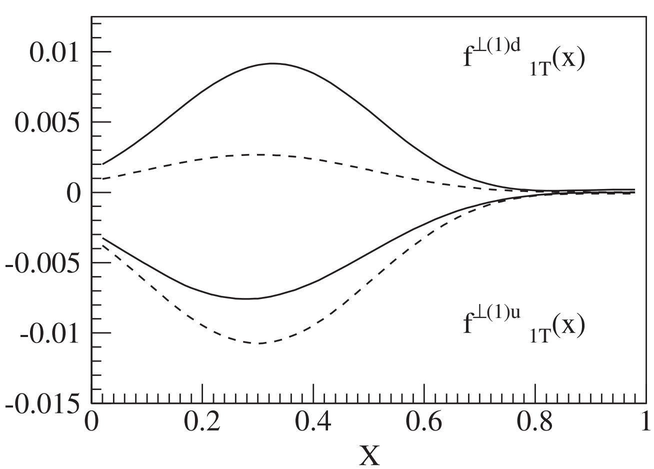

FIG. 2. The quantity $f_{1T}^{\perp(1)q}(x)$, Eq. (10), for the u and d flavour. The dashed curves are the results of the approach of Ref. [14], the full ones those obtained here.

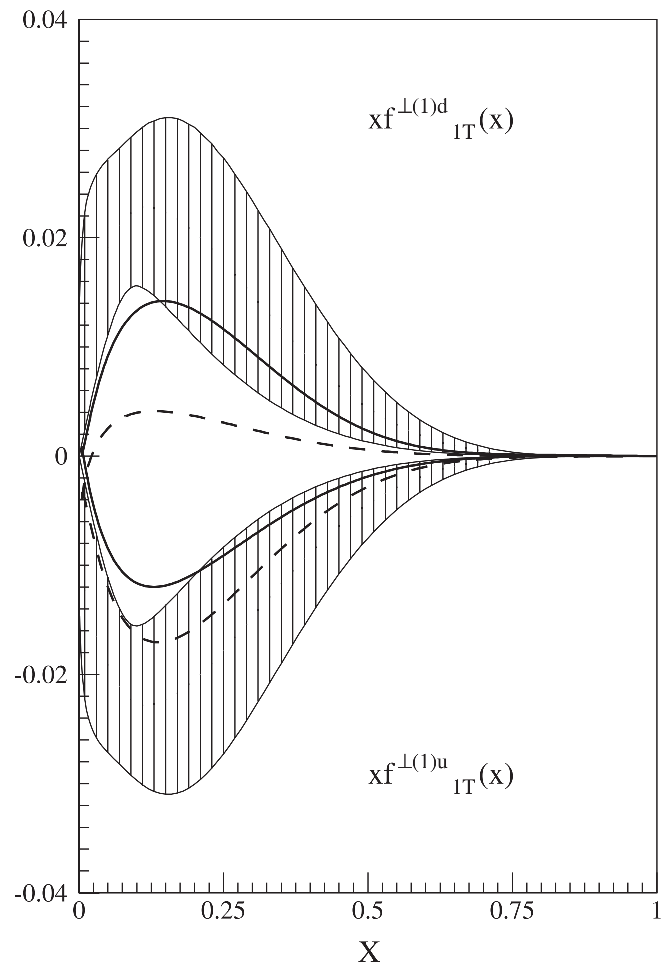

FIG. 3. The same as in Fig. 2, after NLO evolution (see text). The patterned area represents the $1-\sigma$ range of the best fit of the HERMES data proposed in Ref. [11].

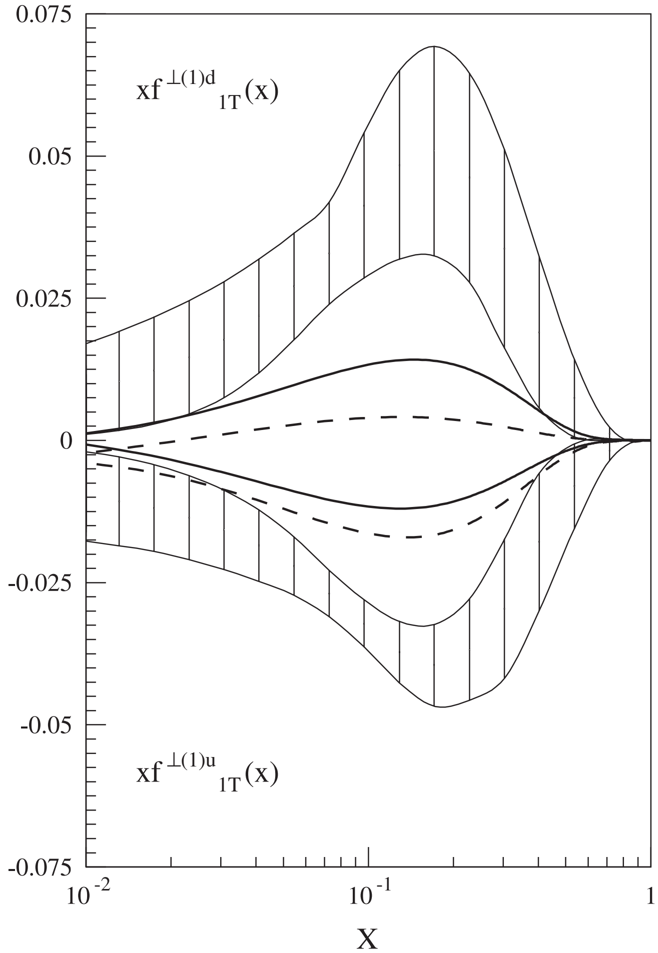

FIG. 4. The same as in Fig. 3, but comparing with the parametrization of the data proposed in Ref. [9] (patterned area).

for the CQM [16], restoring the approximate agreement between the two schemes.

In order to compare the results with the data, one should perform a QCD evolution from the experimental scale, which is, for example, for the HERMES data, $Q^2 = 2.5\ \mathrm{GeV}^2$ [22]. Unfortunately, the evolution of TMDs is still to be understood, although recent developments can be found in Ref. [23]. In order to have an indication of the effect of the evolution, we evolve at NLO the model results assuming, for the moments of the Sivers function, Eq. (10), the same anomalous dimensions of the unpolarized PDFs, as we did in Ref. [16] for the CQM calculation. The parameters of the evolution have been fixed in order to have a fraction $\simeq 0.55$ of the momentum carried by the valence quarks at $0.34\ \mathrm{GeV}^2$, as in typical parametrizations of PDs, starting from a scale of $\mu_0^2 \simeq 0.1\ \mathrm{GeV}^2$ with only valence quarks. The results in Figs. 3 and 4 show an impressive improvement of the agreement with data once the full contribution of Eq. (7) is taken into account. The data are described rather well for both flavors. Comparing this encouraging outcome with that of Ref. [16], one can notice that the Burkardt SR is better fulfilled in the CQM. Most probably this has to do with the fact that the Burkardt SR is associated with transverse momentum conservation and, in the MIT bag model, the proton wave function is not an exact momentum eigenstate. In closing, we can say that, for the first time, it has been established that correct model calculations provide phenomenological successful interpretations of the Sivers function, which are consistent with each other.

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