

Angular momentum decomposition for an electron

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We calculate the orbital angular momentum of the “quark” in the scalar diquark model as well as that of the electron in QED (to order α). We compare the orbital angular momentum obtained from the Jaffe-Manohar decomposition to that obtained from the Ji relation and estimate the importance of the vector potential in the definition of orbital angular momentum.

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I. INTRODUCTION

While the total angular momentum of an isolated system is uniquely defined, ambiguities arise when decomposing the total angular momentum of an interacting multiconstituent system into contributions from various constituents. Moreover, in a gauge theory, switching the gauge may result in shuffling angular momentum between matter and gauge degrees of freedom. In the context of nucleon structure, this gives rise to subtleties in defining these quantities which may have important consequences for the numerical value of the contribution from spin and orbital degrees of freedom to the nucleon spin $s = \frac{1}{2}$.

In the context of hadron structure, it is natural to perform a decomposition of the \hat{z} component of the angular momentum as the \hat{z} component of the quark spin has a partonic interpretation as a difference between parton densities. Indeed, in the light-cone framework, Jaffe and Manohar proposed a decomposition of the form [1]

$$\frac{1}{2} = \frac{1}{2} \sum_q \Delta q + \sum_q \mathcal{L}_q^z + \frac{1}{2} \Delta G + \mathcal{L}_g^z, \quad (1)$$

whose terms are defined as matrix elements of the corresponding terms in the +12 component of the angular momentum tensor

$$M^{+12} = \frac{1}{2} \sum_q q_+^\dagger \gamma_5 q_+ + \sum_q q_+^\dagger (\vec{r} \times i\vec{\partial})^z q_+ + \varepsilon^{+ij} \text{Tr} F^{+i} A^j + 2 \text{Tr} F^{+j} (\vec{r} \times i\vec{\partial})^z A^j. \quad (2)$$

The first and third term in (1) and (2) are the “intrinsic” contributions (no factor of $\vec{r} \times$) to the nucleon’s angular momentum $J^z = +\frac{1}{2}$ and have a physical interpretation as quark and gluon spin, respectively, while the second and fourth term can be identified with the quark/gluon orbital angular momentum (OAM). Here $q_+ \equiv \frac{1}{2} \gamma^- \gamma^+ q$ is the dynamical component of the quark field operators, and light-cone gauge $A^+ \equiv A^0 + A^z = 0$ is implied. The residual gauge invariance is fixed by imposing antiperiodic

boundary conditions $\mathbf{A}_\perp(\mathbf{x}_\perp, \infty^-) = -\mathbf{A}_\perp(\mathbf{x}_\perp, -\infty^-)$ on the transverse components of the vector potential.

Since the quark spin term does not contain any derivatives, its manifest gauge invariance is evident. However, ΔG is also gauge invariant, as it is experimentally accessible. In gauges other than light-cone gauge, it is defined through a nonlocal operator [2]. The net parton OAM

$$\mathcal{L}^z = \sum_q \mathcal{L}_q^z + \mathcal{L}_g^z = \frac{1}{2} - \frac{1}{2} \sum_q \Delta q - \frac{1}{2} \Delta G \quad (3)$$

can be related to differences between observables and is thus also obviously gauge invariant. However, similar to the case of ΔG , a manifestly gauge invariant operator defining \mathcal{L}^z would be nonlocal, reducing to a local expression in light-cone gauge only. For the individual OAMs the situation is more subtle and a detailed discussion can be found in Ref. [2].

