PHYSICAL REVIEW D **79,** 065008 (2009)

Gauged supersymmetries in Yang-Mills theory

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(Received 12 September 2008; published 11 March 2009)

In this paper we show that Yang-Mills theory in the Curci-Ferrari-Delbourgo-Jarvis gauge admits some up to now unknown local linear Ward identities. These identities imply some nonrenormalization theorems with practical simplifications for perturbation theory. We show, in particular, that all renormalization factors can be extracted from two-point functions. The Ward identities are shown to be related to supergauge transformations in the superfield formalism for Yang-Mills theory. The case of nonzero Curci-Ferrari mass is also addressed.

DOI: 10.1103/PhysRevD.79.065008 PACS numbers: 11.15.Bt

I. INTRODUCTION

In Yang-Mills theories, for almost all calculations aside from lattice simulations of gauge-invariant quantities, one needs to fix the gauge. In order to choose a sort of "optimal" gauge fixing among a large number of possibilities, one would like to preserve as many properties of the nonfixed gauge theory as possible. In particular, it is convenient to choose a gauge fixing that preserves Lorentz invariance, the global color symmetry group, the renormalizability of the theory, its locality and Becchi-Rouet-Storat-Tyutin (BRST) symmetry [1–3] seen as a nontrivial subgroup of the gauge symmetry. Of course, one also wants the resulting model to be physically acceptable preserving, in particular, the unitarity. Gauge fixings exist that, at the perturbative level, satisfy all these requirements. The most popular are the linear covariant gauges, including, in particular, the Landau gauge. However, such gauge fixings are ambiguous because of the Gribov copies problem [4–6]. One manifestation of this problem is that if one tries to construct a nonperturbative version of the BRST symmetry on the lattice, the expectation value of gauge-invariant quantities is an undefined $0/0$ expression. This is sometimes called the Neuberger's 0 problem [7,8]. These zeros originate from the compensation in the functional integral of the contributions of pairs of Gribov copies that come with opposite weights. One therefore faces the alternative of either working with a gauge fixing with a Gribov ambiguity or lose one of the above mentioned properties. In fact, recent works propose a third option, and that is to calculate some gauge-invariant quantities without fixing the gauge (see [9] and references therein).

If one chooses the second option, one can, for example,

(i) Use the axial gauge that explicitly breaks Lorentz invariance but does not have a Gribov problem [10].

(ii) Use the maximal Abelian gauge that breaks the global color symmetry group; in this case, the partial gauge fixing to the maximal Abelian subgroup of the gauge group has been proven to avoid the Gribov problem [12].

(iii) Use the absolute Landau gauge by imposing a global extremization condition of a certain functional (see, for example, [13]); however no local action is known to implement this gauge fixing and the very useful BRST symmetry is also lost. Moreover, no efficient algorithm is known to implement that idea in practice.

In this paper we will follow a more heterodox strategy, which consists of taking the Curci-Ferrari (CF) model [14,15] that corresponds to the Yang-Mills theory in a particular gauge, supplemented with a mass for gluons and ghosts. This model is not unitary [15–17] but the presence of the masses lifts the degeneracy of contributions coming from different Gribov copies and therefore regularizes the Neuberger's zero [18–20]. If one studies the model directly at zero mass, one has a standard gauge fixing sometimes called the Curci-Ferrari-Delbourgo-Jarvis (CFDJ) gauge [14,15,21], with all good properties, including unitarity, except that it has a Gribov ambiguity. It is actually possible to have unitarity *and* regularize the $0/0$ expressions by computing physical observables in the massive theory and then taking the limit of vanishing masses as proposed in [18–20].

The mass term can also be seen as a source for the dimension-two composite operator $\frac{1}{2}(A_\mu^a)^2 + \xi_0 \bar{c}^a c^a$ that attracted a lot of attention recently in relation with nonperturbative effects on the behavior of the correlation functions of ghosts and gluons (see, for example, [22–25]).

All these reasons strongly motivate the use of the CF model. However, this model is not widely used in practice, mainly because it seems much more cumbersome than the linear gauges. For instance it has a four-ghosts interaction.

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In this paper we show that this widespread prejudice should be reconsidered. We will show that beside the large symmetry group of the CF model, there exist local transformations that induce very simple variations of the action. Therefore, although these transformations are not symmetries of the full action, one can deduce from them useful linear Ward identities. We show that there are actually underlying symmetries associated with these transformations that clearly appear in the superspace formulation of Yang-Mills theory [21,26–28]. In this formulation, gluons and ghosts are part of a single supervector in a superspace with four bosonic coordinates and two anticommuting Grassmann coordinates. We show that the transformations associated with the new Ward identities are, in fact, supergauge transformations. The associated identities allow us to deduce nonrenormalization theorems reducing the number of independent renormalization factors from five [17] to three. The situation is very close to the gauge-fixed Abelian theories where gauge transformations are not symmetries of the full bare action but allow one to deduce linear Ward identities. This, in turn, implies that gauge-fixing terms are not renormalized. Another question addressed in the present paper is the meaning of the Curci-Ferrari mass in the superspace formulation. We show that it can be seen as a consequence of a curvature in the Grassmannian sector of the superspace. Finally we show that these nonrenormalization theorems imply practical simplifications for perturbation theory: all the renormalization factors can be extracted from diagrams with two external legs only. We should stress that, up to now, a fully satisfying construction of Yang-Mills theory based on a superspace formulation has not been proposed (see for instance [21] for a discussion of the problems encountered). However, in this article, the superfield formalism is mainly used as a way to rewrite in an elegant manner things that have their equivalent in the standard formulation of the Yang-Mills theory (i.e., not expressed in terms of superfields). The possible issue of a consistent quantization of the Yang-Mills theory in the superfield formalism has therefore no impact on the results presented here.

The paper is organized as follows: In Sec. II, we review the model and its symmetries in the massless case. We also derive the new Ward identities. In Sec. III, we analyze the renormalization properties of the model and deduce a nonrenormalization theorem. In Sec. IV we generalize the results of the two previous sections to take into account the CF mass and deduce another nonrenormalization theorem. In Sec. V we analyze the consequences of these results for perturbation theory. In Sec. VI we review the superspace formulation of Yang-Mills theory and interpret in this context the mentioned Ward identities in terms of supergauge transformations. We also give an interpretation of the CF model in the superspace formulation. Finally, we give our conclusions in Sec. VII.

II. THE ACTION AND ITS SYMMETRIES

In this section, we analyze the CF model with vanishing masses, i.e., the Yang-Mills theory in the CFDJ gauge [14,15,21]. We will consider here the model in a four-dimensional Euclidean space without including matter, but most of the results can be generalized to Minkowskian space and the inclusion of matter does not modify the main results. The gauge-fixed Lagrangian reads

$$\mathcal{L} = \mathcal{L}_{\rm YM} + \mathcal{L}_{\rm GF}. \tag{1}$$

$\mathcal{L}_{\rm YM}$ is the Yang-Mills Lagrangian:

$$\mathcal{L}_{\rm YM} = \tfrac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}, \tag{2}$$

$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_0 f^{abc} A^b_\mu A^c_\nu$ is the bare field strength, g_0 is the bare gauge coupling, A_μ is the gauge field, and f^{abc} denotes the structure constants of the gauge group that are chosen completely antisymmetric. $\mathcal{L}_{\rm GF}$ is the gauge-fixing term, which includes a ghost sector. It takes the form:

$$\mathcal{L}_{\rm GF} = \frac{1}{2}\partial_\mu \bar{c}^a (D_\mu c)^a + \frac{1}{2}(D_\mu \bar{c})^a \partial_\mu c^a + \frac{\xi_0}{2} h^a h^a + i h^a \partial_\mu A^a_\mu - \xi_0 \frac{g_0^2}{8} (f^{abc} \bar{c}^b c^c)^2. \tag{3}$$

