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Multiquark baryons and color screening at finite temperature

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We study baryons in $SU(N)$ gauge theories at finite temperature according to the gauge/string correspondence based on IIB string theory. The baryon is constructed out of the D5-brane and N fundamental strings to form a color singlet N-quark bound state. At finite temperature and in the deconfining phase, we could find $k(<N)$-quark baryons. Thermal properties of such k-quark baryons and also of the N-quark baryon are examined. We study the temperature dependence of color screening distance and the Debye length of the baryon of the k quark and the N quark. We also estimate the melting temperature, where the baryons decay into quarks and gluons completely.

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I. INTRODUCTION

In the context of string/gauge theory correspondence the baryons related to the D5-branes has been proposed [1–9] and studied using the Born-Infeld approach [10–12]. In these approaches, the fundamental strings (F string) are dissolved in the D5-brane in order to make a baryon vertex. Here the D5-brane is embedded as a probe in a 10D background in which the conformal invariance is broken by the nontrivial dilaton. As a result of nonconformal invariance, the embedded configurations of the D5-brane generally have cusps, which are the singular points to be cancelled out by some external objects. This is performed by introducing F strings whose surface term cancels out with this cusp singularity. This is represented by the no-force condition between the F strings and the D5-brane. From a viewpoint of the smoothed picture of the baryon configuration, these added F strings could be regarded as the ones which flow from the D5-brane through the cusps.

According to the above idea, an analysis for the baryon configurations has been performed in [13] based on a simple holographic model. At finite temperature and in the quark deconfinement phase, we could find possible bound states of a small number $k(<N)$ of quarks with a D5 vertex. Here we study the thermal properties of these states and also the usual N-quarks baryons. The quarks in the baryon are connected to the vertex through the F string. This F string is common to the one of mesons which are the bound states of a quark and an antiquark. So we expect, for the part related to the quarks in the baryon, that the thermal properties are similar to the ones of the mesons. But the situation is different from the meson case in the following two points: (i) Since the baryon is constructed from N quarks, various configurations are possible in the quark deconfinement phase. They are discriminated by the number of quarks retained in the state. (ii) The second point is the nontrivial configuration of the vertex given by the D5-brane, which is determined by the dynamics of the gauge theory.

These properties depend on the temperature (T). At low temperature, we find four types [(A) $\sim$ (D)] of D5-brane canfigurations. Two of them [(C) and (D)] appear for the first time at finite temperature. When the temperature exceeds a value T_{c_1}, which is given in [13], the solution is reduced to the one type [(A)]. But before arriving at this temperature, any baryon state decays into quarks and gluons since its energy (E) exceeds the sum of the effective mass of free quarks. We call this temperature the melting point, $T_{\rm melt}$, which is slightly smaller than T_{c_1} as estimated below.

The baryon energy E varies with its size. We study here the relation between the size and the energy E of the baryon. As for the size of the baryon, we could measure it as the F string length observed in the real three-dimensional space. The configurations of the F strings are intimately related to the vertex configurations or the D5-brane structure, which varies with the temperature. Through the study of the F string configurations, we find that the quarks in the baryon can not stay far from the vertex exceeding a distance ($L_{\rm max}$) which increases with decreasing T. This is the reflection of the thermal screening of the color force and it is seen by measuring L_{q-v}, the distance of the quarks from the vertex, as a function of E. At low temperature, we find $E - L_{q-v}$ relation as three connected curves. They are a little complicated compared to the case of mesons. The first curve starts from $L_{q-v} = 0$ and ends at $L_{\rm max}$, then it comes back toward the small L_{q-v} side along the second curve with larger E and it stops at

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$L_{q-v} = L_1(>0)$. Then it turns to the larger L_{q-v} again along the third curve with larger E, and it stops at $L_{q-v} = L_2(<L_{\max})$. (See Fig. 5.) This behavior is common to all the baryon configurations of different k''s, and the slopes of the curves of different k's are proportional to k.

However we find a real screening point at $L^*(\ll L_{\max})$, where the baryon decays to the quarks and gluons, before arriving at $L_{\max}$. So we could not see the above complicated properties as stable baryon states, and for $L^* < L_{q-v}$, they decay into the quarks and gluons. When T increases and approaches to T_{melt}, then L^* vanishes to zero and all the baryons melt down.

In Sec. II, we review our model briefly, and the equation of motion for D5-branes is solved to show four types of solutions and to discuss the stability of the baryon. In Sec. III, the balance condition of the forces is given, and in Sec. IV, $E - L_{q-v}$ relations and the estimation of the screening mass are given. In Sec. V, we discuss the stability of the baryon state from the viewpoint of baryon vertex energy and the gauge condensate parameter. The summary and discussions are given in the final section.

II. D5 BARYON VERTEX AT FINITE TEMPERATURE

Here, we consider the baryon configurations in the nonconfining, finite temperature Yang-Mills (YM) theory. Such a model is given by the AdS black hole solution, which represents the high temperature gauge theory. In our theory with dilaton, the corresponding background solution is given as [14]

$$ds^2_{10} = e^{\Phi/2}\left(\frac{r^2}{R^2}[-f^2dt^2 + (dx^i)^2] + \frac{R^2}{r^2f^2}dr^2 + R^2d\Omega_5^2\right). \tag{1}$$

$$f = \sqrt{1 - \left(\frac{r_T}{r}\right)^4}, \qquad e^{\Phi} = 1 + \frac{q}{r_T^4}\log\left(\frac{1}{f^2}\right), \qquad \chi = -e^{-\Phi} + \chi_0. \tag{2}$$

The temperature (T) is denoted by $T = r_T/(\pi R^2)$. The world volume action of the D5-brane is rewritten by eliminating the U(1) flux in terms of its equation of motion as above, then we get its energy as [13][1]

$$U_{D5} = \frac{N}{3\pi^2\alpha'}\int d\theta e^{\Phi/2}f\sqrt{r^2 + r'^2/f^2 + (r/R)^4x'^2} \times \sqrt{V_\nu(\theta)}, \tag{3}$$

where

$$V_\nu(\theta) = D(\nu,\theta)^2 + \sin^8\theta, \tag{4}$$

$$D \equiv D(\nu,\theta) = \left[\frac{3}{2}(\nu\pi - \theta) + \frac{3}{2}\sin\theta\cos\theta + \sin^3\theta\cos\theta\right]. \tag{5}$$

Here, D denotes the electric flux at the θ cross section. And the integration constant ν is expressed as $0 \le \nu = k/N \le 1$, where k corresponds to the number of F strings emerging from one of the poles of the $\mathbf{S}^5$ at $\theta = 0$. Then $N - n$ comes out from the other pole at $\theta = \pi$.