An alternative decomposition [3] of the nucleon spin

$$\frac{1}{2} = \frac{1}{2} \sum_q \Delta q + \sum_q L_q^z + J_g^z \quad (4)$$

into quark spin, quark OAM, and gluon (total) angular momentum is obtained from the expectation value of

$$M^{0xy} = \sum_q \frac{1}{2} q^\dagger \Sigma^z q + \sum_q q^\dagger (\vec{r} \times i\vec{D})^z q + [\vec{r} \times (\vec{E} \times \vec{B})]^z \quad (5)$$

with $i\vec{D} = i\vec{\partial} - g\vec{A}$. Its main advantages are that each term can be expressed as the expectation value of a manifestly gauge invariant local operator and that the quark total angular momentum $J_q^z = \frac{1}{2} \Delta q + L_q^z$ can be related to generalized parton distributions (GPDs), using [3]

$$J_q^z = \frac{1}{2} \int_0^1 dx x [q(x) + E_q(x, 0, 0)], \quad (6)$$

and can thus be measured in deeply virtual Compton scattering (DVCS) or calculated in lattice gauge theory. Its main disadvantage is that both quark OAM L_q^z as well as gluon angular momentum J_g^z contain interactions through the vector potential in the gauge covariant derivative, which complicates their physical interpretation.

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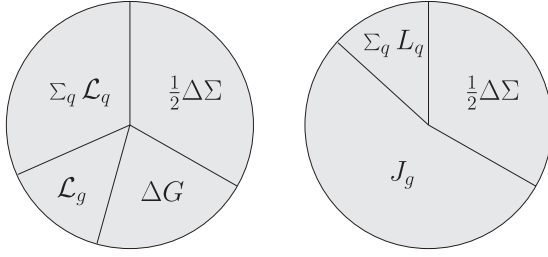


FIG. 1. Schematic comparison between the two decompositions (1) and (4) of the nucleon spin. In general, only $\frac{1}{2}\Delta\Sigma \equiv \frac{1}{2}\Sigma_q\Delta q$ is common to both decompositions.

Since the expectation value of $\bar{q}\gamma^z\Sigma^z q$ vanishes for a parity eigenstate, one can replace $q^\dagger\Sigma^z q \rightarrow \bar{q}\gamma^+\Sigma^z q = q^\dagger\gamma_5 q_+$, i.e. the Δq are common to both decompositions. This is not the case for all the other terms. For example, the angular momenta in these decompositions (1) and (4) are not defined through matrix elements of the same operator and one should not expect them to have the same numerical value (Fig. 1). However, no intuition exists as to how large that difference is.

In the matrix element defining L_q^z , one may make the replacement

$$q^\dagger(\vec{r} \times i\vec{D})^z q = \bar{q}\gamma^0(\vec{r} \times i\vec{D})^z q \rightarrow \bar{q}(\gamma^0 + \gamma^z)(\vec{r} \times i\vec{D})^z q = q^\dagger_+(\vec{r} \times i\vec{D})^z q_+, \quad (7)$$

provided that the expectation value is taken in a parity eigenstate. While the Dirac structure of the operator on the right-hand side of (7) is now the same as that appearing in (2), Eq. (7) still contains the transverse component of the vector potential through the gauge covariant derivative, and therefore, even in light-cone gauge, \mathcal{L}_q^z and L_q^z differ by the expectation value of $q^\dagger_+(\vec{r} \times g\vec{A})^z q_+$. While it has long been realized that in general $\mathcal{L}_q^z \neq L_q^z$, little intuition exists regarding the expected size, or even sign, of the difference between these two definitions for the quark orbital angular momentum. The main purpose of this paper is to address this issue first in the context of a scalar diquark model and then in QED.

II. ORBITAL ANGULAR MOMENTUM IN THE SCALAR DIQUARK MODEL

In a two particle system we introduce center of momentum and relative \perp coordinates as

$$\mathbf{P}_\perp \equiv \mathbf{p}_{1\perp} + \mathbf{p}_{2\perp} \quad (8)$$

$$\mathbf{R}_\perp \equiv x_1\mathbf{r}_{1\perp} + x_2\mathbf{r}_{2\perp} = x\mathbf{r}_{1\perp} + (1-x)\mathbf{r}_{2\perp}$$

$$\mathbf{k}_\perp \equiv x_2\mathbf{p}_{1\perp} - x_1\mathbf{p}_{2\perp} = (1-x)\mathbf{p}_{1\perp} - x\mathbf{p}_{2\perp}$$