Here, c and $\bar{c}$ are ghost and antighosts fields, respectively, and $(D_\mu \varphi)^a = \partial_\mu \varphi^a + g_0 f^{abc} A^b_\mu \varphi^c$ is the covariant derivative for any field φ in the adjoint representation. The main interest of the CFDJ Lagrangian (3) is that the ghost-antighost exchange symmetry is explicit and that it preserves the linear realization of some continuous symmetries [21]. This is not the case if the Lagrange multiplier h^a is introduced, as often done [28], in a nonsymmetric way:

$$\mathcal{L}^{\rm ns}_{\rm GF} = \partial_\mu \bar{c}^a (D_\mu c)^a + \frac{\xi_0}{2} h^a h^a + i h^a \partial_\mu A^a_\mu - i \frac{\xi_0}{2} g_0 f^{abc} h^a \bar{c}^b c^c - \xi_0 \frac{g_0^2}{4} (f^{abc} \bar{c}^b c^c)^2. \tag{4}$$

However, these two versions of the CF model are in fact equivalent: indeed, one obtains (4) by performing the change of variables $ih^a \to ih^a + \frac{g_0}{2} f^{abc} \bar{c}^b c^c$ in (3).

Note that the considered gauge-fixing Lagrangian is different from the more standard linear gauge fixing:

$$\mathcal{L}^{\rm linear}_{\rm GF} = \partial_\mu \bar{c}^a (D_\mu c)^a + \frac{\xi_0}{2} h^a h^a + i h^a \partial_\mu A^a_\mu. \tag{5}$$

One cannot obtain one from the other by a change of variables in the fields. However, all these gauge fixings coincide in the particular case of the Landau gauge limit $\xi_0 \to 0$. In fact, (4) and (5) are identical in this limit.

Let us list the symmetries of the gauge-fixing Lagrangian (3):

(a) The Euclidean symmetries of the spacetime.
(b) The global color symmetry.

(c) The already mentioned ghost conjugation symmetry: $c^a \to \bar{c}^a$, $\bar{c}^a \to -c^a$ without modifying the other fields. This symmetry allows one to obtain most of the relations of this paper by conjugating those explicitly considered.

(d) The continuous symplectic group $SP(2,\mathbb{R})$ [29,30] with generators N, t, and $\bar{t}$ defined by

$$\begin{aligned} &tA^a_\mu = 0, && \bar{t}A^a_\mu = 0, && NA^a_\mu = 0,\\ &tc^a = 0, && \bar{t}c^a = -\bar{c}^a, && Nc^a = c^a,\\ &t\bar{c}^a = c^a, && \bar{t}\bar{c}^a = 0, && N\bar{c}^a = -\bar{c}^a,\\ &th^a = 0, && \bar{t}h^a = 0, && Nh^a = 0. \end{aligned} \tag{6}$$

N is associated with the ghost-number conservation. One observes that A and h are singlets while c and $\bar{c}$ form a doublet of this group. Note that t and $\bar{t}$ have ghost numbers 2 and -2, respectively.

(e) The model is also invariant under the nonlinear BRST and anti-BRST symmetries:

$$\begin{aligned} &sA^a_\mu = (D_\mu c)^a, \qquad \bar{s}A^a_\mu = (D_\mu \bar{c})^a,\\ &sc^a = -\frac{g_0}{2}f^{abc}c^bc^c, \qquad \bar{s}\bar{c}^a = -\frac{g_0}{2}f^{abc}\bar{c}^b\bar{c}^c,\\ &s\bar{c}^a = ih^a - \frac{g_0}{2}f^{abc}\bar{c}^bc^c,\\ &\bar{s}c^a = -ih^a - \frac{g_0}{2}f^{abc}\bar{c}^bc^c,\\ &sih^a = \frac{g_0}{2}f^{abc}\left(ih^bc^c + \frac{g_0}{4}f^{cde}\bar{c}^bc^dc^e\right),\\ &\bar{s}ih^a = \frac{g_0}{2}f^{abc}\left(ih^b\bar{c}^c - \frac{g_0}{4}f^{cde}c^b\bar{c}^d\bar{c}^e\right). \end{aligned} \tag{7}$$

These symmetries satisfy the standard nilpotency property ($s^2 = \bar{s}^2 = \bar{s}s + s\bar{s} = 0$).

We now review the standard procedure to handle these symmetries [28,31,32]. However, let us stress that, contrary to [28] here we use the action (3) that ensures a linear realization of the $SP(2,\mathbb{R})$ group. This implies some differences that are detailed below. In order to deduce Slavnov-Taylor identities for these symmetries, it is necessary to introduce sources for the variations of the fields under BRST and anti-BRST symmetries. Since the symmetry is nilpotent, it is sufficient to introduce sources for $s\varphi^a$, $\bar{s}\varphi^a$, and $s\bar{s}\varphi^a$ for $\varphi^a = A^a_\mu$, c^a, and $\bar{c}^a$ [33]. For completeness, we give here

$$\begin{aligned} s\bar{s}A^a_\mu &= i(D_\mu h)^a + \frac{g_0}{2}f^{abc}(\bar{c}^b(D_\mu c)^c - (D_\mu\bar{c})^bc^c),\\ s\bar{s}c &= -g_0f^{abc}\left(ih^bc^c + \frac{g_0}{4}f^{cde}\bar{c}^bc^dc^e\right),\\ s\bar{s}\,\bar{c} &= -g_0f^{abc}\left(ih^b\bar{c}^c - \frac{g_0}{4}f^{cde}c^b\bar{c}^d\bar{c}^e\right). \end{aligned} \tag{8}$$

Observe that

$$ih^a = (s\bar{c}^a - \bar{s}c^a)/2 \tag{9}$$

so that the variations of h^a can be expressed in terms of the variations of the other fields. Consequently we do not introduce new sources for these variations.

We therefore consider the generating functional:

$$\begin{aligned} &\exp(W[J,\chi,\bar{\chi},R,K,L,\bar{K},\bar{L},M,\alpha,\beta,\bar{\beta}])\\ &\quad = \int \mathcal{D}(A,c,\bar{c},h)\exp\int d^4x(-\mathcal{L} + \mathcal{L}_{\text{sources}}), \end{aligned} \tag{10}$$

where

$$\begin{aligned} \mathcal{L}_{\text{sources}} = {}& J^a_\mu A^a_\mu + \bar{\chi}^ac^a + \bar{c}^a\chi^a + R^ah^a + \bar{K}^a_\mu sA^a_\mu\\ &+ \bar{s}A^a_\mu K^a_\mu + \bar{L}^asc^a + L^a\bar{s}\bar{c}^a\\ &+ M^a(s\bar{c}^a + \bar{s}c^a)/2 + \alpha^a_\mu s\bar{s}A^a_\mu + \bar{\beta}^as\bar{s}c^a\\ &+ s\bar{s}\bar{c}^a\beta^a. \end{aligned} \tag{11}$$

We coupled the variations of the fields to the sources so that R is a singlet and (L^a, $\bar{L}^a$, M^a) a triplet of the $SP(2,\mathbb{R})$ group. In Table I we give the dimensions, ghost numbers, and ghost conjugates of the sources and fields.

Simple Ward identities can be easily derived for linearly realized symmetries (a)–(d). For instance, the Ward identity associated with the symmetry of generator t is

$$\begin{aligned} \int d^4x\Big(&c^a\frac{\delta\Gamma}{\delta\bar{c}^a} + K^a_\mu\frac{\delta\Gamma}{\delta\bar{K}^a_\mu} - 2L^a\frac{\delta\Gamma}{\delta M^a}\\ &- M^a\frac{\delta\Gamma}{\delta\bar{L}^a} + \beta^a\frac{\delta\Gamma}{\delta\bar{\beta}^a}\Big) = 0. \end{aligned} \tag{12}$$

One of the advantages of using the action (3) is that Ward identities for the $SP(2,\mathbb{R})$ group are linear.