As mentioned above, we restricted the solutions to the SO(5) symmetric form of $r = r(\theta)$ here. Although the SO(6) symmetric solution, $r =$ constant, is the simplest one, the less symmetric SO(5) solution considered here contains the SO(6) symmetric one, $r = q^{1/4}$, as shown below. This solution ($r = q^{1/4}$) gives actually the minimum of the vertex energy as shown in Sec. V (see Fig. 8). The baryon is however made by adding N fundamental strings to this vertex with no-force condition (15). In this case, the energy of the baryon is given as the sum of the one of the vertices and the attached F strings. Hence we can show that the baryon with this SO(6) symmetric vertex is not the lowest energy state, and the lowest state is found rather in the baryon with a SO(5) symmetric vertex. Then, it would be meaningful to consider SO(5) symmetric solutions. As for the SO(n) symmetric solutions with smaller $n(\le 4)$, we should examine them if possible. But it will be postponed as the next stage of work for us, and we will find more complicated baryon vertex configurations.

As for the background solution given here, we give the following comment. Here we are considering the finite temperature gauge theory holographically. Hence the supersymmetry of the gauge theory is lost, then one might reconsider whether the state with $q = F^2_{\mu\nu} \neq 0$ is still the ground state of the theory. We think that the above background with $q \neq 0$ could be regarded as the ground state in the present model, and to investigate the above solution is meaningful from the following two viewpoints:

(i) The density values of the bulk action with the background of $q = 0$ and nonzero q are the same in spite of the finite temperature since q does not appear in the action.[2] This would be understood as the remnant of $N = 2$ supersymmetry at $T = 0$. So we cannot judge which state of $q = 0$ or $q \neq 0$ is the ground state from the bulk theory. In other words, our solution with $q \neq 0$ can be said to be the ground state still.

(ii) (ii) On the other hand, from 4D lattice QCD simulations, we could find the observation that the gauge

[1]Here we consider the SO(5) symmetric solution, so only the coordinate θ remains.

[2]Here the 5D reduced action can be given as

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left(R - 3\Lambda - \frac{1}{2}(\partial\Phi)^2 + \frac{1}{2}e^{2\Phi}(\partial\chi)^2\right).$$

condensate disappears at the deconfinement temperature [15]. But, we could point out two possible reasons to use our model with nonzero q even at the finite temperature. (a) Our model is the finite temperature version of "$N = 2$ supersymmetry SU(n) YM theory, where n is large." This point is different from the "QCD" simulated on the lattice. Then there might be the region where our solution is ground state. (b) Second, there would be a finite region of temperature for the finite q ground state if the real transition was not so sharp with respect to q in the real QCD.

A. Embedded solutions of the D5-brane

The vertex configuration is obtained from the above Legendre transformed action (3). Here, for simplicity, we concentrate on the point vertex configuration and $\nu = 0$, which means the quarks come from the cusp at $\theta = \pi$. There is no other cusp. Then, we set $x' = 0$, and we obtain

$$U_{D5} = \frac{N}{3\pi^2\alpha'}\int d\theta e^{\Phi/2} f\sqrt{r^2 + r'^2/f^2}\sqrt{V_0(\theta)}. \tag{6}$$

From this action, the equation of motion of $r(\theta)$ is obtained as follows:

$$\partial_\theta\left(\frac{r'}{\sqrt{r^2f^2 + (r')^2}}\sqrt{V_0(\theta)}\right) - \frac{P(r)}{\sqrt{r^2f^2 + (r')^2}}\sqrt{V_0(\theta)} = 0. \tag{7}$$

$$P(r) = \frac{1}{2e^\Phi}\partial_r(e^\Phi r^2 f^2). \tag{8}$$

The solutions $r(\theta)$ of Eq. (7) are characterized by the prefactor $P(r)$ in the second term. Its behavior depends on the temperature. For comparison, $P(r)$ at $T = 0$ is also shown in Fig. 1 with the one of the various finite temperatures.

At any temperature, $P(r)$ is positive at large r and changes its sign to the negative value for the low temperature cases according to the curves (α) and (β) in the Fig. 1. When we take the value of $r(0)(\equiv r_0)$ at the point of $P > 0$ [$P < 0$], then we obtain the typical configuration (A) [(B)] of the baryon as shown in Fig. 2 where F strings are added according to the condition given below. The details of the boundary $r = r_{\rm max}$, used in Fig. 2, are given in Sec. III B. Here, we notice only the configuration of the vertex part.

In Fig. 2, we assumed circular quark distribution for the sake of simplicity. However, it is possible to consider other complicated positioning as far as the no-force condition is satisfied for the fundamental strings connecting to the quarks. But such configurations are not considered here since it is not effective to understand various properties of the theory in a simple way. A comment on this point is given in Sec. IV A.

For the case of zero temperature, the two types of solutions (A) and (B) correspond to the region of $P > 0$ and $P < 0$, respectively. However, at finite and low temperatures, the behavior of P is not so simple since the second zero point appears at smaller r. When r_0 is chosen near this zero point, we find two typical solutions (C) and (D) for $P < 0$ and $P > 0$, respectively, as shown in Fig. 3. They oscillate once with respect to θ, and these solutions cannot be seen at zero temperature.

At higher temperature, $T > T_{c_1}$, the zero points of $P(r)$ disappear as shown by the curve (γ) in Fig. 1. In this case, only the type (A) solutions are obtained, and several solutions with different r_0 are shown in Fig. 4. We can see that the end point value $r(\pi) \equiv r_c$ of the solutions $r(\theta)$ takes its minimum for some initial value $r(0)$, then it rises when $r(\pi)$ decreases furthermore and approaches the horizon r_T.

Next, we turn to the F strings. In the present finite T model, the effective-quark mass ($\tilde{m}_q$) is finite and it diverges at $T = 0$. In this sense, the present theory is in the quark deconfinement phase for finite T. So, free quarks or independent F strings could exist, and the end points of these F strings touch on the horizon r_T and $r_{\rm max}$. Then, some of the F strings, which were originally attached to the baryon, could escape from the baryon to form such free F strings. And as shown in Fig. 2, we could find $k(<N)$-quarks baryon configuration with extra $N - k$ F strings which connect the vertex and the horizon. These $N - k$ F strings are not observed as quarks in the gauge theory, but they contribute to the energy of the baryon. As a result, some number of $N - k$ quarks disappear from the original baryon, and we would find it as a k-quarks baryon. Then, this implies that we may find color nonsinglet baryons as depicted in Fig. 2.

