D4 brane probes in gauge/gravity duality

Yang Zhou

Institute of Theoretical Physics, Chinese Academy of Science, Beijing 100190, People's Republic of China and Interdisciplinary Center of Theoretical Studies, USTC, Hefei, Anhui 230026, People's Republic of China

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We propose a Dirac-Born-Infeld vertex brane + N_c fundamental strings configuration for a probe baryon in the finite-temperature thermal gauge field via AdS/CFT correspondence. In particular, we investigate properties of this configuration in QCD_4 and warped $AdS_6 \times S^4$. We find that, in the D4–D8 system, a holographic probe baryon can be described as N_c fundamental strings connecting through a vertex D4 brane wrapped on S^4. In QCD_4 background, a closed vertex can exist in a confined phase but cannot exist in a deconfined phase. In the low temperature region, the screening effect still exists in the confined phase like a meson, and the vertex D4 brane dominates the baryon mass. The lower energy state corresponds to the vertex brane closer to the radial cutoff position ($r = r_c$), and the higher energy state corresponds to the vertex brane a little farther away from the cutoff position. The high energy limit of this configuration is just like the unclosed vertex brane configuration in a higher temperature deconfined phase. In warped $AdS_6 \times S^4$ background, a closed vertex can exist in a deconfined phase and the vertex contains a spike, while fundamental strings are relatively short. The screening length should be defined through the distance between the top position of the vertex spike and the boundary.

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I. INTRODUCTION

In recent years, there has been much interest in studying hadrons in strongly coupled QCD in terms of AdS/CFT correspondence. One important topic is the study of holographic baryons in gravity background [1–4]. A holographic baryon in gauge/gravity duality was first introduced by Witten [5], where a baryon is identified with a compact D brane wrapped on a transverse sphere with N_c strings attached to it. Investigations of baryons in AdS/CFT started ten years ago [6,7], in order to find a solution with a compact vertex brane and Dirac-Born-Infeld (DBI) strings, but there are many challenges [6–10]. One big problem is how to get a closed brane solution for a baryon vertex from the DBI + CS (Chern-Simons) action in a certain gravity background which is dual to the gauge field we want to study. We construct a new configuration with a wrapped vertex brane and N_c strings to solve these problems. Furthermore, we investigate properties of this configuration in a probe limit and argue that these properties may have some signals in the quark-gluon plasma (QGP) in the Relativistic Heavy Ion Collider (RHIC) experiment.

Recently, a simple configuration of baryon in a hot strongly coupled super Yang-Mills plasma was proposed [11], where the screening length of a moving baryon with finite velocity was investigated. Screening length and J-E^2 behavior (J is angular momentum and E is baryon mass) of high spin baryons were analyzed [12]. All these investigations are in the framework of thermal super Yang-Mills gauge theory/AdS black hole duality and component quarks are considered as probes. The main results of these works show that baryons in a hot strongly coupled plasma have a screening length, which is similar to the meson case. Boost velocity and angular velocity dependence of the screening length for baryons are both similar to those of mesons. These results are natural and reasonable, because there the vertex brane is treated as a massive point in AdS_5, with an action depending only on the gravity potential.

Generally, the vertex brane is not a point in AdS_5, but a line (trajectory of a wrapped brane on S^5), which has an action with a DBI term and a Chern-Simons term. Though it is difficult to find a good solution for the vertex brane in many gravity backgrounds, in the new configuration we propose, we obtain two kinds of solutions in QCD_4 and $AdS_6 \times S^4$ background, respectively, each of which has one closed vertex brane and N_c hanging strings.[1] From new configurations constructed, we find that baryons have some special properties in the probe limit. An apparent property is that baryon mass may be dominated by the vertex brane, so the "melting picture" of a probe baryon in the quark-gluon plasma is very different from the meson melting.

Another way to see the role of a baryon vertex in gauge/gravity framework is through the finite quark density (or we also say baryon density) D_{color}-D_{flavor} system. The chemical potential of a finite quark density was introduced as a time component of the U(1) gauge field on flavor branes [13], and the chemical potential of isospin density was introduced as a time component of the SU(2) gauge field on flavor branes. Finite quark and isospin density affect embedding of the flavor brane, as well as the phase transition corresponding to meson dissociation [14–16]. In a D_{color}-D_{flavor} system, a flavor brane can have Minkowski embedding or black hole embedding. But in the case of

[1] QCD_4 has the same mean as the one in [6].

finite quark density, it has been argued that D_{flavor} branes have to touch the black hole horizon since the strings connecting the D_{flavor} branes and the horizon can be replaced by the deformation of the D_{flavor} brane [13], and thus there is no Minkowski embedding in the finite quark density case. In this case, a baryon vertex can end hanging strings outside of the horizon, so there is no way for a flavor brane to touch the horizon [2]. Since this argument, we can see that we can safely discuss a baryon in the probe limit in the Minkowski embedding. We should also note that a baryon in our probe limit is different from the baryons in finite density which should be considered as background baryons. In our probe limit, a baryon almost has no backreaction on flavor branes.

To find a suitable D_{color}-D_{flavor} system for discussion, we notice the QCD_4 gravity background from type IIA string theory. The D4 branes background of QCD_4 with Euclidean signature have two compactified directions and D8 branes are flavors [17]. There are two main phases, confinement and deconfinement, in this holographic model. In the present paper, we will investigate the properties of a probe baryon in these two phases, which is an extended work on baryon probe, of the former work [18]. It is believed that the heavy-quark bound state can survive in a quark-gluon plasma at a temperature which is higher than the confinement/deconfinement transition temperature [19]. We want to have a closer look at the baryon configuration in these different phases, especially in the deconfined phase of a hot quark-gluon plasma.

The present paper is organized as follows. In Sec. II, we review different phases of QCD_4 at different temperatures. We also review a point brane + strings model and a DBI brane model of a holographic baryon in phases in QCD_4 background and the main properties of $AdS_6 \times S^4$ background that we will use. In Sec. III, we propose a new baryon configuration in different backgrounds. In Sec. IV, we study the screening length, baryon mass, and interacting energy in QCD_4 background. In the last section, we study the baryon properties and define a new screening length in warped $AdS_6 \times S^4$ background.

II. REVIEW OF DIFFERENT BACKGROUND PHASES AND BARYON PROBE MODEL

A. Different phases of D4 branes background

QCD_4 background.—We summarize different geometry backgrounds from the D4–D8 branes model [17]. The main success of this model is the prediction of the spectrum of low energy hadrons and their dynamics, which are nonperturbative properties of QCD. It is a good choice for the top-down holographic QCD model since it contains chiral fermions and an apparent chiral symmetry breaking mechanism. Gluons are represented in terms of fluctuating modes of open strings on N_c D4 branes. Massless quarks are represented in terms of fluctuation modes of open strings connecting N_c D4 branes and N_f $D8(\overline{D8})$ branes.[2] The supergravity description of N_c D4 branes and the gauge field description of the D4 branes give the gauge/gravity duality. If $N_c \gg N_f$, D8 branes which give the flavor freedom have no backreaction to the background geometry and can be considered as probes. When the excitations on D4 branes are very heavy, there will be a black hole in the bulk, which corresponds to the thermal gauge field on the boundary. At different temperatures, different gravity backgrounds appear [20]. The confining background metric of the Euclidean model at zero temperature is

$$ds^2 = \left(\frac{r}{R}\right)^{3/2}(dt_E^2 + d\vec{x}^2 + f(r)dx_4^2) + \left(\frac{R}{r}\right)^{3/2}\left(\frac{dr^2}{f(r)} + r^2 d\Omega_4^2\right) \tag{2.1}$$