As usual, the Slavnov-Taylor identity [34,35] associated with the BRST symmetry is obtained by performing the change of variables in the functional integral $\varphi \to \varphi + \varsigma s\varphi$ for all fundamental fields φ with a constant Grassmannian parameter ς. One obtains

$$\begin{aligned} \int d^4x\Big\{ &-\frac{\delta\Gamma}{\delta\bar{K}^a_\mu}\frac{\delta\Gamma}{\delta A^a_\mu} - \frac{\delta\Gamma}{\delta\bar{L}^a}\frac{\delta\Gamma}{\delta c^a} + \left(ih^a - \frac{\delta\Gamma}{\delta M^a}\right)\frac{\delta\Gamma}{\delta\bar{c}^a}\\ &- K^a_\mu\frac{\delta\Gamma}{\delta\alpha^a_\mu} + L^a\frac{\delta\Gamma}{\delta\beta^a} + \frac{1}{2}\left(-i\frac{\delta\Gamma}{\delta h^a} - M^a\right)\frac{\delta\Gamma}{\delta\bar{\beta}^a}\Big\} = 0. \end{aligned} \tag{13}$$

A similar equation can be deduced for the anti-BRST symmetry. However, we will not need it here because its information can be obtained by exploiting the ghost conjugation (c). Slavnov-Taylor identities for the BRST and anti-BRST symmetries have been deduced in [28] for the action (4). Here, the use of the action (3) let us use the simple ghost conjugation in order to deduce the identity associated with anti-BRST from that associated with BRST. The physical interpretation of (13) is well known. If one evaluates it for vanishing sources for composite

TABLE I. Canonical dimension, ghost number, and (ghost) conjugation of different fields and sources.

Field/source	A	c	$\bar{c}$	h	α	β	$\bar{\beta}$	K	$\bar{K}$	L	$\bar{L}$	M
Dimension	1	1	1	2	1	1	1	2	2	2	2	2
Ghost no.	0	1	-1	0	0	1	-1	1	-1	2	-2	0
Conjugation	A	$\bar{c}$	$-c$	h	α	$\bar{\beta}$	$-\beta$	$\bar{K}$	$-K$	$\bar{L}$	L	$-M$

operators, it says that Γ is invariant under $A_\mu^a \to A_\mu^a - \varsigma\delta\Gamma/\delta\bar{K}_\mu^a$, $c^a \to c^a - \varsigma\delta\Gamma/\delta\bar{L}^a$, etc. The symmetry transformation itself acquires quantum corrections.

After this review of these well-known symmetries and their consequences, we now come to the deduction of other new Ward identities that are linear and local. The first one is the equation of motion for the Lagrange multiplier h^a. It can be obtained in the usual way by performing an infinitesimal spacetime dependent shift on the h field $ih^a(x) \to ih^a(x) + \hat{\lambda}^a(x)$. This gives

$$\frac{\delta\Gamma}{\delta h^a} = \xi_0 h^a + i[\partial_\mu A_\mu^a + (D_\mu\alpha_\mu)^a - g_0 f^{abc}(\bar{\beta}^b c^c + \bar{c}^c\beta^b)]. \tag{14}$$

This equation means that terms in the effective action including the h field are not renormalized. Note that the nonsymmetric Lagrangian (4) contains terms that couple the h field trilinearly which prevents one to derive a simple equation as (14). Such terms do not exist in Lagrangians (3) and (5) giving tractable equations of motion for h. Another gauge where tractable equations for the (Abelian) Lagrange multiplier can be deduced is the maximal Abelian gauge.

In the case of linear gauge fixing, as well as in maximal Abelian gauge [36,37], another local and linear identity can be deduced from the equation of motion of the antighost field. We find an analogous relation here if we shift the ghost field by a space-dependent term $\delta\bar{c}^a(x) = \bar{\eta}^a(x)$ and simultaneously change the Lagrange multiplier according to $\delta ih^a(x) = \frac{g_0}{2} f^{abc}\bar{\eta}^b(x)c^c(x)$:

$$-\frac{\xi_0}{2}\frac{\delta\Gamma}{\delta\bar{\beta}^a} - \partial_\mu\frac{\delta\Gamma}{\delta\bar{K}_\mu^a} + \frac{\delta\Gamma}{\delta\bar{c}^a} - D_\mu K_\mu^a + g_0 f^{abc}\bigg(-\bar{c}^b L^c + \frac{1}{2}c^b\bigg(-i\frac{\delta\Gamma}{\delta h^c} - M^c\bigg) - \frac{\delta\Gamma}{\delta\bar{K}_\mu^b}\alpha_\mu^c + \frac{\delta\Gamma}{\delta\bar{L}^b}\bar{\beta}^c + \bigg(ih^b - \frac{\delta\Gamma}{\delta M^b}\bigg)\beta^c\bigg) = 0. \tag{15}$$

Here, contrary to what happens in linear gauges, we obtain a third equation by ghost conjugation.

A fourth identity can be deduced by making the following change of variables in the functional integral:

$$\begin{aligned}\delta A_\mu^a(x) &= (D_\mu\lambda(x))^a,\\ \delta c^a(x) &= g_0 f^{abc}c^b(x)\lambda^c(x),\\ \delta\bar{c}^a(x) &= g_0 f^{abc}\bar{c}^b(x)\lambda^c(x),\\ \delta h^a(x) &= g_0 f^{abc}h^b(x)\lambda^c(x),\end{aligned} \tag{16}$$

which gives the identity:

$$\begin{aligned}&\bigg(D_\mu\frac{\delta\Gamma}{\delta A_\mu}\bigg)^a - \partial_\mu\frac{\delta\Gamma}{\delta\alpha_\mu^a}\\ &\quad= g_0 f^{abc}\bigg(c^c\frac{\delta\Gamma}{\delta c^b} + \bar{c}^c\frac{\delta\Gamma}{\delta\bar{c}^b} + K_\mu^c\frac{\delta\Gamma}{\delta K_\mu^b} + \bar{K}_\mu^c\frac{\delta\Gamma}{\delta\bar{K}_\mu^b}\\ &\qquad + h^c\frac{\delta\Gamma}{\delta h^b} + M^c\frac{\delta\Gamma}{\delta M^b} + L^c\frac{\delta\Gamma}{\delta L^b} + \bar{L}^c\frac{\delta\Gamma}{\delta\bar{L}^b}\\ &\qquad + \alpha_\mu^c\frac{\delta\Gamma}{\delta\alpha_\mu^b} + \beta^c\frac{\delta\Gamma}{\delta\beta^b} + \bar{\beta}^c\frac{\delta\Gamma}{\delta\bar{\beta}^b}\bigg).\end{aligned} \tag{17}$$

To our knowledge, no such relation was found in the linear gauge fixing.

Let us stress that these four identities [Eqs. (14) and (15) and its conjugate, and (17)] are not fully independent from the Slavnov-Taylor equation (13). Actually, the change of variable yielding (15) is obtained by commuting the shift of h used to deduce (14) and the BRST transformation that generates (13). Similarly, the transformation (16) is obtained by commuting the anti-BRST transformation with the transformation that leads to (15). Observe also that these four equations look like Ward identities for gauged linear (super)symmetries letting aside some nonhomogeneous terms that play the role of gauge-fixing terms. As mentioned in the Introduction, these terms behave as in Abelian gauge theories where gauge fixing preserves its bare form under the renormalization process. This is very different from Slavnov-Taylor identities, which are nonlinear in Γ and therefore much harder to handle. The obtention of local, linear Ward identities is a nontrivial result and is the heart of the present manuscript.