Finally, in this section, we comment on the role of the parameter q in the dilaton solution given in Eq. (2). It represents the vacuum expectation value of gauge field strength $\langle F_{\mu\nu}F^{\mu\nu}\rangle$ from the AdS/CFT dictionary. When we neglect this parameter by setting $q = 0$, then the end

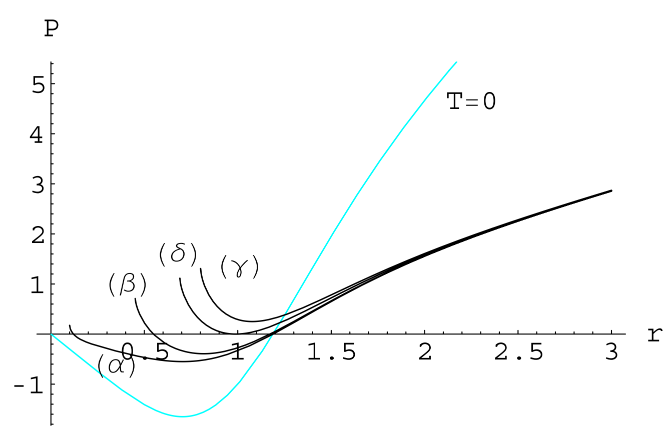

FIG. 1 (color online). $P(r)$ for $q = 2$ and $r_T = (\alpha)0.1$, $(\beta)0.45$, $(\gamma)0.8$, $(\delta)0.689$. The curve (δ) is given at the critical point T_{c_1}. The curve of $T = 0$ shows $3P|_{r_t=0} = 3r(1 - q/r^4)/(1 + q/r^4)$.

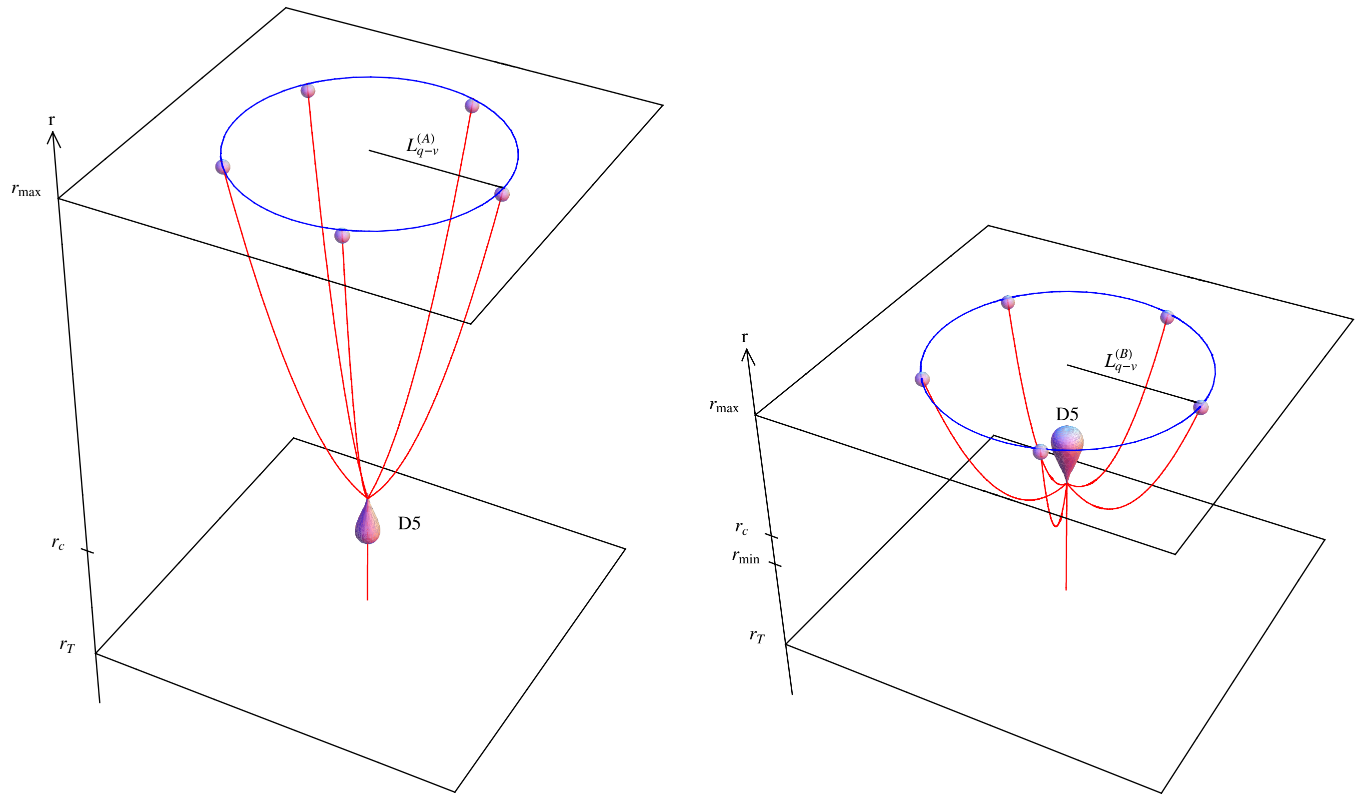

FIG. 2 (color online). Baryons at finite temperature with k quarks ($N - k$ quarks) on the boundary (in the horizon) at $r = r_{\max}$ (r_T). The spindles represent the D5 vertex with one cusp at $r_c = r(\pi)$ since $\nu = 0$. The left panel [right panel] corresponds to the solutions (A) and (C) [(B) and (D)]. Notice that the strings end on the cusp of the D5-brane vertex at r_c in any case. The distance (L_{q-v}) between the vertex and the quarks in the real three space, for example, to x direction, is shown for each baryon configuration. The scale of r is arbitrary.

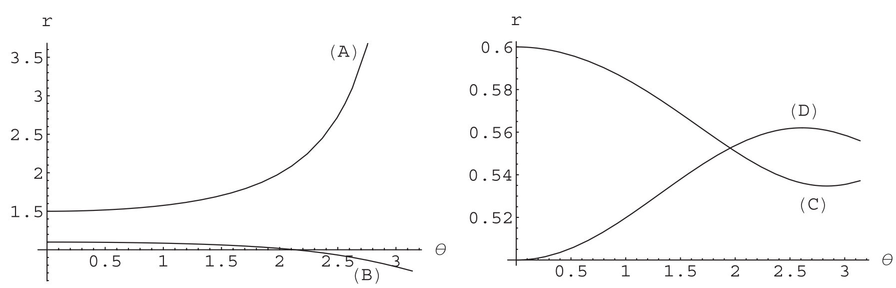

FIG. 3. The four types of solutions of $r(\theta)$ for $q = 2$ at $r_T = 0.45$. The curves represent types A) $r_0 = 1.5$, (B) $r_0 = 1.1$, (C) $r_0 = 0.6$, and (D) $r_0 = 0.5$, respectively.

