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Sigma-model aether

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Theories of low-energy Lorentz violation by a fixed-norm ''aether'' vector field with two-derivative kinetic terms have a globally bounded Hamiltonian and are perturbatively stable only if the vector is timelike and the kinetic term in the action takes the form of a sigma model. Here we investigate the phenomenological properties of this theory. We first consider the propagation of modes in the presence of gravity and show that there is a unique choice of curvature coupling that leads to a theory without superluminal modes. Experimental constraints on this theory come from a number of sources, and we examine bounds in a two-dimensional parameter space. We then consider the cosmological evolution of the aether, arguing that the vector will naturally evolve to be orthogonal to constant-density hypersurfaces in a Friedmann-Robertson-Walker cosmology. Finally, we examine cosmological evolution in the presence of an extra compact dimension of space, concluding that a vector can maintain a constant projection along the extra dimension in an expanding universe only when the expansion is exponential.

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I. INTRODUCTION

Models of fixed-norm vector fields, sometimes called ''aether'' theories, serve a useful purpose as a phenomenological framework in which to investigate violations of Lorentz invariance at low energies [1–7]. For a recent review, see [8]. In a companion paper [9], we argue that almost all such models are plagued by instabilities. For related work on stability in aether theories, see [10–17].

There is one version of the aether theory that is stable under small perturbations and in which the Hamiltonian is globally bounded when only two-derivative terms are included in the action. This model is defined by a kinetic Lagrange density of the form

$$\mathcal{L}_{\sigma}^{\text{kinetic}} = -\tfrac{1}{2}(\nabla_{\mu}A_{\nu})(\nabla^{\mu}A^{\nu}), \tag{1}$$

where A_{μ} is a dynamical timelike four-vector aether field. (The spacelike version has an unbounded Hamiltonian and is unstable.) We refer to the theory defined by this action as ''sigma-model aether,'' due to its resemblance to a theory of scalar fields propagating on a fixed manifold with an internal metric, familiar from studies of spontaneous symmetry breaking. The aether theory is not identical to such a sigma model—in particular in curved space where covariant derivatives act on the vector—but the nomenclature is convenient.

Even though this theory is stable, it has an important drawback. It is conventional in aether models to give the vector field an expectation value by means of a Lagrange multiplier, which enforces the fixed-norm constraint

$$A_{\mu}A^{\mu} = -m^2. \tag{2}$$

We take m^2 to be positive and use a metric signature $(-+++)$, so that this defines a timelike vector field. Despite the convenience of this formulation, it seems likely that a more complete version of the theory would arise as a limit of a theory in which the expectation value is fixed by minimizing a smooth potential of the form $V(A_{\mu}) = \xi(A_{\mu}A^{\mu} + m^2)^2$. As we showed in [9], any such theory would be plagued by ghosts and tachyons. As far as we can tell, therefore, the sigma-model aether theory cannot be derived from models with a smooth potential.

Nevertheless, as it is the only example of a Lorentz-violating aether theory that we are sure is globally well behaved, examining the dynamics and experimental constraints on this model is worthwhile. We undertake such an investigation in this paper.

First we examine the degrees of freedom in this theory, taking into account the mixing with the gravitational field. There are three different massless modes, of spins 0, 1, and 2 in the aether rest frame.[1] Demanding that none of the modes propagate faster than light fixes a unique value for the coupling of the vector field to the Ricci tensor. We use experimental constraints on the preferred frame parameters $\alpha_{1,2}$ in the parametrized post-Newtonian (PPN) expansion to limit the magnitude of the vacuum expectation value, m. The spin-2 mode can propagate subluminally for some values of the vector field/Ricci tensor coupling; in such cases, very tight restrictions on the vacuum expectation

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[1]The lack of rotational symmetry in frames other than the aether rest frame make classification of modes by spin in such frames impossible. But the aether rest frame has rotational symmetry, which allows for the spin classification with respect to this frame.

value, m, due to limits from vacuum Čerenkov radiation of gravitons come into play.