with a dilaton and a 4-form Ramond-Ramond (RR) field strength

$$e^{\phi} = g_s\left(\frac{r}{R}\right)^{3/4}, \qquad F_4 = \frac{2\pi N_c}{V_4}\epsilon_4, \tag{2.2}$$

where $f(r) = 1 - (\frac{r_c}{r})^3$. The duality relation is $R^3 = \pi g_s N_c l_s^3$. g_s, N_c, and l_s are string coupling constant, color number, and string length scale, respectively. $\vec{x} = x_{1,2,3}$, r is the radial coordinate, and Ω_4 is four angle coordinates, in the $x_{5,6,7,8,9}$ space. $V_4 = 8\pi^2/3$ is the volume of the unit S^4 and ϵ_4 is the volume 4-form. To cancel the conical singularity at $r = r_c$, the period of δx_4 in the compactified direction must be

$$\delta x_4 = \frac{4\pi}{3}\left(\frac{R^3}{r_c}\right)^{1/2}. \tag{2.3}$$

The Kaluza-Klein mass is defined as

$$M_{KK} \equiv \frac{2\pi}{\delta x_4} = \frac{3}{2}\left(\frac{r_c}{R^3}\right)^{1/2}. \tag{2.4}$$

In this phase, we can obtain the glueball and meson spectra by computing the fluctuation of background supergravity fields and fields on the flavor branes, respectively. Their spectra are discrete, which shows that the system is confined. We regard the Hawking temperature of the background as the temperature of the thermal field. We have $T = 1/\beta$, where β is the period of Euclidean time. At the high temperature region, the phase is deconfined. The background geometry contains a black hole:

[2] We do not consider these light quarks in the same way as [18] and just consider a probe baryon composed with heavy quarks in our present work.

$$ds^2 = \left(\frac{r}{R}\right)^{3/2}(f(r)dt_E^2 + d\vec{x}^2 + dx_4^2) + \left(\frac{R}{r}\right)^{3/2}\left(\frac{dr^2}{f(r)} + r^2 d\Omega_4^2\right) \quad (2.5)$$

with a dilaton and a 4-form RR field strength

$$e^{\phi} = g_s\left(\frac{r}{R}\right)^{3/4}, \qquad F_4 = \frac{2\pi N_c}{V_4}\epsilon_4, \quad (2.6)$$

where $R^3 = \pi g_s N_c l_s^3$, $f(r) = 1 - (\frac{r_0}{r})^3$. The t_E must have period

$$\delta t_E = \frac{4\pi}{3}\left(\frac{R^3}{r_0}\right)^{1/2} = \frac{1}{T}. \quad (2.7)$$

This background metric has the same form as the confining one (2.1), only with the exchanging of t_E and x_4.

Warped $AdS_6 \times S^4$ background.—Massive type IIA supergravity admits a warped $AdS_6 \times S^4$ vacuum solution, which is expected to be dual to an $\mathcal{N} = 2$, $D = 5$ superconformal Yang-Mills theory. This background is supported by Ramond-Ramond field strengths, in addition to a nonconstant dilaton. The $AdS_6 \times S^4$ metric is warped, with a warped factor which becomes singular on the equator of S^4. Thus the geometry really corresponds to a hemisphere instead of the full S^4. This background arises as the near-horizon geometry of a semilocalized D4–D8 system [21,22]. In the string frame, this solution is given by [23,24]

$$ds_{10}^2 = (\cos\theta)^{-1/3}[ds_{AdS_6}^2 + 2g^{-2}(d\theta + \sin^2\theta d\Omega_3^2)]. \quad (2.8)$$

Four-form field strength and the dilaton take the forms

$$F_{(4)} = \frac{5\sqrt{2}}{6}g^{-3}(\cos\theta)^{1/3}\sin^3\theta d\theta \wedge \Omega_{(3)}, \quad e^{\phi} = (\cos\theta)^{-5/6}, \quad (2.9)$$

where $\Omega_{(3)}$ is a three sphere volume form. The AdS black hole metric is

$$ds_{AdS_6}^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(\Sigma_{i=1}^4 dx^i dx^i), \quad (2.10)$$

where $f(r) = g^2 r^2 - \frac{\mu}{r^3}$, and g and μ are two parameters independent on r.

B. Point brane + hanging strings configuration for baryon

Holographic baryons have configurations of hanging strings attached to a compact vertex brane, while baryons in the boundary field are composed of external heavy quarks. In the probe limit, each component quark attached with a fundamental string is considered as a probe string in the bulk. Since these fundamental strings connect with a vertex brane, we can see this vertex brane as a probe. In recent works [11,12], the configuration of an open strings + compact D5 brane wrapped on the S^5 was analyzed. The background is given by

$$ds^2 = -f(r)dt^2 + \frac{r^2}{R^2}dx_3^2 + \frac{r^2}{R^2}(d\rho^2 + \rho^2 d\theta^2) + \frac{1}{f(r)}dr^2 + R^2 d\Omega_5^2, \quad (2.11)$$

where

$$f(r) = \frac{r^2}{R^2}(1 - \frac{r_0^4}{r^4}).$$

The action of the baryon is the summation of N_c fundamental strings and a D5 brane

$$S_{\text{total}} = \sum_{i=1}^{N_c} S_{\text{string}}^{(i)} + S_{\text{D5}}, \quad (2.12)$$

where the action of D5 is given by a massive point action in the gravity field

$$S_{\text{D5}} = \frac{\mathcal{V}(r_e)\mathcal{T}V_5}{(2\pi)^5\alpha'^3}. \quad (2.13)$$

The screening length of the baryon with finite moving speed in a plasma was computed [11]. High spin baryons in the $AdS_5 \times S^5$ were investigated and the angular velocity dependence of the screening length and J-E^2 behavior were computed numerically [12]. However, all these computations were done in the conformal and supersymmetric background, and the D5 brane is treated as a massive point in AdS_5. The QCD_4 background is a good choice for the nonsupersymmetric and nonconformal backgrounds. We can construct a new configuration with a DBI D4 brane and N_c strings in this background.

C. A brane model for baryon vertex

Ten years ago, people believed that the main information of a holographic baryon was hidden in the vertex brane. We can indeed get interesting information of a holographic baryon from the DBI + CS action of a compact brane. We take a confining background, for example, as follows:

$$ds^2 = \left(\frac{r}{R}\right)^{3/2}(dt_E^2 + d\vec{x}^2 + f(r)dx_4^2) + \left(\frac{R}{r}\right)^{3/2}\left(\frac{dr^2}{f(r)} + r^2 d\Omega_4^2\right), \quad (2.14)$$

where r is the radial coordinate, and the embedding function of the compact D4 brane wrapped on the S^4 can only be determined by $r(\theta)$. And the gauge field on the single D4 brane can also be written as $A(\theta, t)$ for symmetry. The action of the D4 brane obtained from the induced metric is given by [6,7]

$$S_{\rm D4} = -T_4 \int d^5\xi e^{-\tilde{\phi}} \sqrt{-\det(g+F)} + T_4 \int A_{(1)} \wedge G_{(4)}. \tag{2.15}$$

From this action, we can solve the equation of motion for the gauge field and find the embedding function $r(\theta)$.

It is argued that a closed curve $r(\theta)$ corresponds to a real baryon vertex. The vertex brane dynamics can also be solved in a deconfined phase, but there is no closed solution [7]. However, there is abundant evidence to support the fact that heavy-quark bound states can survive in a quark-gluon plasma [19], which is in the deconfined phase from the lattice results. So there is an apparent paradox. To solve this problem, we try the deconfined warped AdS black hole background in Sec. III C in the present paper, where the warped AdS example reflects more faithfully the physics expected in actual QCD.

