PHYSICAL REVIEW D **79,** 065021 (2009)

Spinning electroweak sphalerons

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(Received 5 October 2008; published 20 March 2009)

We present numerical evidence for the existence of stationary spinning generalizations for the static sphaleron in the Weinberg-Salam theory. Our results suggest that, for any value of the mixing angle θ_W and for any Higgs mass, the spinning sphalerons comprise a family labeled by their angular momentum J. For $\theta_W \neq 0$ they possess an electric charge $Q = eJ$, where e is the electron charge. Inside they contain a monopole-antimonopole pair and a spinning loop of electric current, and for large J, a Regge-type behavior. It is likely that these sphalerons mediate the topological transitions in sectors with $J \neq 0$, thus enlarging the number of transition channels. Their action *decreases* with J, which may considerably affect the total transition rate.

DOI: 10.1103/PhysRevD.79.065021 PACS numbers: 11.15.-q, 11.27.+d, 12.15.−y

I. INTRODUCTION

The sphaleron represents one of the best known examples of solitons in the electroweak sector of the standard model. This is a static, purely magnetic solution of the classical field equations describing a localized, globally regular object with a finite mass of order of several TeV [1]. For vanishing mixing angle θ_W sphaleron is spherically symmetric [1], but for $\theta_W \neq 0$ it is only axially symmetric—due to a nonzero magnetic dipole moment [2]. The sphaleron is unstable and relates to the potential barrier between the topological vacua in the theory [1], thereby mediating nonperturbative transition processes that could be relevant for the generation of the baryon number asymmetry of our Universe [3].

In this article we show that the static, purely magnetic sphaleron admits stationary generalizations including an electric field and supporting a nonzero angular momentum J. This reveals new solitonic states and also provides the first explicit example of stationary *spinning* solitons in the standard model. In fact, it seems that spinning systems in classical field theory should generically radiate and therefore cannot be stationary, while spinning not accompanied by radiation should be viewed as something exceptional [4]. For example, the existence of stationary spinning generalizations can be ruled out for the magnetic monopoles [5,6]. A similar no-go statement can also be proven for the sphalerons, but only assuming that they do not depend explicitly on time [6], which is not the case for our solutions below.

The inner structure of the spinning sphaleron shows a monopole-antimonopole pair joined by a Z-string segment and surrounded by a loop of electric current. The momentum circulating along the loops gives rise to the angular momentum, which can be regarded as an electroweak analogue of cosmic vortons [4,7]. For large J the whole system shows a Regge-type behavior, similar to what was suggested long ago by Nambu [8].

For $\theta_W \neq 0$ the spinning sphalerons carry an electric charge $Q = Je$, where e is the electron charge. Since $J \in \mathbb{Z}$ in the full quantum theory, it follows that only solutions with $Q/e \in \mathbb{Z}$ are allowed. The charged, spinning sphalerons comprise therefore a discrete set. It is likely that they mediate the topological transitions in sectors with fixed charge and angular momentum, thus enlarging the number of transition channels. Their energy increases but the action *decreases* with J, which may considerably affect the total transition rate and thus be important for the theory of baryogenesis.

II. WEINBERG-SALAM THEORY

Its bosonic sector is described by the action $S = \frac{1}{g_z^2} \times \int \mathcal{L} d^4x$, where

$$\mathcal{L} = -\frac{1}{4g^2} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4g'^2} Y_{\mu\nu} Y^{\mu\nu} + (D_\mu \Phi)^\dagger D^\mu \Phi - \frac{\beta}{8}(\Phi^\dagger \Phi - 1)^2. \quad (1)$$

Here, $W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + \epsilon_{abc} W^b_\mu W^c_\nu$ and $Y_{\mu\nu} = \partial_\mu Y_\nu - \partial_\nu Y_\mu$, while Φ is a doublet of complex Higgs fields with $D_\mu \Phi = (\partial_\mu - \frac{i}{2} Y_\mu - \frac{i}{2} \tau_a W^a_\mu)\Phi$, where τ_a are the Pauli matrices. All fields and spacetime coordinates have been rendered dimensionless by rescaling, the rescaled gauge couplings are expressed in terms of the Weinberg angle as $g = \cos\theta_W$, $g' = \sin\theta_W$. The mass scale is $g_z \Phi_0$, where Φ_0 is the dimensionfull Higgs field vacuum expectation value, the electron charge is $e = g_z g g'$. The theory is invariant under gauge transformations