$$\mathbf{r}_\perp \equiv \mathbf{r}_{1\perp} - \mathbf{r}_{2\perp} \quad (9)$$

where $x_1 = x$ and $x_2 = 1 - x$ are the momentum fractions

carried by the active quark and the spectator, respectively. For a state with $\mathbf{P}_\perp = 0$, this implies $\mathbf{p}_{1\perp} = -\mathbf{p}_{2\perp} = \mathbf{k}_\perp$, allowing one to replace the OAM operator for particle 1 by $(1-x)$ times the relative OAM in such a state [4]

$$\begin{aligned} \mathcal{L}_1^z &= \mathbf{r}_{1\perp} \times \mathbf{p}_{1\perp} = [\mathbf{R}_\perp + (1-x)\mathbf{r}_\perp] \times \mathbf{k}_\perp \\ &\rightarrow (1-x)\mathbf{r}_\perp \times \mathbf{k}_\perp = (1-x)\mathcal{L}^z. \end{aligned} \quad (10)$$

Here we used that the internal wave function of a bound state satisfies $\langle \mathbf{k}_\perp \rangle = 0$. Likewise one finds that the expectation value of \mathcal{L}_2^z can be replaced by the expectation value of $x\mathcal{L}^z$.

We now use the above decompositions (1) and (4) to calculate the OAM of the ‘‘quark’’ in the scalar diquark model, where the two particle Fock space amplitudes read [5]

$$\begin{aligned} \psi^\uparrow_{+(1/2)}(x, \mathbf{k}_\perp) &= \left(M + \frac{m}{x}\right)\phi(x, \mathbf{k}_\perp^2) \\ \psi^\uparrow_{-(1/2)}(x, \mathbf{k}_\perp) &= -\frac{k^1 + ik^2}{x}\phi(x, \mathbf{k}_\perp^2) \end{aligned} \quad (11)$$

with $\phi = \frac{g\sqrt{1-x}}{M^2 - \frac{k_\perp^2 + m^2}{x} - \frac{k_\perp^2 + \lambda^2}{1-x}}$. Here g is the Yukawa coupling

and $M/m/\lambda$ are the masses of the ‘‘nucleon’’/‘‘quark’’/diquark, respectively. Furthermore x is the momentum fraction carried by the quark and $\mathbf{k}_\perp \equiv \mathbf{k}_{\perp e} - \mathbf{k}_{\perp \gamma}$ represents the relative \perp momentum. The upper wave function index \uparrow refers to the helicity of the ‘‘nucleon’’ and the lower index to that of the quark. With the light-cone wave functions available (11), it is straightforward to compute either \mathcal{L}_q^z or J_q^z , and hence L_q^z from the Ji relation.

This yields for the orbital angular momentum \mathcal{L}_q^z of the ‘‘quark’’

$$\mathcal{L}_q^z = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} (1-x) |\psi^\uparrow_{-(1/2)}|^2. \quad (12)$$

Alternatively one may consider the OAM as obtained from GPDs using the Ji relation (6) as

$$L_q^z = \frac{1}{2} \int_0^1 dx [xq(x) + xE(x, 0, 0) - \Delta q(x)], \quad (13)$$

where

$$\begin{aligned} xq(x) &= Z\delta(1-x) + x \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} [|\psi^\uparrow_{+(1/2)}|^2 + |\psi^\uparrow_{-(1/2)}|^2] \\ \Delta q(x) &= Z\delta(1-x) + \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} [|\psi^\uparrow_{+(1/2)}|^2 - |\psi^\uparrow_{-(1/2)}|^2] \\ xE(x, 0, 0) &= 2Mg^2x \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \\ &\quad \times \frac{(1-x)^2(xm + M)}{[x(1-x)M^2 - (1-x)m^2 - x\lambda^2 - \mathbf{k}_\perp^2]^2} \\ &= \frac{Mg^2}{8\pi^2} \frac{x(1-x)^2(xm + M)}{-x(1-x)M^2 + (1-x)m^2 + x\lambda^2}. \end{aligned} \quad (14)$$