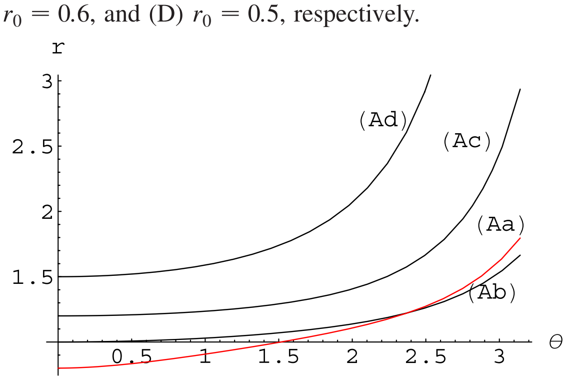

FIG. 4 (color online). The type (A) solutions of $r(\theta)$ for $q = 2$, $r_T = 0.8$, and (Aa) $r_0 = 0.80001$, (Ab) $r_0 = 1.0$, (Ac) $r_0 = 1.2$, (Ad) $r_0 = 1.5$.

point $r(\theta = \pi)$ of the solution diverges to infinity. In other words, we cannot find the cusp at a finite r. This situation is seen for supersymmetric solutions [4] since there was no scale parameter to break the scale invariance of the solutions. In the present case, the dimensionful parameter q plays the role of giving a scale of the cusp point. Another dimensionful parameter r_T, which is proportional to the temperature, does not work as q.

III. NO-FORCE CONDITION AND k-QUARKS BARYON

Full baryon configuration is given by the vertex and F strings. The configurations are determined through the no-force condition at the cusp point of the vertex and the profiles of the vertex and the F strings. They are characterized by the energy of the total system, the baryon energy, and the distance between the quark and the vertex (L_{q-v}).

The tension of the D5-brane at r_c is given as

$$\frac{\partial U_{D5}}{\partial r_c} = NT_{\rm F} e^{\Phi/2} \frac{r_c'}{\sqrt{r_c'^2 + r_c^2 f(r_c)^2}}. \tag{9}$$

As for the F string, the energy is written as

$$U_{\rm F} = kU_{\rm F}^{(1)} + (N-k)U_{\rm F}^{(2)}. \tag{10}$$

Here, $U_{\rm F}^{(1)}$ is the energy of string which extends to the x direction and its action is written as

$$S_{\rm F}^{(1)} = -\frac{1}{2\pi\alpha'} \int dt dx e^{\Phi/2} \sqrt{r_x^2 + f^2(r)(r/R)^4}, \tag{11}$$

where $r_x = \frac{\partial r}{\partial x}$ and the world sheet coordinates are set as (t, x). Then, its energy and r-directed tension at the point $r = r_c$ are obtained as follows:

$$U_{\rm F}^{(1)} = T_{\rm F} \int dx e^{\Phi/2} \sqrt{r_x^2 + f^2(r)(r/R)^4}, \tag{12}$$

$$\frac{\partial U_{\rm F}^{(1)}}{\partial r_c} = T_{\rm F} \frac{r_x}{\sqrt{r_x^2 + (r_c/R)^4 f(r_c)^2}}. \tag{13}$$

$U_{\rm F}^{(2)}$ is the energy of the string extending from horizon r_T to r_c, and it is written as

$$U_{\rm F}^{(2)} = T_{\rm F} \int_{r_T}^{r_c} dr e^{\Phi/2}. \tag{14}$$

Then the no-force condition is obtained as

$$N \frac{r_c'}{\sqrt{r_c'^2 + r_c^2 f(r_c)^2}} + (N-k) = k \frac{r_x}{\sqrt{r_x^2 + (r_c/R)^4 f(r_c)^2}}. \tag{15}$$

A. Lower bound on *k*

From this equation, we can estimate the lower bound for k, which is written as

$$k \geq \frac{N}{2}(1 + Q_5), \qquad Q_5 \equiv \frac{r_c'}{\sqrt{r_c'^2 + r_c^2 f(r_c)^2}}. \tag{16}$$

The value of Q_5 depends on the parameters, but we can see it does not arrive at -1. Then $k > 0$, however, k could decrease and approach zero by choosing the parameters. For example, we find $Q_5 = -.99916$ for $(q, r_0, r_T) = (2.0, 0.6, 0.01)$.

In our model, the F-string configuration must satisfy Eq. (17) with a constant h. This implies $h = 0$ for the string which touches on the horizon r_T since $f(r_T) = 0$. So we obtain a vertical straight line as the configuration. However, it would be possible to obtain a slightly curved string configuration in a more realistic model. Then the configuration of $k = 0$ would be possible in principle at finite temperature. But it would not be realized energetically as a stable state.

B. Energy of *k*-quarks baryon

The energy of baryons is constructed by the F-strings' energy and that of the vertex. In obtaining the F-string configuration from Eq. (12), we introduce the constant h as follows,

$$e^{\Phi/2} \frac{r^4 f^2(r)}{R^4 \sqrt{r_x^2 + (r/R)^4 f^2(r)}} = h. \tag{17}$$

For $r_x > 0(<0)$, k-quarks baryons configuration becomes the type (A)[(B)]. First, consider the solution of (A). In this case, the energy of the F strings is given as

$$U_{\rm F} = T_{\rm F}\left[k \int_{r_c}^{r_{\rm max}} dr \frac{e^{\Phi/2}}{\sqrt{1 - \frac{h^2 R^4}{f^2 e^{\Phi} r^4}}} + (N-k) \int_{r_T}^{r_c} dr e^{\Phi/2}\right]. \tag{18}$$

The second term denotes the F strings stretching between the horizon and r_c. And the distance L_{q-v} is given as

$$L_{q-v}^{(A)} = R^2 \int_{r_c}^{r_{\rm max}} dr \frac{1}{r^2 f \sqrt{\frac{e^{\Phi} r^4 f^2}{h^2 R^2} - 1}}. \tag{19}$$

Here we make the following point. The $r_{\rm max}$ can be introduced as the position of the embedded D7-brane to provide quarks, but it is abbreviated in this article to simplify the problem. So, in the strict sense, $r_{\rm max}$ simply plays the role of the UV cutoff and does not have any other physical meaning. Then the quantities depending on $r_{\rm max}$, like L_{q-v} and $U_{\rm F}$ given above, must be renormalized by an appropriate physical mass scale, for example, some hadron mass. Then their absolute values have no physical meaning at the present stage. On the other hand, $r_{\rm max}$ independent quantities like L^*, tension, $T_{\rm melt}$, and m_D, which are given below, have physical meaning in this article, and they are important quantities in our paper.