$$\Phi \to U\Phi, \qquad \mathcal{W}_\mu \to U(\mathcal{W}_\mu + 2i\partial_\mu)U^{-1}, \quad (2)$$

where $U \in SU(2) \times U(1)$ and $\mathcal{W}_\mu = Y_\mu + \tau^a W^a_\mu$. The electromagnetic field can be defined in a gauge invariant way as $F_{\mu\nu} = \frac{g}{g'} Y_{\mu\nu} - \frac{g'}{g} n_a W^a_{\mu\nu}$ with $n_a = (\Phi^\dagger \tau_a \Phi)/(\Phi^\dagger \Phi)$ [8]. The electric current is $j_\mu = \partial^\nu F_{\nu\mu}$, while the dual of $F_{\mu\nu}$ determines similarly the magnetic current.

III. AXIAL SYMMETRY

Let us split the spacetime coordinates as $x^k = (\rho, z)$ with $k = 1, 2$, and $x^a = (t, \varphi)$. We are interested in stationary, axially symmetric systems for which $K_{(a)} = \partial/\partial x^a$ are the symmetry generators. The existence of these symmetries implies conservation of two Noether charges $\int T^0_\mu K^\mu_{(a)} d^3\mathbf{x}$ which are, respectively, the energy E and angular momentum J for $a = 0, 3$ (the dimensionfull values being $\Phi_0 E/g_z$ and J/g_z^2). The energy-momentum tensor is obtained by varying the Lagrangian (1) with respect to the spacetime metric $T^\mu_\nu = 2g^{\mu\sigma}\partial\mathcal{L}/\partial g^{\sigma\nu} - \delta^\mu_\nu \mathcal{L}$.

The two Killing vectors K_a commute between themselves. Since all the internal symmetries in the theory (1) are gauged, there exists a gauge where the symmetric fields do not depend on x^a [9]. The most general stationary and axially symmetric fields can therefore be chosen in the form $\Phi = \Phi(x^k)$, $\mathcal{W}_\mu = \mathcal{W}_\mu(x^k)$. These can then be consistently truncated by imposing the on-shell conditions $\Im(\Phi) = 0$, $W^2_a = W^1_k = W^3_k = Y_k = 0$, such that the fields can be parametrized as

$$\mathcal{W} = (Y_a + \tau_1 \psi^1_a + \tau_3 \psi^3_a) dx^a + \tau_2 v_k dx^k, \qquad \Phi = \begin{bmatrix} \phi_+ \\ \phi_- \end{bmatrix}. \tag{3}$$

This is in fact a version of the Rebbi-Rossi ansatz [10]. This parametrization defines the reduced field theory for the complex scalars $\psi_a = \psi^1_a + i\psi^3_a$ and $\phi = \phi_+ + i\phi_-$ and a vector v_k living on the 2D space spanned by x^k. The field equations following from (1) and (3) read

$$\frac{1}{\rho}\partial_k(\rho h^{ab}\partial_k Y_b) = 2g'^2 \Im(\phi\lambda^a), \tag{4a}$$

$$\frac{1}{\rho}D_k(\rho h^{ab} D_k \psi_b) = 2g^2\phi^*\lambda^a + \frac{1}{2}(\epsilon^{cd}\psi_c\psi^*_d)\epsilon^{ab}\psi_b, \tag{4b}$$

$$\frac{1}{\rho}\partial_s(\rho V_{sk}) = \Im\{\psi^{a*} D_k\psi_a + g^2\phi^*\mathcal{D}_k\phi\}, \tag{4c}$$

$$\frac{1}{\rho}\mathcal{D}_k(\rho\mathcal{D}_k\phi) + \psi^*_a\lambda^a + i\lambda^*_a Y^a = \frac{\beta}{4}(|\phi|^2 - 1)\phi. \tag{4d}$$