As one may have expected, the wave function renormalization constant

$$Z = 1 - \int_0^1 dx \int \frac{d^2 \mathbf{k}_\perp}{16\pi^3} [|\psi_{+(1/2)}^\dagger|^2 + |\psi_{-(1/2)}^\dagger|^2] \quad (15)$$

cancels in L_q^z , yielding

$$L_q^z = \frac{1}{2} \int_0^1 dx \int \frac{d^2 \mathbf{k}_\perp}{16\pi^3} [(x-1)|\psi_{+(1/2)}^\dagger|^2 + (x+1)|\psi_{-(1/2)}^\dagger|^2] + \frac{1}{2} \int_0^1 dx x E(x, 0, 0). \quad (16)$$

Since some of the above \mathbf{k}_\perp integrals diverge, a manifestly Lorentz invariant Pauli-Villars regularization (subtraction with heavy scalar $\lambda^2 \rightarrow \Lambda^2$) is always understood. Evaluating the above integrals is tedious, but straightforward, and one finds

$$L_q^z = L_e^z \quad (17)$$

as was expected since L_q^z in the scalar diquark model does not contain a gauge field term. However, there is no such identity for the OAM distribution. The distribution of the \hat{z} component of the OAM $\mathcal{L}_q^z(x)$ is defined as in (12), but without the x -integration. A comparison with (13) without x -integration, i.e. comparing $\mathcal{L}_q^z(x)$ with $L_q^z(x) \equiv \frac{1}{2} \times [xq(x) + xE(x, 0, 0) - \Delta q(x)]$ (Fig. 2) shows that, even in a model without gauge fields, $L_q^z(x)$ cannot be identified with the x -distribution of \mathcal{L}_q^z for a longitudinally polarized nucleon [6]. Of course, the difference between the unintegrated orbital angular momenta is expected to become less significant under QCD evolution, and may almost vanish at the experimental scale. However, in the following section we will explicitly demonstrate that in QED the

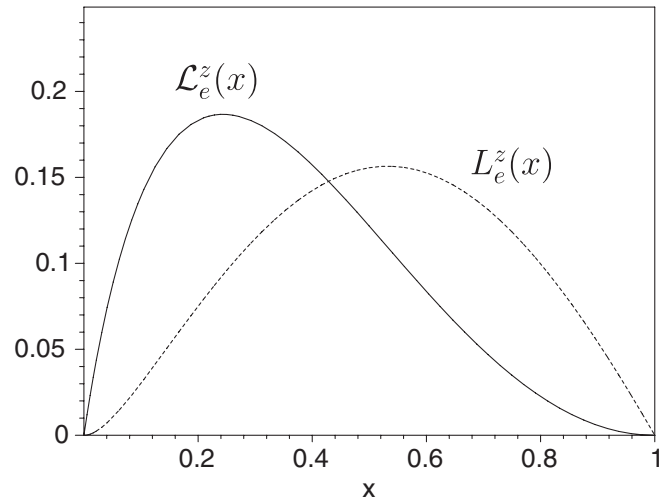


FIG. 2. x distribution of the orbital angular momentum $\mathcal{L}_q^z(x)$ (full line) compared to $L_q^z(x)$ from the unintegrated Ji relation (dotted line) in the scalar diquark model for parameters $\Lambda^2 = 10m^2 = 10\lambda^2$. Both in units of $\frac{g^2}{16\pi^2}$.

difference between the integrated orbital angular momenta is nonzero, $\mathcal{L}_e^z - L_e^z \neq 0$, and that difference does not change under leading order QCD evolution.

III. ORBITAL ANGULAR MOMENTUM IN QED

In QED, there are four polarization states in the $e\gamma$ Fock component. To lowest order, the respective Fock space amplitudes for a dressed electron with $J^z = +\frac{1}{2}$ read

$$\begin{aligned} \Psi_{+(1/2)+1}^\dagger(x, \mathbf{k}_\perp) &= \frac{k^1 - ik^2}{x(1-x)} \phi(x, \mathbf{k}_\perp^2) \\ \Psi_{+(1/2)-1}^\dagger(x, \mathbf{k}_\perp) &= -\frac{k^1 + ik^2}{1-x} \phi(x, \mathbf{k}_\perp^2) \\ \Psi_{-(1/2)+1}^\dagger(x, \mathbf{k}_\perp) &= \left(\frac{m}{x} - m\right) \phi(x, \mathbf{k}_\perp^2) \\ \Psi_{-(1/2)-1}^\dagger(x, \mathbf{k}_\perp) &= 0 \end{aligned} \quad (18)$$