Next, we turn to the configuration of (B). In this case, the string configuration includes a bottom part of the U-shaped configuration and the bottom point is seen at $r = r_{\rm min}$, where we find

$$\left.\frac{dr}{dx}\right|_{r=r_{\rm min}} = 0. \tag{20}$$

Since (17) can be used at any r, we obtain

$$h = e^{\Phi(r_{\rm min})/2} f(r_{\rm min}) \left(\frac{r_{\rm min}}{R}\right)^2 = e^{\Phi(r_c)/2} \frac{r_c^4 f^2(r_c)}{R^4 \sqrt{r_x^2 + (r_c/R)^4 f^2(r_c)}}. \tag{21}$$

From this, $r_{\rm min}$ is determined by r_c, then by r_0. On the other hand, r_x is determined by using r_c from (15). As a result, we obtain the string energy and the distance between the quark and the vertex as follows:

$$U_{\rm F} = T_{\rm F}\Big[k\Big(\int_{r_{\rm min}}^{r_{\rm max}} dr \frac{e^{\Phi/2}}{\sqrt{1-\frac{h^2R^4}{f^2e^{\Phi}r^4}}} + \int_{r_{\rm min}}^{r_c} dr \frac{e^{\Phi/2}}{\sqrt{1-\frac{h^2R^4}{f^2e^{\Phi}r^4}}}\Big) + (N-k)\int_{r_T}^{r_c} dr e^{\Phi/2}\Big], \tag{22}$$

$$L^{(B)}_{q-v} = R^2\Big[\int_{r_{\rm min}}^{r_{\rm max}} dr \frac{1}{r^2 f\sqrt{\frac{e^{\Phi}r^4f^2}{h^2R^4}-1}} + \int_{r_{\rm min}}^{r_c} dr \frac{1}{r^2 f\sqrt{\frac{e^{\Phi}r^4f^2}{h^2R^4}-1}}\Big]. \tag{23}$$

Here we consider the energy of k-quark (for any k) baryon and $N-k$ free quarks to compare the energy with that of the N free quarks which form the baryon as a singlet state. So we add $(N-k)$ free quarks, which are strings stretching between the horizon and $r_{\rm max}$, to the k-quark baryon state. We estimate the energy of the baryon in this way as a function of the distance L_{q-v}. Then the total k-quark baryon energy is obtained as

$$E_k = U_{\rm F} + U_{D5} + (N-k)\tilde{m}_q, \tag{24}$$

where we note that the first two terms $U_{\rm F}$ and U_{D5} are dependent on k, and $\tilde{m}_q$ is the energy of a free quark,

$$\tilde{m}_q = T_{\rm F}\int_{r_T}^{r_{\rm max}} dr e^{\Phi/2}. \tag{25}$$

In the following, we give numerical analyses by setting $r_{\rm max} = 10$ for simplicity. This value changes the energy of the string part or quark mass. In the sense, $r_{\rm max} = 10$ would be set at the position of the embedded D7-brane which provides quarks. But here we do not discuss the embedding of the quark brane.

IV. COLOR SCREENING AND BARYON MELTING

Here we consider the stability of the various baryon configurations in the deconfining thermal medium. Although the baryon is in the deconfining phase some of the baryon states could be stable until the temperature arrives at the critical point where all the baryons melt down to free quarks. This melting temperature is estimated below by comparing the baryon energy and the energy of free quarks.

In the following, we estimate numerically the energy E and the distance between the quark and the vertex, L_{q-v}, for the k-quark baryon ($k \leq N$) in order to cover various cases. The color screening due to the temperature is observed as the existence of a maximum value of L_{q-v} for each baryon states. However, the real maximum value of L_{q-v} is determined energetically by comparing the energy of the baryon state and the one of the summation of the free quarks.

A. For $k = N$ case

Typical $E - L_{q-v}$ curves are shown in Fig. 5 for two temperatures. The beginning point at $L = 0$ corresponds to the solution of $r(\theta)$ with $(r(\pi) =)r_c = r_{\rm max}$ and $(r(0) =)r_0 > r_T$. Then E increases almost linearly with L_{q-v}. We notice two turnover points in these curves, and the second one has not been seen in the case of the mesons. We could see that the second turnover is related to the existence of two zero points of $P(r)$ and two new types of solutions [(C) and (D)] as seen in Sec. II. However, this intriguing behavior of $E - L_{q-v}$ curves is seen in the unstable region of the baryon since the baryon energy in this region is higher than the sum of the free quarks energy, which are shown by the horizontal lines. They cross at $L = L^*$, which depends on the temperature; then the baryon, which has the energy below $N\tilde{m}_q$, is allowed as a stable state. The point L^* is therefore interpreted as the maximum size of the baryon. Within this distance, the color force works on the quark, and the color gauge force is screened outside this distance. As a result, all the baryons decay into quarks and gluons for $L^* < L_{q-v}$.

We notice here that the $r_{\rm max}$ dependence of L^* is very small and we could neglect this dependence. For example, in Fig. 6, we can show the numerical estimation of this $r_{\rm max}$ dependence for the case of Fig. 5. It shows that the physical quantity L^* is almost independent for $r_{\rm max} > 5$.

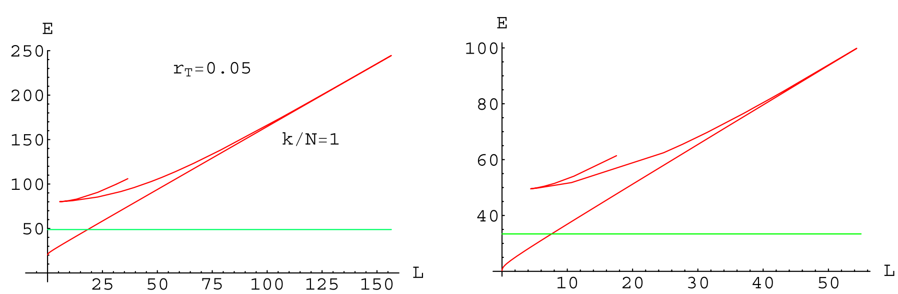

FIG. 5 (color online). Energy of the N-quarks baryon versus L_{q-v} (which is denoted simply by L in the figure) at $r_T = 0.05$ (left panel) and $r_T = 0.1$ (right panel) for $q = 2$ and $r_{\rm max} = 10$. In these figures two turnovers are seen as mentioned in the text. However, the physically allowed region is only the part below the horizontal lines of $N\tilde{m}_q$.

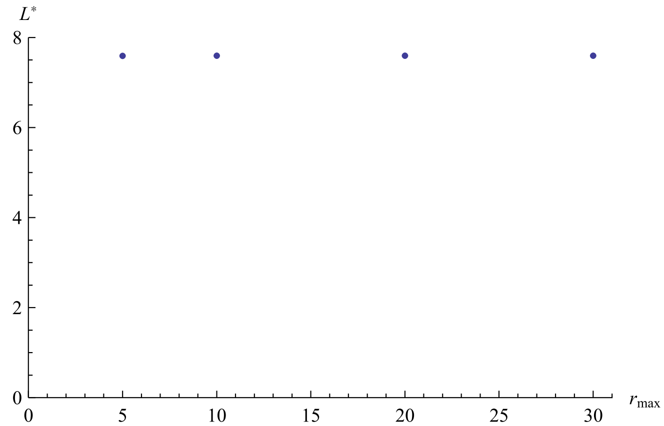

FIG. 6 (color online). The cutoff (r_{max}) dependence of L^* for $q = 2$, $r_T = 0.1$, and $R = 1$.