III. DBI BRANE + N_c STRINGS CONFIGURATION IN DIFFERENT PHASES

In this section, we propose a DBI brane + N_c strings configuration for a holographic baryon probe and investigate its properties in different phases of D4 brane backgrounds.

A. DBI brane + N_c strings in confined QCD$_4$ background

We start from the confined background:

$$ds^2 = \left(\frac{r}{R}\right)^{3/2}(dt_E^2 + d\vec{x}^2 + f(r)dx_4^2) + \left(\frac{R}{r}\right)^{3/2}\left(\frac{dr^2}{f(r)} + r^2 d\Omega_4^2\right). \tag{3.1}$$

This metric with periodic Euclidean time and a compactified space direction x_4 corresponds to a four-dimensional boundary thermal field. We consider a static baryon in the thermal field as hanging open strings attached to a D4 brane wrapped on the S^4. Open strings connect flavor branes and the single compact D4 brane. In this configuration, we denote the world volume coordinates of the D4 brane as $(t, \theta, \alpha, \beta, \gamma)$. And the embedding function is $r = r(t, \theta, \alpha, \beta, \gamma)$, and the U(1) gauge field on the D4 brane is $A_\mu = A_\mu(t, \theta, \alpha, \beta, \gamma)$. The induced metric on the D4 brane is

$$ds_{\rm D4}^2 = -\left(\frac{r}{R}\right)^{3/2} dt^2 + \left(\frac{R}{r}\right)^{3/2}\left[\left(\frac{r'^2}{f(r)} + r^2\right)d\theta^2 + r^2\sin^2\theta d\Omega_3^2\right]. \tag{3.2}$$

The action of the compact D4 brane can be given as

$$S_{\rm D4} = -T_4 \int d^5\xi e^{-\tilde{\phi}} \sqrt{-\det(g+F)} + T_4 \int A_{(1)} \wedge G_{(4)}, \tag{3.3}$$

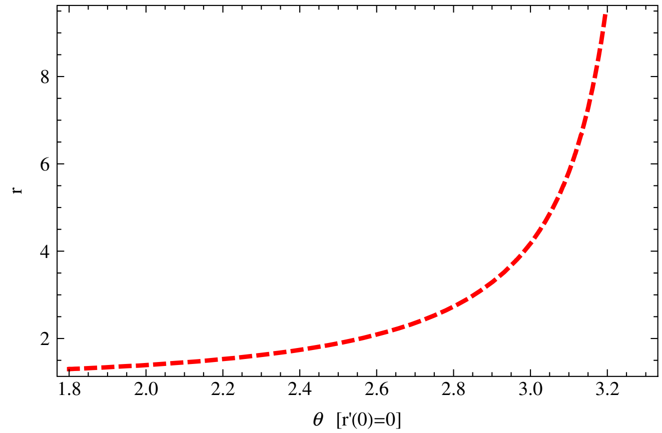

FIG. 1 (color online). Embedding function of the D4 brane in the $r - \theta$ plane.

where $g_{\mu\nu}$ is the induced metric on the D4 brane. $F = dA$, $T_4 = 1/(g_s(2\pi)^4 l_s^5)$ is the D4 brane tension, and the last term is the Wess-Zumino term. We suppose that the D4 brane wrapped on the S^4 has a SO(4) symmetry, and the embedding function and gauge field depend only on (t, θ). For a static configuration, we have $r = r(\theta)$ and $A_t = A_t(\theta)$. We can rewrite the action by performing a Legendre transformation [6][3]:

$$\mathcal{H} = T_4\Omega_3 R^3 \int d\theta \sqrt{r^2 + f(r)^{-1} r'(\theta)^2} \sqrt{D(\theta)^2 + \sin^6\theta}, \tag{3.4}$$

where the displacement D satisfies

$$\partial_\theta D = -3\sin^3\theta, \tag{3.5}$$

which comes from the equation of motion of the gauge field. Solving Eq. (3.5), we get

$$D(\theta) = 3\cos\theta - \cos^3\theta - 2 + 4\nu. \tag{3.6}$$

ν in the constant of integration is a parameter $0 \le \nu \le 1$, which controls the number of Born-Infeld strings emerging from the D4 brane at each pole of the S^4 ($\theta = \pi$ and $\theta = 0$) [6,7]. We find that the spikes at $\theta = 0$ and $\theta = \pi$ have the same asymptotic "tension" as νN_c and $(1-\nu)N_c$ fundamental strings, respectively. We hope that the solution which contains a vertex brane with excited stringlike spikes can be obtained from the total action of the vertex brane. But lots of challenges remain[6,7,10,25,26]. So we always use the force balance condition (FBC) to cancel the singularity of a vertex brane.

Now we give the numerical result in Fig. 1. We set $r'(0) = 0$ and consider $\nu = 0$, which means that N_c fundamental strings all connect with the north pole of S^4. The N_c fundamental strings hang from the flavor brane (we just consider the single flavor). By the following string embed-

[3]Since this is a static solution, we ignore the time component.

ding

$$\tau = t, \quad \sigma = r, \quad \rho = \sqrt{x_1^2 + x_2^2} = \rho(r), \quad x_3 = \text{const}, \tag{3.7}$$

where $x_i (i = 1, 2, 3)$ is the boundary spatial direction, the action can be written as

$$S_{\text{string}} = \frac{1}{2\pi\alpha'} \int dt \int_{r_e}^{r_\Lambda} dr \sqrt{-\det[h_{ab}]}, \tag{3.8}$$

thus the string world sheet Lagrangian density is

$$\mathcal{L} = \sqrt{-\det[h_{ab}]}, \tag{3.9}$$

where

$$h_{ab} = g_{\mu\nu} \frac{\partial x^\mu \partial x^\nu}{\partial \sigma^a \partial \sigma^b}.$$

The total action is

$$S_{\text{total}} = \sum_{i=1}^{N_c} S_{\text{string}}^{(i)} + S_{\text{D4}}. \tag{3.10}$$

To obtain the FBC, we rewrite the action of the D4 brane:

$$S'_{\text{D4}} = \int dt \mathcal{H}. \tag{3.11}$$

Extremizing S_{total} with respect to r_e, we get FBC

$$\sum_{i=1}^{N_c} H^{(i)}|_{r_e} = \Sigma, \tag{3.12}$$

where

$$H^{(i)} \equiv \mathcal{L}^{(i)} - \rho'^{(i)} \frac{\partial \mathcal{L}^{(i)}}{\partial \rho'^{(i)}}, \tag{3.13}$$

$$\Sigma \equiv \frac{2\pi\alpha'}{\mathcal{T}} \frac{\partial S'_{\text{D4}}}{\partial r_e} = 2\pi\alpha' \frac{\partial \mathcal{H}}{\partial r_e}. \tag{3.14}$$

For a given r_0 we can get the value of $r_e = r(\pi)$ from the equation of motion (EOM) of $r(\theta)$. Then we can get the embedding function of N_c strings with the FBC.

We consider a D8 brane as a flavor brane. The interaction between the D8 brane and the background N_c D4 branes makes the D8 brane embed nontrivially in the confined geometry. The DBI action of the D8 brane is

$$S_{\text{D8}} = T_8 \int d\xi^9 \sqrt{-\det(g + F)}, \tag{3.15}$$

where the D8 brane extends $(t, \vec{x}, x_4, \Omega_4)$. The gauge field fluctuations on the flavor brane always correspond to vector mesons in the boundary field. In the present paper, we ignore these fluctuations. After the ansatz of the D8 brane, the embedding function is only determined by $x_4(r)$ and we can obtain the solution by solving the equation of motion with an initial condition. In the D4–D8 system, the heavy quarks have no backreaction to the system, so we ignore the pull force of hanging strings to the flavor brane. As discussed in our former work [12], we can obtain solutions and define the screening length.