Equations (14), (15), and (17) are very simple and have far reaching consequences. However, to our surprise, they have never been addressed before in the CF model. In the next two sections, we discuss the consequences of these relations showing, in particular, that they induce nontrivial nonrenormalization theorems for some quantities.

III. NONRENORMALIZATION THEOREM FOR THE COUPLING

The four new identities derived in the previous section have many consequences on the form of the effective action. As a concrete example, we analyze in this section the implications on the renormalization properties of the model.

The perturbative renormalizability of this model has been proven by considering five renormalization factors [17], including the renormalization of the mass. Recently, however, one of us [38] proved two nonrenormalization theorems that reduce the number of renormalization factors from five to three. We now prove in this section and in the following that these nonrenormalization theorems are, in fact, a direct consequence of the new identities discussed in the previous section.

We follow the standard procedure (see, for example, [11]) of considering terms that can diverge by power counting and constraining them iteratively. In a loop expansion, suppose that all divergences have been renormalized at order $n-1$. Divergent terms that appear at order n in the effective action have couplings with positive or zero dimension. Let us call them $\Delta\Gamma^{(n)}_{\rm div}$, and take an infinitesimal constant ϵ. If one calls $\Gamma^{(n)}_{\rm div} = S + \epsilon\Delta\Gamma^{(n)}_{\rm div}$, then, in four dimensions, the most general form for this functional at order n that satisfies the linear symmetries (a)–(d), takes the form:

$$\begin{aligned}\Gamma^{(n)}_{\rm div}&[A, c, \bar c, h, K, \bar K, L, \bar L, M, \alpha, \beta, \bar\beta]\\ &= -\int d^4x\Big\{Z_L\Big(\bar L^a L^a - \frac{1}{4}M^a M^a\Big) + Z_K \bar K^a_\mu K^a_\mu\\ &\quad + \bar K^a_\mu \tilde s A^a_\mu + \tilde{\bar s} A^a_\mu K^a_\mu + \bar L^a \tilde s c^a + L^a \tilde{\bar s}\bar c^a\\ &\quad + M^a(\tilde s\bar c^a + \tilde{\bar s} c^a)/2\Big\} + \hat\Gamma[A, c, \bar c, h, \alpha, \beta, \bar\beta]. \quad (18)\end{aligned}$$

We introduced the notation $\tilde s$ and $\tilde{\bar s}$ in terms linear in K, $\bar K$, L, $\bar L$, and M in analogy with (11). However, for the moment, $\tilde s A^a_\mu$, $\tilde{\bar s} A^a_\mu$, $\tilde s c^a$, $\tilde{\bar s}\bar c^a$, and $\tilde s\bar c^a$ denote arbitrary operators depending on $\{A, c, \bar c, h, \alpha, \beta, \bar\beta\}$, of dimension two, with the same transformations under linear symmetries as the corresponding bare expressions. In order for $\tilde s$ and $\tilde{\bar s}$ to be the symmetries of $\hat\Gamma$ discussed just below Eq. (13), we complement their definitions [again by analogy with (11) and (7)] by

$$\tilde s i h^a = \frac{1}{2}\frac{\delta\hat\Gamma}{\delta\bar\beta^a}, \quad (19)$$

$$\tilde{\bar s} i h^a = -\frac{1}{2}\frac{\delta\hat\Gamma}{\delta\beta^a}, \quad (20)$$

$$\tilde s\bar c^a - \tilde{\bar s} c^a = 2ih^a. \quad (21)$$

For generic operators, one defines $\tilde s$ as

$$\begin{aligned}\tilde s = \int d^4x\Big\{&\tilde s A^a_\mu(x)\frac{\delta}{\delta A^a_\mu(x)} + \tilde s c^a(x)\frac{\delta}{\delta c^a(x)}\\ &+ \tilde s\bar c^a(x)\frac{\delta}{\delta\bar c^a(x)} + \tilde s h^a(x)\frac{\delta}{\delta h^a(x)}\Big\} \quad (22)\end{aligned}$$

and similarly for $\tilde{\bar s}$. It is now easy to check that, with these definitions, $\tilde s$ and $\tilde{\bar s}$ are symmetries of $\hat\Gamma$.

We now want to solve the Slavnov-Taylor equation (13) together with Eqs. (14), (15), and (17). The calculation is lengthy but straightforward. Some details are given in the Appendix. The resolution simplifies if one introduces the variables

$$\begin{aligned}&\tilde c^a = c^a + Z_L\beta^a, \quad \tilde{\bar c}^a = \bar c^a + Z_L\bar\beta^a, \quad \tilde A^a_\mu = A^a_\mu - Z_K\alpha^a_\mu,\\ &i\tilde h^a = ih^a + \frac{Z_L}{2}((\tilde D_\mu\alpha_\mu)^a + \tilde g f^{abc}(\bar c^b\beta^c - \bar\beta^b c^c)), \quad (23)\end{aligned}$$

where $(\tilde D_\mu\phi)^a = Z\partial_\mu\phi^a + \tilde g f^{abc}\tilde A^b_\mu\phi^c$. Z and $\tilde g$ are at this level arbitrary constants. In term of these variables, the solution reads

$$\begin{aligned}\hat\Gamma = \int d^4x\Big\{&\frac{\tilde Z}{4}\tilde F^a_{\mu\nu}\tilde F^a_{\mu\nu} + \frac{Z_L}{2Y}(\partial_\mu\tilde A^a_\mu)^2\\ &+ \frac{(\tilde D_\mu\tilde{\bar c})^a\partial_\mu\tilde c^a + \partial_\mu\tilde{\bar c}^a(\tilde D_\mu\tilde c)^a}{2Y} - \frac{\tilde g^2(f^{abc}\tilde{\bar c}^b\tilde c^c)^2}{8Y}\\ &+ \frac{\xi_0}{2}\tilde h^a\tilde h^a + i\tilde h^a\partial_\mu\tilde A^a_\mu + \bar\beta^a\tilde g f^{abc}\Big(i\tilde h^b\tilde c^c\\ &+ \frac{\tilde g}{4}f^{cde}\tilde{\bar c}^b\tilde c^d\tilde c^e\Big) + \tilde g f^{abc}\Big(i\tilde h^b\tilde{\bar c}^c - \frac{\tilde g}{4}f^{cde}\tilde c^b\tilde{\bar c}^d\tilde{\bar c}^e\Big)\beta^a\\ &- \frac{Z_L}{4}((\tilde D_\mu\alpha_\mu)^a - \tilde g f^{abc}(\bar\beta^b\tilde c^c - \tilde{\bar c}^b\beta^c))^2\\ &- \alpha^a_\mu\Big(i(\tilde D_\mu\tilde h)^a + \frac{\tilde g}{2}f^{abc}(\tilde{\bar c}^b(\tilde D_\mu\tilde c)^c - (\tilde D_\mu\tilde{\bar c})^b\tilde c^c)\Big)\Big\},\end{aligned} \quad (24)$$

with $\tilde F^a_{\mu\nu} = Z(\partial_\mu\tilde A^a_\nu - \partial_\nu\tilde A^a_\mu) + \tilde g f^{abc}\tilde A^b_\mu\tilde A^c_\nu$ and $Y = 1 - Z_L\xi_0/2$. The action of $\tilde s$ and $\tilde{\bar s}$ on the fields reads

$$\begin{aligned}&\tilde s A^a_\mu = (\tilde D_\mu\tilde c)^a, \qquad \tilde{\bar s} A^a_\mu = (\tilde D_\mu\tilde{\bar c})^a,\\ &\tilde s c^a = -\frac{\tilde g}{2}f^{abc}\tilde c^b\tilde c^c, \qquad \tilde{\bar s}\bar c^a = -\frac{\tilde g}{2}f^{abc}\tilde{\bar c}^b\tilde{\bar c}^c, \quad (25)\\ &\tilde{\bar s} c^a = -ih^a - \frac{\tilde g}{2}f^{abc}\tilde{\bar c}^b\tilde c^c, \qquad \tilde s\bar c^a = ih^a - \frac{\tilde g}{2}f^{abc}\tilde{\bar c}^b\tilde c^c.\end{aligned}$$

Note that Eq. (24) is written in terms of the bare gauge parameter ξ_0. The reason being that Eq. (14) ensures that the h sector of the effective action is not renormalized. Actually, Eqs. (14), (15), and (17) impose two other relations:

$$g_0 = \tilde g Y, \qquad 1 + Z_K = ZY. \quad (26)$$

These equations are at the core of the nonrenormalization theorem (see below).