Finally, we consider the cosmological evolution of the vector field in two different backgrounds. We study the evolution of the timelike vector field in a flat Friedmann-Robertson-Walker (FRW) universe and find that the vector field tends to align to be orthogonal to constant density hypersurfaces. In a background consisting of a timelike dimension, three expanding spatial dimensions, and one compact (nonexpanding) extra spatial dimension, we find that the vector field can evolve to have a nonzero projection in the direction of the compact extra dimension if the large dimensions are de Sitter-like. We take this as evidence that a timelike vector field with the Lagrangian that satisfies the aforementioned theoretical and experimental constraints would not lead to any significant departure from isotropy.

II. EXCITATIONS IN THE PRESENCE OF GRAVITY

We would like to understand the experimental constraints on, and cosmological evolution of, the sigma-model aether theory. For both of these questions, it is important to consider the effects of gravity. But whereas the flat-space model with a kinetic Lagrangian of the form (1) is unique, in curved space there is the possibility of an explicit coupling to curvature. The full action we consider is

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G}R - \frac{1}{2}(\nabla_\mu A_\nu)(\nabla^\mu A^\nu) + \frac{\alpha}{2}R_{\mu\nu}A^\mu A^\nu + \frac{\lambda}{2}(A_\mu A^\mu + m^2)\Big]. \tag{3}$$

Here, λ is the Lagrange multiplier that enforces the fixed-norm constraint (2), α is a dimensionless coupling, $R_{\mu\nu}$ is the Ricci tensor, and R is the curvature scalar. Note that, given the fixed-norm constraint, there are no other scalar operators that could be formed solely from A_μ and the Riemann tensor $R^\rho{}_{\sigma\mu\nu}$. By integrating by parts and using $R_{\mu\nu}A^\mu A^\nu = A^\nu[\nabla_\mu, \nabla_\nu]A^\mu$, this curvature coupling could equivalently be written purely in terms of covariant derivatives of A_μ; the form (3) has the advantage of emphasizing that the new term has no effects in flat space-time.

In [9] we showed that the sigma-model aether theory was stable in the presence of small perturbations in flat space-time; the possibility of mixing with gravitons implies that we should check once more in curved space-time. The equations of motion for the vector field are

$$-\nabla_\mu\nabla^\mu A^\nu = \lambda A^\nu + \alpha R^{\mu\nu}A_\mu, \tag{4}$$

along with the fixed-norm constraint from the equation of motion for λ. Assuming the fixed-norm constraint, the equations of motion can be written in the form

$$\Big(g^{\sigma\nu} + \frac{1}{m^2}A^\sigma A^\nu\Big)(\nabla_\rho\nabla^\rho A_\sigma + \alpha R_{\rho\sigma}A^\rho) = 0. \tag{5}$$

The tensor $(g^{\sigma\nu} + A_\rho A_\nu/m^2)$ acts to take what would be the equation of motion in the absence of the constraint and project it into the hyperplane orthogonal to A_μ.

The Einstein-aether system has a total of 5 degrees of freedom, all of which propagate as massless fields: one spin-2 graviton, one spin-1 excitation, and one spin-0 excitation. Each of these dispersion relations can be written (in the short-wavelength limit) in frame-invariant notation as

$$k_\mu k^\mu = \Big(\frac{1-v^2}{v^2}\Big)\Big(\frac{\bar{A}_\mu k^\mu}{m}\Big)^2, \tag{6}$$

where v is the phase velocity in the aether rest frame. The squared phase velocities of the gravity-aether modes are [6]

$$v_2^2 = \frac{1}{1-8\pi Gm^2(1+\alpha)} \approx 1 + 8\pi Gm^2(1+\alpha) \quad \text{(spin-2)}, \tag{7}$$

$$v_1^2 = \frac{2-8\pi Gm^2(1+\alpha)(1-\alpha)}{2(1-8\pi Gm^2(1+\alpha))} \approx 1 + 4\pi Gm^2(1+\alpha)^2 \quad \text{(spin-0)}, \tag{8}$$

$$v_0^2 = \frac{2-8\pi Gm^2}{(1-8\pi Gm^2(1+\alpha))(2+8\pi Gm^2(1-2\alpha))} \approx 1 + 16\pi Gm^2\alpha \quad \text{(spin-0)}, \tag{9}$$

where G is the gravitational constant appearing in Einstein's action. The approximate equalities hold assuming $8\pi Gm^2 \ll 1$.[2]