Let us turn to the solution of the compact D4 brane we obtained in Fig. 1. We get a very large r_e if we change the initial $r(0)$ to a suitable one. The cusp of the D4 brane will replace the fundamental strings and touch the flavor brane if $r_e \geq r_\Lambda$. But, since we assume that N_c probe quarks (not dynamic) have no backreaction to D8, here N_c hanging strings do not need to be replaced by the deformed D8 brane. It is very different from the case of finite quarks density where the deformed flavor brane always replaces many open strings connected with the horizon. In experiments, component quarks in probe baryons which can survive in the QGP are much heavier than the background quarks or gluons in quark-gluon plasma. There will be some apparent properties of the probe in the medium, especially when we boost or rotate it. These properties are important signals for hot strongly coupled plasma.

B. DBI brane + N_c strings in deconfined QCD_4 background

As the temperature rises up, the D4–D8 system will undergo a first order phase transition, where the gluonic degree of freedom gets deconfined. The corresponding background geometry becomes

$$ds^2 = \left(\frac{r}{R}\right)^{3/2} (f(r)dt_E^2 + d\vec{x}^2 + dx_4^2) + \left(\frac{R}{r}\right)^{3/2} \left(\frac{dr^2}{f(r)} + r^2 d\Omega_4^2\right). \tag{3.16}$$

The investigation of baryon probes in this phase is similar to the investigation in the confined phase. The induced metric on the vertex D4 brane is

$$ds_{\text{D4}}^2 = -\left(\frac{r}{R}\right)^{3/2} f(r)dt^2 + \left(\frac{R}{r}\right)^{3/2} \left[\left(\frac{r'^2}{f(r)} + r^2\right) d\theta^2 + r^2 \sin^2\theta d\Omega_3^2\right]. \tag{3.17}$$

The DBI action of the compact D4 brane is

$$S_{\text{D4}} = -T_4 \int d^5\xi e^{-\tilde{\phi}} \sqrt{-\det(g + F)} + T_4 \int A_{(1)} \wedge G_{(4)}. \tag{3.18}$$

Thus using the same method in the confined case, we obtain the energy function

$$\mathcal{H} = T_4 \Omega_3 R^3 \int d\theta \sqrt{f(r) r^2 + r'^2} \sqrt{D(\theta)^2 + \sin^6\theta}. \tag{3.19}$$

Among the solutions of $r(\theta)$ from the above Lagrangian, we cannot find a closed one. $r(\pi)$ of all solutions run to infinity.[4] But we can still try to get the baryon configuration as in [6] where the background has no x_4 direction and becomes Euclidean effectively. And an effective baryon there has a transformed Lagrangian which is similar to (3.4).

C. DBI brane + N_c strings in warped $AdS_6 \times S^4$

Massive type IIA supergravity admits a warped $AdS_6 \times S^4$ vacuum solution, which is expected to be dual to an $\mathcal{N} = 2$, $D = 5$ superconformal Yang-Mills theory. We study a DBI brane + N_c strings configuration for a baryon in this background. In Eq. (2.8), we note that the metric is singular at $\theta = \frac{\pi}{2}$, thus θ covers $[0, \frac{\pi}{2})$ in a hemisphere instead of the full S^4. We shall study DBI vertex brane + hanging strings configuration in this geometry, which corresponds to a baryon state in $\mathcal{N} = 2$, $D = 5$ superconformal Yang-Mills theory. We denote coordinates of three sphere as (α, β, γ) and world volume coordinates of a vertex D4 brane as $(\tau, \xi^1, \xi^2, \xi^3, \xi^4)$. By the following consistent ansatz that describes the embedding D4 brane

$$\begin{aligned} \tau &= t, \qquad \xi^1 = \theta, \qquad \xi^2 = \alpha, \\ \xi^3 &= \beta, \qquad \xi^4 = \gamma, \qquad r = r(\xi^1), \end{aligned} \tag{3.20}$$

we write the induced metric of the D4 brane

$$\begin{aligned} ds_{\rm D4} = (\cos\xi^1)^{-1/3}\Big[-f(r)d\tau^2 + \Big(\frac{r'^2(\xi^1)}{f(r)} + 2g^{-2}\Big)d^2\xi^1 \\ + 2g^{-2}\sin^2\xi^1 d\Omega_3^2\Big]. \end{aligned} \tag{3.21}$$

The world volume action of the D4 brane contains a DBI term and a Chern-Simons term

$$S_{\rm D4} = S_{\rm DBI} + S_{\rm CS}, \tag{3.22}$$

where

$$\begin{aligned} S_{\rm DBI} &= -T_4 \int d\tau d^4\xi \sqrt{-\det(g+F)} \\ &= -T_4 g_s \Omega_3 \Big(\frac{\sqrt{2}}{g}\Big)^3 \int d\tau d\xi^1 \sin^3(\xi^1) \\ &\quad \times \sqrt{r'(\xi^1) + 2g^{-2}f(r) - \cos(\xi^1)^{2/3}F^2_{t\xi^1}}. \end{aligned} \tag{3.23}$$

The Chern-Simons coupling term is

$$\begin{aligned} S_{\rm CS} &= T_4 \int A \wedge \mathcal{P}(F_4) \\ &= -\frac{5\sqrt{2}}{6} g^{-3}\Omega_3 \int d\tau d\xi^1 A_t \cos(\xi^1)^{1/3}\sin^3(\xi^1), \end{aligned} \tag{3.24}$$

where Ω_3 is the volume of unit S^3. We obtain the vertex D4 brane Lagrangian density along ξ^1

$$\begin{aligned} \mathcal{L}_{\rm D4} &= \sin^3(\xi^1)\cos(\xi^1)^{1/3} \\ &\quad \times \sqrt{[r'(\xi^1) + 2g^{-2}f(r)]\cos(\xi^1)^{-2/3} - F^2_{t\xi^1}} \\ &\quad + \frac{5}{12g_s}A_t. \end{aligned} \tag{3.25}$$

Thus the action of the vertex D4 brane can be written as

$$S = -T_4\Omega_3 g_s \Big(\frac{\sqrt{2}}{g}\Big)^3 \int d\tau \mathcal{L}_{\rm D4}. \tag{3.26}$$

The equation of motion of A_t is given by

$$\frac{\partial \mathcal{L}_{\rm D4}}{\partial F_{t\xi^1}} = -D(\xi^1), \tag{3.27}$$

where

$$\partial_{\xi^1} D(\xi^1) = \frac{5}{12g_s}\sin^3(\xi^1)\cos(\xi^1)^{1/3}. \tag{3.28}$$

To solve $D(\xi^1)$, we denote $y = \sin(\xi^1)$, then $D(\xi^1)$ is given by

$$\begin{aligned} D(y) = \frac{5}{12g_s}\frac{1}{1729}3y\Big[-(1-y^2)^{2/3}(135 + 105y^2 \\ + 91y^4) + 135F\Big[\frac{1}{2}, \frac{1}{3}, \frac{3}{2}, y^2\Big]\Big] + C. \end{aligned} \tag{3.29}$$

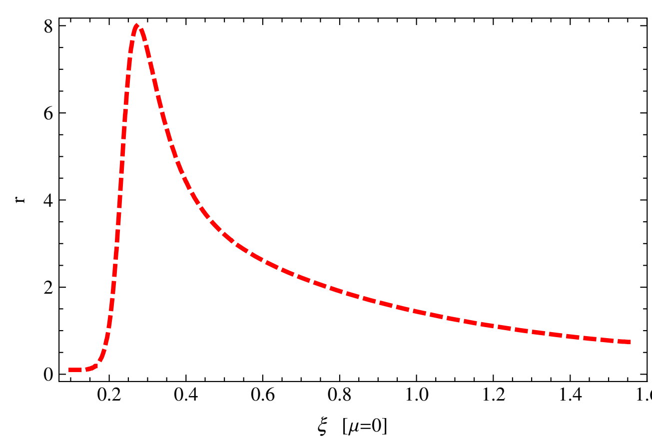

FIG. 2 (color online). Embedding function of the D4 brane in the $r - \xi^1$ plane.

