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All orders analysis of the three dimensional CP^{N-1} model in the $1/N$ expansion

Kiyoshi Higashijima[1,*] and Takahiro Nishinaka[1,†]

[1] *Department of Physics, Graduate School of Science, Osaka University, Toyonaka, Osaka 560-0043, Japan*

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The renormalizability of the three dimensional supersymmetric CP^{N-1} model is discussed in the $1/N$-expansion method, to all orders of $1/N$. The model has N copies of the dynamical field and the amplitudes are expanded in powers of $1/N$. In order to see the effects of supersymmetry explicitly, Feynman rules for superfields are used. All divergences in amplitudes can be eliminated by the renormalizations of the coupling constant and the wave function of the dynamical field to all orders of $1/N$. The beta function of the coupling constant is also calculated to all orders of $1/N$. It is shown that this model has a nontrivial ultraviolet fixed point. The beta function is shown to have no higher order correction in the $1/N$-expansion.

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I. INTRODUCTION

In lower dimensions many quantum field theories are renormalizable. Especially in two dimensions, scalar field theories with arbitrary interactions are renormalizable in perturbation theory. We can easily see this fact by power counting. In two dimensions, a scalar field is dimensionless and any interaction term in the Lagrangian is superrenormalizable.

In three dimensions, however, the situation is different. Since a scalar field now has dimension $1/2$, coupling constants of interaction terms involving more than six scalar fields have negative mass dimensions, implying nonrenormalizability of the theory. In particular, three dimensional nonlinear sigma models are perturbatively nonrenormalizable because they have an infinite number of interaction terms in the Lagrangian.

However, the perturbative nonrenormalizability does not immediately mean that the theory is ill-defined. It only means that we cannot remove divergences order by order in powers of coupling constants. It might be possible to remove divergences by methods other than perturbation theory. To do this, we would have to use some nonperturbative method. Theories which are nonrenormalizable in perturbation theory, but renormalizable by nonperturbative methods, often have interesting nonperturbative phenomena, such as dynamical mass generation, bound states formation, and dynamical generation of the Chern-Simons term. In three dimensions, such nonperturbative phenomena seem to be crucial for the nonperturbative renormalizability. Therefore, it would be interesting to study how these nonperturbative phenomena contribute to the nonperturbative renormalizability of the three dimensional sigma model.

*higashij@het.phys.sci.osaka-u.ac.jp
†nishinaka@het.phys.sci.osaka-u.ac.jp

Actually, some supersymmetric nonlinear sigma models were argued to be renormalizable even in three dimensions by the exact renormalization group method [1]. The supersymmetric CP^{N-1} model is one of the candidates. The CP^{N-1} model is a nonlinear sigma model on the complex projective manifold CP^{N-1}, which was first introduced by Eichenherr [2]. The supersymmetric version of the model was formulated by introducing an auxiliary gauge field [3–5].

The renormalization group method is one of the powerful methods which can reveal the nonperturbative property of the theory. The renormalizability in the renormalization group method is equivalent to the existence of a nontrivial ultraviolet (UV) fixed point of the theory. In the renormalization group analysis in [1], however, the effective action is expanded in powers of derivatives on spacetime and approximated by truncating at the second order of derivatives. Although this approximation is valid in the low energy scale region, it is not obvious in the high energy scale whether the approximation is valid or not.

The existence of the UV fixed point of the three dimensional supersymmetric CP^{N-1} model is also shown by the $1/N$-expansion method up to the next-to-leading order [6,7]. In the $1/N$-expansion, we expand amplitudes in powers of $1/N$ instead of coupling constant, where N is the number of fields involved in the theory. In general, each term of $1/N$ expansion corresponds to a sum of infinite number of Feynman diagrams in perturbation theory. Therefore, the $1/N$-expansion is another powerful nonperturbative method. Indeed, it was argued that the three dimensional nonlinear sigma model and its supersymmetric versions are renormalizable, order by order in the $1/N$ expansion [8,9]. In Refs. [6,7], the beta function of the coupling constant was explicitly evaluated by using Feynman rules in the component field formalism. It was shown that there is no next-to-leading order contribution of $1/N$. Because of supersymmetry, contributions of bosons and fermions cancel each other in the next-to-leading order of $1/N$. There might be, however, contributions of higher orders of $1/N$. Therefore, an all order calculation in $1/N$ is necessary for the complete proof of the existence of the UV fixed point.

In this paper, we study the three dimensional $\mathcal{N} = 2$ supersymmetric CP^{N-1} model in all orders of $1/N$-expansion and show that there is no higher order correction to the beta function in this model. This confirms the existence of the nontrivial UV fixed point to all orders of $1/N$-expansion. We also show explicitly that all divergences can be eliminated by the renormalizations of the coupling constant and the wave function of the dynamical field Φ, namely, the renormalizability in the method of $1/N$-expansion. To show this explicitly we have introduced $\mathcal{N} = 2$ supersymmetry and we use Feynman rules for superfields, which we call "super Feynman rules". For example, a chiral superfield $\Phi(x, \theta, \bar{\theta})$ can be expanded in terms of component fields as

$$\Phi(x, \theta, \bar{\theta}) = \phi(y) + \theta\psi(y) + \frac{1}{2}\theta^2 F(y),$$

where $y^\mu := x^\mu + \frac{i}{2}\bar{\theta}\gamma^\mu\theta$. Therefore, if we know the two-point functions of component fields, we can construct the two-point function of superfield $\Phi(x, \theta, \bar{\theta})$ from them. Using this propagator of superfield, we can explicitly see the cancellation due to supersymmetry.

In Sec. II, we review a part of the argument given in [7]. For the $1/N$-expansion, it is useful to introduce an auxiliary field in the action. Although the auxiliary field has no kinetic term in the classical action, it acquires quadratic terms in the effective action induced by the quantum fluctuations of the dynamical field. When the auxiliary field is introduced, the path integration over the dynamical field becomes a Gaussian integral and can be performed easily. After performing the integration over the dynamical field, we obtain the action with respect to the auxiliary field, which is proportional to N. Therefore, the $1/N$-expansion turns out to be the loop expansion of the auxiliary field. This is the reason for introducing the auxiliary field.

We evaluate the effective action to the leading order of $1/N$ to study the vacuum structure of the model. The model turns out to have two phases, "symmetric phase" and "broken phase." The global $SU(N)$ symmetry is spontaneously broken in the broken phase, while it is unbroken in the symmetric phase. The supersymmetry is unbroken in both the symmetric phase and the broken phase. In the leading order of $1/N$, the effective action has a linear divergence which can be eliminated by the renormalization of the coupling constant.

In Sec. III, we evaluate the propagator of the chiral superfield Φ in the symmetric phase, which can be obtained by combining the propagators of component fields. We call this propagator of superfield "superpropagator." After we modify the chiral superfield Φ by some similarity transformation, the superpropagator can be written by using differential operators on superspace. These differential operators can be obtained by modifying ordinary supercovariant derivatives $D_\alpha, \bar{D}_\alpha$. We call these differential operators "twisted covariant derivatives."

In Sec. IV, we first evaluate one-loop diagrams of the dynamical field, which induce the inverse propagator of the auxiliary field in the effective action. Using the superpropagator of the dynamical field, we can easily calculate the one-loop diagrams by a partial integration over Grassmann coordinates. From the inverse propagator of the auxiliary field, we secondly evaluate the propagator of the auxiliary field V, which can be written in terms of ordinary covariant derivatives D_α, $\bar{D}_\alpha$.

In Sec. V, we study divergent diagrams and the renormalization. We first evaluate the superficial degree of divergence and find that divergent diagrams can be classified into two types. We show that all divergences can be eliminated in each order of $1/N$ by renormalizations of the coupling constant and the wave function of the dynamical field. In the last subsection, we evaluate the beta function of the coupling constant. In the $1/N$-expansion, there is no contribution to the beta function except at the leading order. We find that this model has a nontrivial UV fixed point.

Throughout this paper we work in three dimensions with metric $\eta_{\mu\nu} = \mathrm{diag}(+, -, -)$.

II. CP^{N-1} MODEL

A. Action of the CP^{N-1} model with the auxiliary field

The action of the CP^{N-1} model involves a set of N chiral superfields Φ^j ($j = 1 \sim N$) and one vector superfield V:

$$S = \int d^3x d^4\theta (\Phi^{j\dagger} e^{-V} \Phi^j + cV), \tag{1}$$

where c is a coupling constant and we define

$$\int d^4\theta := \int d^2\theta d^2\bar{\theta}, \qquad \int d^2\theta\theta^2 = \int d^2\bar{\theta}\bar{\theta}^2 = 2.$$

This action has $\mathcal{N} = 2$ supersymmetry, $U(1)$ local gauge symmetry, and a global $SU(N)$ symmetry. In Appendix A, $\mathcal{N} = 2$ supersymmetry in three dimensions is reviewed.

The local gauge transformation is

$$\Phi^j \to e^{i\Lambda}\Phi^j, \qquad \Phi^{j\dagger} \to e^{-i\Lambda^\dagger}\Phi^{j\dagger},$$
$$V \to V + \Lambda + \Lambda^\dagger,$$

where Λ is any chiral superfield. Although V itself is not invariant under this transformation, the following term

$$\int d^2\theta d^2\bar{\theta} V$$

is gauge invariant.

The equation of motion for V

$$\Phi^{j\dagger} e^{-V} \Phi^j = c$$

is solved for the auxiliary field V

$$V = \log(\Phi^{j\dagger}\Phi^j) - \log c.$$

Therefore we can eliminate V from the action:

$$S = c \int d^3x d^4\theta \log(\Phi^{j\dagger}\Phi^j),$$

which reduces to the action with the Fubini-Study metric if we fix the gauge symmetry by $\Phi^N = 1$. Note that $\int d^4\theta \log c = 0$.

For the $1/N$-expansion, the action (1) is more convenient than this action.

In terms of component fields, Φ can be written as

$$\Phi^j(x, \theta, \bar{\theta}) = \phi(x) + \theta\psi(x) + \frac{1}{2}\theta^2 F(x) + \frac{i}{2}(\bar{\theta}\not{\partial}\theta)\phi(x) - \frac{i}{4}\theta^2[\bar{\theta}\not{\partial}\psi(x)] - \frac{1}{16}\theta^2\bar{\theta}^2\partial^2\phi(x)$$

and if we choose the Wess-Zumino gauge, V can be written as

$$V(x, \theta, \bar{\theta}) = \bar{\theta}\not{v}(x)\theta + M(x)\bar{\theta}\theta + \frac{1}{2}[\theta^2\bar{\theta}\lambda(x) + \bar{\theta}^2\theta\bar{\lambda}(x)] + \frac{1}{4}\theta^2\bar{\theta}^2 D(x). \tag{2}$$

Then the action (1) becomes

$$\begin{aligned} S = \int d^3x\{&\partial_\mu\phi^{j*}\partial^\mu\phi^j + i\bar{\psi}^j\not{\partial}\psi^j + F^{j*}F^j \\ &- [i(\phi^{j*}\partial_\mu\phi^j - \phi^j\partial_\mu\phi^{j*}) + \bar{\psi}^j\gamma^\mu\psi^j]v^\mu \\ &+ v^\mu v_\mu\phi^{j*}\phi^j - M^2\phi^{j*}\phi^j - M\bar{\psi}^j\psi^j - D\phi^{j*}\phi^j \\ &+ cD + (\phi^j\bar{\psi}^j\lambda + \phi^{j*}\bar{\lambda}\psi^j)\}. \end{aligned}$$

B. Vacuum structure of the CP^{N-1} model

To investigate the vacuum structure we have to calculate the effective potential. We divide the dynamical fields into the vacuum expectation values and the quantum fluctuations:

$$\phi^j = \phi_c^j + \phi_q^j, \qquad \psi^j = \psi_q^j, \qquad F^j = F_c^j + F_q^j,$$

where

$$\phi_c^j = \langle\phi^j\rangle, \qquad F_c^j = \langle F^j\rangle$$

are constant modes independent of spacetime and $\langle\psi^j\rangle = 0$ because we assume the translation and Lorentz invariance of the vacuum. Quantum fluctuations satisfy $\int \phi_q^i d^3x = \int \psi_q^i d^3x = \int F_q^i d^3x = 0$. Then we perform the path integration over ϕ_q^j, ψ_q^j, F_q^j.

We can first perform the Gaussian integral over F_q^j and find that the effective potential for F^j is $F_c^{j*}F_c^j$. Therefore F^j does not have the vacuum expectation value:

$$F_c^j = 0.$$

Then we integrate out ϕ_q^j and ψ_q^j. Notice that the Lagrangian can be written as

$$\begin{aligned} \mathcal{L} = &\phi_q^{j*}\{-(\partial_\mu + iv_\mu)(\partial^\mu + iv^\mu) - M^2 - D\}\phi_q^j \\ &+ \bar{\psi}_q^j(i\not{\partial} - \not{v} - M)\psi_q^j + \phi_q^j\bar{\psi}_q^j\lambda + \phi_q^{j*}\bar{\lambda}\psi_q^j + cD \\ &- \phi_c^{j*}\{i(\partial_\mu v^\mu) - v^\mu v_\mu + M^2 + D\}\phi_c^j, \end{aligned}$$

where surface terms are ignored. We shift the integration variables ψ_q^j, $\bar{\psi}_q^j$:

$$\begin{aligned} \psi_q'^j &:= \psi_q^j + (i\not{\partial} - \not{v} - M)^{-1}\phi_q^j\lambda, \\ \bar{\psi}_q'^j &:= \bar{\psi}_q^j + \phi^{j*}\bar{\lambda}(i\not{\partial} - \not{v} - M)^{-1}, \end{aligned}$$

then we find

$$\begin{aligned} \mathcal{L} = &\phi_q^{j*}(\nabla_B - \bar{\lambda}\nabla_F^{-1}\lambda)\phi_q^j + \bar{\psi}_q'^j\nabla_F\psi_q'^j + cD \\ &- \phi_c^{j*}\{i(\partial_\mu v^\mu) - v^\mu v_\mu + M^2 + D\}\phi_c^j, \end{aligned}$$

where

$$\begin{aligned} \nabla_B &:= -(\partial_\mu + iv_\mu)(\partial^\mu + iv^\mu) - M^2 - D \\ \nabla_F &:= i\not{\partial} - \not{v} - M. \end{aligned}$$

We perform the Gaussian integration over ϕ_q^j, ψ_q^j and obtain the effective action for the dynamical fields, where the auxiliary fields are treated as the external background fields:

$$\begin{aligned} S_{\text{eff}}(\phi_c; v, M, \lambda, D) = &iN\,\mathrm{Tr}\ln(\nabla_B + \bar{\lambda}\nabla_F^{-1}\lambda) - iN\,\mathrm{Tr}\ln\nabla_F \\ &+ \int d^3x[cD - \phi_c^{j*}\{i(\partial_\mu v^\mu) \\ &- v^\mu v_\mu + M^2 + D\}\phi_c^j]. \end{aligned}$$

To obtain the exact effective potential for both the dynamical fields and the auxiliary fields, we have to perform the path integration over the fluctuations of the auxiliary fields. In this section, we calculate the effective potential in the leading order of the $1/N$-expansion, and we take $c = N/g^2$ in order to make the Lagrangian of order N.

If we take the limit of $N \to \infty$, the path integration over the auxiliary fields can be performed by the saddle point method since the S_{eff} is of order N. In the leading order of $1/N$ expansion, the effective potential is given by the value of S_{eff} at the saddle point.

We take the vacuum expectation values of the auxiliary fields as follows:

$$\langle v^\mu\rangle = \langle\lambda\rangle = 0, \qquad \langle M\rangle = M_c, \qquad \langle D\rangle = D_c,$$

where M_c, D_c are constant fields. Then we find

$$S_{\rm eff} = -\int d^3x V_{\rm eff}$$

$$\begin{aligned}\frac{V_{\rm eff}}{N} &= -i\int^{\Lambda}\frac{d^3k}{(2\pi)^3}\ln(-k^2+M_c^2+D_c^2)\\ &\quad + i\int^{\Lambda}\frac{d^3k}{(2\pi)^3}\,\mathrm{tr}\ln(\not{k}-M_c)+\frac{1}{N}\phi_c^{j*}(M_c^2+D_c^2)\phi_c^j\\ &\quad -\frac{1}{g^2}D_c\\ &= -\frac{1}{6\pi}|M_c^2+D_c|^{3/2}+\frac{1}{6}|M_c|^3\\ &\quad +\frac{1}{N}(M_c^2+D_c)\phi_c^{j*}\phi_c^j+\left(\frac{\Lambda}{2\pi^2}-\frac{1}{g^2}\right)D_c, \end{aligned} \tag{3}$$

where Λ is an ultraviolet cutoff and the last equality is shown in Appendix B. Then we define a renormalized coupling constant g_R to absorb the linear divergence:

$$\frac{\mu}{g_R^2} := \frac{1}{g^2}-\frac{\Lambda}{2\pi^2}+\frac{\mu}{2\pi^2},$$

where μ is a renormalization scale and g_R is dimensionless. Then we define m as follows:

$$\frac{m}{4\pi} := \mu\left(\frac{1}{2\pi^2}-\frac{1}{g_R^2}\right)=\frac{\Lambda}{2\pi^2}-\frac{1}{g^2},$$

which is independent of the renormalization scale μ.

With these definitions, the effective potential can be written as

$$\begin{aligned}\frac{V_{\rm eff}}{N} &= -\frac{1}{6\pi}|M_c^2+D_c|^{3/2}+\frac{1}{6\pi}|M_c|^3\\ &\quad +\frac{1}{N}(M_c^2+D_c)\phi_c^{j*}\phi_c^j+\frac{m}{4\pi}D_c. \end{aligned}\tag{4}$$

The saddle point condition of $S_{\rm eff}$ is

$$\begin{aligned}\frac{1}{N}\frac{\partial V_{\rm eff}}{\partial M_c} &= -2M_c\left(\frac{\epsilon}{4\pi}|M_c^2+D_c|^{1/2}-\frac{1}{4}|M_c|-\frac{1}{N}\phi_c^{j*}\phi_c^j\right)\\ &=0\end{aligned}\tag{5}$$

$$\frac{1}{N}\frac{\partial V_{\rm eff}}{\partial D_c} = -\frac{\epsilon}{4\pi}|M_c^2+D_c|^{1/2}+\frac{1}{N}\phi_c^{j*}\phi_c^j+\frac{m}{4\pi}=0 \tag{6}$$

$$\frac{1}{N}\frac{\partial V_{\rm eff}}{\partial \phi_c^{*i}}=\frac{1}{N}(M_c^2+D_c)\phi_c^i=0,$$

where $\epsilon = \mathrm{sgn}(M_c^2+D_c)$. The first two conditions fix the value of M_c at the saddle point

$$|M_c| = m \quad \text{or} \quad 0.$$

So there are two candidates for the vacuum configuration. We will evaluate the values of $V_{\rm eff}$ at these two configurations.

$|M_c| = m$ *case*: Notice that this case is possible only when $m \geq 0$. Equation (6) can be solved for $\frac{1}{N}\phi_c^{j*}\phi_c^j$ as follows:

$$\frac{1}{N}\phi_c^{j*}\phi_c^j=\frac{\epsilon}{4\pi}|M_c^2+D_c|^{1/2}-\frac{m}{4\pi}. \tag{7}$$

Substituting this and $|M_c| = m$ to (4), we find

$$\frac{V_{\rm eff}}{N}=\frac{1}{12\pi}(|M_c^2+D_c|^{3/2}-m^3).$$

And we can also solve the constraint (6) for $|M_c^2+D_c|^{1/2}$:

$$|M_c^2+D_c|^{1/2}=\left|m+\frac{4\pi}{N}\phi_c^{j*}\phi_c^j\right|.$$

Then we obtain the vacuum energy when ϕ_c^j is kept fixed

$$V_{\rm eff}(\phi_c)=\frac{N}{12\pi}\left(\left|\frac{4\pi}{N}\phi_c^{j*}\phi_c^j+m\right|^3-m^3\right).$$

Assuming $m > 0$, the minimum of this vacuum energy is located at $\phi_c^j = 0$. Then (6) implies $D_c = 0$. Since ϕ_c and D_c are the order parameter of $SU(N)$ and supersymmetry, respectively, both $SU(N)$ and supersymmetry are unbroken in this case. The fact that the minimum vacuum energy is exactly zero also implies supersymmetry is not broken.