These squared mode phase velocities minus the squared speed of light are plotted in Fig. 1 as a function of α. It is clear that the only value of α for which none of the modes propagate superluminally ($v^2 > 1$) is

$$\alpha = -1. \tag{11}$$

We therefore have a *unique* version of a Lorentz-violating aether theory for which the Hamiltonian is bounded below (in flat space) and that is free of superluminal modes when coupled to gravity: the sigma-model kinetic term with an expectation value fixed by a Lagrange-multiplier constraint

[2]The relationship between the parameters in Eq. (3) (α, m^2) and those in Ref. [6] (c_1, c_2, c_3, c_4) is

$$c_1 = 8\pi Gm^2, \qquad -c_2 = c_3 = \alpha 8\pi Gm^2, \qquad c_4 = 0. \tag{10}$$

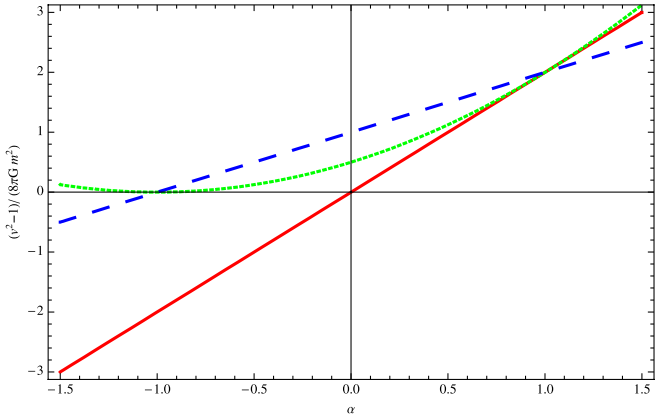

FIG. 1 (color online). Aether rest-frame mode phase velocities squared, v^2, minus the speed of light in units of $8\pi G m^2$ as a function of α. The solid (red) line corresponds to spin-0, the small dashed line (green) to spin-1, and the large dashed line (blue) to spin-2. Only for $\alpha = -1$ do none of the modes propagate faster than light ($v^2 - 1 > 0$).

and a coupling to curvature of the form in (3) with $\alpha = -1$. In what follows, we will generally allow α to remain as a free parameter when considering experimental limits, keeping in mind that models with $\alpha \neq -1$ are plagued by superluminal modes. We will find that the experimental limits on m are actually weakest when $\alpha = -1$.

Before moving on, however, we should note that the existence of superluminal phase velocities does not constitute *prima facie* evidence that the theory is ill behaved. There are two reasons for suspecting that superluminal propagation is bad. First, in [9], we showed that such models were associated with perturbative instabilities: there is always a frame in which small perturbations grow exponentially with time. Second, acausal propagation around a closed loop in space-time could potentially occur if the background aether field were not constant through space [5,13]. But in the presence of gravity, these arguments are not decisive. There now exists a scale beyond which we expect the theory to break down: namely, length scales on the order of $M_{\rm pl}^{-1}$. Perhaps there is some length scale involved in boosting to a frame where the instability is apparent (or, equivalently, in approaching a trajectory that is a closed timelike curve) that is order $M_{\rm pl}^{-1}$.