with $\phi(x, \mathbf{k}_\perp^2) = \frac{\sqrt{2}}{\sqrt{1-x}} \frac{e}{M^2 - \frac{k_\perp^2 + m^2}{x} - \frac{k_\perp^2 + \lambda^2}{1-x}}$.

Using these light-cone wave functions, it is again straightforward to calculate the orbital angular momentum (10) of the electron in the Jaffe-Manohar [1] decomposition

$$\begin{aligned} \mathcal{L}_e^z &= \int_0^1 dx \int \frac{d^2 \mathbf{k}_\perp}{16\pi^3} (1-x) [|\Psi_{+(1/2)-1}^\dagger(x, \mathbf{k}_\perp)|^2 \\ &\quad - |\Psi_{+(1/2)+1}^\dagger(x, \mathbf{k}_\perp)|^2] \end{aligned} \quad (19)$$

Likewise, it is straightforward to evaluate the OAM using the Ji relation

$$L_e^z = \frac{1}{2} \int_0^1 dx [xq_e(x) + xE_e(x, 0, 0) - \Delta q_e(x)] \quad (20)$$

with [5]

$$\begin{aligned} xq_e(x) &= Z\delta(1-x) + x \int \frac{d^2 \mathbf{k}_\perp}{16\pi^3} [|\psi_{+(1/2),+1}^\dagger|^2 \\ &\quad + |\psi_{+(1/2),-1}^\dagger|^2 + |\psi_{-(1/2),+1}^\dagger|^2] \\ \Delta q_e(x) &= Z\delta(1-x) + \int \frac{d^2 \mathbf{k}_\perp}{16\pi^3} [|\psi_{+(1/2),+1}^\dagger|^2 \\ &\quad + |\psi_{+(1/2),-1}^\dagger|^2 - |\psi_{-(1/2),+1}^\dagger|^2] \\ xE_e(x, 0, 0) &= 4m^2 e^2 \int \frac{d^2 \mathbf{k}_\perp}{16\pi^3} \frac{x^2(1-x)^2}{[m^2(1-x)^2 + \lambda^2 x + \mathbf{k}_\perp^2]^2} \\ &= \frac{m^2 e^2}{4\pi^2} \frac{x^2(1-x)^2}{m^2(1-x)^2 + \lambda^2 x}. \end{aligned} \quad (21)$$

Again the wave function renormalization constant

$$Z = 1 - \int_0^1 dx \int \frac{d^2 \mathbf{k}_\perp}{16\pi^3} [|\psi_{+(1/2),+1}^\dagger|^2 + |\psi_{+(1/2),-1}^\dagger|^2 + |\psi_{-(1/2),+1}^\dagger|^2] \quad (22)$$

drops out in (20), yielding

$$\begin{aligned} L_e^z &= \frac{1}{2} \int_0^1 dx \int \frac{d^2 \mathbf{k}_\perp}{16\pi^3} [(x-1)|\psi_{+(1/2),+1}^\dagger|^2 \\ &\quad + (x-1)|\psi_{+(1/2),-1}^\dagger|^2 + (x+1)|\psi_{-(1/2),+1}^\dagger|^2] \\ &\quad + \frac{1}{2} \int_0^1 dx x E_e(x, 0, 0). \end{aligned} \quad (23)$$

Because of the divergent \mathbf{k}_\perp integrals a Pauli-Villars subtraction with $\lambda^2 \rightarrow \Lambda^2$ is understood and $\lambda^2 \rightarrow 0$ at the end of the calculation, while $\Lambda^2 \gg m^2$.