Here, $D_k = \partial_k - iv_k$, $\mathcal{D}_k = \partial_k + \frac{i}{2}v_k$ and $\epsilon^{03} = -\epsilon^{30} = 1/\rho$ with $\lambda_a = 1/4(\phi\psi_a + i\phi^* Y_a)$ also $V_{ik} = \partial_i v_k - \partial_k v_i$. The indices a, b are raised and lowered by the "target space" metric $h_{ab} = \mathrm{diag}(1, -\rho^2)$. The residual symmetry of the ansatz (3) generated by $U = \exp(\frac{i}{2}\xi\tau_2)$ gives rise to the local U(1) symmetry of Eqs. (4),

$$\psi_a \to e^{i\xi}\psi_a, \qquad \phi \to e^{-(1/2)\xi}\phi, \qquad v_k \to v_k + \partial_k\xi. \tag{5}$$

Modulo this symmetry, zero energy fields are given by

$$\mathcal{W}_0 = (\tau^3 - 1)(\omega dt + n d\varphi), \qquad \Phi_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \tag{6}$$

with constant ω, n. Equations (4) also admit a discrete symmetry under $z \to -z$,

$$Y_a \to Y_a, \qquad \psi_a \to -\psi^*_a, \qquad \phi \to \phi^*, \qquad v_k dx^k \to -v_k dx^k. \tag{7}$$

IV. BOUNDARY CONDITIONS

Let $z + i\rho = re^{i\vartheta}$. Finite energy fields should approach (6) for $r \to \infty$, so that one should have $\mathcal{W} = \mathcal{W}_0 + \delta\mathcal{W}$, $\Phi = \Phi_0 + \delta\Phi$. Linearizing Eqs. (4) with respect to $\delta\mathcal{W}$, $\delta\Phi$ gives (in the $\delta\phi_- = 0$ gauge)

$$\delta Y_a + \delta\psi^3_a \sim e^{-m_Z r}, \qquad \delta\phi_+ \sim e^{-m_H r}, \qquad \delta\psi^1_a \sim v_k \sim e^{-m_W r},$$

which correspond, respectively, to the Z, Higgs and W bosons with masses (in units of $g_z\Phi_0$)

$$m_Z = \frac{1}{\sqrt{2}}, \qquad m_H = \sqrt{\beta} m_Z, \qquad m_W = \sqrt{\frac{g^2}{2} - \omega^2}. \tag{8}$$

We see that the W-boson mass gets screened, since having $\omega \neq 0$ is equivalent to a manifest time-dependence of the fields. For the photon field Eqs. (4) give for large r

$$\left(\frac{g}{g'}\delta Y_a - \frac{g'}{g}\delta\psi^3_a\right)dx^a = \frac{Q}{4\pi r}dt + \frac{\mu}{4\pi r}\sin^2\vartheta d\varphi + \ldots, \tag{9}$$

where Q, μ are the electric charge and magnetic moment.

In the gauge (3) the fields depend only on x^k but they have a Dirac string singularity at the z axis. If $n \in \mathbb{Z}$, then this singularity can be removed by the gauge transformation $U = \tau_1 \exp\{\frac{i}{2}\chi(1 - \tau_3)\}$ with $\chi = \omega t + n\varphi \equiv \omega_a x^a$. Applying this to (3) gives

$$\mathcal{W} = (Y_a + \omega_a + \tau_1 K^1_a - \tau_2 K^2_a)dx^a - \tau_3 v_k dx^k + 2i\tau_1 d\tau_1, \qquad \Phi = \tau_1 \begin{bmatrix} \phi_+ \\ e^{i\chi}\phi_- \end{bmatrix} \tag{10}$$

with $\boldsymbol{\tau}_1 = \tau_1 \sin\vartheta \cos\chi + \tau_2 \sin\vartheta \sin\chi + \tau_3 \cos\vartheta$, $\boldsymbol{\tau}_2 = \partial_\vartheta \boldsymbol{\tau}_1$, $\boldsymbol{\tau}_3 = \partial_\chi \boldsymbol{\tau}_1 / \sin\vartheta$, and $K_a^2 + iK_a^1 = e^{i\vartheta}(\psi_a - i\omega_a)$. The Dirac string is now absent, but the fields depend explicitly on t, φ. The boundary conditions at infinity and in the equatorial plane are specified by Eqs. (6) and (7). Transforming (10) to Cartesian coordinates one requires that all term proportional to $1/\rho$ and $1/r$ should vanish at the z axis and at the origin, respectively. At least for $n = \pm 1$ no additional complications at the axis then arise [11] and the fields (10) are everywhere regular. These boundary conditions still allow for a residual gauge freedom (5) with ξ vanishing for $\rho = 0$, for $z = 0$, and for $r = \infty$. This freedom can be fixed by the gauge condition $\partial_k(\rho v_k) = 0$.