Again, in a background flat space-time with a background timelike aether field $\bar{A}_\mu = $ constant, the dispersion relations have the generic form

$$(v^{-2} - 1)(t^\mu k_\mu)^2 = k_\mu k^\mu, \tag{12}$$

where $t_\mu = \bar{A}_\mu/m$ characterizes the 4-velocity of the preferred rest frame. The velocity v^2 is given by Eqs. (7)–(9). In a boosted frame, where $t^\mu = (-\cosh\eta, \sinh\eta\hat{n})$, the frequency is given by

$$\frac{\omega}{|\vec{k}|} = \frac{-(1 - v^{-2})\sinh\eta\cosh\eta(\hat{k}\cdot\hat{n}) \pm \sqrt{1 - (1 - v^{-2})(\cosh^2\eta - \sinh^2\eta(\hat{n}\cdot\hat{k})^2)}}{1 - (1 - v^{-2})\cosh^2\eta}. \tag{13}$$

Let us parametrize the boost in the standard way as

$$\cosh^2\eta = \frac{1}{1 - \beta^2}, \qquad 0 \leq \beta^2 < 1. \tag{14}$$

Then

$$\frac{\omega}{|\vec{k}|} = \frac{-(1 - v^{-2})\beta(\hat{k}\cdot\hat{n}) \pm \sqrt{1 - \beta^2}\sqrt{v^{-2} - \beta^2 + \beta^2(1 - v^{-2})(\hat{n}\cdot\hat{k})^2}}{v^{-2} - \beta^2}. \tag{15}$$

There is a pole in the frequency at $\beta^2 = v^{-2}$. The pole is physical if $v > 1$ and (in the limit as $\hat{n}\cdot\hat{k} \to 0$), as β passes through the pole ($\beta^2 \to \beta^2 > v^{-2}$), the frequency acquires a nonzero imaginary part, which corresponds to growing mode amplitudes. (The frequency becomes imaginary at some $\beta^2 < 1$ as long as $\hat{n}\cdot\hat{k} \neq 1$.) The time scale on which the mode grows is set by $1/\mathrm{Im}(\omega)$. In frames with a boost factor greater than the inverse rest-frame mode speed, $\beta > v^{-1}$, the time scale on which mode amplitudes grow is maximal for modes with wave vectors perpendicular to the boost direction ($\hat{n}\cdot\hat{k} = 0$) and is given by

$$T_{\rm MAX}(\beta) = \frac{1}{|\mathrm{Im}(\omega)|} = |\vec{k}|^{-1}\frac{\sqrt{\beta^2 - v^{-2}}}{\sqrt{1 - \beta^2}} \quad \text{when } v^2 > 1. \tag{16}$$

We generically expect the linearized gravity analysis that led to the propagation speeds in Eqs. (6)–(9) to be valid for wave vectors that are much greater in magnitude than the energy scale set by other energy density in the space-time—generally, the Hubble scale, H. Thus the analysis makes sense for $|\vec{k}|^{-1} \ll H^{-1}$ and (as long