$M_c = 0$ *case*: Substituting (7) and $M_c = 0$ to the effective potential (4), we find

$$V_{\rm eff}(\phi_c)=\frac{1}{12\pi}|D_c|^{3/2}.$$

And by solving the constraint (6) for $|D_c|^{1/2}$ and substituting it to this equation, we obtain the vacuum energy

$$V_{\rm eff}(\phi_c)=\frac{N}{12\pi}\left|\frac{4\pi}{N}\phi_c^{j*}\phi_c^j+m\right|^3.$$

If $m > 0$, the minimum of this vacuum energy is located at $\phi_c^j = 0$ and larger than zero, and therefore the true vacuum is located at $M_c = m$. On the other hand, if $m < 0$, the minimum is located at

$$\phi_c^{j*}\phi_c^j=\frac{N}{4\pi}|m|,$$

then (6) implies $D_c = 0$. Therefore supersymmetry is not broken while $SU(N)$ symmetry is spontaneously broken in this case. The minimum vacuum energy is again exactly zero.

In summary, in the case of $m \geq 0$ which we call the symmetric phase, both supersymmetry and $SU(N)$ are unbroken, and $\phi_c^j = D_c = 0$, $|M_c| = m$ at the vacuum. On the other hand, in the case of $m < 0$ which we call the broken phase, supersymmetry is not broken while $SU(N)$ is spontaneously broken, and $\phi_c^j = \frac{4\pi}{N}|m|$, $M_c = D_c = 0$ at the vacuum.

III. PROPAGATOR OF THE DYNAMICAL FIELD

In this section, we will evaluate the propagator of the dynamical field Φ^j in the symmetric phase. We first evaluate the propagators of the component fields ϕ^j, ψ^j, F^j, and then we construct the propagator of the superfield Φ^j.

A. Propagator of the component fields

In the symmetric phase, we redefine M as follows:

$$M \to M + m$$

so that $\langle M \rangle = 0$. Then in the Lagrangian, the kinetic term for the dynamical field becomes

$$\mathcal{L}_{\rm kin} = \phi^{j*}(-\partial^2 - m^2)\phi^j + \bar{\psi}^j(i\not{\partial} - m)\psi^j + F^{j*}F^j.$$

Notice that although the dynamical field obtained the mass m, neither supersymmetry nor $SU(N)$ symmetry is broken in this phase.

Defining the Green's function

$$\Delta_F(x - x') := (-\partial^2 - m^2 + i\epsilon)^{-1}\delta(x - x'),$$

the propagators of ϕ^j, ψ^j, and F^j can be written as

$$\begin{aligned}\langle \phi^j(x)\phi^{k*}(x')\rangle_0 &= i\delta^{jk}\Delta_F(x - x')\\ \langle \psi^{j\alpha}(x)\bar{\psi}^k_\beta(x')\rangle_0 &= i\delta^{jk}[(i\not{\partial} - m)^{-1}]^\alpha_\beta\delta(x - x')\\ &= i\delta^{jk}(i\not{\partial} + m)^\alpha{}_\beta\Delta_F(x - x')\\ \langle F^j(x)F^{k*}(x')\rangle_0 &= i\delta^{jk}\delta(x - x')\\ &= i\delta^{jk}(-\partial^2 - m^2)\Delta_F(x - x').\end{aligned}$$

All other two-point functions vanish.

B. Superpropagator of the dynamical field

Using these component propagators, we can construct the propagator of the superfield Φ^j which we call the superpropagator.

Since the superfield Φ^j can be written in terms of components as

$$\Phi^j(x, \theta, \bar{\theta}) = \phi^j(y) + \theta\psi^j(y) + \frac{1}{2}\theta^2F^j(y),$$

where $y^\mu := x^\mu + \frac{i}{2}\bar{\theta}\gamma^\mu\theta$, the free-field two-point function of Φ^j becomes as follows:

$$\begin{aligned}\langle \Phi^j(x, \theta, \bar{\theta})\Phi^{\dagger k}(x', \theta', \bar{\theta}')\rangle_0 = &\langle \phi^j(y)\phi^{k*}(y'^\dagger)\rangle_0\\ &+ \langle \theta\psi^j(y)\bar{\theta}'\bar{\psi}^k(y'^\dagger)\rangle_0\\ &+ \frac{1}{4}\theta^2\bar{\theta}'^2\langle F^j(y)F^{k*}(y'^\dagger)\rangle_0.\end{aligned}$$

If we note

$$\langle \theta\psi^j(y)\bar{\theta}'\bar{\psi}^k(y'^\dagger)\rangle_0 = \theta_\alpha\langle \psi^{j\alpha}(y)\bar{\psi}^k_\beta(y')\rangle_0\bar{\theta}'^\beta,$$

then we find

$$\begin{aligned}&\langle \Phi^j(x, \theta, \bar{\theta})\Phi^{\dagger k}(x', \theta', \bar{\theta}')\rangle_0\\ &= i\delta^{jk}\left[1 + \theta(i\not{\partial} + m)\bar{\theta}' + \frac{1}{4}\theta^2\bar{\theta}'^2(-\partial^2 - m^2)\right]\\ &\quad\times \Delta_F(y - y'^\dagger)\\ &= i\delta^{jk}e^{\theta(i\not{\partial}+m)\bar{\theta}'}\Delta_F(y - y'^\dagger).\end{aligned}$$

In the last line, we use the equation $[\theta(i\not{\partial} + m)\bar{\theta}']^2 = \frac{1}{2}\theta^2\bar{\theta}'^2(-\partial^2 - m^2)$, which is shown in Appendix C. Recalling $y^\mu = x^\mu + \frac{i}{2}\bar{\theta}\gamma^\mu\theta$ and noting that $\theta(i\not{\partial} + m)\bar{\theta}' = -\bar{\theta}'(i\not{\partial} - m)\theta$, then we find

$$\begin{aligned}&\langle \Phi^j(x, \theta, \bar{\theta})\Phi^{\dagger k}(x', \theta', \bar{\theta}')\rangle_0\\ &= i\delta^{jk}e^{-\bar{\theta}'(i\not{\partial}-m)\theta+(i/2)\bar{\theta}\not{\partial}\theta+(i/2)\bar{\theta}'\not{\partial}\theta'}\Delta_F(x - x').\end{aligned}$$

In momentum space, this becomes

$$\begin{aligned}&\langle \Phi^j(p, \theta, \bar{\theta})\Phi^{\dagger k}(-p, \theta', \bar{\theta}')\rangle_0\\ &= e^{-\bar{\theta}'(\not{p}-m)\theta+(1/2)\bar{\theta}\not{p}\theta+(1/2)\bar{\theta}'\not{p}\theta'}\frac{i}{p^2 - m^2 + i\epsilon}\delta^{jk}. \qquad (8)\end{aligned}$$

All the propagators of component fields are combined in this superpropagator.

Since we redefine M as $M \to M + m$, the action becomes

$$S = \int d^3xd^4\theta(\Phi^{j\dagger}e^{-m\bar{\theta}\theta}e^{-V}\Phi^j + cV).$$

We define $\tilde{\Phi}^j$, $\tilde{\Phi}^{j\dagger}$ by

$$\tilde{\Phi}^j := e^{-(1/2)m\bar{\theta}\theta}\Phi^j, \qquad \tilde{\Phi}^{j\dagger} := e^{-(1/2)m\bar{\theta}\theta}\Phi^{j\dagger},$$

then we find

$$S = \int d^3xd^4\theta(\tilde{\Phi}^{j\dagger}e^{-V}\tilde{\Phi}^j + cV), \qquad (9)$$

and the propagator of $\tilde{\Phi}^j$ becomes

$$\begin{aligned}&\langle \tilde{\Phi}^j(p, \theta, \bar{\theta})\tilde{\Phi}^{\dagger k}(-p, \theta', \bar{\theta}')\rangle_0\\ &= e^{-\bar{\theta}'(\not{p}-m)\theta+(1/2)\bar{\theta}(\not{p}-m)\theta+(1/2)\bar{\theta}'(\not{p}-m)\theta'}\frac{i}{p^2 - m^2 + i\epsilon}\delta^{jk}. \qquad (10)\end{aligned}$$

Hereafter, we will use $\tilde{\Phi}^j$, $\tilde{\Phi}^{\dagger j}$ as the dynamical fields instead of Φ^j, $\Phi^{\dagger j}$.

C. Twisted covariant derivatives

Since the expression (10) is slightly complicated, we will rewrite it in terms of differential operators on superspace, obtained by twisting D_α, $\bar{D}_\alpha$. We call them the "twisted covariant derivatives."

We first review the propagator of the ordinary massless chiral superfield. Although the Lagrangian

$$\mathcal{L}^{\rm massless}_{\rm chiral} = \int d^4\theta\Phi^\dagger\Phi \qquad (11)$$

does not have any time derivatives, the constraint $\bar{D}_\alpha\Phi = 0$ contains a time derivative and leads to the nontrivial propagation of Φ. From the Lagrangian (11), we can show that the propagator of Φ becomes

$$\langle\Phi(p,\theta,\bar\theta)\Phi^\dagger(-p,\theta',\bar\theta')\rangle_0 = \frac{i}{p^2+i\epsilon}e^{-\bar\theta'\not{p}\theta+(1/2)\bar\theta\not{p}\theta+(1/2)\bar\theta'\not{p}\theta'}. \tag{12}$$

It is known that this is equivalent to the following expression [10–12]:

$$\langle\Phi(p,\theta,\bar\theta)\Phi^\dagger(-p,\theta',\bar\theta')\rangle_0 = \frac{i}{p^2+i\epsilon}\cdot\frac{1}{4}\bar{D}(p)^2D(p)^2\delta^{(4)}(\theta-\theta'), \tag{13}$$

where $D(p)$ and $\bar{D}(p)$ are the covariant derivatives in momentum space:

$$D(p)_\alpha = -\frac{\partial}{\partial\theta^\alpha}+\frac{1}{2}(\bar\theta\not{p})_\alpha,$$
$$\bar{D}(p)_\alpha = -\frac{\partial}{\partial\bar\theta^\alpha}+\frac{1}{2}(\theta\not{p})_\alpha,$$

and we define $\delta^{(4)}(\theta-\theta') := \frac{1}{4}(\theta-\theta')^2(\bar\theta-\bar\theta')^2$. We can show this equivalence of Eqs. (12) and (13) through a straightforward calculation. For the calculation of a loop diagram, the expression (13) is more useful than (12) because we can perform the integration by parts in superspace.

We now look for a constraint for $\tilde\Phi$ such that

$$\bar{E}_\alpha\tilde\Phi = 0.$$

Since $\tilde\Phi(x,\theta,\bar\theta) = e^{-(1/2)m\bar\theta\theta}\Phi(x,\theta,\bar\theta)$, we impose $\bar{E}_\alpha e^{-(1/2)m\bar\theta\theta} = e^{-(1/2)m\bar\theta\theta}\bar{D}_\alpha$, namely

$$\bar{E}_\alpha := e^{-(1/2)m\bar\theta\theta}\bar{D}_\alpha e^{+(1/2)m\bar\theta\theta} = \bar{D}_\alpha+\frac{1}{2}m\theta_\alpha. \tag{14}$$

We define E_α by the same similarity transformation of D_α:

$$E_\alpha := e^{-(1/2)m\bar\theta\theta}D_\alpha e^{+(1/2)m\bar\theta\theta} = D_\alpha+\frac{1}{2}m\bar\theta_\alpha. \tag{15}$$

Secondly, we define another set of differential operators $\bar{H}_\alpha$, H_α as follows:

$$\bar{H}_\alpha := e^{+(1/2)m\bar\theta\theta}\bar{D}_\alpha e^{-(1/2)m\bar\theta\theta} = \bar{D}_\alpha-\frac{1}{2}m\theta_\alpha,$$
$$H_\alpha := e^{+(1/2)m\bar\theta\theta}D_\alpha e^{-(1/2)m\bar\theta\theta} = D_\alpha-\frac{1}{2}m\bar\theta_\alpha.$$

Notice that the sign in front of m is opposite to (14) and (15). We call E_α, $\bar{E}_\alpha$ and H_α, $\bar{H}_\alpha$ the twisted covariant derivatives. Then we can rewrite the expression (10) through a straightforward calculation:

$$\langle\tilde\Phi^j(p,\theta,\bar\theta)\tilde\Phi^{k\dagger}(-p,\theta',\bar\theta')\rangle_0 = \delta^{jk}\frac{i}{p^2-m^2+i\epsilon}\cdot\frac{1}{4}\bar{E}(p)^2H(p)^2\delta^{(4)}(\theta-\theta'), \tag{16}$$

where

$$H(p)_\alpha = D(p)_\alpha-\frac{1}{2}m\bar\theta_\alpha, \qquad \bar{E}(p)_\alpha = \bar{D}(p)_\alpha+\frac{1}{2}m\theta_\alpha.$$

In Appendix D, the derivation of

$$\frac{1}{4}\bar{E}(p)^2H(p)^2\delta^{(4)}(\theta-\theta') = e^{-\bar\theta'(\not{p}-m)\theta+(1/2)\bar\theta(\not{p}-m)\theta+(1/2)\bar\theta'(\not{p}-m)\theta'} \tag{17}$$

is shown in detail.

D. Property of the twisted covariant derivatives

In the previous subsection, we defined the twisted covariant derivatives E_α, $\bar{E}_\alpha$ and H_α, $\bar{H}_\alpha$. We now investigate the property of these differential operators in detail.

We first study E_α and $\bar{E}_\alpha$. We can easily show the anticommutation relations of E_α, $\bar{E}_\alpha$ are those of covariant derivatives:

$$\{E^\alpha,\bar{E}_\alpha\} = i\not{\partial}^\alpha{}_\beta, \qquad \{E^\alpha,E_\beta\} = \{\bar{E}^\alpha,\bar{E}_\beta\} = 0.$$

They are indeed supercovariant derivatives when they act on $\tilde\Phi^j$, $\tilde\Phi^{j\dagger}$. To see this explicitly, recall the definition of the twisted chiral superfield $\tilde\Phi^j = e^{-(1/2)m\bar\theta\theta}\Phi^j$. Under the infinitesimal supersymmetry transformation, the chiral superfield Φ transforms as $\Phi^j \to (1+\xi Q+\bar\xi\bar{Q})\Phi^j$ where ξ, $\bar\xi$ are the transformation parameters. So the transformation law for the twisted chiral superfield $\tilde\Phi^j$ becomes as follows:

$$\tilde\Phi^j \to e^{-(1/2)m\bar\theta\theta}(1+\xi Q+\bar\xi\bar{Q})e^{+(1/2)m\bar\theta\theta}\tilde\Phi^j. \tag{18}$$

If we define

$$R_\alpha := e^{-(1/2)m\bar\theta\theta}Q_\alpha e^{+(1/2)m\bar\theta\theta} = Q_\alpha+\frac{i}{2}m\bar\theta_\alpha$$
$$\bar{R}_\alpha := e^{-(1/2)m\bar\theta\theta}\bar{Q}_\alpha e^{+(1/2)m\bar\theta\theta} = \bar{Q}_\alpha-\frac{i}{2}m\theta_\alpha,$$

the transformation law (18) can be written as $\tilde\Phi^j \to (1+\xi R+\bar\xi\bar{R})\tilde\Phi^j$. We call R_α, $\bar{R}_\alpha$ twisted supercharges, which of course satisfy the anticommutation relations

$$\{R^\alpha,\bar{R}_\beta\} = -i\not{\partial}^\alpha{}_\beta = \{Q^\alpha,\bar{Q}_\beta\},$$
$$\{R^\alpha,R_\beta\} = \{\bar{R}^\alpha,\bar{R}_\beta\} = 0.$$

Supersymmetry transformations for twisted chiral superfields $\tilde\Phi^j$, $\tilde\Phi^{j\dagger}$ are generated by R_α, $\bar{R}_\alpha$.

We can show explicitly that twisted covariant derivatives E_α, $\bar{E}_\alpha$ anticommute with R_α, $\bar{R}_\alpha$:

$$\{E^\alpha,\bar{R}_\beta\} = \{\bar{E}^\alpha,R_\beta\} = \{E^\alpha,R_\beta\} = \{\bar{E}^\alpha,\bar{R}_\beta\} = 0.$$

Therefore E_α, $\bar{E}_\alpha$ are indeed supercovariant derivatives when they act on $\tilde{\Phi}^j$, $\tilde{\Phi}^{j\dagger}$. We now find that the twisted chiral condition $\bar{E}_\alpha \tilde{\Phi}^j = 0$ is supercovariant. The twisted antichiral condition for $\tilde{\Phi}^{j\dagger}$ becomes

$$E_\alpha \tilde{\Phi}^{j\dagger} = 0$$

and also supercovariant.

On the other hand, another set of twisted covariant derivatives H_α and $\bar{H}_\alpha$ do not anticommute with R_α and $\bar{R}_\alpha$. The anticommutation relations are

$$\{H^\alpha, \bar{R}_\beta\} = -im\delta^\alpha{}_\beta, \qquad \{\bar{H}^\alpha, R_\beta\} = im\delta^\alpha{}_\beta$$
$$\{H^\alpha, R_\beta\} = \{\bar{H}^\alpha, \bar{R}_\beta\} = 0$$

and therefore H_α, $\bar{H}_\alpha$ are not supercovariant derivatives when they act on $\tilde{\Phi}^j$, $\tilde{\Phi}^{j\dagger}$. However, since the only difference between H_α, $\bar{H}_\alpha$ and E_α, $\bar{E}_\alpha$ is the sign in front of m, H_α, $\bar{H}_\alpha$ are supercovariant derivatives when they act on

$$e^{+(1/2)m\bar{\theta}\theta}\Phi^j, \qquad e^{+(1/2)m\bar{\theta}\theta}\Phi^{j\dagger},$$

while $\tilde{\Phi}^j = e^{-(1/2)m\bar{\theta}\theta}\Phi^j$ and $\tilde{\Phi}^{j\dagger} = e^{-(1/2)m\bar{\theta}\theta}\Phi^{j\dagger}$. So there are two ways of "twisting" and we can define two sets of twisted superfields (twisted supercharges) and twisted covariant derivatives which are distinguished by the sign in front of m.

Anticommutation relations among supercovariant derivatives in different sets are as follows:

$$\{E^\alpha, \bar{H}_\beta\} = (i\not{\partial} - m)^\alpha{}_\beta, \qquad \{H^\alpha, \bar{E}_\beta\} = (i\not{\partial} + m)^\alpha{}_\beta$$
$$\{E^\alpha, H_\beta\} = \{\bar{E}^\alpha, \bar{H}_\beta\} = 0,$$

which will be used frequently in Appendix E to show useful formulae for loop calculations.

In the following, we will show a useful formula for the propagator of the dynamical field.

