PHYSICAL REVIEW D **79,** 067503 (2009)

Internal structure of a Maxwell-Gauss-Bonnet black hole

S. Alexeyev*
Sternberg Astronomical Institute, Lomonosov Moscow State University, Universitetsky Prospekt, 13, Moscow 119991, Russia

A. Barrau†
Laboratoire de Physique Subatomique et de Cosmologie, UJF-INPG-CNRS, 53, avenue des Martyrs, 38026 Grenoble cedex, France and Institut des Hautes Etudes Scientifiques, 35, route de Chartres, 91440, Bures-sur-Yvette, France

K. A. Rannu‡
Sternberg Astronomical Institute, Lomonosov Moscow State University, Universitetsky Prospekt, 13, Moscow 119991, Russia
(Received 19 January 2009; published 26 March 2009)

The influence of the Maxwell field on a static, asymptotically flat, and spherically-symmetric Gauss-Bonnet black hole is considered. Numerical computations suggest that if the charge increases beyond a critical value, the inner determinant singularity is replaced by an inner singular horizon.

DOI: 10.1103/PhysRevD.79.067503 PACS numbers: 04.70.Dy, 04.50.Kd

The study of new physical effects induced by the four-dimensional, low-energy effective string action with second order curvature correction has been an important topic in black hole physics during the last three decades (see, e.g., [1]). The internal structure of black holes described by the action

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}[m_{\rm pl}^2(-R + 2\partial_\mu\phi\partial^\mu\phi) - e^{-2\phi}F_{\mu\nu}F^{\mu\nu} + \lambda e^{-2\phi}S_{\rm GB}], \tag{1}$$

where $m_{\rm pl}$ is the Plank mass, ϕ is the dilaton field, R is the scalar curvature, $S_{\rm GB} = R_{ijkl}R^{ijkl} - 4R_{ij}R^{ij} + R^2$ is the Gauss-Bonnet term, $F_{\mu\nu}F^{\mu\nu}$ is the Maxwell field, and λ is the string coupling constant, has been investigated in [2]. The influence of the magnetic charge of the black hole on the behavior of the metric functions was considered and it was shown that there exists a ‘‘critical value’’ of the charge beyond which the influence of the Maxwell term becomes more important than the Gauss-Bonnet one. The inner determinant singularity at $r = r_s$ is then replaced by a smooth local minimum.

In this paper, we focus on the behavior of the curvature invariant $R_{ijkl}R^{ijkl}$ near this critical point and in the vicinity of the main singularity at $r = r_x$.

Considering a static, asymptotically flat and spherically-symmetric black hole solution, we focus on the following metric:

$$ds^2 = \Delta dt^2 - \frac{\sigma^2}{\Delta}dr^2 - f^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{2}$$

where Δ, σ, and f are functions that depend on the radial coordinate r only. To simplify the problem, only the magnetic charge will be taken into account. Therefore, for the Maxwell tensor $F_{\mu\nu}$, one can use the ansatz $F = q\sin\theta d\theta \wedge d\varphi$ [3]. The corresponding field equations in the Garfincle-Horowitz-Strominger (GHS) gauge ($\sigma(r) = 1$) are as follows:

$$m_{\rm pl}^2[ff'' + f^2(\phi')^2] + 4e^{-2\phi}\lambda[\phi'' - 2(\phi')^2]\Delta(f')^2 - 1 + 4e^{-2\phi}\lambda\phi' 2\Delta f' f'' = 0, \tag{3}$$

$$m_{\rm pl}^2[1 + \Delta f^2(\phi')^2 - \Delta' ff' - \Delta(f')^2] + 4e^{-2\phi}\lambda\Delta'\phi'[1 - 3\Delta(f')^2] - -e^{-2\phi}q^2f^{-2} = 0, \tag{4}$$

$$m_{\rm pl}^2[\Delta'' f + 2\Delta' f' + 2\Delta f'' + 2\Delta f(\phi')^2] + 4e^{-2\phi}\lambda[\phi'' - 2(\phi')^2]2\Delta\Delta' f' + +4e^{-2\phi}\lambda\phi' 2[(\Delta')^2 f' + \Delta\Delta'' f' + \Delta\Delta' f''] - 2e^{-2\phi}q^2f^{-3} = 0, \tag{5}$$

$$-2m_{\rm pl}^2[\Delta' f^2\phi' + 2\Delta ff'\phi' + \Delta f^2\phi''] + 4e^{-2\phi}\lambda[(\Delta')^2(f')^2 + \Delta\Delta''(f')^2 + +2\Delta\Delta' f' f'' - \Delta''] - 2e^{-2\phi}q^2f^{-2} = 0. \tag{6}$$