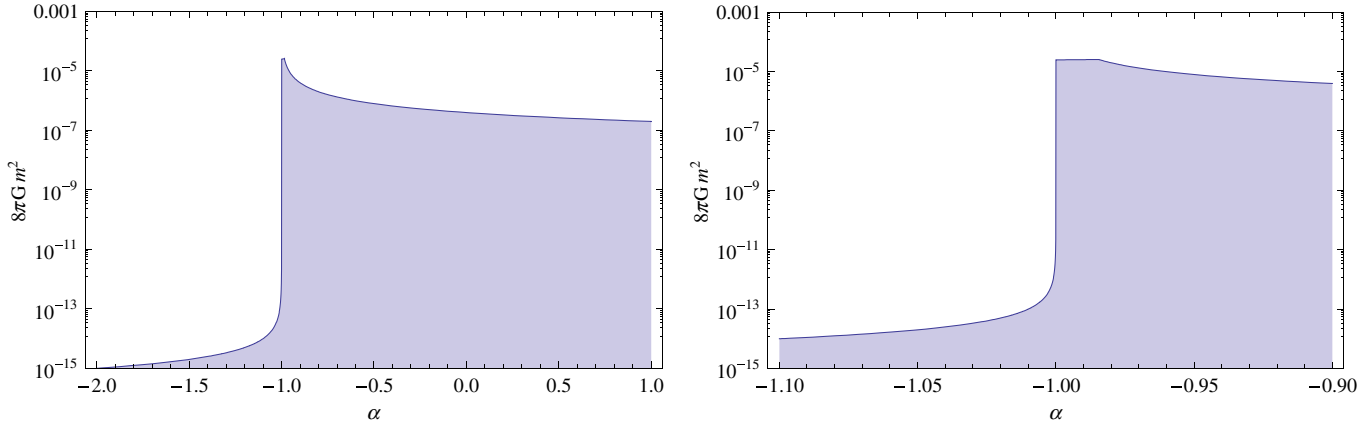

FIG. 2 (color online). Parameter space allowed (shaded region) by constraints from Čerenkov radiation and PPN. The strongest constraint in the $\alpha < -1$ region is from Eq. (17), and for most of the $\alpha > -1$ region the strongest constraint is from the second inequality in Eq. (18). The plot on the right is a blowup of the small range of α for which the first constraint in Eq. (18) is strongest—when $\alpha = -1$ to within a couple of parts in 100.

as $1 - \beta^2$ is not infinitesimal) there will be instabilities on time scales less than the inverse Hubble scale and (unless $\beta^2 - v^{-2}$ is infinitesimal) greater than $M_{\rm Pl}^{-1}$.

Thus, not only could superluminal propagation speeds lead to closed timelike curves and violations of causality, but the existence of instabilities on an unremarkable range of less-than-Hubble-radius time scales in boosted frames indicates that such superluminal propagation speeds lead to instabilities. If $v > 1$, it appears as if instabilities can be accessed without crossing some scale threshold beyond which we would expect the model to break down.

III. EXPERIMENTAL CONSTRAINTS

We now apply existing experimental limits to the sigma-model aether theory, keeping for the moment α as well as m^2 as free parameters. Direct coupling of the aether field to standard model fields fits into the framework of the ''Lorentz-violating extension'' of the standard model considered in Ref. [2]. Such couplings are very tightly constrained by various experiments (for a discussion of experimental constraints, see Ref. [18]). The relevant limit from gravitational Čerenkov radiation in [13] translates to[3]

$$-8\pi G m^2(1+\alpha) < 1 \times 10^{-15}. \tag{17}$$

Limits on PPN parameters give the strongest constraints on α and m^2 when $\alpha \approx -1$ (since the constraint in Eq. (17) is automatically satisfied). The preferred frame parameters must satisfy $|\alpha_1| < 10^{-4}$ and $|\alpha_2| < 10^{-7}$ [19]. We have the limits [8]

$$\begin{aligned} &|\alpha_1| \approx |4\alpha^2(8\pi G_N m^2)| < 10^{-4} \quad \text{and} \\ &|\alpha_2| \approx |(\alpha+1)(8\pi G_N m^2)| < 10^{-7}, \end{aligned} \tag{18}$$

where G_N is the gravitational constant as measured in our solar system or tabletop experiments. This gravitational constant is related to the parameter in the action G by [5]

$$G_N = \frac{G}{1 - 4\pi G m^2}. \tag{19}$$

If we require that all modes have phase speeds v that satisfy $v^2 \leq 1$, then we must have $\alpha = -1$ and

$$8\pi G_N m^2 < 10^{-4} \qquad (\alpha = -1). \tag{20}$$

All relevant constraints (allowing modes to have larger than unity phase velocities) are summarized in Fig. 2. Constraints from big bang nucleosynthesis [5] are significantly weaker than the PPN and Čerenkov constraint above.

IV. COSMOLOGICAL EVOLUTION

We now turn to the evolution of the sigma-model aether field in a cosmological background. It is usually assumed in the literature that the aether preferred frame coincides with the cosmological rest frame—i.e., that in Robertson-Walker coordinates, a timelike aether field has zero spatial components, or a spacelike aether field has zero time component. Under this assumption, there has been some analysis of cosmological evolution in the presence of aether fields [14,20–22]. Cosmological alignment in a de Sitter background was considered in [23]. Evolution of vector field perturbations in a more general context, including the effect on primordial power spectra, was considered in [7,24].