The action of $\tilde{s}$ on h can be deduced from Eq. (19). We just give here the expression at vanishing sources for the composite operators:

$$2iY\tilde{s}h^a = i\tilde{g}Y^2 f^{abc}h^b c^c - \xi_0 \frac{Z_L^2}{4}\tilde{g}f^{abc}\partial_\mu A_\mu^b c^c + \frac{\tilde{g}^2}{4}f^{abc}f^{cde}\bar{c}^b c^d c^e - Z_L(\tilde{D}_\mu \partial_\mu c)^a. \quad (27)$$

An analogous formula can be derived for $\bar{\tilde{s}}h$.

A straightforward calculation shows that $\tilde{s}$ and $\bar{\tilde{s}}$ are nilpotent on shell, i.e., when one imposes the equations of motion for the fields h, c, and $\bar{c}$. Actually $\tilde{s}$ and $\bar{\tilde{s}}$ can be decomposed in a sum of an off-shell nilpotent symmetry that has the form of the bare symmetry (7) up to multiplicative factors and two trivial symmetries with generators:

$$r_1 c = r_1\bar{c} = r_1 A_\mu = 0, \qquad r_1 h^a = -f^{abc}\frac{\delta\hat{\Gamma}}{\delta h^b}c^c, \quad (28)$$

and

$$r_2 A_\mu = r_2 c = 0, \qquad r_2\bar{c}^a = -i\frac{\delta\hat{\Gamma}}{\delta h^a}, \qquad r_2 ih^a = -\frac{\delta\hat{\Gamma}}{\delta\bar{c}^a}. \quad (29)$$

These generators vanish when one imposes the equations of motion. This is consistent with the on-shell nilpotency of $\tilde{s}$ and $\bar{\tilde{s}}$.

Note that there appears in Γ terms that were not present in the bare action described in Sec. II. There are terms with two powers of the sources or more and also a term proportional to $(\partial_\mu A_\mu^a)^2$. In order to make the theory renormalizable, one needs to include such terms in the bare action. Fortunately, it is not necessary to perform again the analysis with this new action. Indeed, the precise form of the bare action is not necessary to deduce Slavnov-Taylor identities. All that is needed is that the bare action satisfies the Slavnov-Taylor identities [39]. Therefore, the form of Γ given in Eqs. (18), (24), and (25) is stable under renormalization. Let us comment that the term in $(\partial_\mu A_\mu^a)^2$ can be eliminated by a shift proportional to $\partial_\mu A_\mu^a$ of the Lagrange multiplier.

We now make contact with the perturbative results and concentrate on the A, c, $\bar{c}$ sector once the Lagrange multiplier has been eliminated by its equation of motion. The standard parametrization (see, for instance, [25]) of the effective action reads

$$\hat{\Gamma} = \int d^4x\bigg\{\frac{1}{2Z_c}(\partial_\mu\bar{c}^a\check{D}_\mu c^a + \check{D}_\mu\bar{c}^a\partial_\mu c^a) + \frac{1}{2\xi_0 Z_\xi}(\partial_\mu A_\mu^a)^2 - \frac{Z_\xi\xi_0 g_0^2}{8Z_g^2 Z_A Z_c^2}(f^{abc}\bar{c}^b c^c)^2 + \frac{1}{4Z_A}\check{F}_{\mu\nu}^a\check{F}_{\mu\nu}^a\bigg\}, \quad (30)$$

with

$$\check{D}_\mu c^a = \partial_\mu c^a + \frac{g_0}{Z_g\sqrt{Z_A}}f^{abc}A_\mu^b c^c,$$
$$\check{D}_\mu\bar{c}^a = \partial_\mu\bar{c}^a + \frac{g_0}{Z_g\sqrt{Z_A}}f^{abc}A_\mu^b\bar{c}^c, \quad (31)$$
$$\check{F}_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + \frac{g_0}{Z_g\sqrt{Z_A}}f^{abc}A_\mu^b A_\nu^b.$$

Comparison with Eq. (24)—where h is eliminated by its equations of motion—yields, together with Eq. (26), the following relations:

$$Z_A = Z^{-2}\tilde{Z}^{-1}, \qquad Z_c = YZ^{-1},$$
$$Z_\xi = Y, \qquad Z_g = YZ^2\tilde{Z}^{1/2}. \quad (32)$$

One then easily deduces the nonrenormalization theorem:

$$Z_g = Z_A^{-1/2}Z_c^{-1}Z_\xi^2. \quad (33)$$

We postpone to Sec. V the discussion of this equation together with another nonrenormalization theorem to be proven in the next section.

IV. THE MASSIVE CASE

As said in the Introduction, Curci and Ferrari proposed a very natural generalization of Yang-Mills theory in this particular gauge [14,15]. One can add a mass term for the ghosts and gluons that preserves BRST-like symmetries:

$$\mathcal{L}_m = m_0^2(\tfrac{1}{2}(A_\mu^a)^2 + \xi_0\bar{c}^a c^a). \quad (34)$$

The theory remains renormalizable; however, nilpotency of the BRST symmetry is lost and, as a result, the model is no longer unitary [15–17] (for a review of the effect of this mass see [32]). The study performed in the previous sections is generalized here to include the mass term (34). We show that the modifications to Eqs. (14), (15), and (17) are very simple. The other striking result is that no independent renormalization factor is needed to renormalize the mass term.

Let us start by discussing the symmetry content of the theory in the presence of the mass term. All the linear symmetries (a)–(d) are preserved. On the contrary the action is not invariant under the original BRST and anti-BRST transformations (7), but is invariant under modified transformations [29], $s_m = s + m_0^2 s_1$ and $\bar{s}_m = \bar{s} + m_0^2\bar{s}_1$, with

$$s_1 c^a = s_1\bar{c}^a = s_1 A_\mu^a = 0, \qquad \bar{s}_1 c^a = \bar{s}_1\bar{c}^a = \bar{s}_1 A_\mu^a = 0,$$
$$s_1 ih^a = c^a, \qquad \bar{s}_1 ih^a = \bar{c}^a. \quad (35)$$

As already mentioned the new BRST and anti-BRST symmetry transformations are no longer nilpotent. Their algebra becomes [29,30]

$$s_m^2 = m_0^2 t, \qquad \bar{s}_m^2 = m_0^2\bar{t}, \qquad \{s_m, \bar{s}_m\} = -m_0^2 N. \quad (36)$$

The Curci-Ferrari mass term induces a change in the Slavnov-Taylor equation. The right-hand side of (13) is not zero anymore and must be replaced by a term proportional to m_0^2:

$$m_0^2 \int d^4x \left(i \frac{\delta\Gamma}{\delta h^a} c^a + \alpha_\mu^a \frac{\delta\Gamma}{\delta \bar{K}_\mu^a} - 2\bar{\beta}^a \frac{\delta\Gamma}{\delta \bar{L}^a} + 2 \frac{\delta\Gamma}{\delta M^a} \beta^a \right). \tag{37}$$

Slight modifications must be introduced to the new Ward identities described in Sec. II. Equation (14) is actually not modified because the mass term (34) is independent of h. In the second identity, Eq. (15), one must add $-\xi_0 m_0^2 c^a$ to the left-hand side. Finally, in Eq. (17), one must add $-m_0^2 \partial_\mu A_\mu^a$ to the left-hand side.