The behavior of the metric functions and of the dilatonic field near the horizon are described by a simple Taylor expansion [4]

$$\begin{aligned} \Delta &= d_1x + d_2x^2 + O(x^2),\\ f &= f_0 + f_1x + f_2x^2 + O(x^2),\\ e^{-2\phi} &= e^{-2\phi_0} + \phi_1x + \phi_2x^2 + O(x^2), \end{aligned} \tag{7}$$

where ($x = r - r_h$, $\ll 1$).

Without the Gauss-Bonnet term, the Gibbons-Maeda–Garfinkle-Horowitz-Strominger solution (GM-GHS) [3] should be recovered as the basic solution of the Einstein equations with the dilaton and Maxwell terms.

*alexeyev@sai.msu.ru
†aurelien.barrau@cern.ch
‡melruin1986@gmail.com

This solution is given by

$$ds^2 = \left(1 - \frac{2M}{r}\right)dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r\left(r - \frac{q^2 \exp(2\phi_0)}{M}\right)d\Omega, \quad (8)$$
$$\exp(-2\phi) = \exp(-2\phi_0) - \frac{q^2}{Mr},$$

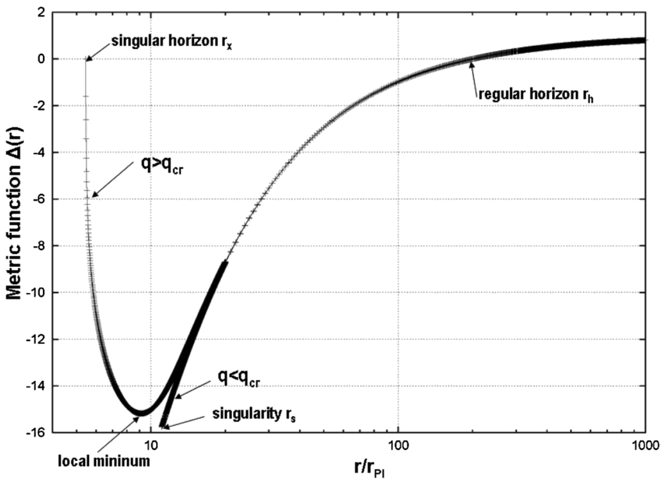

FIG. 1. Metric function Δ as a function of the radial coordinate r for $q = 21.50 < q_{\rm cr}$ (left curve) and $q = 24.81 > q_{\rm cr}$ (right curve) when $r_h = 200.0$ Planck units.

TABLE I. Black hole critical charge $q_{\rm cr}$ as a function of the mass M.

M	5.0	10.0	20.0	50.0	60.0	70.0	90.0	100.0
$q_{\rm cr}$	4.53	7.20	11.42	21.03	23.75	26.32	31.13	33.39

where M stands for the black hole mass. In the limit $\lambda \to 0$, the solution of Eqs. (3)–(6) at infinity should coincide with Eq. (8).

In order to determine the two metric functions and the dilatonic field, three equations are required. Among the four Eqs. (3)–(6), only Eqs. (3), (5), and (6), which contain the second derivative of the metric functions and the dilaton, are used. In contrast, Eq. (4), which contains the first derivative only, is considered as a constraint to check the solution.

To solve the system (3)–(6), the equations are rewritten using $E = e^{-2\phi}$ instead of the dilaton itself. Furthermore, the case $\lambda = 1$ is considered. In the chosen metric gauge, the squared Riemann tensor is given by

$$R_{ijkl}R^{ijkl} = \Delta''^2 + 4\Delta'^2 \frac{f'^2}{f^2} + 8\Delta^2 \frac{f''^2}{f^2} + 8\Delta\Delta' \frac{f' f''}{f^2} + \frac{4}{f^4} - 8\Delta \frac{f'^2}{f^4} + 4\Delta^2 \frac{f'^4}{f^4}. \quad (9)$$

The main difficulty in solving the system numerically is the fact that the metric function Δ has a coordinate singularity at the event horizon, making the numerical calculation ‘‘through’’ the horizon intricate. This is why the computation process was divided into two parts. First, the GM-GHS solution (8) was taken as the initial condition at infinity. Solutions for the metric functions and the dilaton outside the event horizon were found. Then, the results near the horizon were taken as new initial conditions.