Note that the only difference between E_α, $\bar{E}_\alpha$ and H_α, $\bar{H}_\alpha$ is the sign in front of m. Therefore, by replacing $\bar{E}_\alpha$ and H_α with $\bar{H}_\alpha$ and E_α in Eq. (17), we find

$$\frac{1}{4}\bar{H}(p)^2 E(p)^2 \delta^{(4)}(\theta - \theta') = e^{-\bar{\theta}'(\not{p}+m)\theta + (1/2)\bar{\theta}(\not{p}+m)\theta + (1/2)\bar{\theta}'(\not{p}+m)\theta'}. \qquad (19)$$

If we replace θ, $\bar{\theta}$ with θ', $\bar{\theta}'$, we find

$$\frac{1}{4}\bar{H}'(p)^2 E'(p)^2 \delta^{(4)}(\theta' - \theta) = e^{-\bar{\theta}(\not{p}+m)\theta' + (1/2)\bar{\theta}'(\not{p}+m)\theta' + (1/2)\bar{\theta}(\not{p}+m)\theta}, \qquad (20)$$

where $E'(p)$, $\bar{H}'(p)$ stand for twisted covariant derivatives with θ', $\bar{\theta}'$. Recalling the definition of twisted covariant derivatives

$$\bar{H}'(p)_\alpha = -\frac{\partial}{\partial \bar{\theta}'^\alpha} + \frac{1}{2}(\theta'\not{p})_\alpha - \frac{1}{2}m\theta'_\alpha,$$
$$E'(p)_\alpha = -\frac{\partial}{\partial \theta'^\alpha} + \frac{1}{2}(\bar{\theta}'\not{p})_\alpha + \frac{1}{2}m\bar{\theta}'_\alpha,$$

we can exchange $\bar{H}'(p)$ and $E'(p)$ by the following replacement: $\theta' \leftrightarrow \bar{\theta}'$, $m \to -m$. Therefore by replacing $\theta \leftrightarrow \bar{\theta}$, $\theta' \leftrightarrow \bar{\theta}'$, $m \to -m$ in Eq. (20), we find

$$\frac{1}{4}E'(p)^2\bar{H}'(p)^2\delta^{(4)}(\theta' - \theta) = e^{-\theta(\not{p}-m)\bar{\theta}' + (1/2)\theta'(\not{p}-m)\bar{\theta}' + (1/2)\theta(\not{p}-m)\bar{\theta}}.$$

Then, at last, using the fact that $\theta\not{p}\bar{\theta} = -\bar{\theta}\not{p}\theta$ and $m\theta\bar{\theta} = m\bar{\theta}\theta$, we obtain the following result:

$$\frac{1}{4}E'(-p)^2\bar{H}'(-p)^2\delta^{(4)}(\theta' - \theta) = e^{-\bar{\theta}'(\not{p}-m)\theta + (1/2)\bar{\theta}'(\not{p}-m)\theta' + (1/2)\bar{\theta}(\not{p}-m)\theta}.$$

Since the right-hand side is the same as that of (17), Eq. (16) can be written as

$$\begin{aligned}\langle\tilde{\Phi}^j(p,\theta,\bar{\theta})\tilde{\Phi}^{k\dagger}(-p,\theta',\bar{\theta}')\rangle_0 &= \delta^{jk}\frac{i}{p^2 - m^2 + i\epsilon}\cdot\frac{1}{4}\bar{E}(+p)^2 H(+p)^2\delta^{(4)}(\theta - \theta') \\ &= \delta^{jk}\frac{i}{p^2 - m^2 + i\epsilon}\cdot\frac{1}{4}E'(-p)^2\bar{H}'(-p)^2\delta^{(4)}(\theta - \theta').\end{aligned} \qquad (21)$$

We will use this formula in the calculation of the loop diagrams of the twisted chiral superfield.

IV. PROPAGATOR OF THE AUXILIARY FIELD

In the previous section, we studied the propagator of the dynamical field. In this section, we will investigate that of the auxiliary field. Although the auxiliary field has no kinetic term in the classical level, the effective action contains the quadratic term of the auxiliary field induced by quantum effects of the dynamical field. We use the quadratic term induced by one-loop diagrams of the dynamical field, which is the leading order of $1/N$, as a kinetic term in order to calculate the propagator of the auxiliary field. Therefore we have to calculate the one-loop diagram of the dynamical field in order to obtain the propagator of the auxiliary field in the large N-expansion.

A. One-loop diagram of the dynamical field

Since the action of the theory is

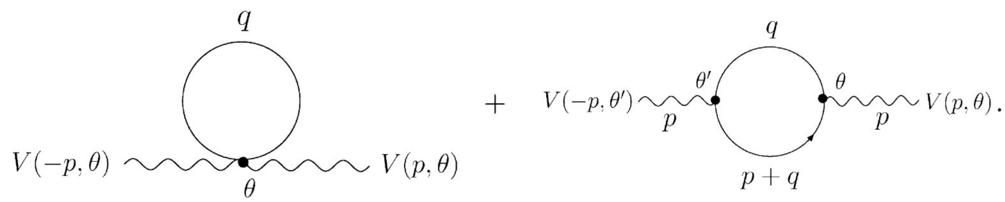
$$S = \int d^3x d^4\theta(\tilde{\Phi}^{j\dagger}e^{-V}\tilde{\Phi}^j + cV) \qquad (j = 1 \sim N), \quad (22)$$

the auxiliary field V does not have the kinetic term at the tree level. If we perform the path integration over $\tilde{\Phi}$, $\tilde{\Phi}^\dagger$, we obtain the effective action $S_{\rm eff}$ when the auxiliary field

V is treated as the external background field while the dynamical fields $\tilde{\Phi}$, $\tilde{\Phi}^\dagger$ fluctuate:

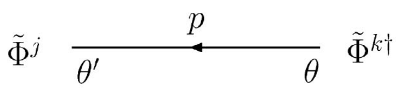
$$S_{\rm eff} = S + \frac{1}{2}\int d^3x d^4\theta V(iG^{-1})V + \cdots.$$

The quadratic term of the auxiliary field defines the inverse propagator of it, which comes from one-loop diagrams of the dynamical field:

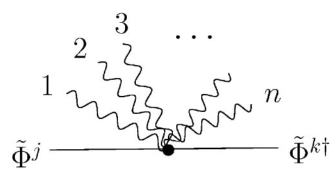


(23)

Feynman rules are as follows



$$= \delta^{jk}\frac{i}{p^2 - m^2 + i\epsilon}\cdot\frac{1}{4}\bar{E}(p)^2H(p)^2\delta^{(4)}(\theta-\theta'),$$



$$= i(-1)^n\delta^{jk}.$$

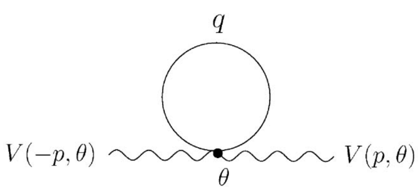
Then the contribution of the first diagram in (23) can be evaluated as

$$\int\frac{d^3p}{(2\pi)^3}\int d^4\theta V(-p,\theta)\cdot i(-1)^2\int\frac{d^3q}{(2\pi)^3}\frac{iN}{q^2-m^2+i\epsilon}\left[\frac{1}{4}\bar{E}(q)^2H(q)^2\delta^{(4)}(\theta-\theta')\right]\bigg|_{\theta'=\theta,\bar{\theta}'=\bar{\theta}}\cdot V(p,\theta).$$

Since $\delta^{(4)}(\theta-\theta') = \frac{1}{4}(\theta-\theta')^2(\bar{\theta}-\bar{\theta}')^2$, we can show

$$\left[\frac{1}{4}\bar{E}(q)^2H(q)^2\delta^{(4)}(\theta-\theta')\right]\bigg|_{\theta'=\theta,\bar{\theta}'=\bar{\theta}} = \frac{1}{16}\left[\frac{\partial}{\partial\bar{\theta}^\alpha}\frac{\partial}{\partial\bar{\theta}_\alpha}\frac{\partial}{\partial\theta^\beta}\frac{\partial}{\partial\theta_\beta}(\theta-\theta')^2(\bar{\theta}-\bar{\theta}')^2\right]\bigg|_{\theta'=\theta,\bar{\theta}'=\bar{\theta}} = 1. \tag{24}$$

Therefore the contribution of the first diagram in (23) becomes

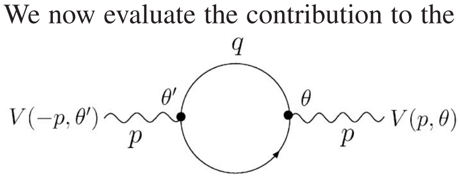


$$= -\int\frac{d^3p}{(2\pi)^3}\int d^4\theta\ V(-p,\theta)\cdot\int\frac{d^3q}{(2\pi^3)}\frac{N}{q^2-m^2+i\epsilon}\cdot V(p,\theta). \tag{25}$$

In the right-hand side, the momentum integral over q has a linear divergence. So we here regularize the integral, for instance by introducing a momentum cut-off.

B. Integration by parts at vertices

We now evaluate the contribution to the effective action from the second diagram in (23):



$$= N\int\frac{d^3p}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\int d^4\theta\, d^4\theta'\, V(-p,\theta')\cdot(-i)^2\frac{i}{(p+q)^2-m^2+i\epsilon}$$

$$\times\frac{i}{q^2-m^2+i\epsilon}\left[\frac{1}{4}\bar{E}'(p+q)^2H'(p+q)^2\delta^{(4)}(\theta'-\theta)\right]\left[\frac{1}{4}\bar{E}(q)^2H(q)^2\delta^{(4)}(\theta-\theta')\right]\cdot V(p,\theta), \tag{26}$$

where $\bar{E}'(p+q)$ and $H'(p+q)$ are twisted covariant derivatives with θ', $\bar{\theta}'$ and momentum $p+q$. Using Eq. (21), we can rewrite the term in the first bracket:

$$\frac{1}{4}\bar{E}'(p+q)^2 H'(p+q)^2 \delta^{(4)}(\theta'-\theta) = \frac{1}{4}E(-p-q)^2\bar{H}(-p-q)^2\delta^{(4)}(\theta-\theta'). \tag{27}$$

In the following, we perform the θ, $\bar{\theta}$ integration by parts, then we can apply $E(-p-q)^2\bar{H}(-p-q)^2$ to the second bracket and $V(p,\theta)$ in Eq. (26). We first note the partial integration rule for covariant derivatives [12]:

$$\int d^4\theta\{D(p)_\alpha A\}B = -\int d^4\theta(-1)^{|A|}A\{D(-p)_\alpha B\},$$

where $|A|=1$ for Grassmann-odd A and $|A|=0$ for Grassmann-even A. Note that the signs in front of $-\frac{\partial}{\partial\theta^\alpha}$ and $\frac{1}{2}(\bar{\theta}\not{p})_\alpha$ become opposite after the integration by parts in the definition of the covariant derivative $D(p)_\alpha = -\frac{\partial}{\partial\theta^\alpha}+\frac{1}{2}(\bar{\theta}\not{p})_\alpha$. Namely, through the integration by parts, $D(p)_\alpha$ becomes $D(-p)_\alpha$ and $\bar{D}(p)$ becomes $\bar{D}(-p)$ as well.

We now recall the definition of twisted covariant derivatives:

$$E(p)_\alpha = D(p)_\alpha + \frac{1}{2}m\bar{\theta}_\alpha,$$

$$\bar{E}(p)_\alpha = \bar{D}(p)_\alpha + \frac{1}{2}m\theta_\alpha,$$

$$H(p)_\alpha = D(p)_\alpha - \frac{1}{2}m\bar{\theta}_\alpha,$$

$$\bar{H}(p)_\alpha = \bar{D}(p)_\alpha - \frac{1}{2}m\theta_\alpha.$$

Then we find in the similar way that $E(p)_\alpha$ becomes $H(-p)_\alpha$ and $\bar{H}(p)_\alpha$ becomes $\bar{E}(-p)_\alpha$ through the integration by parts.

If we take the following operator

$$\int\frac{d^3p}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\int d^4\theta\{E(-p-q)_\alpha\tilde{\Phi}^\dagger(-p-q)\}V(p)\tilde{\Phi}(q), \tag{28}$$

we can integrate by parts and move $E(-p-q)_\alpha$ to $V(p)\tilde{\Phi}(q)$ by substituting $H(p+q)_\alpha$ for it:

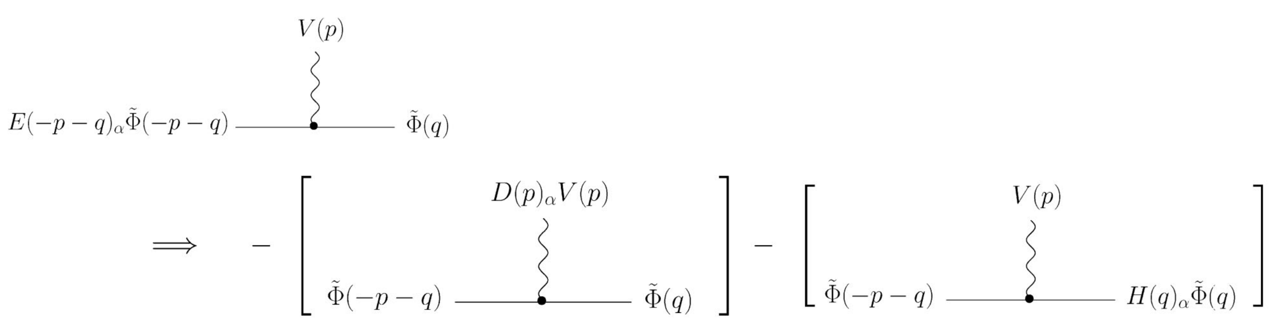
$$-\int\frac{d^3p}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\int d^4\theta\tilde{\Phi}^\dagger(-p-q)\cdot H(p+q)_\alpha \times\{V(p)\tilde{\Phi}(q)\}. \tag{29}$$

Then we can distribute $H(p+q)_\alpha$ to $V(p)$ and $\tilde{\Phi}(q)$ as

$$H(p+q)\{V(p)\tilde{\Phi}(q)\} = \{D(p)V(p)\}\tilde{\Phi}(q) + V(p)\times\{H(q)\tilde{\Phi}(q)\}, \tag{30}$$

where we should recall $H(p+q)_\alpha = -\frac{\partial}{\partial\theta^\alpha}+\frac{1}{2}(\bar{\theta}\not{p})_\alpha+\frac{1}{2}(\bar{\theta}\not{q})_\alpha - \frac{1}{2}m\bar{\theta}_\alpha$ and $D(p)_\alpha = -\frac{\partial}{\partial\theta^\alpha}+\frac{1}{2}(\bar{\theta}\not{p})_\alpha$. We apply the Leibniz rule for $-\frac{\partial}{\partial\theta^\alpha}$ and distribute $\frac{1}{2}(\bar{\theta}\not{p})_\alpha$ to $V(p)$ and $\frac{1}{2}(\bar{\theta}\not{q}) - \frac{1}{2}m\bar{\theta}_\alpha$ to $\tilde{\Phi}(q)$. The momenta in the covariant derivatives should be chosen as the momenta of the fields on which they act. We frequently use this kind of integration by parts at interaction vertices. Since each interaction vertex contains one pair of $\tilde{\Phi}$ and $\tilde{\Phi}^\dagger$, through the integration by parts, we move $\frac{1}{2}m\bar{\theta}_\alpha$ or $\frac{1}{2}m\theta_\alpha$ in twisted covariant derivatives from $\tilde{\Phi}$ to $\tilde{\Phi}^\dagger$, or vice versa. We never distribute $\frac{1}{2}m\bar{\theta}_\alpha$, $\frac{1}{2}m\theta_\alpha$ to the auxiliary field V.

Therefore, the rules for partial integration are as follows: twisted covariant derivatives E_α, $\bar{E}_\alpha(H_\alpha, \bar{H}_\alpha)$ are replaced by another set of twisted covariant derivatives H_α, $\bar{H}_\alpha(E_\alpha, \bar{E}_\alpha)$ when they act on the dynamical fields $\tilde{\Phi}$, $\tilde{\Phi}^\dagger$, while they act as ordinary covariant derivatives D_α, $\bar{D}_\alpha$ on the auxiliary field. The momenta in covariant derivatives should be chosen as the momenta of the fields on which they act. We can draw the result of the integration by parts (28)–(30) as follows:



Then we perform θ, $\bar{\theta}$ integration by parts in Eq. (26). Recall the term in the first bracket has been rewritten as (27). We apply, through the integration by parts, $E(p-q)^2\bar{H}(p-q)^2$ in (27) to the second bracket and $V(p,\theta)$ in (26). We will obtain 16 terms if we perform the integration by parts straightforwardly. To avoid complicated expressions, we rewrite the products of twisted covariant derivatives in (27) as follows:

$$\frac{1}{4}E(-p-q)^2\bar{H}(-p-q)^2$$
$$= \frac{1}{4}\bar{H}(-p-q)^2E(-p-q)^2 - \bar{H}(-p-q)$$
$$\times\{(-\not{p}-\not{q})+m\}E(-p-q) + \{(-p-q)^2 - m^2\}, \tag{31}$$

which is shown in Appendix E (see proposition E-2). Noting that the order of Es and $\bar{H}$s is reversed, we see that many terms vanish through the integration by parts. For instance, if we integrate by parts and move $\bar{H}(-p-q)_\alpha$ in the first term, it acts as $\bar{E}(q)_\alpha$ on the second bracket and as $\bar{D}(p)_\alpha$ on $V(p)$. However, since the second bracket in (26) already has $\bar{E}^2$, it vanishes when $\bar{E}(q)_\alpha$ acts on it. Similarly, we will easily find many terms vanish through the integration by parts if we use the above equation.

We first evaluate the contribution of the third term in the right-hand side of (31). Since this term has no covariant derivatives, we can easily evaluate it by substituting $\{(-p-q)^2 - m^2\}\delta^{(4)}(\theta-\theta')$ for the first bracket in (26):

$$N\int\frac{d^3p}{(2\pi^3)}\frac{d^3q}{(2\pi)^3}\int d^4\theta d^4\theta' V(-p,\theta')\frac{1}{q^2-m^2+i\epsilon}$$
$$\cdot\left[\delta^{(4)}(\theta-\theta')\frac{1}{4}\bar{E}(q)^2H(q)^2\delta^{(4)}(\theta-\theta')\right]\cdot V(p,\theta)$$
$$= +\int\frac{d^3p}{(2\pi^3)}\int d^4\theta V(-p,\theta')\cdot\int\frac{d^3q}{(2\pi)^3}\frac{N}{q^2-m^2+i\epsilon}$$
$$\cdot V(p,\theta),$$

where we should recall Eq. (24). This exactly cancels the linearly divergent contribution of (25).