Here, we relax the aforementioned assumption. We determine the dynamical evolution of the aether alignment with respect to constant density hypersurfaces of flat FRW

[3]Reference [13] uses the same parameters as in [6,8], thus the translation between our parameters and the parameters used in [6,8,13] is as stated in (10).

backgrounds, assuming that the aether field has a negligible effect on the form of the background geometry. We will show that a homogeneous timelike vector field tends to align in the presence of a homogeneous cosmological fluid such that its rest frame coincides with the rest frame of the cosmological fluid.

Take the background space-time to be that of a flat FRW cosmology,

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2). \tag{21}$$

We take the equation state of the cosmological fluid to be $p_{\rm fluid} = w\rho_{\rm fluid}$. The Friedmann equation then implies

$$a(t) = t^{2/3(1+w)} \tag{22}$$

for $w \neq -1$, and

$$a(t) = e^{Ht}, \qquad H = \text{constant} \tag{23}$$

for $w = -1$. We assume that m^2/M_P^2 is small, so that the backreaction of the vector field on the FRW geometry will be small, and the evolution of the vector field will be well approximated by its evolution in the FRW background.

Suppose the vector field is homogeneous. This is a reasonable assumption given that the background space-time is homogeneous and therefore should only affect the time evolution of the vector field. We may use the rotational invariance of the FRW background to choose coordinates such that the x-axis is aligned with the spatial part of the vector field. Then, without loss of generality, $A_0 = m\cosh(\phi(t))$ and $A_x = ma(t)\sinh(\phi(t))$. In this case the equations of motion reduce to

$$\phi''(t) + 3H(t)\phi'(t) + [H^2(t) + \alpha H'(t)]\sinh(2\phi(t)) = 0, \tag{24}$$

where $H(t) = a'(t)/a(t)$. Expanding to first order in the angle ϕ, for $w \neq -1$ we have

$$\phi'' + \left[\frac{2}{(1+w)t}\right]\phi' + \left[\frac{8 - 12\alpha(1+w)}{9(1+w)^2t^2}\right]\phi = 0. \tag{25}$$

It is a simple exercise to show that ϕ behaves as a damped oscillator for all $-1 < w < 1$ and $\alpha < \frac{2}{3(1+w)}$. For the case of a constant Hubble parameter ($w = -1$),

$$\phi(t) = Ae^{-Ht} + Be^{-2Ht}. \tag{26}$$

One can see even for large $\phi(t)$ that $|\phi(t)|$ generically decreases when $-1 < w < 1$ and $\alpha < \frac{2}{3(1+w)}$ because, since $\sinh(\phi) = -\sinh(-\phi)$, the essential features of the full equation mirror those of the linearized equation.

We conclude that a timelike vector field will generically tend to align to be purely timelike in the rest frame of the cosmological fluid, thereby restoring isotropy of the cosmological background. We do not examine the case of a spacelike aether field, since that is perturbatively unstable.

V. EXTRA DIMENSIONS

Consider now the evolution of the vector field in a background space-time with metric

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2) + dr^2. \tag{27}$$

This metric is the local distance measure for a space-time in which the infinite spatial dimensions expand as a usual flat FRW metric, for general equation of state parameter w as discussed in the previous section, and a compact extra dimension with coordinate r does not expand. A scenario in which a spacelike aether is aligned completely along the compact fifth extra dimension was considered in [25].

The equations of motion are once again

$$(g^{\sigma\nu} + A^\sigma A^\nu/m^2)(\nabla_\rho\nabla^\rho A_\sigma + \alpha R_{\rho\sigma}A^\rho) = 0 \tag{28}$$

and $A_\mu A^\mu = -m^2$. Consider homogeneous configurations where, without loss of generality,

$$A_0 = m\cosh\phi(t), \qquad A_x = a(t)m\sinh\phi(t)\cos\theta(t),$$
$$A_y = A_z = 0, \quad \text{and} \quad A_r = m\sinh\phi(t)\sin\theta(t). \tag{29}$$