The behavior of metric the functions Δ and f, together with the dilaton exponent $e^{-2\phi}$, was investigated under the event horizon of the black hole. It differs significantly depending on the black hole charge. If the charge is zero or small the metric function Δ is defined only for $r > r_s$ (r_s being smaller than the event horizon radius r_h). In this case, there exist two mathematical branches: one is physi-

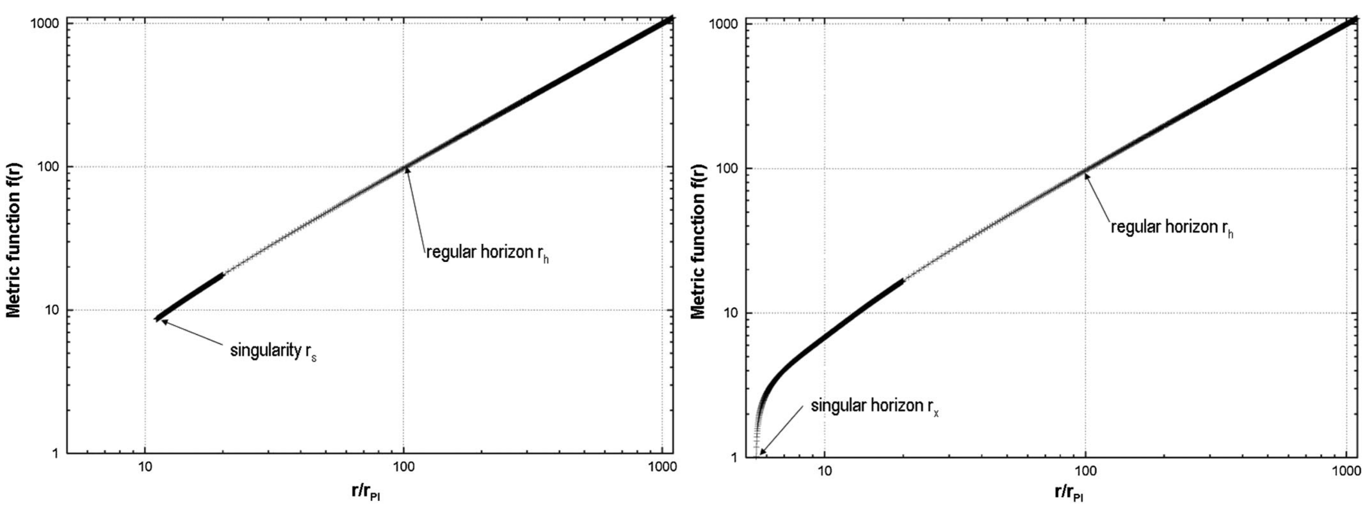

FIG. 2. Metric function f as a function of the radial coordinate r for $q = 21.50 < q_{\rm cr}$ (left plot) and $q = 24.81 > q_{\rm cr}$ (right plot) when $r_h = 200.0$ Planck units.

cal (and displayed in Fig. 1), ranging from $r = r_s$ to infinity, and the other one is an artifact, ranging from $r = r_s$ to $r = r_x$. If the value of the charge is larger than a critical value $q_{\rm cr}$, the inner singularity does not exist anymore and, as it can be seen in Fig. 1 (right), Δ exhibits a local minimum. When the black hole charge increases from zero, a phase transition occurs at $q = q_{\rm cr}$ such that the inner singularity disappears (being relpaced by a local minimum for Δ) and an inner horizon forms at r_x. This is the main difference between the considered solution and the GM-GHS case.

The values of the critical charge have been numerically computed for different masses and are given in Table I.

The behavior of the metric function $f(r)$ (Fig. 2) and $e^{-2\phi(r)}$ (Fig. 3) outside the horizon are analogous to the GM-GHS case. For large values of the radial coordinate, $f(r) \sim r$. When $q < q_{\rm cr}$ these functions are monotonic from $r = r_s$ to infinity. However, when $q > q_{\rm cr}$, they are defined in a wider interval $[r_x, \infty]$. The metric function $f(r)$ vanishes for $r = r_x$, together with $e^{-2\phi}$. This underlines that for $r \to r_x$, the influence of the Maxwell term becomes subdominant when compared to the Gauss-Bonnet one.