The evaluation of the above integrals is again straightforward, yielding

$$\begin{aligned} \mathcal{L}_e^z &= -\frac{\alpha}{2\pi} \int_0^1 dx (1-x^2) \log \frac{(1-x)^2 m^2 + x\Lambda^2}{(1-x)^2 m^2 + x\lambda^2} \\ &\xrightarrow[\lambda \rightarrow 0]{\Lambda \rightarrow \infty} -\frac{\alpha}{4\pi} \left[\frac{4}{3} \log \frac{\Lambda^2}{m^2} - \frac{2}{9} \right] \end{aligned} \quad (24)$$

and

$$\begin{aligned} L_e^z &= -\frac{\alpha}{4\pi} \int_0^1 dx (1+x^2) \left[\log \frac{(1-x)^2 m^2 + x\Lambda^2}{(1-x)^2 m^2 + x\lambda^2} \right. \\ &\quad \left. - \frac{(1-x)^2 m^2}{(1-x)^2 m^2 + x\lambda^2} + \frac{(1-x)^2 m^2}{(1-x)^2 m^2 + x\Lambda^2} \right] \\ &\xrightarrow[\lambda \rightarrow 0]{\Lambda \rightarrow \infty} -\frac{\alpha}{4\pi} \left[\frac{4}{3} \log \frac{\Lambda^2}{m^2} + \frac{7}{9} \right]. \end{aligned} \quad (25)$$

Both \mathcal{L}_e^z and L_e^z are negative, regardless of the value of Λ^2 (as long as $\Lambda^2 > \lambda^2$). In the case of \mathcal{L}_e^z the physical reason is a preference for the emission of photons/gluons with the spin parallel (as compared to antiparallel) to the original quark spin [7]—resulting more likely in a state with negative OAM. In fact, when $x \rightarrow 0$, this preference reflects the more general principle of helicity retention [8], which favors the lead parton (i.e. the parton carrying most of the momentum) to carry a spin as close as possible to that of the parent. It is also encoded in the evolution equations derived in Ref. [6]. The divergent parts of \mathcal{L}_e^z and L_e^z are the same so that their difference is UV finite (Fig. 3)

$$\mathcal{L}_e^z - L_e^z \xrightarrow[\lambda \rightarrow 0]{\Lambda \rightarrow \infty} \frac{\alpha}{4\pi}. \quad (26)$$

Applying these results to a (massive) quark with $J^z = +\frac{1}{2}$ yields to $\mathcal{O}(\alpha_s)$

$$\mathcal{L}_q^z - L_q^z = \frac{\alpha_s}{3\pi}. \quad (27)$$

In QCD, the gluon spin is experimentally accessible, but the gluon OAM \mathcal{L}_g^z is not. On the other hand, the gluon (total) angular momentum J_g^z appearing in the Ji decomposition is accessible, either indirectly (by subtraction, using quark GPDs from lattice QCD and/or DVCS), or directly, using by calculating gluon GPDs on a lattice and/

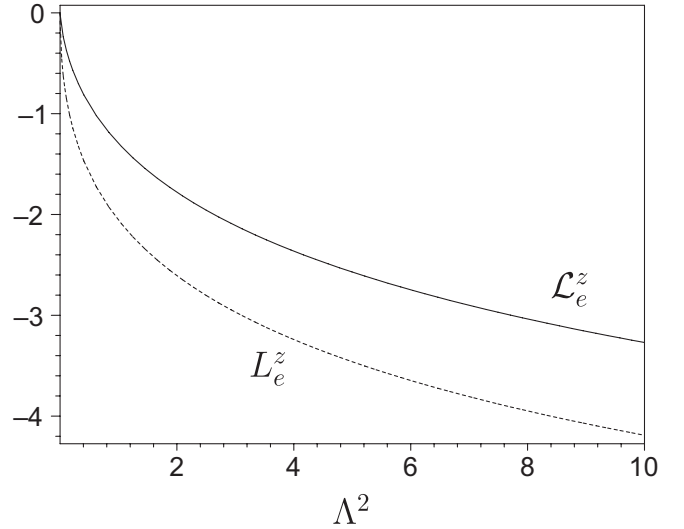


FIG. 3. Cutoff dependence of \mathcal{L}_e^z (full line) and L_e^z (dotted line). Both in units of $\frac{\alpha}{4\pi}$.