[4]We do not consider this kind of unclosed solutions here due to the following reasons. One is that for not very heavy probe quarks these solutions are usually not regarded as baryons since quarks and gluons are deconfined in this phase. Another reason is that we do not know how to analyze properties of baryon probes in detail by these solutions, though we can also think that for very heavy probe quarks, this D4 brane "spike" by itself represents a bundle of N strings and these solutions can be regarded as baryons within which the quarks have coalesced and are no longer individually discernible [6].

The Legendre transformation of the Lagrangian can help to eliminate the gauge field. We obtain an energy function of the embedding coordinate $r(\xi^1)$ only:

$$\mathcal{H} = -T_4\Omega_3 g_s\left(\frac{\sqrt{2}}{g}\right)^3 \int d\xi^1 \sqrt{[r'(\xi^1) + 2g^{-2}f(r)]\cos(\xi^1)^{-2/3}[\sin^6(\xi^1)\cos(\xi^1)^{2/3} + D^2]}. \tag{3.30}$$

From this Lagrangian, we get the EOM of $r(\xi^1)$

$$\frac{\partial \mathcal{H}}{\partial r} - \partial_{\xi^1}\frac{\partial \mathcal{H}}{\partial r'} = 0. \tag{3.32}$$

To see the result quickly, we solve the EOM numerically in Fig. 2. From Fig. 2, we find that for $\xi^1 \in [0, \frac{\pi}{2})$, r has finite values. Since the hemisphere geometry corresponds to the background with $\theta \in [0, \frac{\pi}{2})$, ξ^1 can only take values between 0 and $\frac{\pi}{2}$. The warped AdS black hole corresponds to a deconfined gauge field theory, so we obtain closed vertex D4 brane solutions from the DBI + CS action naturally.

IV. BARYON PROPERTIES IN QCD$_4$ BACKGROUND

Baryon probes in quark-gluon plasma were first investigated in works [11,12,27], where the background is $AdS_5 \times S^5$ dual to the $\mathcal{N} = 4$ supersymmetric Yang-Mills gauge field, and the vertex D5 brane is just a point in the AdS space in [11,12]. In the D4–D8 system, we consider that the trajectory of the vertex D4 brane is not a trivial point and the embedding function depends on $r(\theta)$. We are interested in the properties of the baryon probe of this configuration. We analyze the screening length and baryon mass behavior in this section.

A. Screening length and baryon melting

In the former work [11,12], the screening length of a baryon was defined as the largest value of the boundary quark separation when we change the position of the bottom of hanging strings r_e. From our solutions, we find that the trajectory of the vertex brane in the bulk radial direction is a finite line. We defined the position of a cusp of the vertex brane as the furthest position which can be probed by the baryon. Thus, we obtain the r_e dependence of the quark separation l_q and get the value of screening length l_s.

We consider boosted and rotating quarks and use the following background[5]:

$$ds^2 = \left(\frac{r}{R}\right)^{3/2}(-dt^2 + dx_3^2) + \left(\frac{r}{R}\right)^{3/2}(d\rho^2 + \rho^2 d\theta^2) + \left(\frac{R}{r}\right)^{3/2}\frac{dr^2}{f(r)}. \tag{4.1}$$

[5]Corresponding hanging strings rotate rigidly in the bulk, which is like the meson case in work [18] and different from the strings with rotating ends in work [28].

The metric is invariant when it is boosted in the x_3 direction. So properties of a baryon are independent of the wind in the x_3 direction. We now focus on baryon configurations rotating in the $x_1 - x_2$ plane at angular velocity ω. For symmetry, each string can be described by $\rho(r)$. By the following consistent ansatz:

$$\tau = t, \qquad \sigma = r, \qquad \theta = \omega t, \qquad \rho = \rho(r). \tag{4.2}$$

We can write the action of a single string as

$$S_{\text{string}} = \frac{1}{2\pi\alpha'}\int dt \int_{r_e}^{r_\Lambda} dr \sqrt{-\det[h_{ab}]}. \tag{4.3}$$

h_{ab} can be read from the induced metric

$$ds^2 = \left(\frac{r}{R}\right)^{3/2}(\rho^2\omega^2 - 1)dt^2 + \left[\left(\frac{r}{R}\right)^{3/2}\rho'^2 + \left(\frac{R}{r}\right)^{3/2}\frac{1}{f}\right]dr^2. \tag{4.4}$$

The string world sheet Lagrangian density is

$$\mathcal{L}_{\text{string}} = \left(\frac{r}{R}\right)^{3/2}\sqrt{(1-\rho^2\omega^2)\left(\rho'^2 + \frac{R^3}{r^3 - r_c^3}\right)}. \tag{4.5}$$

To solve the equation of motion

$$\left[\frac{\partial}{\partial\rho(r)} - \frac{\partial}{\partial r}\frac{\partial}{\partial\rho'(r)}\right]\mathcal{L} = 0. \tag{4.6}$$

We need two boundary conditions $\rho(r_e)$ and $\rho'(r_e)$. The constraint at r_e can be obtained from the FBC in Eq. (3.11):

$$\mathcal{L} - \rho'\frac{\partial\mathcal{L}}{\partial\rho'}\bigg|_{r_e} = \frac{2\pi\alpha'}{N_c}\frac{\partial\mathcal{H}}{\partial r_e}. \tag{4.7}$$

The FBC turns out to be

$$\left(\frac{r}{R}\right)^{3/2}\frac{(\frac{R^3}{r^3-r_c^3})}{\sqrt{(\rho'^2 + \frac{R^3}{r^3-r_c^3})}}\bigg|_{r_e} = \frac{8\pi\alpha' T_4\Omega_3 R^3}{N_c}\frac{f^{-1}r'}{\sqrt{r^2 + f^{-1}r'^2}}\bigg|_{r=r(\theta=\pi)}. \tag{4.8}$$

Finally, by analyzing the solutions we define the screening length as the critical value of the boundary quark separation and find the ω dependence of screening length l_s. The screening length l_s shows that if heavy quarks have enough kinetic energy, they may break away from each other. It means that baryons dissociate.

To obtain the screening length of the DBI brane + strings configuration, we calculate the r_e dependence of l_q numerically which has been discussed more carefully in

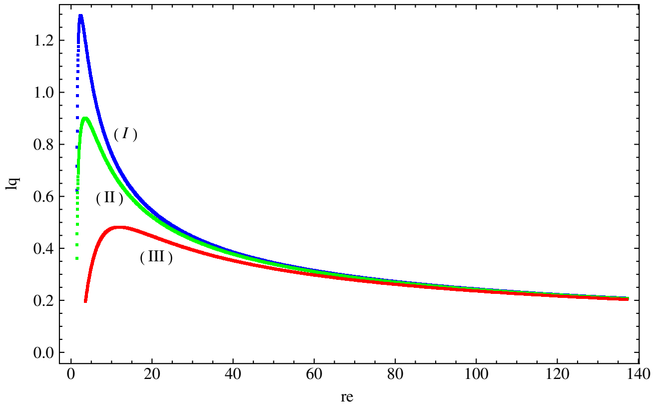

FIG. 3 (color online). r_e dependence of l_q at different values of ω. Curves I, II, and III correspond to $\omega = 0$, 0.5, and 1, respectively. Values of l_q and r_e are determined assuming r_0 as a unit.