Then we evaluate the contribution of the second term in (31). We first apply $\bar{H}(-p-q)$ to the second bracket and $V(p,\theta)$ in (26). However, for the reason mentioned before, the second bracket vanishes if we apply $\bar{H}(-p-q)$ on it as $\bar{E}(q)$. So we apply $\bar{H}(-p-q)$ to $V(p,\theta)$ as $\bar{D}(p)$ through the integration by parts. On the other hand, the twisted covariant derivative $E(-p-q)$ can be distributed to both the second bracket and $V(p,\theta)$ in (26). But if we operate it to the second bracket, we obtain the following factor:

$$\delta^{(4)}(\theta-\theta')H(q)_\alpha\bar{E}(q)^2H(q)^2\delta^{(4)}(\theta-\theta'), \tag{32}$$

and using the commutation relation of $[H(q)_\alpha, \bar{E}(q)^2] = -2[\bar{E}(q)(\not{q}-m)]_\alpha$ (again see Appendix E) we can rewrite this as

$$-2\delta^{(4)}(\theta-\theta')[\bar{E}(q)(\not{q}-m)]_\alpha H(q)^2\delta^{(4)}(\theta-\theta').$$

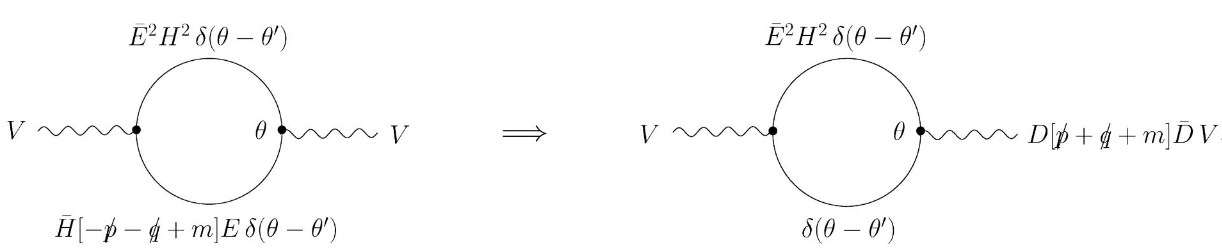
We find this is zero due to the property of Grassmann variables. Since $\delta^{(4)}(\theta-\theta') = \frac{1}{4}(\theta-\theta')^2(\bar{\theta}-\bar{\theta}')^2$, the term containing two delta functions vanishes unless we have four derivatives $(\frac{\partial}{\partial\theta^\alpha})^2(\frac{\partial}{\partial\bar{\theta}^\alpha})^2$ between them. Therefore, a nonzero contribution of the second term in (31) comes only from the term where $\bar{H}(-\not{p}-\not{q}+m)E$ is applied to the auxiliary field. Noting that with Grassmann-even fields A, B

$$\int d^4\theta\{\bar{H}(-p)(-\not{p}+m)E(-p)A\}\cdot B$$
$$= +\int d^4\theta\{(-\not{p}+m)^\alpha{}_\beta E(-p)^\beta A\}\cdot\{\bar{E}(p)_\alpha B\}$$
$$= -(-\not{p}+m)^\alpha{}_\beta\int d^4\theta A\cdot\{H(p)^\beta\bar{E}(p)_\alpha B\}$$
$$= -(-\not{p}-m)_\beta{}^\alpha\int d^4\theta A\cdot\{H(p)^\beta\bar{E}(p)_\alpha B\}$$
$$= \int d^4\theta A\cdot\{H(p)(\not{p}+m)\bar{E}(p)B\}, \tag{33}$$

the contribution of the second term of (31) becomes

$$-N\int\frac{d^3p}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\int d^4\theta d^4\theta' V(-p,\theta')\delta^{(4)}(\theta-\theta')$$
$$\times\left[\frac{1}{4}\bar{E}(q)^2H(q)^2\delta^{(4)}(\theta-\theta')\right]\frac{1}{(p+q)^2-m^2+i\epsilon}$$
$$\times\frac{1}{q^2-m^2+i\epsilon}D(p)\{\not{p}+\not{q}+m\}\bar{D}(p)V(p,\theta). \tag{34}$$

We can graphically express this integration by parts as follows:



Recalling the Eq. (24) again, we evaluate (34) as follows:

$$-N\int\frac{d^3p}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\int d^4\theta V(-p,\theta)\frac{1}{(p+q)^2-m^2+i\epsilon}\times\frac{1}{q^2-m^2+i\epsilon}D(p)\{\not p+\not q+m\}\bar{D}(p)V(p,\theta).$$

When we use the identity

$$\int\frac{d^3q}{(2\pi)^3}\frac{\not q}{[(p+q)^2-m^2+i\epsilon](q^2-m^2+i\epsilon)}=\int\frac{d^3q}{(2\pi)^3}\frac{-\frac{1}{2}\not p}{[(p+q)^2-m^2+i\epsilon](q^2-m^2+i\epsilon)},$$

which is easily shown by shifting the integration variables $q\to -q-p$ in the left-hand side, and we obtain the following result:

$$-iN\int\frac{d^3p}{(2\pi)^3}\int d^4\theta V(-p,\theta)D(p)\left(\frac{\not p}{2}+m\right)\bar{D}(p)V(p,\theta)\cdot\frac{1}{4\pi}I(p^2)^{-1}, \tag{35}$$

where we define $I(p^2)^{-1}$ as

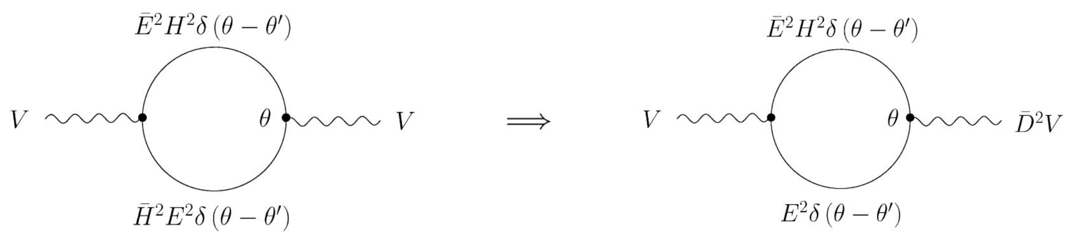
$$I(p^2)^{-1}:=\frac{4\pi}{i}\int\frac{d^3q}{(2\pi)^3}\frac{1}{(p+q)^2-m^2+i\epsilon}\times\frac{1}{q^2-m^2+i\epsilon}=\frac{\arctan\sqrt{\frac{-p^2}{4m^2}}}{\sqrt{-p^2}}.$$

The second equality is shown in Appendix F. This definition of $I(p^2)^{-1}$ is the same as that in [7].

We now evaluate the contribution of the first term in (31)

$$\frac{1}{4}\bar{H}(-p-q)^2E(-p-q)^2\delta^{(4)}(\theta-\theta').$$

We integrate by parts and apply $\frac{1}{4}\bar{H}^2E^2$ to the second bracket and $V(p,\theta)$ in (26). For the same reason as before, we first apply $\bar{H}(-p-q)^2$ only to $V(p,\theta)$ as $\bar{D}(p)^2$:



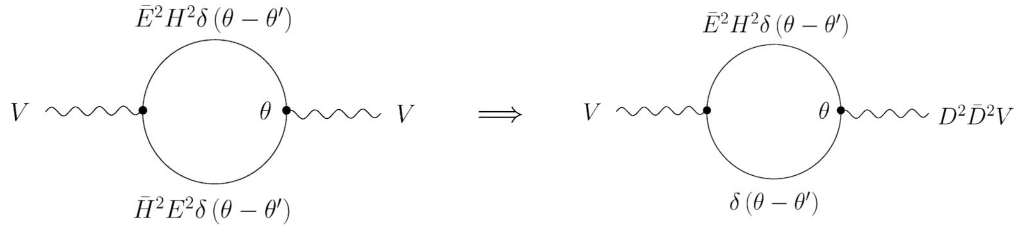
When we moreover apply $E^2(-p-q)$ to the second bracket (the upper chiral propagator in the above picture) and $V(p,\theta)$ in (26), we should note the term (32) vanishes as before and the term

$$\delta^{(4)}(\theta-\theta')H(q)^2\bar{E}(q)^2H(q)^2\delta^{(4)}(\theta-\theta')$$

also vanishes. This is because we have at most two derivatives between two delta functions since we can show $H(q)^2\bar{E}(q)^2H(q)^2=(q^2-m^2)H(q)^2$. We do not have nonzero contributions unless there are four derivatives between two delta functions. Therefore again, the nonzero contribution of the first term in (31) comes only from the term where $\frac{1}{4}\bar{H}^2E^2$ is applied to the auxiliary field:

$$\begin{aligned}
&N\int\frac{d^3p}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\int d^4\theta d^4\theta' V(-p,\theta')\cdot\frac{1}{(p+q)^2-m^2+i\epsilon}\frac{1}{q^2-m^2+i\epsilon}\delta^{(4)}(\theta-\theta')\left[\frac{1}{4}\bar{E}(q)^2H(q)^2\delta^{(4)}(\theta-\theta')\right]\\
&\quad\times\frac{1}{4}D(p)^2\bar{D}(p)^2V(p,\theta)\\
&=N\int\frac{d^3p}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\int d^4\theta V(-p,\theta)\cdot\frac{1}{(p+q)^2-m^2+i\epsilon}\frac{1}{q^2-m^2+i\epsilon}\cdot\frac{1}{4}D(p)^2\bar{D}(p)^2V(p,\theta)\\
&=iN\int\frac{d^3p}{(2\pi)^3}\int d^4\theta V(-p,\theta)\frac{1}{4}D(p)^2\bar{D}(p)^2V(p,\theta)\cdot\frac{1}{4\pi}I(p^2)^{-1}.
\end{aligned}\tag{36}$$

We can graphically express this integration by parts as follows:



In summary, we can evaluate Eq. (26) by using the formula (31) for one of the chiral propagators. The contribution of the third term of (31) cancels the contribution of (25),; the contributions of the second and third term are given by (35) and (36). Considering all the contributions, the quadratic terms of the auxiliary field in the effective action become as follows:

$$\frac{1}{2}\int\frac{d^3p}{(2\pi)^3}\int d^4\theta V(-p,\theta)(iG^{-1})V(p,\theta) = \frac{N}{2}\int\frac{d^3p}{(2\pi)^3}\int d^4\theta V(-p,\theta)\left\{\frac{1}{4}D(p)^2\bar{D}(p)^2 - \frac{1}{2}D(p)\not{p}\bar{D}(p) - mD(p)\bar{D}(p)\right\} \times V(p,\theta)\cdot\frac{1}{4\pi}I(p^2)^{-1} = \frac{N}{2}\int\frac{d^3p}{(2\pi)^3}\int d^4\theta V(-p,\theta)\left\{\frac{1}{4}D(p)\bar{D}(p)^2D(p) - mD(p)\bar{D}(p)\right\}V(p,\theta)\cdot\frac{1}{4\pi}I(p^2)^{-1}, \tag{37}$$

where the last equality is shown in Appendix G. This inverse propagator of the auxiliary field is the same as that of the super Yang-Mills field except for the mass term $mD\bar{D}$ and nonlocal factor $I(p^2)^{-1}$.

C. Propagator of the auxiliary field

In order to derive the propagator of the auxiliary field from (37), we have to evaluate the inverse of the differential operator in it. To do so, we first study the algebra of D_α and $\bar{D}_\alpha$ in detail. Because of the fact that $D^2 = \bar{D}^2 = 0$, all the differential operators composed of D_α and $\bar{D}_\alpha$ can be written as linear combinations of the following six operators (similar to the case of $\mathcal{N} = 1$ in four dimensions [10]), namely, a set of projection operators of chiral and antichiral superfield

$$P_2 := \frac{\bar{D}^2D^2}{4p^2}, \qquad P_1 := \frac{D^2\bar{D}^2}{4p^2},$$

and other four operators:

$$P_+ := -\frac{iD^2}{2\sqrt{-p^2}}, \qquad P_- := -\frac{i\bar{D}^2}{2\sqrt{-p^2}},$$

$$P_T := -\frac{D\bar{D}^2D}{2p^2}, \qquad P_D := -\frac{iD\bar{D}}{\sqrt{-p^2}},$$

where we omit explicitly writing the momentum dependence of covariant derivatives. Note that the term $D\not{p}\bar{D}$ can be written as a linear combination of P_T and P_1. Through a straightforward calculation, we can show that

$$P_1 + P_2 + P_T = 1. \tag{38}$$

The multiplication rules of these operators are indicated in Table I, where the blanks mean zero. We show a part of this table explicitly in Appendix G.

We now want to derive the inverse of

$$\frac{1}{4}D\bar{D}^2D - mD\bar{D} = -\frac{p^2}{2}P_T - im\sqrt{-p^2}P_D \tag{39}$$

by using Table I. But this operator is noninvertible because this annihilates arbitrary antichiral superfields (indeed also annihilates arbitrary chiral superfields). Note here that

$$D\bar{D} = \bar{D}D + \{D_\alpha, \bar{D}^\alpha\} = \bar{D}D + i\,\mathrm{tr}(\not{\partial}) = \bar{D}D.$$

This singularity is of course due to the gauge symmetry. So we need to introduce a gauge-fixing term to define the inverse of (39). We here introduce the following supersymmetric gauge-fixing term in the action

$$S_{\rm GF} = \frac{N}{2\alpha}\int\frac{d^3p}{(2\pi)^3}\int d^4\theta V(-p,\theta)\cdot\frac{1}{8}[D^2\bar{D}^2 + \bar{D}^2D^2]V(p,\theta)\cdot\frac{1}{4\pi}I(p^2)^{-1},$$

where we omit writing momenta p of covariant derivatives

TABLE I. The multiplicative property of operators.

Left\right	P_1	P_2	P_+	P_-	P_T	P_D
P_1	P_1		P_+			
P_2		P_2		P_-		
P_+		P_+		P_1		
P_-	P_-		P_2			
P_T					P_T	P_D
P_D					P_D	P_T

explicitly. With this gauge fixing, the inverse propagator of the auxiliary field is given by

$$\frac{1}{2}\int\frac{d^3p}{(2\pi)^3}d^4\theta V(-p)(iG^{-1})V(p)+S_{\rm GF}$$
$$=\frac{N}{2}\int\frac{d^3p}{(2\pi)^3}\int d^4\theta V(-p,\theta)\nabla_V V(p,\theta)\cdot\frac{1}{4\pi}I(p^2)^{-1},$$

where

$$\nabla_V=\frac{1}{4}D\bar{D}^2D-mD\bar{D}+\frac{1}{8\alpha}(D^2\bar{D}^2+\bar{D}^2D^2)$$
$$=-\frac{P_T}{2}-im\sqrt{-p^2}P_D+\frac{p^2}{2\alpha}(P_1+P_2).$$

Then we can evaluate the inverse of ∇_V. Indeed, by supposing $\nabla_V^{-1}=aP_1+bP_2+cP_++dP_-+eP_T+fP_D$, we can easily show

$$\nabla_V\nabla_V^{-1}=\frac{p^2}{2\alpha}(aP_1+bP_2)+\frac{p^2}{2\alpha}(cP_++dP_-)$$
$$-\frac{1}{2}(p^2e+2im\sqrt{-p^2}f)P_T$$
$$-\frac{1}{2}(p^2f+2im\sqrt{-p^2}e)P_D.$$

If we impose $a=b=\frac{\alpha}{p^2}$, $c=d=0$, $e=-\frac{2}{p^2-4m^2}$, and $f=-\frac{4im}{\sqrt{-p^2}(p^2-4m^2)}$,

$$\nabla_V\nabla_V^{-1}=P_1+P_2+P_T=1.$$

Therefore the inverse operator of ∇_V is

$$\nabla_V^{-1}=-\frac{2}{p^2-4m^2}\left(P_T+\frac{2im}{\sqrt{-p^2}}P_D\right)+\frac{2\alpha}{p^2}(P_1+P_2)$$
$$=\frac{1}{p^2-4m^2}\cdot\frac{D\bar{D}^2D-4mD\bar{D}}{p^2}$$
$$+\frac{\alpha}{2p^4}(D^2\bar{D}^2+\bar{D}^2D^2).$$

Using this inverse operator, the superpropagator of the auxiliary field V can be written as

$$\langle V(-p,\theta',\bar{\theta}')V(p,\theta,\bar{\theta})\rangle_0=\frac{4\pi i}{N}I(p^2)\cdot\nabla_V^{-1}\delta^{(4)}(\theta-\theta'). \tag{40}$$

Note that this propagator has a pole at $p^2=4m^2$, which implies that a one-particle state of the auxiliary field is a bound state of the dynamical field.

If we expand this propagator in components, we obtain propagators of component fields. However, it leads to a complicated expression to expand (40) straightforwardly since the auxiliary superfield $V(p,\theta,\bar{\theta})$ has many unphysical component fields which can be eliminated if we choose the nonsupersymmetric gauge such as the Wess-Zumino gauge. We can, nevertheless, easily obtain the propagators of v_μ and M by taking the coefficient of $\bar{\theta}'\theta'\bar{\theta}\theta$ in the expansion of (40), namely

$$\langle v_\mu(-p)v_\nu(p)\rangle_0=\frac{4\pi i}{N}I(p^2)\bigg\{\frac{1}{p^2-4m^2}\times\bigg[-\eta_{\mu\nu}+\left(1+\alpha\cdot\frac{p^2-4m^2}{p^2}\right)\times\frac{p_\mu p_\nu}{p^2}-\frac{2mi}{p^2}\epsilon_{\mu\nu\rho}p^\rho\bigg]\bigg\} \tag{41}$$

$$\langle M(-p)M(p)\rangle=\frac{4\pi i}{N}I(p^2)\frac{1}{p^2-4m^2}. \tag{42}$$

These propagators of component fields coincide with the result in [7]. The derivations of these expressions are shown in Appendix H.

V. DIVERGENT DIAGRAMS AND RENORMALIZATION

In this section, we investigate divergent diagrams and the renormalizability. We first study the superficial degree of divergence and show that there are two types of divergent diagrams. We can prove all divergences can be eliminated by renormalizations of the coupling constant g and the wave function of the dynamical field $\tilde{\Phi}$.

A. Superficial degree of divergence

We first evaluate the superficial degree of divergence. Recall the superpropagators of the dynamical field and the auxiliary field

$$\langle\tilde{\Phi}^{\dagger k}(-p,\theta',\bar{\theta}')\tilde{\Phi}^j(p,\theta,\bar{\theta})\rangle_0=\delta^{jk}\frac{i}{p^2-m^2+i\epsilon}\cdot\frac{1}{4}\bar{E}^2H^2\delta^{(4)}(\theta-\theta')\langle V(-p,\theta',\bar{\theta}')V(p,\theta,\bar{\theta})\rangle_0$$
$$=\frac{4\pi i}{N}I(p^2)\left[\frac{1}{p^2-4m^2}\cdot\frac{D\bar{D}^2D-4mD\bar{D}}{p^2}+\frac{\alpha}{2p^2}(D^2\bar{D}^2+\bar{D}^2D^2)\right]\delta^{(4)}(\theta-\theta'),$$

where momenta of covariant derivatives are all equal to p. Postponing the discussion on momentum dependence of covariant derivatives, we can evaluate high energy behaviors of above superpropagators as follows:

$$\langle\tilde{\Phi}^{\dagger k}(-p,\theta',\bar{\theta}')\tilde{\Phi}^{j}(p,\theta,\bar{\theta})\rangle_0 \sim \frac{1}{p^2}\times \bar{E}^2 H^2$$
$$\langle V(-p,\theta',\bar{\theta}')V(p,\theta,\bar{\theta})\rangle_0 \sim \frac{\sqrt{-p^2}}{p^4}\times (D\bar{D}^2 D \quad \text{or} \quad mD\bar{D} \quad \text{or} \quad D^2\bar{D}^2 \quad \text{or} \quad \bar{D}^2 D^2).$$

Note that

$$I(p^2) = \frac{\sqrt{-p^2}}{\arctan\sqrt{-\frac{p^2}{4m^2}}} \sim \sqrt{-p^2}$$

at high energy.

Then we evaluate high energy behaviors of covariant derivatives. In any loop diagram, we can integrate by parts and reduce the number of integrations over Grassmann coordinates by virtue of the delta function $\delta^{(4)}(\theta-\theta')$. Then the final expression of Grassmann integrations on each loop has a factor

$$\delta^{(4)}(\theta-\theta')(\text{product of covariant derivatives})\delta^{(4)}(\theta-\theta')$$

in the integrand. As we have seen in the previous section, however, this factor will give a vanishing result unless there are four derivatives between two delta functions. What can be obtained if we have six covariant derivatives between two delta functions? The answer turns out to be zero when we note

$$D^2\bar{D}^2D^2 = [D^2,\bar{D}^2]D^2 = 4p^2D^2,$$

since we have only two derivatives between delta functions. How about the case in which we have eight derivatives between delta functions? In such a case, we obtain a factor

$$D^2\bar{D}^2D^2\bar{D}^2 = 4p^2D^2\bar{D}^2$$

and this gives a nonzero contribution. Similarly, if we have 12 covariant derivatives, we obtain $(D^2\bar{D}^2)^3 = (4p^2)^2D^2\bar{D}^2$. Therefore we find that $D^2\bar{D}^2 \sim p^2$ unless they are used to differentiate a delta function $\delta^{(4)}(\theta-\theta')$. In every loop, we use one $D^2\bar{D}^2$ to differentiate a delta function in the formula

$$\int d^4\theta'\delta^{(4)}(\theta-\theta')\left[\frac{1}{4}D^2\bar{D}^2\right]\delta^{(4)}(\theta-\theta') = 1.$$

This formula is easily shown in the same way as (24). When the diagram contains L loops, L sets of $D^2\bar{D}^2$ are used to differentiate delta functions, reducing the degree of divergence by $2L$.