One easily checks that the divergent part (18) of the solution of the modified Slavnov-Taylor equation now reads $\Gamma_{m,\mathrm{div}}^{(n)} = \Gamma_{\mathrm{div}}^{(n)} + m_0^2 \Gamma_{1,\mathrm{div}}^{(n)}$ with

$$\Gamma_{1,\mathrm{div}}^{(n)} = \int d^4x \left(\frac{\frac{1}{2}(\tilde{A}_\mu^a)^2 + Z\xi_0 \bar{\tilde{c}}^a \tilde{c}^a}{ZY} - \frac{Z_K}{2} (\tilde{\alpha}_\mu^a)^2 + 2Z_L \bar{\tilde{\beta}}^a \tilde{\beta}^a \right). \tag{38}$$

Note that the renormalization of the mass term does not require any new renormalization factor. This leads to another nonrenormalization theorem. If one compares the previous equation at zero sources with the standard parametrization of the mass term [25],

$$\int d^4x \frac{m_0^2}{Z_m} \left(\frac{A_\mu^a A_\mu^a}{2Z_A} + \frac{\xi_0 Z_\xi}{Z_A Z_c} \bar{c}^a c^a \right), \tag{39}$$

by identification of the A^2 terms one deduces that

$$Z_m Z_A = ZY. \tag{40}$$

The $\bar{c}c$ term does not give new information. Using the identifications (32), one obtains another nonrenormalization theorem:

$$Z_\xi^2 = Z_m Z_A Z_c. \tag{41}$$

V. CONSEQUENCES FOR PERTURBATION THEORY

The two nonrenormalization theorems presented in previous sections have far reaching consequences for practical perturbative calculations. First of all, they imply that one has to calculate as many renormalization factors in CFDJ gauge as in linear gauges. Moreover, all these renormalization factors can be extracted from the 2-point function of gluons alone. In fact, one possible set of independent renormalization factors are Z_m (the renormalization for the composite operator $A_\mu^a A_\mu^a$), Z_A, and Z_ξ that can all be extracted from the zero momentum, transverse, and longitudinal parts at order p^2 of the quoted correlation function. Other choices may even be more convenient in practice since some of these renormalization factors can be extracted from the 2-ghost function that has simpler kinematics. In any case, there is no need to calculate 3-point or higher vertices, contrary to what is required in linear gauges. The price to pay is very small: there is a 4-ghost vertex, but the required total number of diagrams seems to be always smaller than that in linear gauges. For example, the 1-loop beta function for pure gauge can be extracted from three diagrams only. So, in what concerns perturbative calculations, once nonrenormalization theorems are exploited, CFDJ gauge is as competitive as linear gauges (same number of renormalization factors) and might even be more convenient (all renormalization factors can be extracted from 2-point functions).

To conclude, let us make three final remarks. First, these two renormalization factors have been found previously by one of us. However, the proof of these nonrenormalization theorems presented in [38] requires extensive use of equations of motions because it is formulated without the introduction of the Lagrange multiplier field h. The physical content of these identities is therefore hidden. Here, these relations are shown to be consequences of the new Ward identities. Second, one can check that the 3-loop renormalization factors [25] satisfy the two nonrenormalization theorems (33) and (41). Actually, it was observed in [40] that the 3-loops renormalization factors satisfy the identity (41) without giving a general proof. Finally, for the Landau gauge ($\xi = 0$), $Z_\xi = 1$ and one recovers the well-known nonrenormalization theorem for the coupling constant [35] as well as the more recent one for the mass [41].

VI. SUPERSPACE INTERPRETATION

A. Flat superspace

It has been shown in the 1980s that reinterpreting the theory in a superspace enables one to give a geometric meaning to the symmetries of the model, in particular, to BRST and anti-BRST symmetries [21,26–28]. We review here the superfield formalism and subsequently reinterpret the new Ward identities described in the previous section in this context.

In the following, we consider a $4+2$-dimensional superspace, with four standard bosonic coordinates, noted x^μ, and two Grassmannian—anticommuting—coordinates θ and $\bar{\theta}$: $\theta^2 = \bar{\theta}^2 = \theta\bar{\theta} + \bar{\theta}\theta = 0$. The (super)fields are now functions of x^μ, θ, and $\bar{\theta}$. In the following, capital indices vary on the four bosonic directions μ and on the two Grassmannian directions: for instance $x^M = (x^\mu, \theta, \bar{\theta})$. Because of the Grassmannian character of θ and $\bar{\theta}$, the Taylor expansion in powers of these variables gives a finite number of terms:

$$f(x^\mu, \theta, \bar{\theta}) = f_{00}(x^\mu) + \theta f_{10}(x^\mu) + \bar{\theta} f_{01}(x^\mu) + \bar{\theta}\theta f_{11}(x^\mu), \tag{42}$$

with $f_{ij}(x^\mu) = \partial^i_\theta \partial^j_{\bar\theta} f(x^\mu, \theta, \bar\theta)|_{\theta=\bar\theta=0}$. Observe that the derivatives with respect to θ and $\bar\theta$ are nilpotent, just as BRST and anti-BRST symmetries. It is actually possible to make this analogy stronger, if one writes a four-dimensional field ϕ and its BRST/anti-BRST variations as a 4 + 2-dimensional superfield Φ

$$\Phi(x^M) = \phi(x^\mu) + \bar\theta s\phi(x^\mu) - \theta\bar s\phi(x^\mu) + \bar\theta\theta s\bar s\phi(x^\mu), \tag{43}$$

where now it is clear that s and $\bar s$ act on the superfield as $\partial_{\bar\theta}$ and $-\partial_\theta$, respectively.

Note moreover that the vectorial superfields, similar to the gauge field, have 4 + 2 components $\mathcal{A}^\mu$, $\mathcal{A}^\theta$, and $\mathcal{A}^{\bar\theta}$, which have ghost numbers 0, 1, and −1, respectively. One can therefore merge the four-dimensional gauge field, the ghost, antighost, and all BRST/anti-BRST variations of these fields in a unique vectorial superfield:

$$\begin{aligned} \mathcal{A}^{\mu}(x^M) &= A^\mu + \bar\theta s A^\mu - \theta\bar s A^\mu + \bar\theta\theta s\bar s A^\mu, \\ \mathcal{A}^\theta(x^M) &= c + \bar\theta s c - \theta\bar s c + \bar\theta\theta s\bar s c, \\ \mathcal{A}^{\bar\theta}(x^M) &= \bar c + \bar\theta s\bar c - \theta\bar s\,\bar c + \bar\theta\theta s\bar s\,\bar c, \end{aligned} \tag{44}$$

where we have omitted the color index and the bosonic space variable. The BRST/anti-BRST symmetries can therefore be interpreted as the invariance under translation in the Grassmannian directions. It is important to understand at this level that the components θ, $\bar\theta$, and $\theta\bar\theta$ of the fields $\mathcal{A}^\mu$, $\mathcal{A}^\theta$, and $\mathcal{A}^{\bar\theta}$ are not independent of the $\theta = \bar\theta = 0$ part of the fields. Indeed, these are explicit functions of A^μ, c, $\bar c$, and h, as given in Eqs. (7) and (8). Consequently, the superfield is constrained and cannot be used as it stands in a functional integral. These constraints are sometimes called "transversality conditions" [21,26–28,42]. Besides this difficulty, another problem in order to construct a consistent superfield formulation for Yang-Mills theory has been the nonexistence for Yang-Mills theory of a superrotation symmetry mixing bosonic and fermionic coordinates [21].