The $\nu = 0$ equation of motion [Eq. (28)] reads

$$[\tfrac{1}{2}(5 - \cos2\theta)(H^2(1+\alpha) + \alpha H') - 2\alpha H^2\cos^2\theta - (\theta')^2] \times \sinh2\phi + 6H\phi' + 2\phi'' = 0. \tag{30}$$

When $\theta'^2 \ll H^2$, we can treat θ as being essentially constant and then the above equation determines the evolution of ϕ. Numerical simulations indicate that ϕ decays to zero, whatever the value of θ, if $-1 < \alpha < \frac{2}{3(1+w)}$. One can see the decay of ϕ (given the bounds on α) explicitly by expanding about $\phi = 0$ and $\theta =$ constant when ϕ is small.

If H is constant (i.e. the noncompact dimensions are de Sitter-like) and the vector field is aligned entirely along the timelike dimension and the compact dimension (so $\theta = \pi/2$), then the equation of motion for $\phi(t)$ is

$$\phi''(t) + 3H\phi'(t) + \tfrac{3}{2}(1+\alpha)H^2\sinh(2\phi(t)) = 0, \tag{31}$$

the solution to which is

$$\phi(t) = A_+e^{-\alpha_+Ht/2} + A_-e^{-\alpha_-Ht/2}, \tag{32}$$

where

$$\alpha_\pm = 3(1 \pm \sqrt{1 - \tfrac{4}{3}(1+\alpha)}), \tag{33}$$

when $|\phi(t)| \ll 1$. If $1 + \alpha > 0$ then ϕ decays to zero. If $\alpha = -1$, ϕ decays to a (generically nonzero) constant, and ϕ can grow with time if $\alpha < -1$. It is interesting to see that, for the case where no perturbative modes propagate superluminally—the case where $\alpha = -1$—the fixed-norm vector field can evolve during a de Sitter expansion phase so that it has a nonzero component in the compact fifth dimension while otherwise aligning so that isotropy is restored in the rest frame of the cosmological fluid. However, when the Universe enters a phase of expansion

where $a(t) = t^{2/3(1+w)}$ and w is strictly greater than -1 (and less than 1), then the component of the vector field in the fifth dimension will decay away.

VI. CONCLUSIONS

We investigated the dynamics of and limits on parameters in a theory with a fixed-norm timelike vector field whose kinetic term takes the form of a sigma model. We argued in a companion paper [9] that such sigma-model theories are the only aether models with two-derivative kinetic terms and a fixed-norm vector field for which the Hamiltonian is bounded below.

In the presence of gravity, the action for sigma-model aether is

$$S_A = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G}R - \frac{1}{2}(\nabla_\mu A_\nu)(\nabla^\mu A^\nu) + \frac{\alpha}{2}R_{\mu\nu}A^\mu A^\nu + \frac{\lambda}{2}(A_\mu A^\mu + m^2)\Big]. \quad (34)$$

We showed that the five massless degrees of freedom in the linearized theory will not propagate faster than light only if $\alpha = -1$, and we argued that faster-than-light degrees of freedom generically lead to instabilities on less-than-Hubble-length time scales. In the special case $\alpha = -1$, the vacuum expectation value m^2 must be less than about $10^{-4}M_p^2$, where M_p is the Planck mass, in order to comply with limits on the PPN preferred frame parameter α_2. Relaxing the $\alpha = -1$ assumption, we summarized the strongest limits on the parameters $\{\alpha, m\}$ (from gravitational Čerenkov radiation and the PPN preferred frame parameters) in Fig. 2.

We also showed that the aether field tends to dynamically align such that it is orthogonal to constant density hypersurfaces for the theoretically and experimentally relevant portion of the parameter space. The dynamics forces the rest frame of the aether and that of the perfect fluid dominating the cosmological evolution to coincide. Finally, we showed that the dynamics allows for the possibility of a nonzero spatial component in a nonexpanding fifth dimension during a de Sitter era. Even a spatial component in a nonexpanding fifth dimension will decay away during non-de Sitter eras, e.g., in a matter- or radiation-dominated universe. We take this as evidence that aether fields with well-behaved semiclassical dynamics will not lead to any significant departure from isotropy.

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