Of course, the value of L^* depends on the temperature. The values for various temperatures are plotted in Fig. 7. The numerical results show that L^* is proportional to $1/r_T$ or $1/T$, and it approaches to zero at some limiting temperature. Above this temperature, any baryon state is not allowed.

The value of L^* should be related to the Debye screening. In the gauge theory, the Debye screening has been considered the correction to the gluon propagator. Then the potential between two colored particles separating r is given by the form $e^{-m_D r}/r$. However, in the present model, the force at short distance is given by the strong coupling conformal field theories (CFT) and a long range force which leads to linear potential. And we find a linearlike potential near L^* even if it is still not so large. So the potential screened by the thermal effect is just this linear-like potential, which is considered as the tail of the linear potential seen at large L_{q-v} in the case of $T = 0$. When we denote this tail as $V_0(L)$ at zero temperature, it is modified at finite temperature as,

$$V \equiv E - N\tilde{m}_q = V_0(L)e^{-m_D(T)L}, \tag{26}$$

where $m_D(T)$ denotes the Debye mass. Using this formula, the numerical curves of $E - L$ are fitted by this form, and we obtain $m_D(T)$. However we used here $V_{T_{\rm min}}(L)$ instead of $V_0(L)$, where $V_{T_{\rm min}}(L)$ denotes the potential of the lowest temperature. So we corrected the data of $m_D(T)$ obtained by shifting the temperature $T \to T'$ such that we could obtain $m_D(T') \to 0$ for $T' \to 0$. The results are shown in the left-hand side of Fig. 7.

Then we could find the relation $L^* = c/m_D(T)$ with a definite constant $c = 6.8$. And we obtain the result that $m_R(T)$ increases linearly with the temperature T.

In [16], using the AdS/CFT correspondence, Bak, Karch and Yaffe examined the behavior of correlators of Polyakov loops and other operators in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory at nonzero temperature. And the implications for Debye screening in that strongly coupled non-Abelian plasma, and comparisons with available results for thermal QCD, were discussed. The connection with our result is that the proportionality $m_D \propto T$ is common with [16]. In [17], however, m_D/T is predicted to decrease slowly with T/T_c on the basis of nonperturbative computation in the deconfined phase of QCD, using the method called background perturbation theory. This prediction is in good agreement with the data of lattice QCD [18]. In order to reproduce this decrease, string-string interactions should be considered in the string theory side. This is a future problem.

Here we give a comment on the other possible configurations to be considered. We have assumed that the quarks are on the same circle. In order to investigate the stability of this configuration, we compare it with the one of the two circles configuration (see Fig. 8). In the latter configurations, we assumed that quarks are divided into two groups with equal numbers. Then the no-force condition is given as

$$N\frac{r_c'}{\sqrt{r_c'^2 + r_c^2 f(r_c)^2}} + (N-k) = \frac{(k/2)r_x^{(1)}}{\sqrt{r^{(1)}2_x + (r_c/R)^4 f(r_c)^2}} + \frac{(k/2)r_x^{(2)}}{\sqrt{r_x^{(2)2} + (r_c/R)^4 f(r_c)^2}}, \tag{27}$$

where $c \equiv r_x^{(1)}/r_x^{(2)}$ is assumed as a constant. When c varies

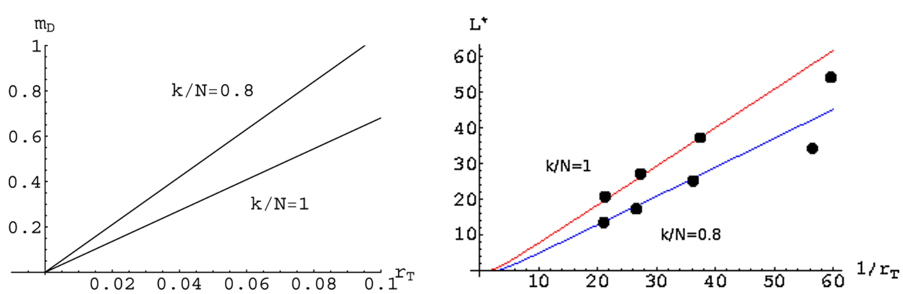

FIG. 7 (color online). The left-hand side shows the Debye screening mass m_D estimated for $k/N = 1$ and 0.8. The line is fitted as $m_D = 6.81r_T$ ($m_D = 10.5r_T$) for $k/N = 1$ ($k/N = 0.8$). In the right-hand figure, melting points L^* are shown by the curves for $k/N = 1$ and 0.8 baryon versus $1/r_T$ at $q = 2$ and $r_{max} = 10$. The dots represent the superposition of the data for m_D given in the left-hand figure. They are given as $L^* = 6.8/m_D$ for both $k/N = 1$ and 0.8.

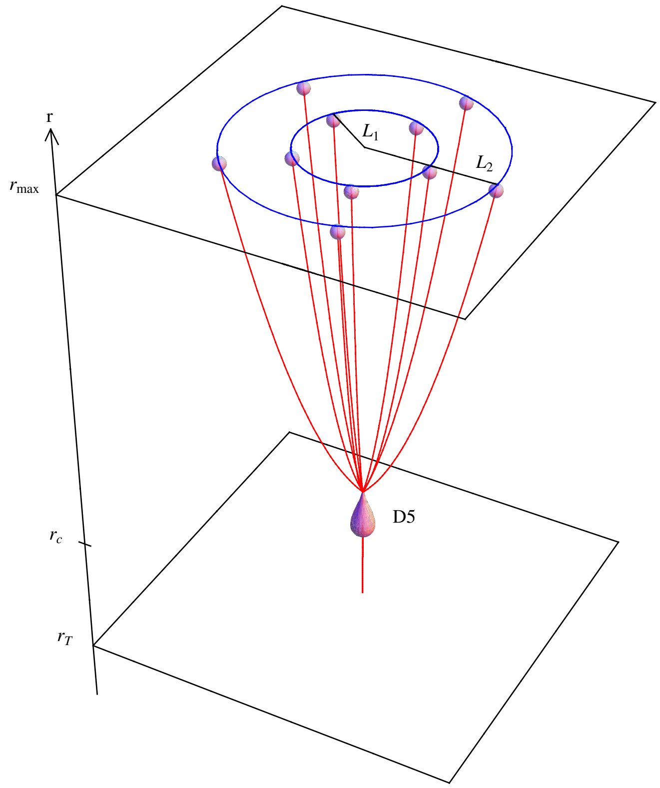

FIG. 8 (color online). Baryon configuration on two circles with radius L_1 and L_2.