The symmetries t and $\bar t$ and N given in Eq. (6) also have a simple geometric interpretation in superspace: they correspond to the invariance under "rotations" in the Grassmannian directions.

The Lagrangian is easily recast in terms of superspace and superfield. One finds for instance [21]

$$\mathcal{L}_{\rm GF} = -\int d\theta d\bar\theta \frac{1}{2}\mathcal{A}^M g_{MN}\mathcal{A}^N, \tag{45}$$

with g a metric in the superspace, defined as

$$g_{MN} = \begin{cases} \delta_{\mu\nu} & \text{if } M=\mu, N=\nu, \\ -\xi_0/2 & \text{if } M=\theta, N=\bar\theta, \\ \xi_0/2 & \text{if } M=\bar\theta, N=\theta, \\ 0 & \text{otherwise.} \end{cases} \tag{46}$$

The gauge-fixing term appears formally as a mass term in the theory. Observe that ξ_0 appears as a different normalization of the bosonic and fermionic coordinates that can be reabsorbed by a change of variables, in the same way as the speed of light can be eliminated in Minkowskian space.

The source term can also be written as [21]

$$\mathcal{L}_{\rm sources} = \int d\theta d\bar\theta \mathcal{A}^M g_{MN}\mathcal{J}^N \tag{47}$$

with

$$\begin{aligned} \mathcal{J}^\mu &= \alpha^\mu - \bar\theta K^\mu - \bar K^\mu\theta + \bar\theta\theta J^\mu, \\ \frac{\xi_0}{2}\mathcal{J}^\theta &= \beta + \theta\frac{M-iR}{2} + \bar\theta L + \bar\theta\theta\chi, \\ \frac{\xi_0}{2}\mathcal{J}^{\bar\theta} &= \bar\beta - \bar\theta\frac{M+iR}{2} - \theta\bar L + \bar\theta\theta\bar\chi. \end{aligned} \tag{48}$$

The Yang-Mills term does not have such a nice superspace expression. One can however write it as [21]

$$\mathcal{L}_{\rm YM} = \int d\theta d\bar\theta \frac{1}{4}\bar\theta\theta(\mathcal{F}^a_{\mu\nu})^2 \tag{49}$$

with

$$\mathcal{F}^{a}_{\mu\nu} = \partial_\mu\mathcal{A}^a_\nu - \partial_\nu\mathcal{A}^a_\mu + g_0 f^{abc}\mathcal{A}^b_\mu\mathcal{A}^c_\nu. \tag{50}$$

After this review of the supersace formalism, let us now come to the interpretation of the supergauge symmetries described in Sec. II. The infinitesimal gauge transformations can actually be written in the very concise form:

$$\delta\mathcal{A}^a_M = \partial_M\Lambda^a + g_0 f^{abc}\mathcal{A}^b_M\Lambda^c, \tag{51}$$

where Λ is an arbitrary function of x^M. This transformation has exactly the same form as a standard gauge transformation in Yang-Mills theory. To make contact with the expressions of Sec. II, we just need to write the Taylor expansion of Λ in powers of θ and $\bar\theta$:

$$\Lambda(x^M) = \lambda(x^\mu) + \bar\theta\eta(x^\mu) + \bar\eta(x^\mu)\theta + \bar\theta\theta\hat\lambda(x^\mu). \tag{52}$$

This transformation not only gives the right gauge transformation of the physical fields A, c, $\bar c$, and h, but also gives consistent gauge variations for their BRST and anti-BRST variations. Moreover, the Ward identities have a very natural interpretation. Indeed, the Yang-Mills part of the action (49) is manifestly invariant under the supergauge transformation. The gauge-fixing term (45) breaks this symmetry. However its variation under (52) is linear in the field,

$$\delta\mathcal{L}_{\rm GF} = -\int d\theta d\bar\theta \mathcal{A}^{a,M}\partial_M\Lambda^a, \tag{53}$$

and one can therefore deal with it in the corresponding Ward identity.

B. Curved superspace

If the superspace formulation of the massless CFDJ model has been known for quite some time, the corresponding formulation for the massive CF model has never been addressed before. This is the aim of this section. The important observation in this respect is that the BRST and anti-BRST transformations s_m and $\bar{s}_m$ (that were associated with translations in the Grassmannian sector in the massless case) do not anticommute. Their anticommutator is indeed proportional to m_0^2 times a rotation in the Grassmannian coordinates. This is very similar to what happens when one studies the commutation relations of the rotations of the sphere in the limit of infinite radius, where the sphere approaches a plane. At leading order in the curvature, two rotations can be interpreted as translations (that commute) and the third corresponds to the rotation of the plane. We therefore expect that the theory in the presence of a mass term is associated with a superfield theory in a curved superspace, with curvature proportional to m_0.

The calculations in a curved superspace require the introduction of a formalism similar to the one of general relativity. Actually all the standard formulas in a curved space have their superspace equivalent that differ by some signs. We followed the formalism and conventions of [43,44], except that we work with left derivatives. In particular, we consider the supercovariant derivative of a superverctor $\mathcal{V}^N$:

$$\mathcal{D}_M \mathcal{V}^N = \partial_M \mathcal{V}^N + \Gamma^N_{MP} \mathcal{V}^P \tag{54}$$

with Christoffel symbols

$$\Gamma^C_{AB} = \frac{(-1)^{bc}}{2}((-1)^{ab+b}\partial_B g_{AD} + (-1)^b \partial_A g_{BD} - (-1)^{d(a+b)+d}\partial_D g_{AB}) g^{DC}. \tag{55}$$

Here and below, the lowercase letters are 1 if the associated uppercase is fermionic and 0 otherwise. The covariant derivative $\mathcal{D}_M$ should not be confused with the derivative D_μ associated with the gauge group, which we used up to now.

As in standard Riemann geometry, superspace symmetries are described by the Killing vectors that satisfy the equation

$$\mathcal{D}_M \mathcal{X}_N + (-1)^{mn} \mathcal{D}_N \mathcal{X}_M = 0. \tag{56}$$

Taking the Lie bracket of two Killing vectors $\mathcal{X}$ and $\mathcal{Y}$,

$$[\mathcal{X}, \mathcal{Y}]^M = \mathcal{X}^P \partial_P \mathcal{Y}^M - \mathcal{Y}^P \partial_P \mathcal{X}^M \tag{57}$$

gives another Killing vector. The corresponding algebra is the Lie algebra of the isometry group of the superspace. Moreover, the Killing vectors generate the infinitesimal field transformations under isometries again by the Lie bracket:

$$\mathcal{A}^M \to \mathcal{A}^M + \epsilon[\mathcal{X}, \mathcal{A}]^M. \tag{58}$$

In the following we consider the metric

$$g_{MN} = \begin{cases} \delta_{\mu\nu} & \text{if } M = \mu, N = \nu, \\ -\frac{\xi_0}{2}(1 + m_0^2\bar{\theta}\theta) & \text{if } M = \theta, N = \bar{\theta}, \\ \frac{\xi_0}{2}(1 + m_0^2\bar{\theta}\theta) & \text{if } M = \bar{\theta}, N = \theta, \\ 0 & \text{otherwise.} \end{cases} \tag{59}$$

Observe first that it identifies with (46) in the limit $m_0 \to 0$. Moreover, it is compatible with Poincaré and symplectic symmetry groups but does not respect the translation invariance in Grassmannian coordinates. A similar superspace was considered in [45] in a pretty different context context. From Eq. (55) one can deduce that the nonzero Christoffel symbols are

$$\Gamma^\theta_{\theta\bar{\theta}} = -\Gamma^\theta_{\bar{\theta}\theta} = -m_0^2\theta, \qquad \Gamma^{\bar{\theta}}_{\theta\bar{\theta}} = -\Gamma^{\bar{\theta}}_{\bar{\theta}\theta} = -m_0^2\bar{\theta}. \tag{60}$$

Using the expression for the scalar curvature of Ref. [43], one finds that the superspace has a finite and homogeneous scalar curvature $R = -12m_0^2/\xi$.