V. ANGULAR MOMENTUM

If $g, g' \neq 0$, then using Eqs. (4) one can represent T^0_φ as a total derivative [5,6]

$$T^0_\varphi = \frac{n}{gg'}\frac{1}{\rho}\partial_k(\rho F_{0k}) + \ldots, \tag{11}$$

the dots denoting the terms that vanishes upon integration. As a result, choosing $n = 1$,

$$J = \int T^0_\varphi d^3\mathbf{x} = \frac{1}{gg'}\oint \vec{\mathcal{E}} d\vec{S} = \frac{Q}{gg'}, \tag{12}$$

where $\vec{\mathcal{E}}$ is the electric field. So far we have used the relativistic units where $\hbar = c = 1$, but let us return for a moment to the standard units where the electron charge is $e = c\hbar g_z g g'$. Dividing Eq. (12) by $c\hbar g_z^2$ gives then the relation for the dimensionfull quantities,

$$J = Q/e \tag{13}$$

with J expressed in units of $\hbar$. Since $J \in \mathbb{Z}$ in the full quantum theory, it follows that only solutions with $Q/e \in \mathbb{Z}$ are allowed. The sphaleron charge is therefore quantized and the charged, spinning sphalerons comprise a discrete family.

VI. THE $\theta_W = 0$ LIMIT

When $g = 1$ and $g' = 0$ the right-hand side of Eq. (4a) vanishes and so the U(1) amplitudes are constant and equal to their asymptotic values, $Y_a = -\omega_a$. The U(1) part of the gauge field (10) then vanishes. Let us consider the spherically symmetric sphaleron [1],

$$\begin{aligned} \mathcal{W} &= (w(r) - 1)(\boldsymbol{\tau}_3 d\vartheta - \boldsymbol{\tau}_2 \sin\vartheta d\varphi), \\ \Phi &= \boldsymbol{\tau}_1 \begin{bmatrix} h(r) \\ 0 \end{bmatrix}, \end{aligned} \tag{14}$$

which is a particular case of the axially symmetric field (10) with $\omega = 0$, $n = 1$. Equations (4) reduce then to

$$\begin{aligned} w'' &= \frac{w(w^2-1)}{r^2} + \frac{1}{2}h^2(w+1), \\ h'' + \frac{2}{r}h' &= \frac{(w+1)^2}{2r^2}h + \frac{\beta}{4}(h^2-1)h, \end{aligned} \tag{15}$$

whose globally regular solution exists for any $\beta \geq 0$ [1] and has the profile shown in Fig. 1.

The strategy now is to vary ω by keeping $n = 1$. If $\omega \neq 0$ then the solution should be sought within the full ansatz (10), but for $|\omega| \ll 1$ it is expected to be close to the sphaleron (14). Specifically, in the gauge (3) where the fields do not depend on t, φ one should have $\mathcal{W} = \mathcal{W}_s + \delta\mathcal{W}$, $\Phi = \Phi_s + \delta\Phi$, where $\mathcal{W}_s$, Φ_s is the sphaleron fields (14) transformed to the gauge (3), while $\delta\mathcal{W}$, $\delta\Phi$ are of order ω. Inserting this into Eqs. (4) and linearizing with respect to $\delta\mathcal{W}$, $\delta\Phi$ reveal that one can consistently choose

$$\delta\mathcal{W} = (-\omega + \tau_1 \delta\psi_0^1 + \tau_3 \delta\psi_0^3)dt, \qquad \delta\Phi = 0, \tag{16}$$

with $\delta\psi_0^1 + i\delta\psi_0^3 = -e^{-i\vartheta}(H_+(r)\sin\vartheta + iH_-(r) \times \cos\vartheta)/r$. The variables in the equations then separate,