By counting $D\bar{D} \sim \not{p}$, we then obtain the complete behavior of superpropagators at high energy as

$$\langle\tilde{\Phi}^{\dagger k}(-p,\theta',\bar{\theta}')\tilde{\Phi}^{j}(p,\theta,\bar{\theta})\rangle_0 \sim 1$$
$$\langle V(-p,\theta',\bar{\theta}')V(p,\theta,\bar{\theta})\rangle_0 \sim \frac{1}{\sqrt{-p^2}},$$

and the degree of divergence has to be reduced by $2L$ if the diagram has L loops. Then we have the superficial degree of divergence d as

$$d = 3L - P_V - 2L,$$

where L denotes the number of loops and P_V denotes the number of propagators of the auxiliary field. The first term comes from the fact that each loop has three momentum integrations. The last term comes from the fact that we use four covariant derivatives at each loop to differentiate a delta function. Using the relation $L = (P_V + P_\Phi) - V + 1$, where P_Φ is the number of propagators of the dynamical field and V denotes the number of vertices, we find

$$d = P_\Phi - V + 1. \tag{43}$$

Then we should notice that all vertices in this theory contain exactly one $\tilde{\Phi}^\dagger\tilde{\Phi}$. This means that

$$V = P_\Phi + \frac{E_\Phi}{2}, \tag{44}$$

where E_Φ denotes the number of external lines of $\tilde{\Phi}$. The formula (44) can be shown as follows. Noting the symmetry of $\tilde{\Phi}\to e^{i\alpha}\tilde{\Phi}$, $\tilde{\Phi}^\dagger \to e^{-i\alpha}\tilde{\Phi}^\dagger$, we find that the internal

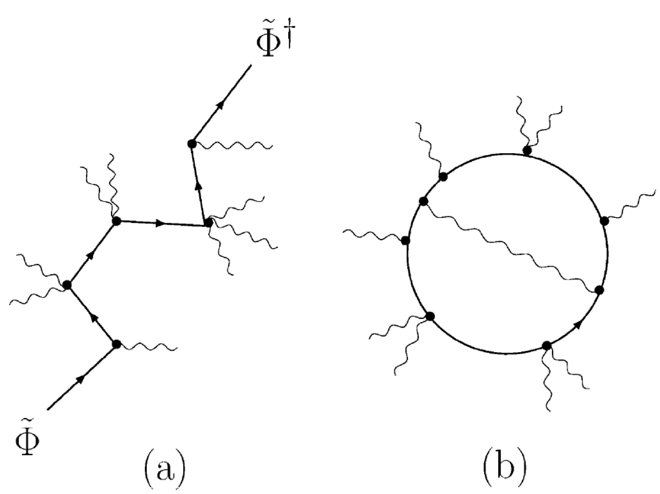


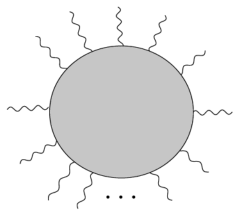
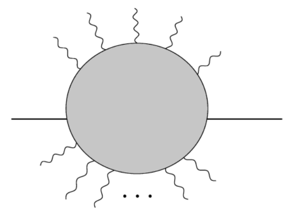
FIG. 1. Typical examples of two types of $\tilde{\Phi}$-lines.

lines of $\tilde{\Phi}$ are not branched. This means there are two types of $\tilde{\Phi}$ lines. The first type goes from one external line to another external line without branches: (a) The second type is an internal circle of $\tilde{\Phi}$ which has no external lines; (b) Typical examples of these two types are indicated in Fig. 1. We now count the number of vertices on the line of $\tilde{\Phi}$. For type (a), we easily find $V = P_\Phi + 1 = P_\Phi + \frac{E_\Phi}{2}$. On the other hand, for type (b), $V = P_\Phi$ is satisfied. But this can be also written as $V = P_\Phi + \frac{E_\Phi}{2}$ because $E_\Phi = 0$ for type (b). Since both types (a) and (b) satisfy (44), diagrams containing both types also satisfy (44).

Combining (43) and (44), we find the final result:

$$d = 1 - \frac{E_\Phi}{2}.$$

Therefore there are only two types of divergent diagrams:

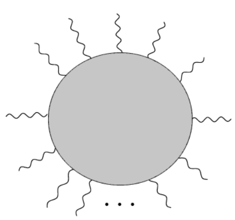
 $: d = 1$, $\tilde{\Phi}$  $\tilde{\Phi}^\dagger$ $: d = 0.$ (45)

In the following, we study these two types of diagrams in detail. We will show that all the divergences can be absorbed into the bare coupling constant and the wave function of $\tilde{\Phi}$.

B. Renormalization of the coupling constant

In this subsection, we study the amplitudes without external $\tilde{\Phi}$-lines shown in the left diagram in (45). Since its superficial degree of divergence is 1, it may contain linear and logarithmic divergences. To see these divergence explicitly, one might expand the amplitude in powers of the external momentum p^μ in the same way as in the ordinary field theory. However, since we now work in the superfield perturbation theory, each field has Grassmann coordinates in addition to spacetime coordinates. Therefore we have to expand the amplitude in powers of D_α, $\bar{D}_\alpha$ as well as p^μ. The reason for expanding it by D_α, $\bar{D}_\alpha$ rather than $\frac{\partial}{\partial\theta^\alpha}$, $\frac{\partial}{\partial\bar{\theta}^\alpha}$ is supersymmetry.

Although there are no terms linear in p^μ due to the Lorentz invariance, there may be terms linear in D^2, $\bar{D}^2$ or $\bar{D}D$, which can be logarithmically divergent:

 $= a\Lambda + \log\Lambda\left(b\bar{D}D + cD^2 + d\bar{D}^2\right) +$ finite terms,

where a, b, c and d are constants independent of external momenta and Grassmann coordinates. The differential operators $\bar{D}D$, D^2, $\bar{D}^2$ acts on external auxiliary fields and independent of internal momenta. Terms linear in p^2, p^4 or $p^2\bar{D}D$ are included in finite terms.

Therefore, the effective action might need counter terms of the form

$$\int d^3x \int d^4\theta[\alpha_n V^n + \beta_n V^{n-2}(\bar{D}V)(DV) + \gamma_n V^{n-2}(DV)^2 + \delta_n V^{n-2}(\bar{D}V)^2], \qquad (46)$$

where n is a positive integer and α_n, β_n, γ_n, δ_n are constants. When $n = 1$, the second, third, and fourth terms should be considered as $\bar{D}DV$, D^2V, and $\bar{D}^2V$, respectively.

If we assume the existence of a gauge invariant regularization, γ_n and δ_n must be zero because operators $V^{n-1}D^2V$, $V^{n^1}\bar{D}^2V$ are not gauge invariant. Similarly, we can show $\beta_n = 0$ unless $n \leq 2$ and $\alpha_n = 0$ unless $n = 1$. However, we can explicitly show these results by analyzing loop integrations without assuming the existence of a gauge invariant regularization.

1. Operators of the form $V^{n-1}\bar{D}DV$, $V^{n-1}D^2V$, $V^{n-1}\bar{D}^2V$

We first show explicitly that β_n, γ_n, $\delta_n = 0$ for all n. Note that all covariant derivatives acting on external fields come from partial integrals over Grassmann coordinates. In order to obtain operators $V^{n-1}\bar{D}DV$, $V^{n-1}D^2V$, $V^{n-1}\bar{D}^2V$, we have to move two covariant derivatives from propagators to external fields through integrations by parts. Suppose there are k propagators in the diagram.

Since every term in every propagator has four covariant derivatives except for $mD\bar{D}$ in the superpropagator of the auxiliary field, $4k-2$ covariant derivatives remain in loops after moving two covariant derivatives to external fields, assuming there is no $mD\bar{D}$ in the diagram. We perform integrations over Grassmann coordinates and shrink all Grassmann loops using the formula

$$\int d^4\theta' \delta^{(4)}(\theta-\theta')\left[\frac{1}{4}D^2\bar{D}^2\right]\delta^{(4)}(\theta-\theta')=1$$

or the similar one which has $E^2\bar{H}^2$ instead of $D^2\bar{D}^2$. We use $4L$ covariant derivatives to shrink all Grassmann loops when the diagrams has L loops. Then $4(k-L)-2$ covariant derivatives remain. But, since all Grassmann loops have been shrunk, these $4(k-L)-2$ derivatives must be changed into internal momenta by using the anticommutation relation

$$[D^2, \bar{D}^2] = 4q^2 - 4\bar{D}\not{q}D$$

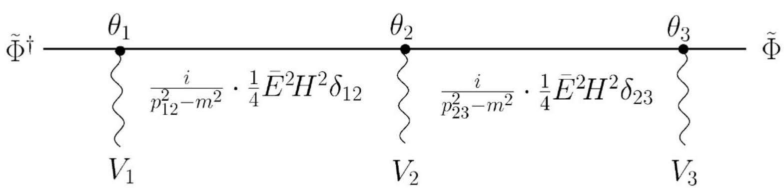
unless they vanish for the reason that there are less than four derivatives between two delta functions. Therefore, if we move two covariant derivatives to external fields, at most $4(k-L)-2$ covariant derivatives are changed into $q^{2(k-L)-1}$, where q is a typical internal momentum. Recall here that $D^2\bar{D}^2 \sim p^2$. However, assuming Lorentz invariance, this factor has to be written as $(q^2)^{k-L}\not{q}$, which contains an odd number of internal momenta. Notice that each propagator is invariant under $p\to -p$ except for covariant derivatives and vertex factors are independent of momenta. Then we find that the total integrand is an odd function of internal momenta. We know that the degree of divergence is reduced at least by 1 if the Feynman integrand is an odd function of internal momenta. Then these integrals are not divergent because their original superficial degree of divergence is zero.

Let us consider what happens if we have some $mD\bar{D}$s in the diagram. Noting that we count all of $D^2\bar{D}^2$, $D\bar{D}^2D$, $mD\bar{D}$ as $\sim p^2$ when we evaluate the superficial degree of divergence, we find that the degree of divergence is again reduced at least by 1 if the diagram contains some $mD\bar{D}$s.

It is proved that in the effective action there is no quantum correction to the operator of the form of $V^{n-1}D^2V$, $V^{n-1}\bar{D}^2V$, or $V^{n-1}\bar{D}DV$.

2. Operators of the form V^n

We now show explicitly that α_n in (46) vanishes unless $n=1$. Recall that no V^2 term arose when we evaluated the inverse propagator of the auxiliary field in subsection IV B. The reason for this is the cancellation between (25) and the contribution of the third term in (31). The contribution of the third term of (31) is a part of (26). The contribution proportional to V^2 induced by the partial integration over Grassmann coordinates in (26) exactly canceled (25) which is also proportional to V^2. This cancellation is due to the fact that the dynamical field is a chiral superfield. To see this, we examine the following diagram:

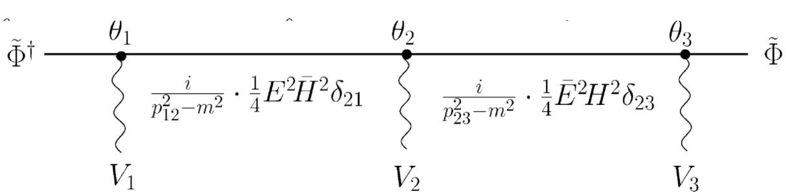


, (47)

where δ_{ij} stands for $\delta^{(4)}(\theta_i-\theta_j)$ and p_{ij} denotes the momentum which flows from θ_i to θ_j. Therefore p_{ji} is equal to $-p_{ij}$. When we write $\bar{E}^2H^2\delta_{ij}$, the covariant derivatives in front of δ_{ij} stand for covariant derivatives with Grassmann coordinates θ_i, $\bar{\theta}_i$ and momentum p_{ij}. On the other hand, if we write $\bar{E}^2H^2\delta_{ji}$, they are covariant derivatives with θ_j, $\bar{\theta}_j$, p_{ji}. We distinguish δ_{ij} from δ_{ji}. Then we can write the formula (27) as

$$\frac{1}{4}\bar{E}^2H^2\delta_{12}=\frac{1}{4}E^2\bar{H}^2\delta_{21}.$$

By using this formula, we can rewrite (47) as

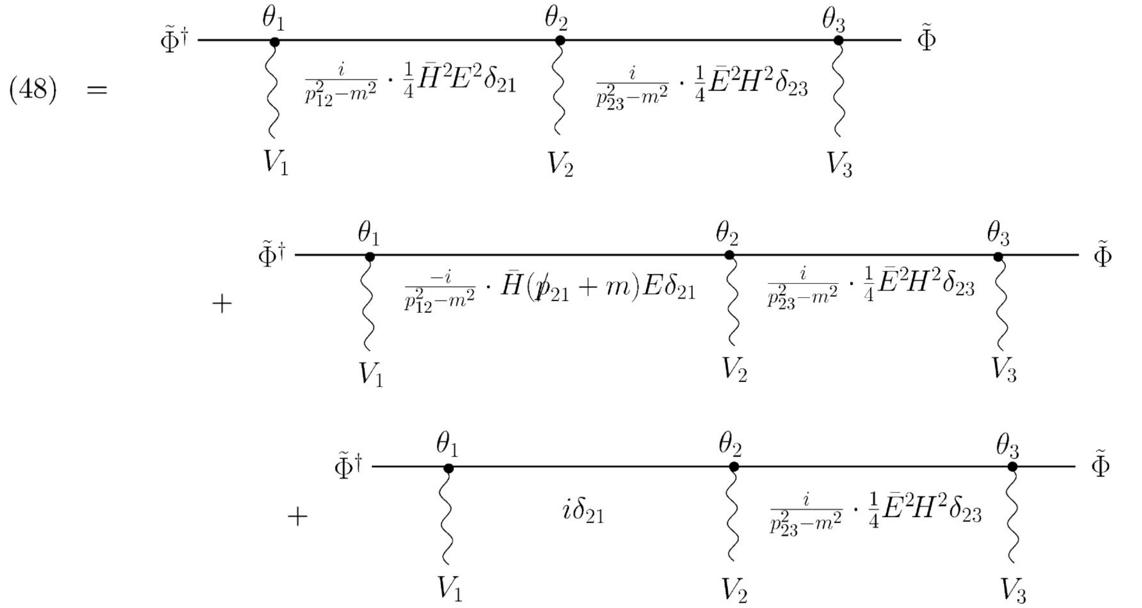


. (48)

Then we use the formula

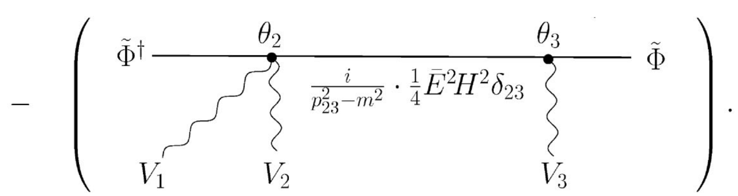
$$\frac{1}{4}E^2\bar{H}^2\delta_{21}=\frac{1}{4}\bar{H}^2E^2\delta_{21}-\bar{H}(\not{p}_{21}+m)E\delta_{21}+(p_{21}^2-m^2)\delta_{21},$$

which is easily shown by the commutation relations of E^2 and $\bar{H}^2$. We can graphically express this formula as follows:



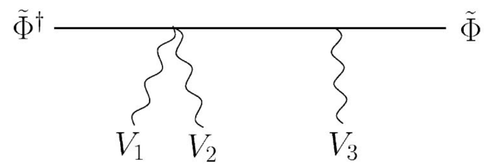
$$. \tag{49}$$

In the third term in the right-hand side, we can easily perform the integration over θ_1. Then we obtain the following contribution



$$. \tag{50}$$

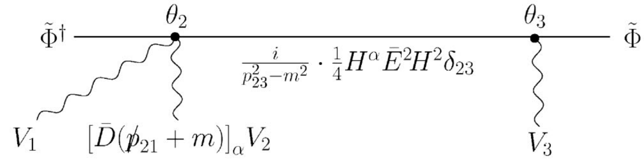
In order to understand the minus sign, we should recall that the vertex factor is $i(-1)^n$ when the vertex is attached to n lines of auxiliary field. Although performing the integration over θ_1 does not change the number of lines of auxiliary field, it reduces the number of vertices by one, leaving a factor i which has been attached to the annihilated vertex. Moreover, there is another i in front of δ_{21}. The minus sign in (50) comes from these two factors of i. Then the contribution from the third term in (49) exactly cancels that of the following diagram:



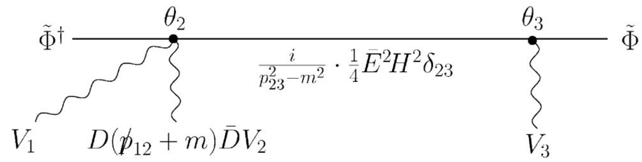
$$. \tag{51}$$

We now consider the remaining terms, namely, the first and second terms in (49). Performing the partial integration over θ_2, we can show as before that all vanish except for contributions in which all covariant derivatives between θ_1 and θ_2 are applied to external auxiliary fields. For instance, see the second term in (49). If we move $\bar{H}_\alpha$ from the left chiral propagator to the right one exchanging it for $\bar{E}_\alpha$, it vanishes because $\bar{E}_\alpha\bar{E}^2 = 0$. We have to apply $\bar{H}_\alpha$ to V_2 to obtain a nonzero contribution. Then we move E_α in the left chiral propagator and perform the integration over θ_1 by virtue of δ_{21}. The result is as follows:

$$\frac{-i}{p_{12}^2 - m^2} \times \Bigg[$$



$+$

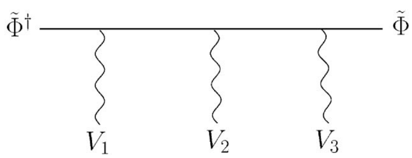


$\Bigg],$

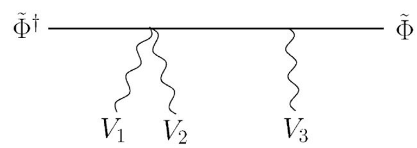
where we use the similar equation as (33).

In the second term in the bracket, all covariant derivatives moved to the external auxiliary field V_2. On the other hand, in the first term, H_α is applied to the other chiral propagator. This H_α can, however, move to V_1 or V_2 if we again perform the partial integration over θ_2. Noting that $E_\alpha \tilde{\Phi}^\dagger = 0$, we find that the contribution of applying H_α to the external $\tilde{\Phi}^\dagger$ vanishes. So we move it only to external auxiliary fields exchanging it for D_α. The result is the same if there is an another chiral propagator instead of the external $\tilde{\Phi}^\dagger$ because $E_\alpha < \tilde{\Phi}^\dagger(p, \theta, \bar{\theta})\tilde{\Phi}(-p, \theta', \bar{\theta}')>_0 = 0$. This vanishing occurs due to the fact that the dynamical field is a (twisted) chiral superfield. Therefore, through the integration by parts, the nonzero contributions of the second term in (49) come only when all the covariant derivatives are applied to external auxiliary fields V_1, V_2.

In the same way, we can show that nonzero contributions of the first term in (49) arise only when all the covariant derivatives are applied to external auxiliary fields, by performing a partial integration over θ_2 to move $\bar{H}^2 E^2$ in front of δ_{21}. Therefore, all nonzero contributions from the first and second terms in (49) have at least one covariant derivative applied to external auxiliary fields. Only the third term has no covariant derivative applied to external auxiliary fields but it was canceled by the contribution of (51). We can express the whole argument given above by the following simple graphical equation:



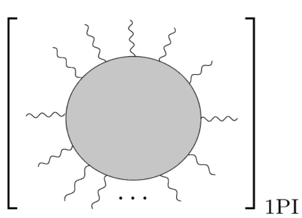
$+$



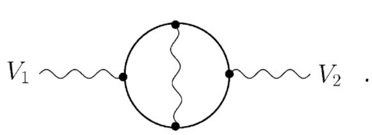
$\sim \quad 0,$

where $\sim$ means that both sides are equal up to terms with at least one covariant derivative applied to external fields.