[12]. The results with different ω are shown in Fig. 3. We choose the maximal value of l_q (also the critical value) as the screening length for baryons. And we think that only the right part of each curve beside the highest point contains the points which correspond to real baryon configurations. The ω dependence of l_s can be given in Fig. 4.

B. Baryon mass and interaction potential

Now we want to give the definition of the baryon mass and interaction potential. In a very general way, baryon mass is given by summation of the energy of N_c strings and the vertex brane.

$$E_{\rm total} = N_c E_{\rm string} + E_{\rm D4}, \tag{4.9}$$

where

$$E_{\rm string} = \omega \frac{\partial L}{\partial \omega} - L, \qquad E_{\rm D4} = \mathcal{H}. \tag{4.10}$$

The string Lagrangian is

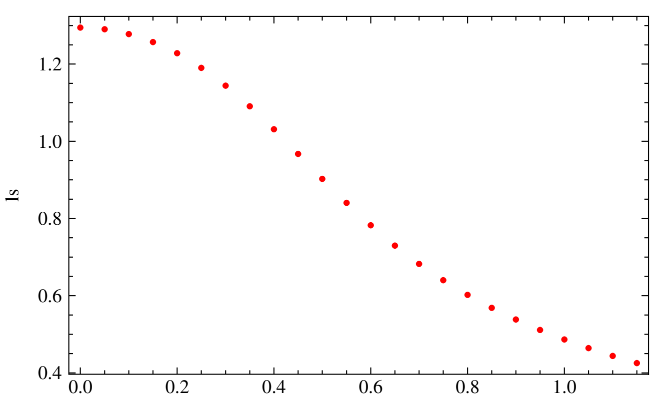

FIG. 4 (color online). ω dependence of l_s.

$$L = \frac{1}{2\pi\alpha'} \int_{r_e}^{r_\Lambda} dr \left(\frac{r}{R}\right)^{3/2} \sqrt{(1 - \rho^2\omega^2)\left(\rho'^2 + \frac{R^3}{r^3 - r_c^3}\right)}. \tag{4.11}$$

In order to obtain the interaction potential, we analyze the free quarks case, in which N_c strings hang from the boundary to r_c and the compact D4 brane is almost wrapped on the $r = r_0$. The so-called interaction potential is given by subtracting the energy of the free strings and the corresponding vertex brane. Assuming the radial position of D8 is r_Λ, the radial distance of r_c and D8 is $r_\Lambda - r_c$, and the energy of a single quark is given by[6]

$$E_q = \frac{1}{2\pi\alpha'} \int_{r_c}^{r_\Lambda} dr, \tag{4.12}$$

and the energy of initial brane which is very close to the "cutoff position"($r = r_c$) is given by

$$\mathcal{H}_0 = T_4 \Omega_3 R^3 r_0 \int d\theta \sqrt{D(\theta)^2 + \sin^6\theta}. \tag{4.13}$$

Then the interaction potential of the baryon is[7]

$$E_I = E_{\rm total} - N_c E_q - \mathcal{H}_0. \tag{4.14}$$

We give the numerical results about l_q dependence of E_I at different values of ω in Fig. 5. Figure 5 shows that we should choose the low energy branch for a given l_q. Comparing with Fig. 3, we find that the low energy branch in Fig. 5 corresponds to the left branch in Fig. 3. It implies that we should choose the smaller r_e for a given l_q. The main reason is that a larger r_e corresponds to a larger D4 brane energy. Actually, for a given r_0 larger than a critical value, the D4 brane will be elongated to the boundary D8 brane and the length of the hanging strings becomes zero. Thus we cannot obtain a closed brane configuration, which is similar to the deconfined case. When the background becomes closer to a deconfined one, quarks will try to run away from each other. In our confined background, it corresponds to the vertex D4 brane elongated to attach to the boundary brane.

C. Discussion

After a simple discussion of point brane + N_c strings in $AdS_5 \times S^5$ in a former work [12], we extend the investigation to a new configuration of a DBI brane + N_c strings in QCD_4 background. Our investigation implies that some properties such as screening length still exist in the new configuration. But the baryon mass and interaction poten-

[6]Actually, there is no free quark in this confined phase. To calculate the interaction energy in the string picture, we should subtract a mass of N_c quarks, which are not real physical objects in this confined phase but a reference.

[7]We should note that the interaction potential here is different from the binding energy; the rigid rotating effect of strings and the vertex brane contribute more energy to make E_I positive.

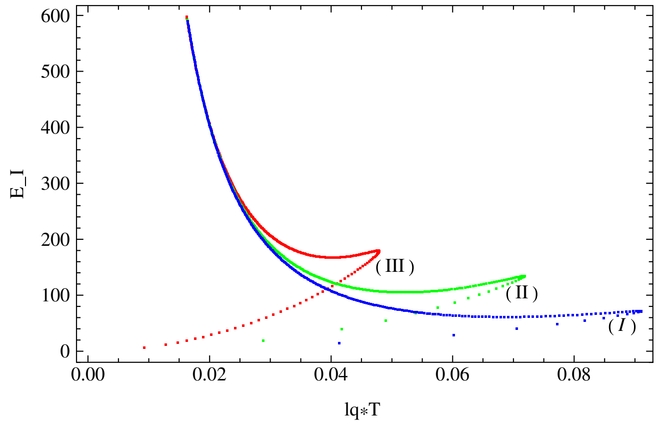

FIG. 5 (color online). l_q dependence of the interaction potential of the N_c quarks. Curves I, II, and III correspond to $\omega = 0.3$, 0.5, and 0.8, respectively. For simplicity, the value of E_I in the figure is actually $E_I * (2\pi\alpha'/N_c)$ in our paper.

tial are very different from the simple model with a point vertex brane in the AdS bulk. In the QCD$_4$ background, there is no horizon and we find that the minimal energy state corresponds to the pointlike D4 brane in the AdS gravity background, connected to N_c hanging strings with the largest length. It appears very different from the N_c strings + point D5 brane as before [11,12], where the energy of the D5 brane depends only on r_e. The energy of the D4 brane dominates the interaction potential in this case.

V. BARYON PROPERTIES IN WARPED AdS$_6 \times S^4$ BACKGROUND

After analyzing the properties of the holographic baryon probe in QCD$_4$ background, we want to see properties of our solution in warped AdS$_6 \times S^4$. We consider boosted and rotating quarks and use the following background:

$$ds^2 = (\cos\theta)^{-1/3}\Big[-\Big(g^2r^2 - \frac{\mu}{r^3}\Big)dt^2 + r^2dx_3^2 + \frac{dr^2}{(g^2r^2 - \frac{\mu}{r^3})} + r^2(d\rho^2 + \rho^2 d\theta^2)\Big]. \tag{5.1}$$

If we stand in the rest frame of quarks, we will feel a moving plasma wind. We now focus on baryon configurations rotating in the $x_1 - x_2$ plane with angular velocity ω in the plasma moving with a wind velocity $v = -\tanh\eta$ in the x_3 direction. For symmetry, each of the strings can be described by $\rho(r)$. By the following consistent ansatz:

$$\tau = t, \qquad \sigma = r, \qquad \theta = \omega t, \qquad \rho = \rho(r), \qquad x_3 = x_3(\sigma). \tag{5.2}$$