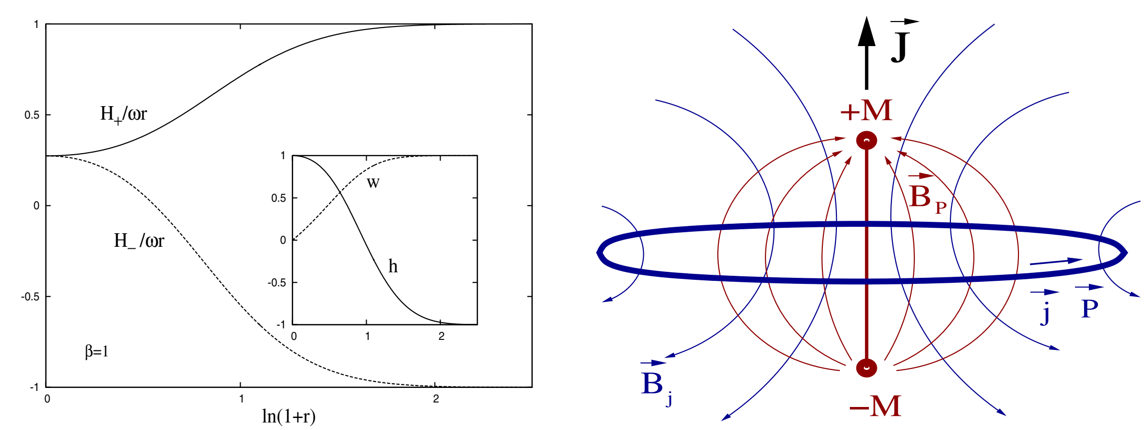

FIG. 1 (color online). Left: perturbative solutions of Eqs. (17) and the background sphaleron profiles. Right: schematic charge-current distribution inside the spinning sphaleron.

$$\left(\frac{d^2}{dr^2} - \frac{w^2+1}{q_\mp r^2} - \frac{h^2}{2}\right)H_\pm + \frac{2w}{q_\mp r^2}H_\mp = \mp\omega\frac{rh^2}{2}, \quad (17)$$

with $q_\pm = 2/(3 \pm 1)$. For $\omega \neq 0$ the source term in these equations forces $H_\pm$ to be nonzero. Numerically solving these equations gives a globally regular solution $H_\pm(r)$ (see Fig. 1) for which $H_\pm/r \to \pm\omega$ as $r \to \infty$, so that asymptotically $\delta\psi_0 \to i\omega$ as it should. Passing back to the globally regular gauge (10), this perturbative solution describes a slow rotational excitation of the sphaleron. The angular momentum is obtained by linearizing $\int T^0_\varphi d^3\mathbf{x}$,

$$J = 2\pi\omega\int_0^\infty h^2\left(1 + \frac{1}{3r}H_+ - \frac{2}{3r}H_-\right)r^2 dr, \quad (18)$$

which evaluates, e.g., to $J = 22.7\omega$ for $\beta = 1$.

Summarizing, choosing a nonzero value of ω in Eqs. (10) breaks the spherical symmetry of the sphaleron down to the axial one, generates an electric field, and produces an angular momentum. If ω is small then J is small, the spinning configuration is only slightly nonspherical and can be perturbatively described by Eqs. (16)–(18). For larger ω deviations from spherical symmetry become large and one needs to integrate the full system of partial differential equations (PDE's)(4) to construct the solutions.

We have performed our numerical calculations using the elliptic PDE solver FIDISOL based on the iterative Newton-Ralphson method [12]. We integrated a suitably discretized version of Eqs. (4) with the described above boundary conditions. Starting from the spherically symmetric sphaleron for $\theta_W = \omega = 0$ and increasing ω our numerics give nonperturbative, axially symmetric solutions. The perturbative results are recovered for small ω. However, as ω grows, deviations from the spherical symmetry, as well as J and the energy E increase, although the perturbative description seems to be still applicable as long as $J/\omega \approx$ const (see Figs. 2 and 3). It seems that there is a maximal value, ω_{max}, beyond which no localized solutions exist. Although it is difficult to approach this value numerically, it appears that $\omega^2_{max} = g^2/2$, in which case the effective W-boson mass m_W defined by Eq. (8) vanishes, leading to a delocalization of the field configuration.