In order to verify that it does correspond to the CF model we calculated the most general Killing vector, obtaining

$$\begin{aligned} \mathcal{X}^\mu &= a^\mu + R^{\mu\nu}x^\nu, \\ \mathcal{X}^\theta &= \alpha(1 + m_0^2\bar{\theta}\theta) + \bar{\theta}\beta - \theta\delta, \\ \mathcal{X}^{\bar{\theta}} &= \bar{\alpha}(1 + m_0^2\bar{\theta}\theta) + \bar{\beta}\theta + \bar{\theta}\delta. \end{aligned} \tag{61}$$

The part proportional to a^μ corresponds to translations and the one proportional to $R^{\mu\nu} = -R^{\nu\mu}$ to rotations in bosonic coordinates. The parts proportional to $\bar{\beta}$ and β correspond to the symmetries t and $\bar{t}$, respectively, while the part proportional to δ corresponds to the ghost number. Finally, the parts proportional to $\bar{\alpha}$ and α correspond to BRST and anti-BRST symmetries, respectively (observe that they become translations when $m_0 \to 0$). By a straightforward calculation one can verify that the Lie bracket of the Killing vectors generates the Lie algebra of symmetries of the CF model as described in Sec. IV. It is also an easy task to verify that the Killing vectors generate the right field transformations for the fields A, c, $\bar{c}$ and h as defined in Sec. IV. Finally, one can verify that the only renormalizable Lagrangian compatible with the symmetries of the curved superspace is that of the CF model.

VII. CONCLUSION

In the present paper, we have shown that the CFDJ gauge fixing of Yang-Mills theory verifies four nontrivial local and linear Ward identities. This result has many consequences. First, it allows the deduction of two nonrenormalization theorems that reduces the number of independent renormalization factors from five to three. Consequently, in perturbation theory, one has to calculate as many renormalization factors as in linear gauges. Moreover, as discussed in Sec. V, all these renormalization factors can be extracted from the 2-point functions alone. We expect that this simplifies considerably the perturbative calculations in Yang-Mills theory.

Another important result of the present paper is that the obtained Ward identities can be interpreted in the superfield formalism for Yang-Mills theory as consequences of supergauge transformations. The generalization to the theory with a CF mass term is simple and it is shown to be equivalent in the superfield formalism to a curvature of the superspace in the Grassmannian coordinates. Up to now, however, the superfields are constrained by the so-called transversality condition. As a consequence, one cannot use them as they stand in a functional integral. Let us stress that the existence of this supergauge symmetry reinforces our conviction that the superfield formalism is of prime importance in this field. This pushes one to look for the description of the model in terms of unconstrained superfields. More precisely, one would like to build a theory in which the transversality constraints appear dynamically (very much like the ϕ^4 potential of the Landau-Ginzburg action that imposes, in the low-temperature limit, a hard-spin constraint) and are not imposed at hand as external constraints. Such a realization would be a good starting point to build a fully consistent quantization of the Yang-Mills theory in the superspace. This work is in progress.

ACKNOWLEDGMENTS

We thank B. Delamotte, M. Reisenberger, and G. Tarjus for useful discussions. M. T. thanks the IFFI for hospitality, where most of this work was done. We acknowledge support of PEDECIBA and the ECOS program. N. W. acknowledges support of the PDT Uruguayan program. LPTMC is UMR 7600 of CNRS.

APPENDIX: SOLVING SLAVNOV-TAYLOR IDENTITY

In this Appendix we give some details of the derivation of Eqs. (22) and (24). We first substitute the expression (18) into the Slavnov-Taylor identity (13) and analyze the terms quadratic in the sources K, L, $\bar{K}$, $\bar{L}$, and M. One easily finds that $\tilde{s}A$ and $\tilde{s}c$ do not depend arbitrarily on c, A, β, and α, but only through $\tilde{c}^a$ and $\tilde{A}^a_\mu$ [see Eq. (23)].

If one now studies the terms linear in K, L, $\bar{K}$, $\bar{L}$, and M, one finds four independent constraints. The two relations

$$\tilde{s}^{\,2}A^a_\mu = 0, \qquad \tilde{s}^2 c^a = 0, \tag{A1}$$

give the nilpotency in a particular sector. One also finds

$$\tilde{s}\,\bar{\tilde{s}}\,A^a_\mu(x) = -Z_K\frac{\delta\hat{\Gamma}}{\delta A^a_\mu(x)} - \frac{\delta\hat{\Gamma}}{\delta\alpha^a_\mu(x)} - \frac{Z_L}{2}\Big(\tilde{D}_\mu\frac{\delta\hat{\Gamma}}{\delta h}\Big)^a. \tag{A2}$$

Adding to this equations its conjugate, one deduces

$$\{\tilde{s},\bar{\tilde{s}}\}A^a_\mu = 0, \tag{A3}$$

which again expresses the nilpotency in another sector. The fourth relation reads

$$\tilde{s}\,\bar{\tilde{s}}\,\bar{c}^a = -Z_L\frac{\delta\hat{\Gamma}}{\delta c^a} + \frac{\delta\hat{\Gamma}}{\delta\beta^a} - i\frac{\tilde{g}Z_L}{2}\frac{\delta\hat{\Gamma}}{\delta h^b}f^{abc}\bar{\tilde{c}}^c. \tag{A4}$$

Now, the most general operators of dimension two, respecting Lorentz invariance, global color invariance, the symmetry (6), ghost-number conservation, the definition (19), and nilpotency (A1), are written in (25).

Equations (A2) and (A4) take a simpler form if one introduces the variable $\tilde{h}^a$ defined in Eq. (23). Taking as independent variables $\tilde{A}_\mu$, $\tilde{c}$, $\bar{\tilde{c}}$, $\tilde{h}$, α_μ, β, and $\bar{\beta}$ one deduces

$$\tilde{s}\,\bar{\tilde{s}}\,\bar{c}^a = \frac{\delta\hat{\Gamma}}{\delta\beta^a}, \qquad \tilde{s}\,\bar{\tilde{s}}\,A^a_\mu = -\frac{\delta\hat{\Gamma}}{\delta\alpha^a_\mu}. \tag{A5}$$

Note that at this level we have explicit expressions for the $\tilde{s}$ and $\bar{\tilde{s}}$ variations of fields A, c, and $\bar{c}$ but not h. However, the left-hand sides of Eqs. (A5) can be computed without knowledge of the variations of h. Therefore, we have an explicit expression for the derivatives of $\hat{\Gamma}$ with respect to α and β (and by conjugation of $\bar{\beta}$). One can then integrate these trivial differential equations and obtain the dependence of $\hat{\Gamma}$ on these variables. As a result, we only need to find the part of $\hat{\Gamma}$ that does not depend on the sources. The dependence on h (and then on $\tilde{h}$) is trivially deduced from (14). The remaining part is obtained by imposing the invariance of $\hat{\Gamma}$ under $\tilde{s}$. One finally obtains the result (24).

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