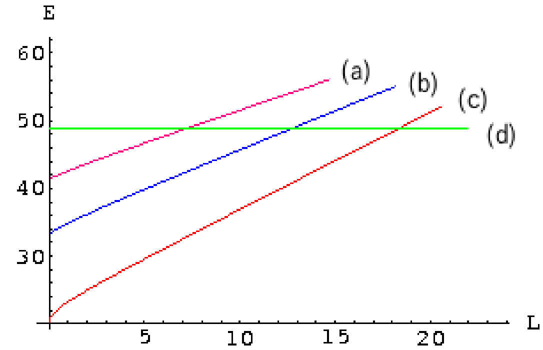

FIG. 9 (color online). Energy of the k-quarks baryon versus L_{q-v} for $r_T = 0.05$. The curves represents $k/N = 1$ (a), 0.8 (b), and 0.67 (c). And the flat line (d) represents $N\tilde{m}_q = 48.8066$ for $r_{max} = 10$.

from 1 to 2, 3, 4 the radius of two circles are changed. The energies of these configurations for fixed r_0 and r_{max} are given in the Table I. The total energies vary only a little bit as c varies. However it should be emphasized that a single circle configuration has the lowest energy. Thus an assumption of the single circle configuration is justified.

B. For $k < N$ case

In this subsection, we discuss the properties discussed above for the baryon with $k < N$. Several $E - L_{q-v}$ curves for different k baryons are shown in Fig. 9. The melting points L^* are obtained from these results, and they are shown for $k/N = 0.8$ in Fig. 7. Compared to the case of $k/N = 1$, it starts from a larger $1/r_T$ (smaller temperature) with a smaller grade in the $L^* - 1/r_T$ plane. For smaller k, the starting point moves to larger $1/r_T$ with smaller slope. Then we could find small k baryon at low temperature as stated in Sec. III A.

However, the energy of k baryon is larger than the k' baryon for $k < k'$ at the same L. Then the k baryon ($k < N$) is always observed as an exited state of the N-quark baryon. One more point to be noted is that our model is in the deconfinement phase as far as the temperature is finite. However, the confinement phase would be realized at a finite temperature, in other words, the critical temperature of quark confinement and deconfinement must be finite. In this sense, the lower bound of k would be existing at some finite value. But this bound is not given here.

V. BARYON STABILITY

We discuss, here, more about the stability of the baryon from two viewpoints in order to get physical insights. Since the vertex is an important part of the baryon, it would be important to see its role for the baryon stability. Another important ingredient in our model is the gauge condensate which is the main source of the confinement force. So we like to see its role in the deconfinement phase considered here.

A. Vertex energy and stability

First, we give a comment from the viewpoint of the vertex energy. In [14], a comment on the stability of the baryon at finite temperature has been given through the

TABLE I. Various energies and string lengths of configurations for $c = r_x^{(1)}/r_x^{(2)} = 1, 2, 3, 4$. r_0 is an initial value of r. (The final value of r is r_{max}.) E, E_2, E_3, and E_4 are the total energies for $c = 1, 2, 3,$ and 4. L is a corresponding length to $c = 1$ and L_1:L_2 is each corresponding value of length to $c = 2, 3, 4$. The other parameters are set to $q = 2$, $k/N = 1$, $r_T = 0.05$, and $r_{max} = 20$.

r_0	E (L)	E_2 (L_1:L_2)	E_3 (L_1:L_2)	E_4 (L_1:L_2)
1.3	21.0915	21.0933	21.0953	21.0968
	(0.087 4267)	(0.063 526:0.107 572)	(0.049 5745:0.116 267)	(0.040 3862:0.120 839)
1.2	22.3686	22.3697	22.3710	22.3721
	(0.535 681)	(0.514 419:0.557 827)	(0.504 027:0.569 335)	(0.497 848:0.576 406)

vertex energy without any inner structure. We give a similar kind of comment on this point in terms of the energy of the vertex with the structure given above. Before doing it, we first notice that the energy of the vertex is without structure. Assuming the baryon is at r, the energy is given from Eq. (6) by setting $r' = 0$ as

$$U_{D5} = \tilde{N} e^{\Phi/2} r f(r), \tag{28}$$

where $\tilde{N} = \frac{N}{3\pi^2\alpha'} \int d\theta \sqrt{V_0(\theta)}$ denotes a r-independent constant. Then we find

$$P(r) \propto \frac{\partial U_{D5}}{\partial r}. \tag{29}$$

From this, we can say that the zero point of $P(r)$ is needed to find a minimum of U_{D5}, and we actually find such a point at low energy. However, there is a temperature T_{c_1} above which the zero point disappears, then the vertex becomes unstable as mentioned in [14]. However, this statement should be modified when the structure of the vertex is taken into account.

By substituting the solution of (7) into (3), we obtain the vertex energy U_{D5} as a function of r_0. In this case, we find the minimum even if the temperature is taken to be larger than T_{c_1}, as seen in Fig. 10. Then we need another criterion to see the stability of the baryon.

The new criterion can be stated as follows. The stability of the baryon is assured when its energy is smaller than the sum of free effective-quark mass, $N\tilde{m}_q$,because the baryon decays to free quarks when its energy exceeds $N\tilde{m}_q$.

The energy of the baryon is given by the sum of E_{D5} and F strings. Then the minimum value of the baryon energy is given by $E_{D5}^{\rm min}$ when the F strings part vanishes due to $L = 0$. Then, the stability of the baryon is found by comparing $E_{D5}^{\rm min}$ with $N\tilde{m}_q$. The value of $E_{D5}^{\rm min}$ is given by the D5 energy with a configuration of $r_T < r(o) < r < r(\pi) = r_{\rm max}$. In this configuration, the F strings are reduced to a point at r_c. The higher energy states are obtained by adding F strings with finite length to the D5-brane with $r(\pi) < r_{\rm max}$ and $r(0) \geq r_T$.

In Fig. 11, the minimum energy of the N-quark baryon (E_{D5}) and $N\tilde{m}_q$ are plotted as functions of r_T. We find the melting point ($T_{\rm melt}$) near T_{c_1}, above which E_{D5} exceeds $N\tilde{m}_q$. Then the baryon decays to quarks and gluons. It is an interesting point that this temperature is very near to T_{c_1}. This temperature $T_{\rm melt}$ depends sensitively on the gauge condensate q, so q is also related to the stability of the baryon state. We discuss this point in the following subsection.

B. Gauge condensate and baryon at high temperature

As shown in Fig. 11, $T_{\rm melt}$ increases with q. Then an unstable baryon at a temperature can be stabilized by increasing the gauge condensate q. This point is understood in terms of the role of the gauge condensate in the gauge theory.