VII. THE CASE OF $\theta_W \neq 0$

In this case the described above features remain qualitatively the same, but the U(1) amplitudes Y_a are no longer constant and the solutions support a long-range electromagnetic field (9) characterized by the electric charge Q and magnetic dipole moments μ. If $\omega \to 0$ then $Q, J \to 0$ but μ remains finite (see Fig. 3), the solutions then becoming static and axially symmetric [2].

Our numerics indicate that $J \approx C(\beta, \theta_W)\omega E^2$ and so for $\omega \to \omega_{max}$ one has $J \sim E^2$. A similar Regge-type behavior was predicted long ago by Nambu [8] for the dumbbell—monopole-antimonopole pair (MAP) connected by a Z-string segment and spinning around its center of mass.

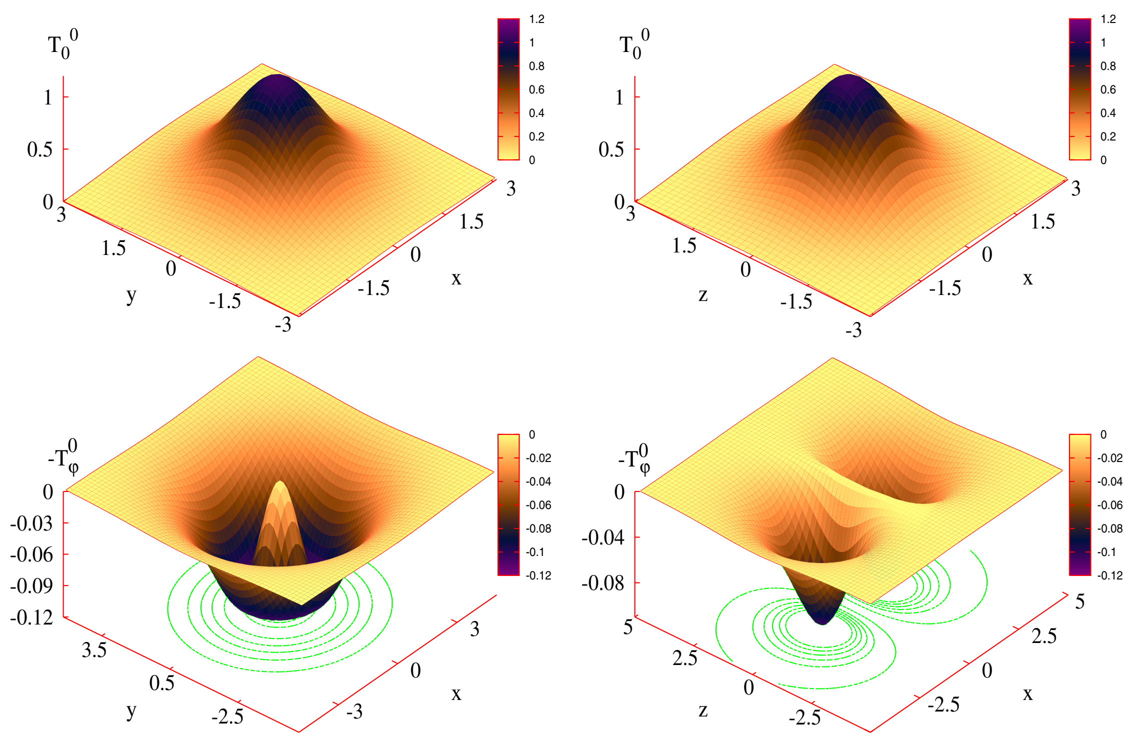

FIG. 2 (color online). The energy density T^0_0 and the negative (to better see the structure) angular momentum density $-T^0_\varphi$ shown in the $z = 0$ and $y = 0$ planes for the spinning sphaleron with $\sin^2\theta_W = 0.23$, $\beta = 2$, $\omega = 0.433$.