The behavior of the curvature invariant $R_{ijkl}R^{ijkl}$ under the event horizon of the black hole was also studied and it was confirmed that $R_{ijkl}R^{ijkl} \to \infty$ for $r \to r_s$ when $q < q_{\rm cr}$. The situation when the black hole charge reaches its critical value and the metric function Δ begins to exhibit a local minimum instead of a singularity at $r = r_s$ was considered in more detail. It was checked that in this case the value of the curvature invariant does not diverge anymore. It is therefore obvious that the local minimum of the metric function $\Delta(r)$ is intrinsically nonsingular.

When $q > q_{\rm cr}$ (i.e. when the r_s singularity vanishes), the important point is r_x, where f vanishes. It was numerically checked that the curvature invariant diverges at this point. So, it can be conjectured that $r = r_x$ becomes a singular horizon inside the black hole. When $q < q_{\rm cr}$, this horizon belongs to the nonphysical branch of the considered system of equations. Near the singular horizon r_x (when $q > q_{\rm cr}$), the curvature invariant diverges significantly more rapidly than near the singularity r_s (when $q < q_{\rm cr}$).

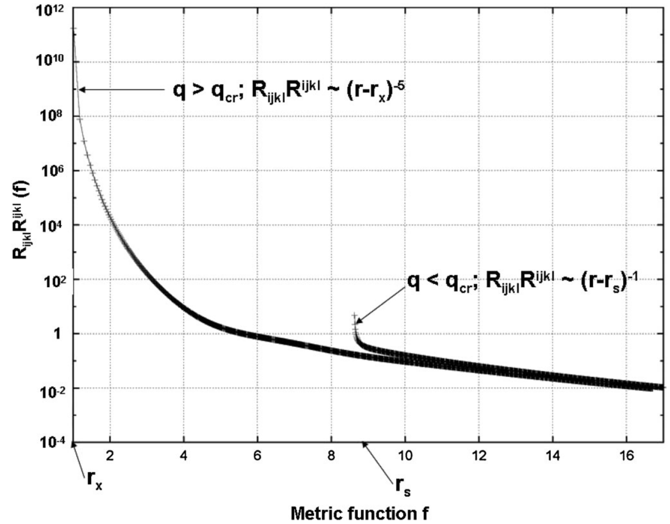

FIG. 4. Curvature invariant $R_{ijkl}R^{ijkl}$ as a function of the metric function f for $q = 21.50 < q_{\rm cr}$ (left curve) and $q = 24.81 > q_{\rm cr}$ (right curve), with $r_h = 200.0$ Planck units.

In Fig. 4, the curvature invariant is shown as a function the of metric function f which, in the chosen metric, has the intuitive meaning of the radius of a two-sphere. The asymptotic behavior of the metric function f can be expressed as [2]

$$f(r \to r_s) = f_s + f_{s2}(r - r_s) + f_{s3}(\sqrt{r - r_s})^3 + \cdots,$$
$$f(r \to r_x) = f_x + f_{x1}\sqrt{r - r_x} + f_{x2}(r - r_x) + \cdots, \quad (10)$$

where f_i are expansion coefficients.

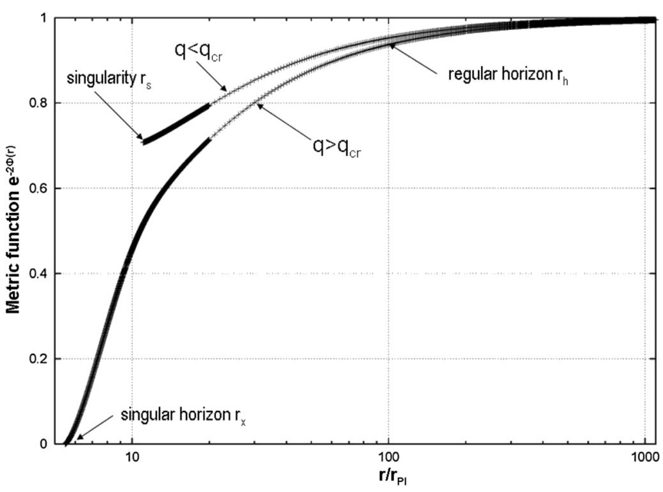

FIG. 3. Dilatonic exponent $e^{-2\phi}$ as a function of the radial coordinate r for $q = 21.50 < q_{\rm cr}$ (upper curve) and $q = 24.81 > q_{\rm cr}$ (lower curve) when $r_h = 200.0$ Planck units.