We can write the action of a single string as

$$S_{\rm string} = \frac{1}{2\pi\alpha'}\int dt \int_{r_e}^{r_\Lambda} dr\sqrt{-\det[h_{ab}]} = \frac{1}{2\pi\alpha'}\int dt \int_{r_e}^{r_\Lambda} dr\mathcal{L}_{\rm string}. \tag{5.3}$$

h_{ab} can be read from the induced metric on the string world sheet (more details can be read from the Appendix). All hanging strings attach to the highest point of the vertex solution in the r direction, and then the FBC turns out to be

$$L - \rho'\frac{\partial L}{\partial \rho'}\Big|_{r_e} = \frac{2\pi\alpha'(\cos\theta)^{1/3}}{N_c}\frac{\partial\mathcal{H}}{\partial r_e}, \tag{5.4}$$

where $r_e = r_{\rm top}$ [with $r'(\xi^1) = 0$] in the brane solution in Fig. 2. To calculate the $\frac{\partial\mathcal{H}}{\partial r_e}$, we rewrite the energy function of the D4 brane as

$$\mathcal{H} = -T_4\Omega_3 g_s\Big(\frac{\sqrt{2}}{g}\Big)^3\int dr\sqrt{[1 + 2g^{-2}f\xi'^1(r)]\cos(\xi^1)^{-2/3}[\sin^6(\xi^1)\cos(\xi^1)^{2/3} + D^2]}. \tag{5.5}$$

We denote

$$\mathcal{L} = \sqrt{[1 + 2g^{-2}f(r)\xi'^1(r)]\cos(\xi^1)^{-2/3}[\sin^6(\xi^1)\cos(\xi^1)^{2/3} + D^2]}. \tag{5.6}$$

Then the force in the r direction at the highest point is given by

$$\mathcal{L} - \xi'^1\frac{\partial\mathcal{L}}{\partial\xi'^1} = \sqrt{\cos(\xi^1)^{-2/3}[\sin^6(\xi^1)\cos(\xi^1)^{2/3} + D^2]}. \tag{5.7}$$

The FBC supplies the constraint between r_e and $\rho'(r_e)$. For a given $r(\xi^1 = 0)$, we can obtain a maximum value of r in a vertex brane solution, and then we can get the string solutions. By analyzing these string solutions in this AdS$_6 \times S^4$ background, we plot the curve of r_e dependence of l_q by the numerical calculation in Fig. 6. We note that there is no maximal value of boundary quark separation. There are two reasons for this phenomenon: one is that hanging strings in this AdS$_6 \times S^4$ background do not have similar behaviors as in AdS$_5 \times S^5$; the other is that the spike of the vertex brane eliminates the critical behavior of the hanging strings. We note that the spike solution of

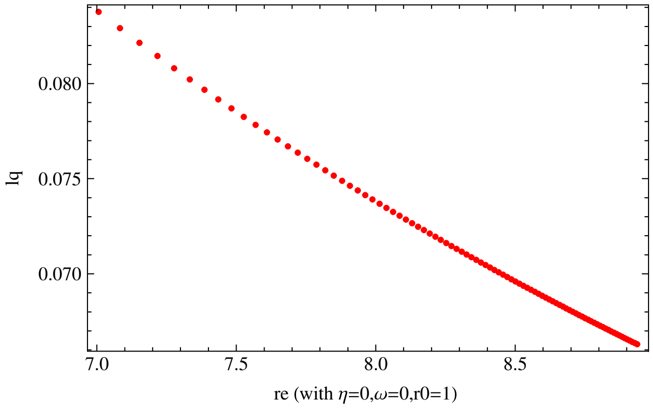

FIG. 6 (color online). r_e dependence of l_q in the $\mathrm{AdS}_6 \times S^4$ background.

the vertex D4 brane tries to replace the hanging strings. This is a signal to show that, in this configuration, the vertex brane dominates the properties of baryons.

In this case, we should choose another way to define a "critical length" (which can be used to judge whether the baryon is physical, but very different from the usual screening length). We note in the QCD_4 deconfined phase, baryon vertex solutions always touch the boundary brane directly and cannot form closed ones; thus we argue that this touching process corresponds to a baryon dissociation in a deconfined medium.[8] From this point of view, we can consider the distance between the top of the vertex brane and the bottom of the boundary brane as another parameter to judge whether baryons dissociate.

In the $\mathrm{AdS}_6 \times S^4$ background, through analyzing the solutions of the D4 vertex brane, we indeed find a maximal value of r_{top} [$r_{\mathrm{top}} = r_e$ is the radial top position of each solution $r(\theta)$] when we change initial conditions $r(\xi^1 = 0)$.[9] Thus, we can define the maximal top position $l_m = \max\{r_{\mathrm{top}}\}$ as a critical position. And the critical distance between l_m and the boundary is $l_c = r_\Lambda - l_m$. We can use this critical distance to judge whether a baryon state can exist due to the following reason. Among these vertex solutions with a spike in the $\mathrm{AdS}_6 \times S^4$ background, we can see from Fig. 6 that sizes of baryons are always very small compared with Fig. 3. Baryons in this background almost stand in a spatial point because the spikes of the vertex brane replace hanging strings to some extent. In this case, the vertex brane dominates the main properties of the baryons. We can use baryon energy to judge whether it is physical or not physical. From the numerical results, we find that larger r_{top} corresponds to larger baryon energy, and a solution with l_m has the largest energy. Beyond this l_m, there is no solution with a spike, so we discard those solutions here. Thus l_m gives an upper limit of baryon energy, which is like the usual screening analysis of holographic baryons. We can understand this point from the rough relation $E_m \sim \frac{1}{r_\Lambda - l_m}$, where E_m is the maximal energy of baryons. To avoid possible confusion, we wish to emphasize that l_m or l_c are not lengths in the gauge theory, but radial parameters in the gravity dual.

Our investigation shows that vertex branes dominate properties of baryons in these configurations. So the velocity dependence of baryon screening should be obtained by studying the vertex brane, and the high spin baryon should be described by the vertex brane with inner J charge.

VI. CONCLUSION AND DISCUSSION

How to understand the confinement and calculate the hadron spectrum are considered as the two biggest problems in QCD (or nonperturbative QCD exactly). So far we know little about the nonperturbative world and have almost no general powerful tool to study the strong coupling problem. AdS/CFT correspondence, which is usually called gauge/gravity duality in general, is believed to be a useful framework to study these problems. On the experiment side, many people believe there exists a QGP state in the RHIC, which is a strongly coupled quark and gluon thermal state like a fluid, investigated in many works [29–32]. How to describe this QGP and understand the strongly coupled behavior is still a problem, though it is very useful for solving confinement and the hadron spectrum problem. It is believed that the heavy-quark bound state can be alive in a QGP, including the J/ψ meson and some multiquark bound states. We call these multiquark bound states baryons, though they may be different from baryons we see when they survive within a QGP. Using a meson or a baryon as a probe is the simplest method to study the properties of the strongly coupled quark-gluon state.

In the gauge/gravity duality framework, we calculate properties of the probe in the classic gravity background. From the strong/weak duality, we know these results are always suitable for the probe in the strongly coupled background on the field side. A lot of works have been done on the meson spectrum and meson melting process in different gauge/gravity systems. When we consider a baryon in the present work, we find the following interesting results:

(1) In the D4–D8 system, a holographic probe baryon can be described as N_c fundamental strings that connect through a vertex D4 brane wrapped on S^4.

[8] Usually, we know that there are no baryon states in the deconfined phase, since quarks and gluons are deconfined. From the holographic solutions, we see that solutions in the deconfined phase are always unclosed. Thus we think that even if quarks can form a state with the unclosed solution, it is not a baryon.