The role of q as a source of quark confinement force has been made clear. At zero temperature with finite q, we find the linear potential between quark and antiquark in the mesons [19], and also between the vertex and the quark in the baryons [13]. The tension of this linear potential is proportional to $q^{1/2}$ in both cases. On the other hand, the role of the temperature is to screen the color force and to prevent making the bound states of quarks. The highly exited state has a large size, then it would be destabilized by the screening effect at high temperature. However, this temperature would depend on the magnitude of the force or the value of q.

Actually, we find that the strength of the linear potential preserved even in the short range is proportional to $q^{1/2}$, so the force to make the baryon at finite T is also proportional to $q^{1/2}$. Then we need a higher temperature to destroy the baryon formed at larger q. Since the effects of T and q are opposite, then the melting point $T_{\rm melt}$ increases with q.

This implies that the baryon configuration constructed by the vertex and F strings can not be obtained for $q = 0$.

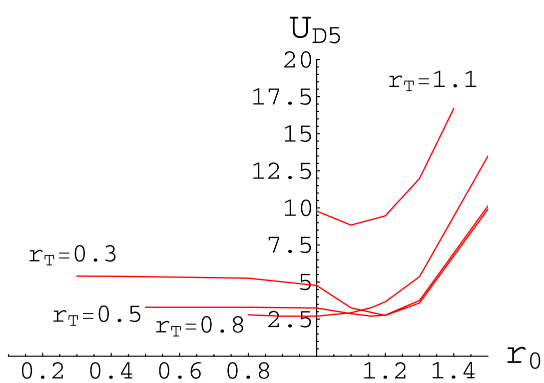

FIG. 10 (color online). The vertex energy U_{D5} versus r_0. The minimum of U_{D5} exists near $r_0 = q^{1/4}$. For $r_T \geq 0.8$, the minimum increases rapidly.

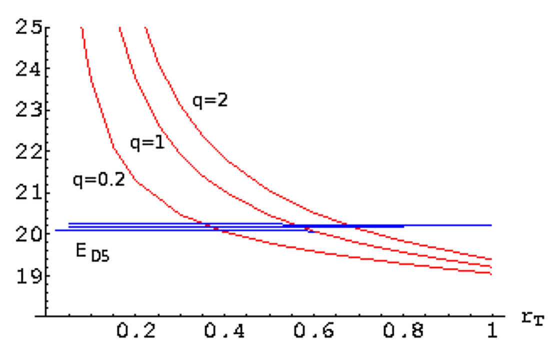

FIG. 11 (color online). The T dependence of the minimum energy of the N-quark baryon (E_{D5}) are shown by the top, middle, and the bottom lines for $q = 2.0$, 1.0 and 0.2, respectively. The three curves show $N\tilde{m}_q$ for $q = 2.0$, 1.0, and 0.2, and $r_{\rm max} = 10$.

In other words, the gauge condensate is essential to find the $k(\leq N)$-quark baryon at finite temperature. On the other hand, mesons are expected to be observed even in the deconfinement high temperature phase of $q = 0$ [14].

This is understood also from the vertex energy given by (28) which is represented for $q = 0$ as follows:

$$U_{D5} = \tilde{N}\sqrt{r^2 - \frac{r_T^4}{r^2}}. \tag{30}$$

This implies that the vertex is not stable and it vanishes into the horizon. But this result is given for the structureless vertex [14]. When the structure is considered, the end point of the vertex increases infinitely toward the boundary when we solve the configuration of the D5-brane, $r(\theta)$. The reason is that the mass scale to retain r_c at a finite value is given by q. For $q = 0$, the configurations given above do not exist. In other words, the confinement force given at zero temperature should be retained at finite temperature in order to get the $k(\leq N)$-quark baryon. This point should be verified by the experiments at the finite temperature.

VI. SUMMARY AND DISCUSSION

The baryon is studied here from a holographic approach. It has a complicated structure since it is constructed by the N quarks and their vertex, which would not be a point but an extended object. From the holographic viewpoint, the baryon vertex is identified with the D5-brane and its configuration is obtained by solving the equation of motion given by D5-brane action which includes N fundamental string flux. At zero temperature, this vertex has two typical structures. When we observe it in our real space-time, its shape is a pointlike or one-dimensionally extended object. From the 10D supergravity side, both configurations have a further complicated structure due to the extra six-dimensional coordinates.

For simplicity, here, we study the baryon with the vertex, which appears as a point in the 4D space-time, at finite temperature. In our model, the quarks are not confined in the sense that the effective free quark mass is finite. In this sense, the theory is in the deconfinement phase. Then, the fundamental strings could touch on the horizon and the baryon configuration with the $k(<N)$ quark is possible in this phase. The configuration of the k-quark baryon is formed by the D5-brane vertex and k fundamental strings which connect the cusp of the vertex and the boundary. The boundary is set here at the finite radial coordinate of the extra dimension as a cutoff point of the string energy or the position of the flavor quark brane. And the remaining $N - k$ fundamental strings are connecting the vertex and the horizon. Then the k-quark baryons with $k \leq N$ in a thermal medium are studied here.

The structure of the vertex becomes a little rich at finite temperature since two extra new solutions of vertex configuration are found. They are not seen at zero temperature. However, we find that these new configurations cannot be observed since they are unstable against the decay into the free quarks. This instability is studied through the calculation of the baryon energy for its fixed size L. The size is defined by the distance between the vertex and a quark. We assumed that the quarks in the baryon are distributed maximally symmetric, then we impose the no-force condition given at the connecting point of the fundamental strings and the vertex. The energy of the k-quark baryon is obtained in this way and it is compared with the one of the free k quarks.

From these analyses, the baryon energy is smaller than the one of the free quarks at small L. And we find a point $L = L^*$, where the baryon energy exceeds the one of the free quarks. Then the baryon with large size $(L^*<)L$ decays into the free quarks, we call this point the melting point. The size depends on the vertex configuration through the no-force condition, and the configurations newly found at finite temperature give a larger size than L^*.

The value of L^* can also be related to the Debye length, which was originally introduced for the Coulomb potential in electromagnetism, and it depends on the temperature. This is estimated here by multiplying the Gauss damping factor to the linearlike potential, which appear as the tail of the zero temperature linear potential. And we find the expected behavior of the Debye mass gap, which is proportional to the temperature. This temperature dependence is seen for all the states of the k-quark baryon, but small k-quark baryons are restricted to low temperature.

Furthermore, we find that the baryon stability at finite temperature is supported by the gauge condensate q. The thermal medium destabilizes the baryon by the screening effect of the color force, but q support the color force to make a linear potential to retain the baryon at a not so high temperature, and we could observe the baryon even at finite temperature.

ACKNOWLEDGMENTS

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