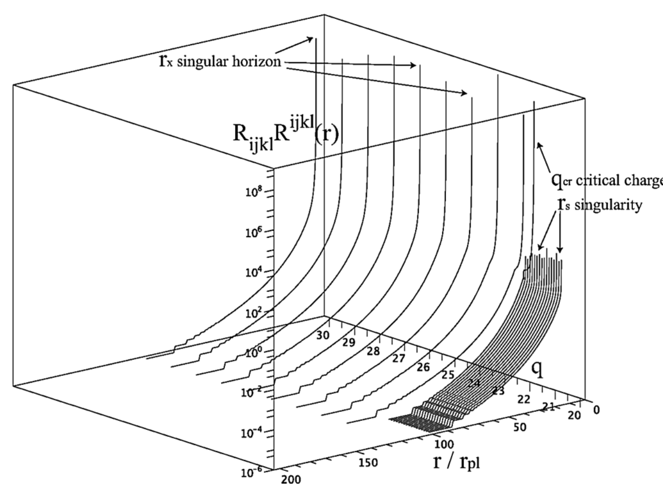

FIG. 5. Three-dimensional dependence of the curvature invariant $R_{ijkl}R^{ijkl}$ against the charge q and the radial coordinate r for $r_h = 200.0$.

So, the divergence of the curvature invariant in terms of the metric function f is given by

$$R_{ijkl}R^{ijkl} \sim \mathrm{const}_1 \times (f - f_s)^{-1} \quad \text{for } f \to f_s, \quad R_{ijkl}R^{ijkl} \sim \mathrm{const}_2 \times (f - f_x)^{-5} \quad \text{for } f \to f_x. \tag{11}$$

The three-dimensional dependence of the curvature invariant as a function of the charge q and the radial coordinate r is given in Fig. 5.

This establishes the internal structure of a Maxwell-Gauss-Bonnet black hole. It can also be noticed that the regularization of the internal structure, which is expected by some models of "cosmological natural selection" [5] and is predicted by loop quantum gravity [6], does not happen in Gauss-Bonnet gravity, even for highly charged black holes.

S. A. would like to thank the Russian Foundation for Basic Research for the support via Grant No. 07-02-01034-a.

[1] B. Zwiebach, Phys. Lett. B **156**, 315 (1985); E. Poisson, Classical Quantum Gravity **8**, 639 (1991); D. Witt, Phys. Rev. D **38**, 3000 (1988); J. T. Wheeler, Nucl. Phys. **B268**, 737 (1986); **B273**, 732 (1986); S. Mignemi and N. R. Stewart, Phys. Rev. D **47**, 5259 (1993); P. Kanti, N. E. Mavromatos, J. Rizos, K. Tamvakis, and E. Winstanley, Phys. Rev. D **54**, 5049 (1996); T. Torii, H. Yajima, and K. Maeda, Phys. Rev. D **55**, 739 (1997); S. Alexeyev *et al.*, Classical Quantum Gravity **19**, 4431 (2002); A. Barrau, S. Alexeyev, and J. Grain, Phys. Lett. B **584**, 114 (2004).

[2] S. O. Alexeyev and M. V. Pomazanov, Phys. Rev. D **55**, 2110 (1997); S. O. Alexeyev and M. V. Sazhin, Gen. Relativ. Gravit. **30**, 1187 (1998); S. O. Alexeyev, M. V. Sazhin, and M. V. Pomazanov, Int. J. Mod. Phys. D **10**, 225 (2001).

[3] G. W. Gibbons and K. Maeda, Nucl. Phys. **B298**, 741 (1988); D. Garfincle, G. Horowitz, and A. Strominger, Phys. Rev. D **43**, 3140 (1991); **45**, 3888 (1992).

[4] S. O. Alexeyev and M. V. Pomazanov, arXiv:gr-qc/9706066v2.

[5] L. Smolin, Physica A (Amsterdam) **340**, 705 (2004).

[6] M. Bojowald *et al.*, Phys. Rev. Lett. **95**, 091302 (2005).