[9] We ignore solutions with $r_{\mathrm{top}} > \max\{r_{\mathrm{top}}\}$, because there is no spike in these solutions. This seems to be the result from the numerical calculation, as we have not found any physical reasons.

(2) In the QCD_4 background, a closed vertex can exist in a confined phase but cannot exist in a deconfined phase. In the low temperature region, a screening effect still exists in the confined phase like a meson and the vertex D4 brane dominates the baryon mass. The lower energy state corresponds to a vertex brane closer to the cutoff position ($r = r_c$) and the higher energy state corresponds to a vertex brane closer to the boundary (or flavor brane exactly). We think it is reasonable, because the high energy limit of this configuration is just like the unclosed vertex brane configuration in a higher temperature deconfined phase.

(3) In the warped $AdS_6 \times S^4$ background, a closed vertex can exist in a deconfined phase and the vertex contains a spike, and the fundamental strings are relatively short. The screening length should be defined through the distance between the top position of the vertex spike and the boundary.

Related extended works can be done in the future. One is finding more evidence from the experiment data to support a live multiquark bound state in the quark-gluon plasma. Another is calculating some special parameters of QGP through a baryon probe and comparing them with experiment data. A clear picture of baryon melting is needed and the energy loss of the baryon probe is also a very interesting unsolved problem [33–37].

ACKNOWLEDGMENTS

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APPENDIX

We denote coordinates in the rest frame of quarks as (t', x'_3), and then we have

$$\begin{aligned} dt &= dt' \cosh\eta - dx'_3 \sinh\eta, \\ dx_3 &= -dt' \sinh\eta + dx'_3 \cosh\eta. \end{aligned} \tag{A1}$$

The boosted metric is given by

$$ds^2 = (\cos\theta)^{-1/3}\bigg[Adt^2 + 2Bdtdx_3 + Cdx_3^2 + \frac{dr^2}{(g^2r^2 - \frac{\mu}{r^3})} + r^2(d\rho^2 + \rho^2 d\theta^2)\bigg], \tag{A2}$$

where

$$\begin{aligned} A &= -\Big(g^2r^2 - r^2 - \frac{\mu}{r^3}\Big)\cosh^2\eta - r^2; \\ B &= \Big(g^2r^2 - r^2 - \frac{\mu}{r^3}\Big); \\ C &= -\Big(g^2r^2 - r^2 - \frac{\mu}{r^3}\Big)\sinh^2\eta + r^2. \end{aligned} \tag{A3}$$

In the warped $AdS_6 \times S^4$ background, h_{ab} can be read from the induced metric

$$ds_{\tau,\sigma} = (\cos\theta)^{-1/3}\bigg[(A + \sigma^2\rho^2\omega^2)d\tau^2 + 2Bx'_3 d\tau d\sigma + \Big(Cx'^2_3 + \frac{1}{g^3\sigma^2 - \frac{\mu}{\sigma^3}} + \sigma^2\rho'^2\Big)d\sigma^2\bigg]. \tag{A4}$$

The string Lagrangian density

$$\mathcal{L}_{\text{string}} = (\cos\theta)^{-1/3}\sqrt{(A + \sigma^2\rho^2\omega^2)\Big(Cx'^2_3 + \frac{1}{g^2\sigma^2 - \frac{\mu}{\sigma^3}} + \sigma^2\rho'^2\Big) - B^2x'^2_3}. \tag{A5}$$

When the wind velocity $v = 0$ ($\eta = 0$), then $x'_3(\sigma) = 0$. The Lagrangian becomes

$$\mathcal{L}_{\eta=0} = (\cos\theta)^{-1/3}\sqrt{\Big(-g^2\sigma^2 + \frac{\mu}{\sigma^3} + \sigma^2\rho^2\omega^2\Big)\Big(\frac{1}{g^2\sigma^2 - \frac{\mu}{\sigma^3}} + \sigma^2\rho'^2\Big)}. \tag{A6}$$

When angular velocity $\omega = 0$, the Lagrangian becomes

$$\mathcal{L}_{\omega=0} = (\cos\theta)^{-1/3}\sqrt{\Big(\frac{1}{g^2\sigma^2 - \frac{\mu}{\sigma^3}} + \sigma^2\rho'^2\Big) + (AC - B^2)x'^2_3}. \tag{A7}$$

The EOM of x_3

$$(\cos\theta)^{1/3}\frac{\partial\mathcal{L}_{\text{string}}}{\partial x'_3} = \frac{[(A + \sigma^2\rho^2\omega^2)C - B^2]x'_3}{\sqrt{(A + \sigma^2\rho^2\omega^2)(Cx'^2_3 + \frac{1}{g^2\sigma^2 - \frac{\mu}{\sigma^3}} + \sigma^2\rho'^2) - B^2x'^2_3}} = F, \tag{A8}$$

where F is a constant. The new Lagrangian containing no $x'_3(\sigma)$ is given by

$$L = \sqrt{\frac{F^2(\frac{1}{g^2\sigma^2 - \frac{\mu}{\sigma^3}} + \sigma^2\rho'^2)}{(A + \sigma^2\rho^2\omega^2)C - B^2 - F^2} + (A + \sigma^2\rho^2\omega^2)\left(\frac{1}{g^2\sigma^2 - \frac{\mu}{\sigma^3}} + \sigma^2\rho'^2\right)}. \tag{A9}$$

All hanging strings attach to the highest point, and then the FBC turns out to be

$$L - \rho' \frac{\partial L}{\partial \rho'}\bigg|_{r_e} = \frac{2\pi\alpha'(\cos\theta)^{1/3}}{N_c}\frac{\partial \mathcal{H}}{\partial r_e}, \tag{A10}$$

where $r_e = r_{\max}$ [with $r'(\xi^1) = 0$] in the brane solution. To calculate the $\frac{\partial \mathcal{H}}{\partial r_e}$, we rewrite the new Lagrangian of the D4 brane as

$$\begin{aligned}\mathcal{H} &= -T_4\Omega_3 g_s\left(\frac{\sqrt{2}}{g}\right)^3 \int d\xi^1 \sqrt{[r'(\xi^1) + 2g^{-2}f(r)]\cos(\xi^1)^{-2/3}[\sin^6(\xi^1)\cos(\xi^1)^{2/3} + D^2]} \\ &= -T_4\Omega_3 g_s\left(\frac{\sqrt{2}}{g}\right)^3 \int dr \sqrt{[1 + 2g^{-2}f(r)\xi'^1(r)]\cos(\xi^1)^{-2/3}[\sin^6(\xi^1)\cos(\xi^1)^{2/3} + D^2]}.\end{aligned} \tag{A11}$$

We denote

$$\mathcal{L} = \sqrt{[1 + 2g^{-2}f(r)\xi'^1(r)]\cos(\xi^1)^{-2/3}[\sin^6(\xi^1)\cos(\xi^1)^{2/3} + D^2]}. \tag{A12}$$

$$\xi'^1(r) = \frac{\partial \xi^1}{\partial r}.$$

Then the force in the r direction at the highest point is given by

$$\mathcal{L} - \xi'^1\frac{\partial \mathcal{L}}{\partial \xi'^1} = \frac{\sqrt{\cos(\xi^1)^{-2/3}[\sin^6(\xi^1)\cos(\xi^1)^{2/3} + D^2]}}{\sqrt{1 + 2g^{-2}f\xi'^1}}\Bigg|_{\xi'^1=0} = \sqrt{\cos(\xi^1)^{-2/3}[\sin^6(\xi^1)\cos(\xi^1)^{2/3} + D^2]}. \tag{A13}$$

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