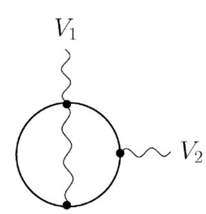
Recalling the purpose of this subsection, in the following, we consider only one-particle irreducible amplitudes with no external chiral superfields and no covariant derivatives applied to external auxiliary fields:



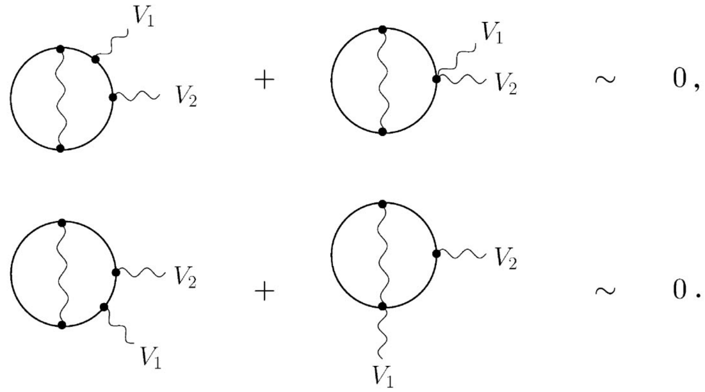
For instance, consider the following amplitude with two external auxiliary fields



This induces an order $1/N^2$ correction to the inverse propagator of the auxiliary field. By the same argument as above, we can show this amplitude cancels the following one:



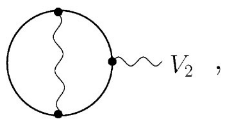
except for terms with at least one covariant derivative applied to external auxiliary fields. Note that covariant derivatives do not act on the internal line of the auxiliary field, since V_1 is attached on the propagator of dynamical fields. Shifting the vertex attached to V_1 along the chiral loop in a clockwise direction, we can also show that



These six amplitudes are given by inserting a vertex

(52)

into the following diagram



(53)

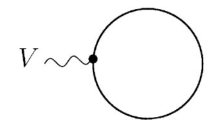
where there are six possible ways of insertion and summing up all these amplitudes leads to a vanishing result. Similarly, all possible insertions of (52) into the following diagrams

(54)

also give vanishing results.

Considering all possible insertions into (53) and (54), we can obtain all amplitudes of order $1/N^2$ with two external auxiliary fields. This means that in the effective action there is no quantum correction to the operator V^2 in order $1/N^2$. We can also prove that amplitudes of order $1/N^m$ with two external auxiliary fields vanish when we sum them up, with an arbitrary positive integer m, except for terms with at least one covariant derivative applied to external fields.

In the same way, if we fixed the number of external auxiliary fields and the order of $1/N$, except for only one case, we can show that all contributions vanish if we sum them up, up to terms which have at least one covariant derivatives applied to external fields. For example, if the diagram has n external auxiliary fields and we consider the contribution of order $1/N^m$, we choose one external auxiliary field and consider all diagrams of order $1/N^m$ without it. Then we consider all possible insertions of the chosen external auxiliary field into them. The insertions have to be made at propagators or vertices on loops of $\tilde{\Phi}$. Since any chiral loop has the same number of propagators and vertices, it gives a vanishing result to sum up all insertions. The only exception is the following one-loop amplitude:



(55)

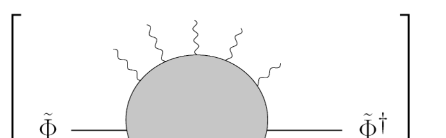
Since this is only one amplitude of order N with one external auxiliary field, it has no counterpart to cancel. However, this contribution was already considered when we evaluated the vacuum structure of the theory in subsection II B. It gave a linearly divergent contribution to be eliminated by the renormalization of the coupling constant.

In summary, it is proved that no counterterms of the form of V^n are necessary, except for the case $n = 1$. Namely, α_n in (46) vanishes unless $n = 1$. In the case $n = 1$, there is a linearly divergent term proportional to N but it is canceled by a counterterm induced by the renormalization of the coupling constant. Notice here that the counterterm is also proportional to N.

We now find that all divergent amplitudes of the left type in (45) become finite, at each order of the $1/N$-expansion, only by renormalizing the coupling constant.

C. Wave function renormalization of $\tilde{\Phi}$

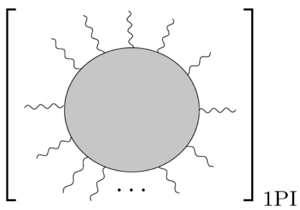
We now show that all divergences from diagrams of the second type in (45)



can be eliminated by the renormalization of the wave function of $\tilde{\Phi}$. Since its superficial degree of divergence is zero, it may contain logarithmic divergences. If we expand the amplitudes in powers of covariant derivatives and external momenta, only the lowest order, which has no external momenta and no covariant derivative acting on external superfields, can be divergent. Therefore, in this subsection, we only consider terms with no covariant derivatives acting on external superfields.

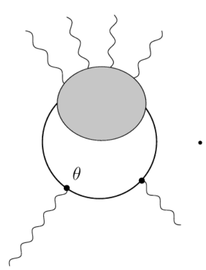
Notice here that we have to expand amplitudes in powers of E_α, $\bar{E}_\alpha$ acting on external $\tilde{\Phi}$, $\tilde{\Phi}^\dagger$ as well as in powers of D_α, $\bar{D}_\alpha$ acting on external V. The reason for this is supersymmetry. Differential operators E_α, $\bar{E}_\alpha$ are supercovariant when they act on $\tilde{\Phi}$, $\tilde{\Phi}^\dagger$, while D_α, $\bar{D}_\alpha$ are supercovariant when they act on V. In order to study divergent amplitudes, it is enough to investigate amplitudes which have no E_α, $\bar{E}_\alpha$ acting on external $\tilde{\Phi}$, $\tilde{\Phi}^\dagger$ and no D_α, $\bar{D}_\alpha$ on external V.

We have already seen that the following type of amplitudes



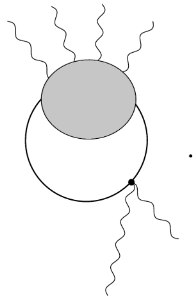
(56)

has no divergence. Especially when we neglect terms with covariant derivatives acting on external superfields, it led to a vanishing result to sum up all the diagrams of the above type. The reason for this was as follows. Suppose the following diagram and take one external auxiliary field, which is always attached to a loop of a twisted chiral superfield:



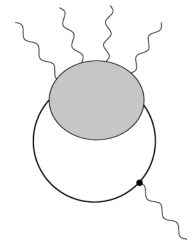
(57)

When we perform a partial integration at a vertex θ where the chosen external auxiliary field is attached, we use the formula (48) at a chiral propagator next to the vertex. Keeping only the contribution with no D_α, $\bar{D}_\alpha$ acting on external auxiliary fields, we find that it exactly cancels another diagram



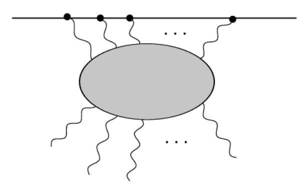
(58)

These two diagrams can be obtained by inserting one external auxiliary field in the following diagram:

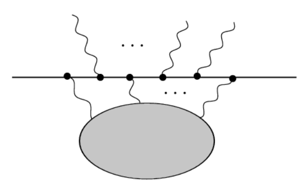


The diagram (57) can be obtained by inserting an external V into the chiral propagator in the loop while the diagram (58) can be obtained by inserting it into the vertex in the loop. The diagrams obtained by these two insertions cancel each other. Since any loop has the same number of propagators and vertices, the contributions of diagrams obtained by moving one external auxiliary line along the chiral loop cancel each other. This kind of cancellation occurs when other external lines and all internal lines are fixed. Therefore, considering all diagrams of the form (56) leads to a vanishing result.

In the same way, it gives a vanishing result to sum up all the diagrams of the following form

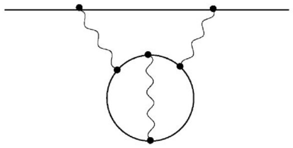


when we neglect terms with covariant derivatives acting on external superfields. But we cannot use the same argument to evaluate the following diagrams



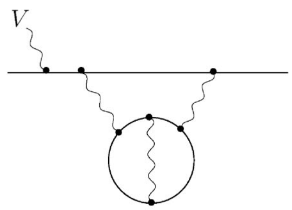
because there is no external auxiliary field attached to an internal chiral loop. All external auxiliary fields are attached to the chiral line which connects two external twisted chiral superfields. In this case, even if we neglect terms with covariant derivatives acting on external superfields and move one external auxiliary field along the chiral line, with other external lines and all internal lines being fixed, it gives a nonvanishing result to sum up all contributions. The reason for this is as follows. The chiral line has one more vertex than propagators, and it has also two external chiral lines. Then, there are two types of insertion of the chosen external auxiliary field: (A) insertion in a vertex on the chiral line; (B) insertion in a chiral propagator or an external chiral line. Each diagram of type (A) has a counterpart of type (B) to cancel out. However, since there are one more diagram of type (B) than that of (A), without cancellation when we sum up all insertions of the chosen external auxiliary field, only one diagram of type (B) remains.

For instance, suppose the following diagram:

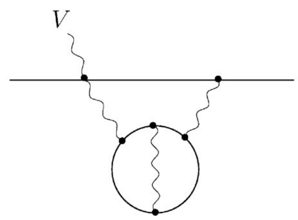


$$. \qquad (59)$$

We insert one external auxiliary field in the above diagram. Since the chiral loop in the above diagram has four vertices and four propagators, considering all insertion of the external auxiliary field in the chiral loop, we obtain a vanishing result up to terms with covariant derivatives acting on external superfields. On the other hand, the chiral line in the above diagram has two vertices, one propagator, and two external lines. We first find that the following two diagrams vanish when they are summed:

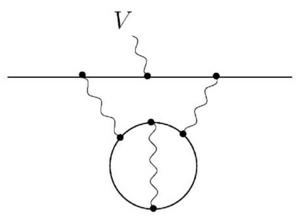


$+$

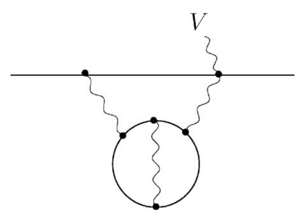


$\sim \quad 0,$

where we neglect terms with covariant derivatives acting on external superfields. In the same way, we find that the sum of the following diagrams vanishes:

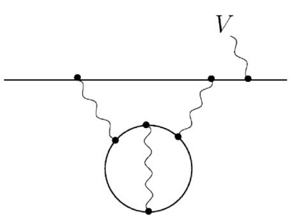


$+$



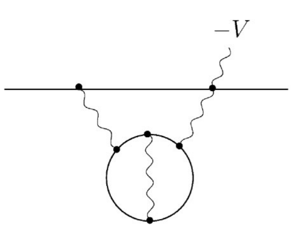
$\sim \quad 0.$

The following diagram, however, remains:



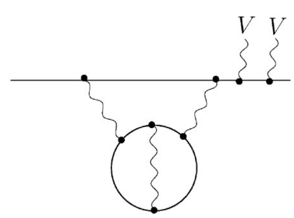
$$. \qquad (60)$$

This diagram does not have a counterpart to cancel. Therefore, summing up all diagrams obtained by inserting one external auxiliary field in the diagram (59), all diagrams cancel out each other except for the diagram (60). If we again perform the partial integration, we find the remaining diagram (60) is equivalent to



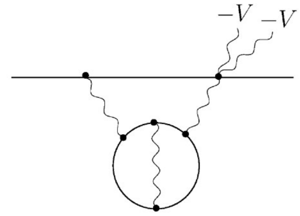
up to terms with covariant derivatives acting on external superfields.

When we insert two external auxiliary fields in (59), we fix the first external auxiliary field and consider all insertion of the second external auxiliary field. Then we find that only one diagram remains, in which the second external field is inserted in the external line of $\tilde{\Phi}$. We now consider all insertion of the first external auxiliary field and again find that only one diagram remains. The remaining diagram is as follows:



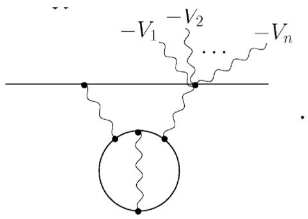
.

Notice here that if there is at least one external auxiliary field inserted in the internal chiral loop, we can move it along the chiral loop and obtain a vanishing result. Performing partial integration, we find the above remaining diagram is equivalent to



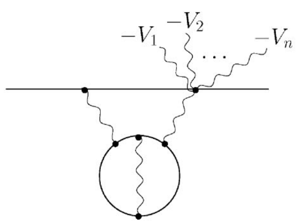
up to terms with covariant derivatives acting on external superfields.

In the same way, when we insert n external auxiliary fields in the diagram (59), considering all insertions, all diagrams cancel each other except for one diagram:

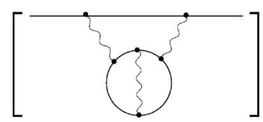


. (61)

In order to obtain an amplitude from this diagram, we have to perform all integration over Grassmann coordinates of remaining vertices as well as internal momenta. In doing so, we neglect terms with covariant derivatives acting on external superfields because such terms have no divergence and we are only interested in divergent terms. Particularly, we neglect terms with D_α, $\bar{D}_\alpha$ acting on external auxiliary fields. Therefore, in order to study divergent terms, we can rewrite (61) as



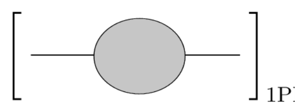
$\sim \quad (-V_1)(-V_2)\cdots(-V_n) \times$



,

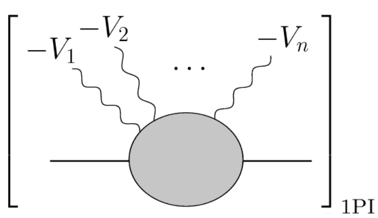
where $\sim$ means both sides are equivalent up to terms with covariant derivatives acting on external superfields. In the right-hand side, n external auxiliary fields are just multiplied by the diagram (59) which has no external auxiliary fields.

In general, any one-particle irreducible diagram with n external auxiliary fields and one pair of external $\tilde{\Phi}$, $\tilde{\Phi}^\dagger$, can be obtained by insertion of n external auxiliary fields in a diagram of the form

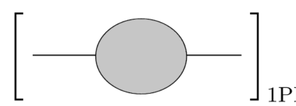


. (62)

If we choose one diagram of the above form and consider all insertions of n external auxiliary fields in it, the result is equivalent to multiplication of the chosen diagram by $V_1 V_2 \cdots V_n$, neglecting terms with covariant derivatives acting on external auxiliary fields. Namely,

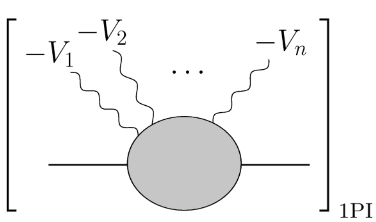


$\sim \quad (-V_1)(-V_2)\cdots(-V_n) \times$



. (63)

We now find that all divergences included in the left-hand side of (63) can be eliminated by a renormalization of the wave function of $\tilde{\Phi}$. Indeed, Eq. (63) implies that all logarithmic divergences included in diagrams of the form



are canceled by a counter term of the form

$$\tilde{\Phi}^{j\dagger} e^{-V} \tilde{\Phi}^j = \sum_{n=0}^{\infty} \frac{1}{n!} \tilde{\Phi}^{j\dagger} (-V)^n \tilde{\Phi}^j,$$

namely by a renormalization of the wave function of the dynamical field.

Note that Eq. (63) is satisfied for any internal diagram in the shaded circle. Therefore, it is also satisfied at each order of $1/N$. Then divergences are eliminated at each order of $1/N$ by the renormalization.

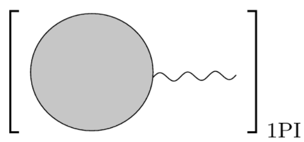
D. Beta function of the coupling constant

We have shown that all divergences in the $1/N$-expansion can be eliminated by the renormalizations of the coupling constant and the wave function of the twisted chiral superfield. In this subsection, we evaluate the beta function of the coupling constant g_R.

In Sec. II, we defined g_R by

$$\frac{\mu}{g_R^2} := \frac{1}{g^2} - \frac{\Lambda}{2\pi^2} + \frac{\mu}{2\pi^2}, \tag{64}$$

so that the linear divergence from a one-loop diagram of the dynamical field (55) is eliminated. In the above definition, μ is a renormalization scale and Λ is a momentum cutoff. In subsection V B, we showed that there is no more divergence from diagrams of the form



.

Therefore we need no more renormalization of the coupling constant. Then we can treat g_R defined by (64) as a renormalized coupling constant correct in all orders of $1/N$-expansion.

We now evaluate the beta function of g_R. Differentiating both sides of (64) by μ, we obtain

$$\frac{1}{g_R^2} - \frac{2\mu}{g_R^3}\cdot\frac{dg_R}{d\mu} = \frac{1}{2\pi^2},$$

where we should note that g and Λ are independent of μ but g_R depends on μ. If we define $\beta(g_R) := \mu\frac{dg_R}{d\mu}$, we find

$$\beta(g_R) = \frac{1}{2}g_R - \frac{1}{4\pi^2}g_R^3.$$

This beta function is shown in Fig. 2. This vanishes when $g_R = 0, \sqrt{2}\pi$. We find that this theory has one ultraviolet fixed point at $g_R = \sqrt{2}\pi$.

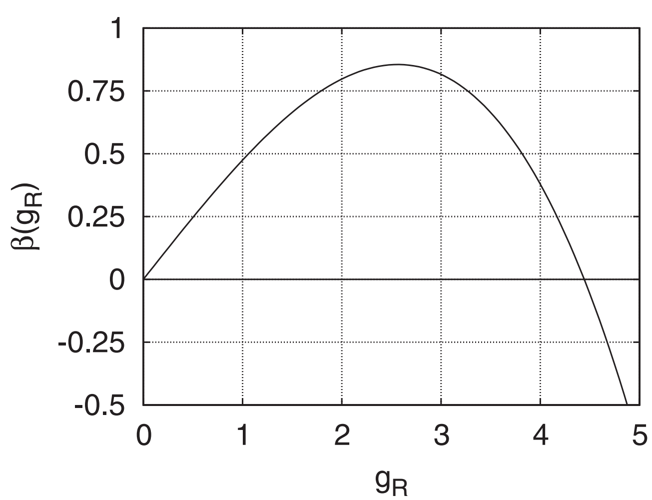


FIG. 2. Beta function of the coupling constant.

VI. CONCLUSIONS

In this paper, we have studied a three dimensional CP^{N-1} model in the method of $1/N$-expansion. This model has $\mathcal{N} = 2$ supersymmetry, $U(1)$ gauge symmetry, and global $SU(N)$ symmetry. For the $1/N$-expansion, it is useful to use the Lagrangian with the auxiliary field V. Using the super Feynman rules, we have derived the superpropagator of the auxiliary field induced by quantum effects of the dynamical field. Then we have proved that all divergences in amplitudes can be eliminated in each order of $1/N$ by renormalizations of the coupling constant and the wave function of the dynamical field. We have also shown that there is no contribution to the beta function except in the leading order of $1/N$. This model has been shown to have a nontrivial ultraviolet fixed point. These arguments are valid in all orders of $1/N$-expansion.

ACKNOWLEDGMENTS

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APPENDIX A: $\mathcal{N} = 2$ SUSY IN THREE DIMENSIONS

The smallest supersymmetry algebra in three dimensions has one Majorana (real) spinor of supercharges. It has two real degrees of freedom. So $\mathcal{N} = 2$ supersymmetry in three dimensions has one Dirac (complex) spinor of supercharges. It has four real degrees of freedom. Therefore the dimensional reduction of the $\mathcal{N} = 1$ supersymmetry in four dimensions gives the $\mathcal{N} = 2$ supersymmetry in three dimensions.

The superspace has coordinates x^μ, θ^α, and $\bar{\theta}^\alpha$ where $\mu = 0, 1, 2$ and $\alpha = 1, 2$. Here θ^α is a two-component Dirac spinor and $\bar{\theta}^\alpha$ is the complex conjugate of θ^α.