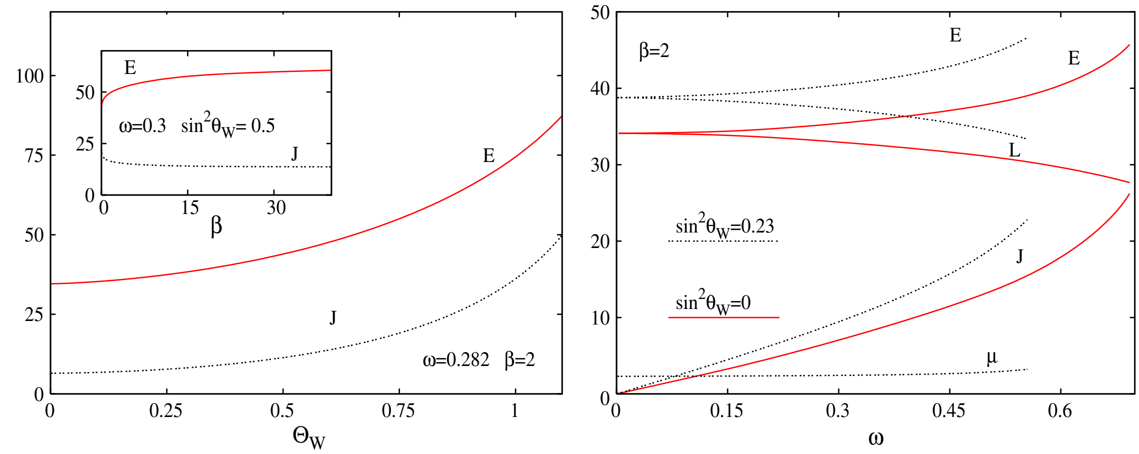

FIG. 3 (color online). Parameters of the spinning solutions.

This suggests similarities with the dumbbell scenario, and the electric and magnetic current distributions for our solutions reveal indeed a MAP, but also an electric current loop encircling it, as schematically shown in Fig. 1. Following [13] one can show that the MAP members have magnetic charges $\pm 4\pi g'/g$. The current loop seems to be stabilized by the MAP field, producing at the same time the Biot-Savart field that props the MAP up. The fields of the MAP and of the loop create together the sphaleron dipole moment [13], while for $\omega \neq 0$ there is also a momentum circulating along the loop and creating the angular momentum J directed *along* the MAP. Therefore, it is the loop that spins inside the sphaleron and not the MAP, the whole system then resembling somewhat a vorton: vortex loop stabilized by the centrifugal force [4,7]. This picture, however, can only be qualitative, since the electromagnetic field is not uniquely defined off the Higgs vacuum [8,13].

The static sphaleron is a saddle point solution relating to the top of the potential barrier between the topological vacua. The spinning sphalerons determine additional critical points of the action, and, by continuity, at least for small J, it is likely (but technically difficult to show) that these are also saddle points with one negative mode. Each of them presumably relates to the potential barrier separating the minimum energy states in sectors with fixed $Q = eJ$, as, for example, asymptotic states of N spin-one W bosons with the charge $Q = Ne$ and with $J = N$. The barrier transition amplitude is then determined by the sphaleron action density $L = \int(-\mathcal{L})d^3\mathbf{x}$, which *decreases* with J (see Fig. 3). The sphaleron-mediated transitions might therefore be enhanced in channels with nonzero charge and angular momentum.

The fact that instead of just one saddle point of the action there are many of them increases the number of transition channels. For example, one can argue that in hot electroweak plasma the topological transitions in ZZ collisions, say, are mediated by the $J = 0$ sphaleron, those in $ZW^{\pm}$ collisions—by the $J = Q/e = \pm 1$ sphaleron, and so on. To get the total transition rate one should sum over all channels, which may considerably affect the standard one-channel result [3]. Of course, detailed calculations are necessary, since there could be competing effects, as, for example, the Coulombian repulsion preventing the formation of charged sphalerons. However, such a repulsion could perhaps be overcome by the weak force or by the plasma screening effects. In any case, the fact that there are many of them suggests that the overall contribution of the charged sphalerons may be important.

We have checked that the multisphaleron, sphaleron-antisphaleron and vortex ring solutions with $n \neq 1$ [14] also admit spinning generalizations. For $\theta_W \neq 0$ they have $Q = neJ$. Charged sphalerons were also discussed perturbatively [15], and, after the preprint of the present paper was released, nonperturbatively in Ref. [16].

ACKNOWLEDGMENTS

The work of E. R. was supported by ANR Grant No. NT05-1_42856 “Knots and Vortons.” It is a pleasure to thank G. Semenoff for discussions.

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