or deeply virtual J/ψ production. Even though $\frac{1}{2}\Delta G$ and J_g^z belong to two incommensurable decompositions of the nucleon spin, one may thus be tempted to consider the difference between these two quantities, hoping to learn something about gluon OAM. Subtracting (1) from (4), it is straightforward to convince oneself that

$$J_g^z - \frac{1}{2}\Delta G = \mathcal{L}_g^z + \sum_q (\mathcal{L}_q^z - L_q^z), \quad (28)$$

i.e. numerically $J_g^z - \frac{1}{2}\Delta G$ differs from \mathcal{L}_g^z by the same amount that $\sum_q \mathcal{L}_q^z$ differs from $\sum_q L_q^z$. In our QED example, with

$$\begin{aligned} \Delta\gamma &= \int_0^1 dx \int \frac{d^2 \mathbf{k}_\perp}{16\pi^3} [|\psi_{+(1/2),+1}^\dagger|^2 - |\psi_{+(1/2),-1}^\dagger|^2 \\ &\quad + |\psi_{-(1/2),+1}^\dagger|^2] \end{aligned} \quad (29)$$

being the photon spin contribution, one thus finds (for $\lambda \rightarrow 0$, $\Lambda \rightarrow \infty$)

$$J_\gamma^z - \frac{1}{2}\Delta\gamma = \mathcal{L}_\gamma^z + \frac{\alpha}{4\pi}. \quad (30)$$

As was the case in (26), $\frac{\alpha}{4\pi}$ appears to be a small correction, but one needs to keep in mind that for an electron J_γ^z , $\Delta\gamma$, and \mathcal{L}_γ^z are also only of order α .

IV. DISCUSSION AND SUMMARY

We have studied both the Jaffe/Manohar, as well as the Ji decomposition of angular momentum in the scalar diquark model, as well as for an electron in QED to order α . As expected, both decompositions yield the same numerical value for the fermion OAM in the scalar diquark model, but not in QED. This calculation demonstrates explicitly that the presence of the vector potential in the manifestly gauge invariant local operator for the OAM does indeed contrib-

ute significantly to the numerical value of the OAM. While the numerical value for difference between the fermion OAM in these two decompositions in QED appears to be small ($\frac{\alpha}{4\pi}$), one should keep in mind that the OAM itself is of the same order α . Moreover, applying the same calculation to a massive quark in QCD yields a contribution from the vector potential term to the angular momentum of the quark of $\frac{\alpha_s}{3\pi}$.

The sign of the contribution to the angular momentum arising from the vector potential is also significant in light of recent lattice results for the contributions from the u and d quark OAM to the nucleon spin [9], yielding $L_u^z < 0$ and $L_d^z > 0$. The signs of the lattice results are thus exactly opposite to what one would have expected on the basis of relativistic quark models, such as the bag model, where the OAM arises from the lower Dirac component and its expectation value is thus positively correlated to the expectation value of the quark spin. While the lattice results still neglect insertions of the operator into disconnected quark loops, this does not affect $L_u^z - L_d^z$, and the sign of that difference should be reliable. Virtual gluons emitted from a quark tend to have their helicity aligned with that of

the parent quark, which implies that the quark acquires OAM in the direction opposite to its spin from virtual gluon emission [7]. This observation is encoded in the evolution equations derived in Ref. [6], and can also be read off from the cutoff dependence of the quark OAM depicted in Fig. 3. The negative OAM resulting from QCD evolution has been used in Ref. [10] to numerically explain the apparent discrepancy between the lattice result [9] and quark-model-based intuition. Our main result, Eqs. (26) and (27) adds to that effect in the sense that the vector potential also adds a contribution to the OAM that is in the opposite direction from the quark spin. Such a shift would imply $\mathcal{L}_u^z > L_u^z$ and $\mathcal{L}_d^z < L_d^z$, moving \mathcal{L}_q^z closer to the quark-model-based intuitive expectation than L_q^z .

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