1. Gamma matrices and Dirac spinor

We use the metric $\eta_{\mu\nu} = \mathrm{diag}(+, -, -)$ and gamma matrices

$$\gamma^0 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \qquad \gamma^1 = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix},$$
$$\gamma^2 = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}.$$

These matrices satisfy the anticommutation relations $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$ and the identity

$$\gamma^\mu\gamma^\nu = \eta^{\mu\nu} + i\epsilon^{\mu\nu\rho}\gamma_\rho,$$

where $\epsilon^{\mu\nu\rho}$ is a totally antisymmetric tensor so that $\epsilon^{012} = +1$.

Spinors with upper and lower indices are related through the antisymmetric tensor C:

$$C^{\alpha\beta} = C_{\alpha\beta} = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}_{\alpha\beta}, \qquad C^{\alpha\beta}C_{\beta\gamma} = \delta^{\alpha}_{\gamma}$$
$$\psi_\alpha = C_{\alpha\beta}\psi^\beta, \qquad \psi^\alpha = C^{\alpha\beta}\psi_\beta.$$

We use the following summation convention:

$$\psi\chi := \psi_\alpha\chi^\alpha = -\chi^\alpha\psi_\alpha = \chi_\alpha\psi^\alpha = \chi\psi$$
$$\bar\psi\,\bar\chi := \bar\psi_\alpha\bar\chi^\alpha = -\bar\chi^\alpha\bar\psi_\alpha = \bar\chi_\alpha\bar\psi^\alpha = \bar\chi\,\bar\psi$$
$$\bar\psi\chi := \bar\psi_\alpha\chi^\alpha = -\chi^\alpha\bar\psi_\alpha = \chi_\alpha\bar\psi^\alpha = \chi\bar\psi.$$

The gamma matrices have the following index structure:

$$(\gamma^\mu)^\alpha{}_\beta \qquad \bar\psi\gamma^\mu\chi := \bar\psi_\alpha(\gamma^\mu)^\alpha{}_\beta\chi^\beta.$$

Since this γ^μ satisfies $(\gamma^\mu)_\alpha{}^\beta := C_{\alpha\gamma}C^{\beta\delta}(\gamma^\mu)^\gamma{}_\delta = (\gamma^\mu)^\beta{}_\alpha$, we find the identity $\bar\psi\gamma^\mu\chi = -\chi\gamma^\mu\bar\psi$.

We also find the following identities:

$$(\psi\chi)^\dagger = (C_{\alpha\beta}\psi^\beta\chi^\alpha)^\dagger = C_{\beta\alpha}\bar\chi^\alpha\bar\psi^\beta = \bar\chi\,\bar\psi = \bar\psi\,\bar\chi$$
$$(\bar\psi\chi)^\dagger = (C_{\alpha\beta}\bar\psi^\beta\chi^\alpha)^\dagger = C_{\beta\alpha}\bar\chi^\alpha\psi^\beta = \bar\chi\psi = \psi\bar\chi$$
$$(\bar\psi\gamma^\mu\chi)^\dagger = \bar\chi\gamma^\mu\psi.$$

Notice here that $\bar\psi_\alpha := C_{\alpha\beta}\bar\psi^\beta \neq \overline{C_{\alpha\beta}\psi^\beta}$ because $C^*_{\alpha\beta} = -C_{\alpha\beta} = C_{\beta\alpha}$.

2. Supersymmetry algebra and covariant derivatives

A supersymmetry transformation in the superspace

$$x^\mu \to x' = x^\mu + \frac{i}{2}(\bar\xi\gamma^\mu\theta - \bar\theta\gamma^\mu\xi),$$
$$\theta^\alpha \to \theta'^\alpha = \theta^\alpha + \xi^\alpha, \qquad \bar\theta^\alpha \to \bar\theta'^\alpha = \bar\theta + \bar\xi^\alpha$$

is generated by the differential operators

$$Q^\alpha := i\left[\frac{\partial}{\partial\theta_\alpha} + \frac{i}{2}(\partial\!\!\!/\bar\theta)^\alpha\right],$$
$$\bar Q^\alpha := -i\left[\frac{\partial}{\partial\bar\theta_\alpha} + \frac{i}{2}(\partial\!\!\!/\theta)^\alpha\right],$$

namely

$$e^{i(\xi Q - \bar\xi\bar Q)}F(x,\theta,\bar\theta) = F(x',\theta',\bar\theta').$$

Supercharges Q^α and $\bar Q^\alpha$ satisfy the following anticommutation relations:

$$\{Q^\alpha, \bar Q_\beta\} = -i\partial^\alpha{}_\beta \qquad \{Q^\alpha, Q_\beta\} = \{\bar Q^\alpha, \bar Q_\beta\} = 0.$$

We define the covariant derivatives:

$$D_\alpha := -\frac{\partial}{\partial\theta^\alpha} + \frac{i}{2}(\bar\theta\partial\!\!\!/)_\alpha, \qquad \bar D_\alpha := -\frac{\partial}{\partial\bar\theta^\alpha} + \frac{i}{2}(\theta\partial\!\!\!/)_\alpha,$$

and we find

$$\{D^\alpha, \bar D_\beta\} = i\partial^\alpha{}_\beta \qquad \{D^\alpha, D_\beta\} = \{\bar D^\alpha, \bar D_\beta\} = 0.$$

With these definitions, supercharges Q^α, $\bar Q^\alpha$ and covariant derivatives D^α, $\bar D^\alpha$ anticommute.

3. Chiral and vector superfield

Since $\bar D_\alpha$ and Q^α, $\bar Q^\alpha$ anticommute, the chirality constraint

$$\bar D_\alpha\Phi(x,\theta,\bar\theta) = 0$$

is consistent with supersymmetry transformations. The expressions for D_α and $\bar D_\alpha$ in terms of $y^\mu := x^\mu + \frac{i}{2}\bar\theta\gamma^\mu\theta$, θ^α, $\bar\theta^\alpha$ are

$$D_\alpha = -\frac{\partial}{\partial\theta^\alpha} + i(\bar\theta\gamma^\mu)_\alpha\frac{\partial}{\partial y^\mu}, \qquad \bar D_\alpha = -\frac{\partial}{\partial\bar\theta^\alpha}.$$

We can therefore expand Φ in powers of θ:

$$\begin{aligned}\Phi(x,\theta,\bar\theta) &= \phi(y) + \theta\psi(y) + \frac{1}{2}\theta^2F(y)\\ &= \phi(x) + \theta\psi(x) + \frac{1}{2}\theta^2F(x) + \frac{i}{2}(\bar\theta\partial\!\!\!/\theta)\phi(x)\\ &\quad - \frac{i}{4}\theta^2[\bar\theta\partial\!\!\!/\psi(x)] - \frac{1}{16}\theta^2\bar\theta^2\partial^2\phi(x).\end{aligned}$$

The superfield $\Phi^\dagger$ satisfies the constraint $D_\alpha\Phi^\dagger = 0$.

Note that there are no chiral spinors in three dimensions. Although we call Φ chiral superfield, ψ is a Dirac spinor.

A vector superfield V satisfies the constraint

$$V^\dagger = V$$

and has the expansion

$$\begin{aligned}V(x,\theta,\bar\theta) &= C(x) + [\theta\eta(x) + \bar\theta\,\bar\eta(x)] + \frac{1}{2}[\theta^2f(x)\\ &\quad + \bar\theta^2f^*(x)] + \bar\theta v\!\!\!/(x)\theta + M(x)\bar\theta\theta\\ &\quad + \frac{1}{2}\theta^2\bar\theta[\lambda(x) - i\partial\!\!\!/\psi] + \frac{1}{2}\bar\theta^2\theta[\bar\lambda(x) + i\partial\!\!\!/\bar\psi]\\ &\quad + \frac{1}{4}\theta^2\bar\theta^2\left[D(x) + \frac{1}{4}\partial^2C(x)\right],\end{aligned}$$

where C, v^μ, M and D are real.

APPENDIX B: CALCULATION OF THE EFFECTIVE POTENTIAL

In propsition B-1, we show the last equality in (9).

1. Proposition B-1

$$\begin{aligned}&-i\int^\Lambda\frac{d^3k}{(2\pi)^3}\ln(-k^2 + M_c^2 + D_c^2)\\ &+i\int^\Lambda\frac{d^3k}{(2\pi)^3}\operatorname{tr}\ln(k\!\!\!/ - M_c)\\ &= -\frac{1}{6\pi}|M_c^2 + D_c|^{(3/2)} + \frac{1}{6\pi}|M_c|^3 + \frac{\Lambda}{2\pi^2}D_c. \qquad \text{(B1)}\end{aligned}$$

In order to show this, we first note $\int^\Lambda\frac{d^3k}{(2\pi)^3}\operatorname{tr}\ln(k\!\!\!/ - M_c) = \int^\Lambda\frac{d^3k}{(2\pi)^3}\ln(-k^2 + M_c^2)$.

Then we perform the Wick rotation in the left-hand side (LHS):

$$(\text{LHS}) = \int^{\Lambda} \frac{d^3k_E}{(2\pi)^3} \ln(k_E^2 + M_c^2 + D_c^2) - \int^{\Lambda} \frac{d^3k_E}{(2\pi)^3} \ln(k_E^2 + M_c^2).$$

We now combine two integrals as follows:

$$\int_{M_c^2}^{M_c^2+D_c} dm^2 \int^{\Lambda} \frac{d^3k_E}{(2\pi)^3} \frac{1}{k_E^2 + m^2} = \frac{\Lambda}{2\pi^2} D_c - \frac{1}{2\pi^2} \int_{M_c^2}^{M_c^2+D_c} dm^2 \int_0^{\Lambda} dK \frac{m^2}{K^2 + m^2}. \tag{B2}$$

Notice that the first term is linearly divergent while the second term has no divergence. Therefore we take the limit $\Lambda \to \infty$ at the second term. We find

$$(\text{LHS}) = \frac{\Lambda}{2\pi^2} D_c - \frac{1}{4\pi} \int_{M_c^2}^{M_c^2+D_c} dm^2 |m| = (\text{RHS}).$$

Alternatively, we can interpret the left-hand side of (B1) as a vacuum zero-point energy. Namely, if we take the limit $\Lambda \to \infty$ and perform the contour integral over k^0, then we obtain

$$(\text{LHS}) = \int \frac{d^2k}{(2\pi)^2} \sqrt{\vec{k}^2 + M_c^2 + D_c} - \int \frac{d^2k}{(2\pi)^2} \sqrt{\vec{k}^2 + M_c^2}.$$

The first term in the right-hand side (RHS) is a zero-point energy of ϕ, and the second is that of ψ. These include a linear divergence proportional to D.

APPENDIX C: FIERZ TRANSFORMATIONS

In this appendix, we show some useful formulae for spinor calculations. We can first show

$$(\theta\xi)(\theta\chi) = -\frac{1}{2}\theta^2(\xi\chi) \tag{C1}$$

through a straightforward calculation. Substituting $\bar{\theta}$ for both ξ and χ, we find

$$(\theta\bar{\theta})^2 = -\frac{1}{2}\theta^2\bar{\theta}^2.$$

On the other hand, if we substitute $\gamma^\mu\bar{\theta}$ for ξ and $\partial_\mu\psi$ for χ in (C1), we obtain the equation

$$(\theta\gamma^\mu\bar{\theta})(\theta\partial_\mu\psi) = -\frac{1}{2}\theta^2 C_{\alpha\beta}(\gamma^\mu\bar{\theta})^\beta(\partial_\mu\psi)^\alpha = \frac{1}{2}\theta^2(\bar{\theta}\not{\partial}\psi).$$

Substituting $\gamma^\mu\bar{\theta}$ for ξ and $\gamma^\nu\bar{\theta}$ for χ in Eq. (C1), we can show $(\theta\gamma^\mu\bar{\theta})(\theta\gamma^\nu\bar{\theta}) = \frac{1}{2}\theta^2(\bar{\theta}\gamma^\mu\gamma^\nu\bar{\theta})$. Using the equation $\gamma^\mu\gamma^\nu = \eta^{\mu\nu} + i\epsilon^{\mu\nu\rho}\gamma_\rho$ and the fact that $\epsilon^{\nu\mu\rho} = -\epsilon^{\mu\nu\rho}$, we find

$$(\theta\gamma^\mu\bar{\theta})(\theta\gamma^\nu\bar{\theta}) = \frac{1}{2}\theta^2\bar{\theta}^2\eta^{\mu\nu}.$$

At last, if we substitute $(i\not{\partial} + m)\bar{\theta}$ for both ξ and χ in (C1), we obtain

$$[\theta(i\not{\partial} + m)\bar{\theta}]^2 k = -\frac{1}{2}C_{\alpha\beta}[(i\not{\partial} + m)\bar{\theta}]^\beta[(i\not{\partial} + m)\bar{\theta}]^\alpha = \frac{1}{2}\bar{\theta}^2(-\partial^2 - m^2).$$

We use this equation in Sec. III B to construct the superpropagator of the dynamical field.

In the following, we will show the Fierz transformation for general two-by-two complex matrices. In general, any two-by-two complex matrix Γ can be expanded by the following four matrices:

$$\Gamma^\mu := \gamma^\mu (\mu = 0, 1, 2), \qquad \Gamma^3 := i\mathbf{1},$$

such that $\Gamma = c_A\Gamma^A$ where the capital index A runs over 0, 1, 2, 3. Note the relation

$$\text{tr}[\Gamma_A\Gamma_B] = 2\eta_{AB},$$

where $\eta_{AB} := \text{diag}(+ - - -)$ and $\Gamma_A := \eta_{AB}\Gamma^B$. Then we find $c_A = \frac{1}{2}\text{tr}[\Gamma_A\Gamma]$ and therefore

$$\Gamma = \frac{1}{2}\text{tr}[\Gamma_A\Gamma]\Gamma^A. \tag{C2}$$

If we take an another two-by-two matrix Γ', the product of the matrix elements $\Gamma_{ab}\Gamma'_{cd}$ can be treated as the (a, b)-element of a two-by-two matrix fixing the indices b, c. Then we use the above relation (C2):

$$\Gamma_{ab}\Gamma'_{cd} = \frac{1}{2}\sum_{e,f}[(\Gamma_A)_{ef}\Gamma_{fb}\Gamma'_{ce}](\Gamma^A)_{ad} = \frac{1}{4}\text{tr}(\Gamma_B\Gamma'\Gamma_A\Gamma)(\Gamma^B)_{cb}(\Gamma^A)_{ad}.$$

In the last equality, we again used the relation (C2).

APPENDIX D: SUPERPROPAGATOR WITH TWISTED COVARIANT DERIVATIVES

In this appendix, we will explicitly show the calculation to rewrite the superpropagator of the dynamical field (10) into (16) which is written in terms of the twisted covariant derivatives. We first show a proposition.

1. Proposition D-1

$$H^2(\theta - \theta')^2 = -4\exp\left\{-\frac{1}{2}[\bar{\theta}(i\not{\partial} - m)(\theta - \theta')]\right\}. \tag{D1}$$

The proof is straightforward. Noting that $H_\alpha = -\frac{\partial}{\partial\theta} + \frac{1}{2}[\bar{\theta}(i\not{\partial} - m)]_\alpha$, we see

$$H^2(\theta-\theta')^2 = \left\{-\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta_\alpha} + [\bar\theta(i\not\partial - m)]_\alpha \frac{\partial}{\partial\theta_\alpha} + \frac{1}{4}\bar\theta^2(\partial^2 + m^2)\right\}(\theta-\theta')^2,$$

where we should note that $C_{\alpha\beta}\frac{\partial}{\partial\theta^\beta} = -\frac{\partial}{\partial\theta_\alpha}$. We can easily show

$$-\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta_\alpha}(\theta-\theta')^2 = -4,$$

$$[\bar\theta(i\not\partial - m)]_\alpha \frac{\partial}{\partial\theta_\alpha}(\theta-\theta')^2 = 2\bar\theta(i\not\partial - m)(\theta-\theta'),$$

and then we find

$$H^2(\theta-\theta')^2 = -4\left\{1 - \frac{1}{2}\bar\theta(i\not\partial - m)(\theta-\theta') - \frac{1}{16}\bar\theta^2(\theta-\theta')^2(\partial^2+m^2)\right\}.$$

Noting that $[-\frac{1}{2}\bar\theta(i\not\partial - m)(\theta-\theta')]^2 = -\frac{1}{8}\bar\theta^2(\theta-\theta')^2 \times (\partial^2+m^2)$, we can prove the statement.

Using this, we can show Eq. (17) in page (11).

2. Proposition D-2

$$\frac{1}{4}\bar E^2 H^2 \delta^{(4)}(\theta-\theta') = \exp\left[-\bar\theta'(i\not\partial - m)\theta + \frac{1}{2}\bar\theta(i\not\partial - m)\theta + \frac{1}{2}\bar\theta'(i\not\partial - m)\theta'\right].$$

In the above equation, we define $\delta^{(4)}(\theta-\theta') = \frac{1}{4}\times(\theta-\theta')^2(\bar\theta-\bar\theta')^2$. The proof is again straightforward but needs a large amount of calculation. We first use proposition D-1 and obtain

$$\bar E^2 H^2 \delta^{(4)}(\theta-\theta') = -\bar E^2\left\{\exp\left[-\frac{1}{2}\bar\theta(i\not\partial - m)(\theta-\theta')\right]\cdot(\bar\theta-\bar\theta')^2\right\}.$$

Since we can expand $\bar E^2$ as

$$\bar H^2 = -\frac{\partial}{\partial\bar\theta^\alpha}\frac{\partial}{\partial\bar\theta_\alpha} + [\theta(i\not\partial + m)]_\alpha\frac{\partial}{\partial\bar\theta_\alpha} + \frac{1}{4}\theta^2(\partial^2+m^2),$$

we find

$$\begin{aligned}\bar E^2 H^2 \delta^{(4)}(\theta-\theta') = &-\exp\left[-\frac{1}{2}\bar\theta(i\not\partial - m)(\theta-\theta')\right]\bar E^2(\bar\theta-\bar\theta') + \left\{\frac{\partial}{\partial\bar\theta^\alpha}\frac{\partial}{\partial\bar\theta_\alpha}\exp\left[-\frac{1}{2}\bar\theta(i\not\partial - m)(\theta-\theta')\right]\right\}(\bar\theta-\bar\theta')^2 \\ &+ 2\left\{\frac{\partial}{\partial\bar\theta^\alpha}\exp\left[-\frac{1}{2}\bar\theta(i\not\partial - m)(\theta-\theta')\right]\right\}\frac{\partial}{\partial\bar\theta_\alpha}(\bar\theta-\bar\theta')^2 \\ &- [\bar\theta(i\not\partial + m)]_\alpha\left\{\frac{\partial}{\partial\bar\theta_\alpha}\exp\left[-\frac{1}{2}\bar\theta(i\not\partial - m)(\theta-\theta')\right]\right\}(\bar\theta-\bar\theta')^2,\end{aligned} \tag{D2}$$

where we use the Leibniz rule for the derivative with respect to $\bar\theta$. We can easily show

$$\begin{aligned}\bar E^2(\bar\theta-\bar\theta')^2 &= -4 - 2(\bar\theta-\bar\theta')(i\not\partial - m)\theta - \frac{1}{4}\theta^2(\bar\theta-\bar\theta')^2(\partial^2+m^2)\frac{\partial}{\partial\bar\theta^\alpha}\frac{\partial}{\partial\bar\theta_\alpha}\exp\left[-\frac{1}{2}\bar\theta(i\not\partial - m)(\theta-\theta')\right] \\ &= -\frac{1}{4}(\theta-\theta')^2(\partial^2+m^2)\exp\left[-\frac{1}{2}\bar\theta(i\not\partial - m)(\theta-\theta')\right]\frac{\partial}{\partial\bar\theta^\alpha}\exp\left[-\frac{1}{2}\bar\theta(i\not\partial - m)(\theta-\theta')\right] \\ &= -\frac{1}{2}[(\theta-\theta')(i\not\partial + m)]_\alpha\exp\left[-\frac{1}{2}\bar\theta(i\not\partial - m)(\theta-\theta')\right].\end{aligned}$$

Therefore the Eq. (D2) becomes

$$\bar E^2 H^2 \delta^{(4)}(\theta-\theta') = \exp\left[-\frac{1}{2}\bar\theta(i\not\partial - m)(\theta-\theta')\right]\cdot 4\left\{1 + \frac{1}{2}(\bar\theta-\bar\theta')(i\not\partial - m)(2\theta-\theta') - \frac{1}{16}(\bar\theta-\bar\theta')^2(2\theta-\theta')^2(\partial^2+m^2)\right\}.$$

We can rewrite this equation as

$$\bar E^2 H^2 \delta^{(4)}(\theta-\theta') = 4\exp\left[-\frac{1}{2}\bar\theta(i\not\partial - m)(\theta-\theta')\right]\times\exp\left[+\frac{1}{2}(\bar\theta-\bar\theta')(i\not\partial - m)(2\theta-\theta')\right].$$

Then, at last, we can prove the statement.

APPENDIX E: USEFUL FORMULAE WITH TWISTED COVARIANT DERIVATIVES

In this appendix, we will show some useful formulae for loop calculations involving twisted covariant derivatives, which are shown only by using the anticommutation relation of them.

1. Proposition E-1

$$[\bar{E}^2, H_\alpha] = 2[\bar{E}(i\not{\partial} - m)]_\alpha,$$
$$[\bar{E}^2, H^\alpha] = -2[(i\not{\partial} + m)\bar{E}]^\alpha$$
$$[\bar{H}^2, E_\alpha] = 2[\bar{H}(i\not{\partial} + m)]_\alpha,$$
$$[\bar{H}^2, E^\alpha] = -2[(i\not{\partial} - m)\bar{H}]^\alpha.$$

We can easily find the anticommutation relations $\{H^\alpha, \bar{E}_\beta\} = (i\not{\partial} + m)^\alpha{}_\beta$ and $\{E^\alpha, \bar{H}_\beta\} = (i\not{\partial} - m)^\alpha{}_\beta$. Using these, this proposition can be shown through a direct calculation.

By using this proposition, we can show the following important formula.

2. Proposition E-2

$$\begin{aligned}[\bar{E}^2, H^2] &= 4(-\partial^2 - m^2) - 4H(i\not{\partial} + m)\bar{E} \\ [E^2, \bar{H}^2] &= 4(-\partial^2 - m^2) - 4\bar{H}(i\not{\partial} + m)E.\end{aligned} \tag{E1}$$

We first show the upper equation. The second term $H(i\not{\partial} + m)\bar{E}$ in the right-hand side means $H_\alpha(i\not{\partial} + m)^\alpha{}_\beta H^\beta$. When we rewrite the left-hand side as $H_\alpha[\bar{E}^2, H^\alpha] + [\bar{E}^2, H_\alpha]H^\alpha$ and use the proposition E-1, the left-hand side becomes

$$\begin{aligned}&-2H(i\not{\partial} + m)\bar{E} + 2\bar{E}(i\not{\partial} - m)H \\ &= -4H(i\not{\partial} + m)\bar{E} + 2(i\not{\partial} - m)^\alpha{}_\beta \cdot \{\bar{E}_\alpha, H^\beta\}.\end{aligned}$$

Then using the anticommutation relations of $\bar{E}_\alpha$, H^β, we can prove the statement. The lower equation of (E1) is proved in the same way.

APPENDIX F: $I(p^2)^{-1} = \frac{\arctan\sqrt{\frac{-p^2}{4m^2}}}{\sqrt{-p^2}}$

We here explicitly show the equation

$$I(p^2)^{-1} = \frac{\arctan\sqrt{\frac{-p^2}{4m^2}}}{\sqrt{-p^2}},$$

where the definition of $I(p^2)^{-1}$ is

$$\begin{aligned}I(p^2)^{-1} := \frac{4\pi}{i}\int\frac{d^3q}{(2\pi)^3}\frac{1}{(p+q)^2 - m^2 + i\epsilon} \\ \times\frac{1}{q^2 - m^2 + i\epsilon}.\end{aligned}$$

We first introduce a Feynman parameter:

$$\begin{aligned}I(p^2)^{-1} = \frac{4\pi}{i}\int_0^1 dx\int\frac{d^3k}{(2\pi)^3} \\ \times\frac{1}{[(k - xp)^2 - \Delta(p^2; x) + i\epsilon]^2},\end{aligned}$$

where $\Delta(p^2; x) := -x(1 - x)p^2 + m^2$. Then we shift the integration variable k as $k \longrightarrow k + xp$ and perform the Wick rotation:

$$I(p^2)^{-1} = \frac{2}{\pi}\int_0^1 dx\int_0^\infty dK\frac{K^2}{[K^2 + \Delta(p^2; x)]^2}.$$

We can easily perform this integral by changing the integration variable as $K = \sqrt{\Delta(p^2; x)}\tan\theta$. The result is

$$I(p^2)^{-1} = \frac{2}{\pi}\int_0^1 dx\frac{\pi}{4\sqrt{\Delta(p^2; x)}} = \frac{\arctan\sqrt{-\frac{p^2}{4m^2}}}{\sqrt{-p^2}}.$$

APPENDIX G: D ALGEBRA

We here study the algebra of supercovariant derivatives D_α, $\bar{D}_\alpha$ especially in momentum space. Recall that we can obtain D_α, $\bar{D}_\alpha$ from twisted covariant derivatives E_α, $\bar{E}_\alpha$, H_α, $\bar{H}_\alpha$ imposing $m = 0$. Therefore, from proposition E-1 and E-2, we find in momentum space that

$$[\bar{D}^2, D^\alpha] = -2(\not{p}\bar{D})^\alpha, \qquad [\bar{D}^2, D_\alpha] = 2(\bar{D}\not{p})_\alpha,$$

and

$$[\bar{D}^2, D^2] = 4p^2 - 4D(\not{p})\bar{D}. \tag{G1}$$

Moreover, we can show the following proposition.

1. Proposition G-1

$$\begin{aligned}\bar{D}^2D^2 - 2\bar{D}\not{p}D = D\bar{D}^2D, \\ D^2\bar{D}^2 - 2D\not{p}\bar{D} = D\bar{D}^2D,\end{aligned} \tag{G2}$$

where $D\bar{D}^2D := D_\alpha\bar{D}^2D^\alpha$.

The proof is straightforward. By using the anticommutation relation of D_α and $\bar{D}^\alpha$, we can rewrite $\bar{D}^2D^2$ as $D\bar{D}^2D + 2\bar{D}\not{p}D$ Therefore the left equation in (G2) is proved. The right equation can be shown in the similar way.

We now define the following projection operators:

$$P_1 := \frac{D^2\bar{D}^2}{4p^2}, \qquad P_2 := \frac{\bar{D}^2D^2}{4p^2}. \tag{G3}$$

Suppose Φ is an arbitrary chiral superfield. Then we can show $P_1\Phi = 0$ and

$$P_2\Phi = \left(P_1 + \frac{[\bar{D}^2, D^2]}{4p^2}\right)\Phi = \Phi.$$

In the last equality, we use Eq. (G1). In the same way, we can show that $P_1\Phi^\dagger = \Phi^\dagger$ and $P_2\Phi^\dagger = 0$ for arbitrary antichiral superfield $\Phi^\dagger$. Therefore, P_1 and P_2 are projection operators to antichiral and chiral superfield, respectively.

In addition to (G3), we can define the following four Lorentz invariant operators from D_α and $\bar{D}_\alpha$:

$$P_+ := -\frac{iD^2}{2\sqrt{-p^2}}, \qquad P_- := -\frac{i\bar{D}^2}{2\sqrt{-p^2}},$$
$$P_T := -\frac{D\bar{D}^2 D}{2p^2}, \qquad P_D := -\frac{i\bar{D}D}{\sqrt{-p^2}}.$$

Since $D_\alpha D^2 = \bar{D}_\alpha \bar{D}^2 = 0$, any other differential operator composed of D_α, $\bar{D}_\alpha$ can be written as a linear combination of these six operators. Note that $D\not{p}\bar{D}$ can be written as a linear combination of P_2 and P_T using proposition G-1.

Moreover, we can show the following useful formula.

2. Proposition G-2

$$P_1 + P_2 + P_T = 1.$$

The proof is straightforward. Recalling the definition of projection operators, we can show that

$$P_1 + P_2 + P_T = \frac{1}{4p^2}(D^2\bar{D}^2 + \bar{D}^2 D^2 - 2D\bar{D}^2 D)$$
$$= \frac{1}{2p^2}(D\not{p}\bar{D} + \bar{D}\not{p}D).$$

In the last equality, we use proposition G-1. We moreover rewrite this as

$$P_1 + P_2 + P_T = \frac{1}{2p^2}\not{p}^\beta{}_\alpha\{D^\alpha, \bar{D}_\beta\} = \frac{1}{2p^2}\,\mathrm{tr}(\not{p}^2) = 1.$$

Then the statement has been proved.

The multiplication rules of these projection operators are indicated in Table I. The table is, for the most part, the same as that in [10,12]. Here we have, however, a new operator P_D. Below, we will show the multiplication property of it.

We first note that since

$$P_D = -\frac{i\bar{D}D}{\sqrt{-p^2}} = -\frac{iD\bar{D}}{\sqrt{-p^2}},$$

we obtain a vanishing result if we multiply P_D by P_1, P_2, P_+, or P_-. Next we will show that $P_D^2 = P_T$. By using the Fierz identity, we see

$$\delta^\alpha{}_\beta \delta^\gamma{}_\delta = \frac{1}{4}\,\mathrm{tr}[\Gamma_A\Gamma_B]\Gamma^{B\gamma}{}_\beta\Gamma^{A\alpha}{}_\delta = \frac{1}{2}\Gamma^{A\gamma}{}_\beta\Gamma_A{}^\alpha{}_\delta$$
$$= \frac{1}{2}(\gamma^{\mu\gamma}{}_\beta\gamma_\mu{}^\alpha{}_\delta + \delta^\gamma{}_\beta\delta^\alpha{}_\delta).$$

We can therefore show that

$$P_D^2 = \frac{1}{2p^2}D_\alpha\bar{D}^\beta\bar{D}_\gamma D^\delta \delta^\gamma{}_\beta\delta^\alpha{}_\delta = \frac{1}{2p^2}D_\alpha\bar{D}^\beta\bar{D}_\beta D^\alpha$$
$$= -\frac{1}{2p^2}D\bar{D}^2 D = P_T.$$

We will now show that $P_T P_D = P_D$. First note that

$$P_T P_D = -\frac{i}{2(-p^2)^{3/2}}(D\bar{D}^2 D)(\bar{D}D)$$
$$= \frac{i}{(-p^2)^{3/2}}(D\not{p}\bar{D})(\bar{D}D)$$
$$= \frac{i}{(-p^2)^{3/2}}D_\alpha\bar{D}^\beta\bar{D}_\gamma D^\delta\not{p}^\alpha{}_\beta\delta^\gamma{}_\delta. \tag{G4}$$

Then use the Fierz identity as follows:

$$\not{p}^\alpha{}_\beta\delta^\gamma{}_\delta = \frac{1}{4}\,\mathrm{tr}[\Gamma_A\not{p}\Gamma_B]\Gamma^{B\gamma}{}_\beta\Gamma^{A\alpha}{}_\delta.$$

Substituting this for (G4), we see that the factor $\bar{D}^\beta\Gamma^{B\gamma}{}_\beta\bar{D}_\gamma$ vanishes if $\Gamma^B = \gamma^\mu$. The reason for this is $\bar{D}\gamma^\mu\bar{D} = 0$. Nonzero contribution, therefore, occurs if and only if $\Gamma^B = i\mathbf{1}$, namely,

$$P_T P_D = \frac{i}{4(-p^2)^{3/2}}D_\alpha\bar{D}^\beta\bar{D}_\gamma D^\delta\,\mathrm{tr}[\not{p}\gamma_\mu]\delta^\gamma{}_\beta\gamma^{\mu\alpha}{}_\delta$$
$$= \frac{i}{(-p^2)^{3/2}}(\bar{D}\not{p}^2 D) = P_D.$$

In the same way, we can show the equation $P_D P_T = P_D$.

APPENDIX H: PROPAGATORS OF AUXILIARY COMPONENT FIELDS

We here derive the expressions (41) and (42) from the superpropagator of the auxiliary field

$$\langle V(-p,\theta',\bar{\theta}')V(p,\theta,\bar{\theta})\rangle_0 = \frac{4\pi i}{N}I(p^2)\cdot\nabla_V^{-1}\delta^{(4)}(\theta-\theta'), \tag{H1}$$

where

$$\nabla_V^{-1} = \frac{1}{p^2 - 4m^2}\cdot\frac{D\bar{D}^2 D - 4mD\bar{D}}{p^2}$$
$$+ \frac{\alpha}{2p^4}(D^2\bar{D}^2 + \bar{D}^2 D^2). \tag{H2}$$

We will first expand the left-hand side of (H1) in components. Choosing the Wess-Zumino gauge, the auxiliary field V can be written as

$$V = \bar{\theta}\not{v}\theta + M\bar{\theta}\theta + \frac{1}{2}\bar{\theta}^2\theta\lambda + \frac{1}{2}\theta^2\bar{\theta}\,\bar{\lambda} + \frac{1}{4}\theta^2\bar{\theta}^2 D.$$

Then we see that the propagators of M and v^μ can be obtained by taking terms proportional to $\bar{\theta}'\theta'\bar{\theta}\theta$ in (H1), namely,

$$\langle V(-p,\theta',\bar{\theta}')V(p,\theta,\bar{\theta})\rangle_0|_{O(\bar{\theta}'\theta'\bar{\theta}\theta)}$$
$$= \langle\bar{\theta}'[\not{v}(-p) + M(-p)]\theta'\bar{\theta}[\not{v}(p) + M(p)]\theta\rangle_0. \tag{H3}$$

In the right-hand side of (H1), we can show that

$$D^2\bar{D}^2\delta^{(4)}(\theta-\theta')|_{O(\bar{\theta}'\theta'\bar{\theta}\theta)} = \bar{D}^2D^2\delta^{(4)}(\theta-\theta')|_{O(\bar{\theta}'\theta'\bar{\theta}\theta)} = (\bar{\theta}\not{p}\theta)(\bar{\theta}'\not{p}\theta'), \tag{H4}$$

$$D(\not{p}+2m)\bar{D}\delta^{(4)}(\theta-\theta')|_{O(\bar{\theta}'\theta'\bar{\theta}\theta)} = p^2(\theta\theta')(\bar{\theta}\bar{\theta}') + m(\bar{\theta}\not{p}\bar{\theta}')(\theta\theta') + m(\theta\not{p}\theta')(\bar{\theta}\bar{\theta}'). \tag{H5}$$

By using the Fierz identity, we obtain

$$(\theta\theta')(\bar{\theta}\bar{\theta}') = -\frac{1}{2}(\theta\gamma^\mu\bar{\theta})(\bar{\theta}'\gamma_\mu\theta') - \frac{1}{2}(\theta\bar{\theta})(\bar{\theta}'\theta'),$$

$$(\bar{\theta}\not{p}\bar{\theta}')(\theta\theta') = -\frac{i}{2}\epsilon_{\mu\rho\nu}p^\rho(\bar{\theta}\gamma^\mu\theta)(\theta'\gamma^\nu\bar{\theta}') - \frac{1}{2}(\bar{\theta}\not{p}\theta)(\theta'\bar{\theta}') - \frac{1}{2}(\bar{\theta}\theta)(\theta'\not{p}\bar{\theta}'),$$

$$(\theta\not{p}\theta')(\bar{\theta}\bar{\theta}') = -\frac{i}{2}\epsilon_{\mu\rho\nu}p^\rho(\bar{\theta}\gamma^\mu\theta)(\theta'\gamma^\nu\bar{\theta}') + \frac{1}{2}(\bar{\theta}\not{p}\theta)(\theta'\bar{\theta}') + \frac{1}{2}(\bar{\theta}\theta)(\theta'\not{p}\bar{\theta}').$$

Then Eq. (H5) becomes

$$D(\not{p}+2m)\bar{D}\delta^{(4)}(\theta-\theta')|_{O(\bar{\theta}'\theta'\bar{\theta}\theta)} = \frac{p^2}{2}[(\bar{\theta}\gamma^\mu\theta)(\bar{\theta}'\gamma_\mu\theta') - (\bar{\theta}\theta)(\bar{\theta}'\theta')] - im\epsilon_{\mu\nu\rho}p^\rho(\bar{\theta}\gamma^\mu\theta)(\bar{\theta}'\gamma^\nu\theta'). \tag{H6}$$

From (H2), (H4), and (H6) and proposition G-1, we can evaluate terms proportional to $\bar{\theta}'\theta'\bar{\theta}\theta$ in (H1) as follows:

$$\langle V(-p,\theta',\bar{\theta}')V(p,\theta,\bar{\theta})\rangle_0|_{O(\bar{\theta}'\theta'\bar{\theta}\theta)} = (\bar{\theta}'\gamma^\mu\theta')(\bar{\theta}\gamma^\nu\theta)\times\frac{4\pi i}{N}I(p^2)\times\left\{\frac{1}{p^2-4m^2}\left(-\eta_{\mu\nu}+\frac{p_\mu p_\nu}{p^2}\left(1-\frac{\alpha}{p^2}(p^2-4m^2)\right)+\frac{2mi}{p^2}\epsilon_{\mu\nu\rho}p^\rho\right)\right\} + (\bar{\theta}'\theta')(\bar{\theta}\theta)\times\frac{4\pi i}{N}I(p^2)\frac{1}{p^2-4m^2}.$$

Comparing this to (H3), we can show that

$$\langle v^\mu(-p)v^\nu(p)\rangle_0 = \frac{4\pi i}{N}I(p^2)\left\{\frac{1}{p^2-4m^2}\left(-\eta_{\mu\nu}+\frac{p_\mu p_\nu}{p^2}\times\left(1-\frac{\alpha}{p^2}(p^2-4m^2)\right)+\frac{2mi}{p^2}\epsilon_{\mu\nu\rho}p^\rho\right)\right\},$$

$$\langle M(-p)M(p)\rangle_0 = \frac{4\pi i}{N}I(p^2)\frac{1}{p^2-4m^2},$$

$$\langle v^\mu(-p)M(p)\rangle_0 = 0.$$

These coincide with the results in [7].

[1] K. Higashijima and E. Itou, Prog. Theor. Phys. **110**, 563 (2003).
[2] H. Eichenherr, Ph.D. thesis, Heidelberg University, 1978; Nucl. Phys. **B146**, 215 (1978).
[3] E. Witten, Nucl. Phys. **B149**, 285 (1979).
[4] A. D' Adda, P. Di Vecchia, and M. Lüscher, Nucl. Phys. **B152**, 125 (1979).
[5] K. Higashijima and M. Nitta, Prog. Theor. Phys. **103**, 635 (2000); **103**, 833 (2000); in *Proceedings of Confinement 2000* (World Scientific, Singapore, 2001), p. 279; in *Proceedings of ICHEP2000* (World Scientific, Singapore, 2001), p. 1368.
[6] T. Inami, Y. Saito, and M. Yamamoto, Prog. Theor. Phys. **103**, 1283 (2000).
[7] K. Higashijima, E. Itou, and M. Tsuzuki, arXiv:hep-th/0505056.
[8] Ya. Aref'eva, E. R. Nissimov, and S. J. Pacheva, Commun. Math. Phys. **71**, 213 (1980).E. R. Nissimov and S. J. Pacheva, Bulgarian Journal of Physics **6**, 610 (1979).
[9] E. R. Nissimov and S. J. Pacheva, Lett. Math. Phys. **5**, 67 (1981); **5**, 333 (1981).
[10] J. Wess and J. Bagger, *Supersymmetry and Supergravity*, Princeton Series in Physics (Princeton University, Princeton, NJ, 1992).
[11] M. T. Grisaru, W. Siegel, and M. Roček, Nucl. Phys. **B159**, 429 (1979).
[12] S. J. Gates, Jr., M. T. Grisaru, M. Roček, and W. Siegel, *Superspace or One Thousand and One Lessons in Supersymmetry* (Benjamin, New York, 1983).