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Worldline casting of the stochastic vacuum model and nonperturbative properties of QCD: General formalism and applications

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The stochastic vacuum model for QCD, proposed by Dosch and Simonov, is fused with a worldline casting of the underlying theory, i.e. QCD. Important nonperturbative features of the model are studied. In particular, contributions associated with the spin-field interaction are calculated, and the validity of both the loop equations and of the Bianchi identity is explicitly demonstrated. As an application, a simulated meson-meson scattering problem is studied in the Regge kinematical regime. The process is modeled in terms of the helicoidal Wilson contour along the lines introduced by Janik and Peschanski in a related study based on an AdS/CFT-type approach. Working strictly in the framework of the stochastic vacuum model and in a semiclassical approximation scheme, the Regge behavior for the scattering amplitude is demonstrated. Going beyond this approximation, the contribution resulting from boundary fluctuations of the Wilson loop contour is also estimated.

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I. INTRODUCTION

The confrontation of nonperturbative issues associated with dynamical processes constitutes a problem of great importance which definitely merits attention if QCD is to attain the status of a complete and fully self-consistent theory. Clearly, the most concrete advancement in formulating a nonperturbative casting of QCD is traced to Wilson's proposal [1], which paved the way for the lattice formulation of gauge field theories in general. Remarkable results have been produced, especially in relation to the study of static properties of hadrons [2], finite temperature properties of the theory, etc.

On the analytical front, important theoretical progress, relevant to nonperturbative, dynamical explorations of QCD, has been achieved within the context of the loop equations [3–6], while, in recent years, (super)string theory has, through the AdS/CFT conjecture [7–9], opened new pathways for approaching the nonperturbative domain of QCD, *albeit* in the sense of some supersymmetric version of the theory and within the context of unified schemes.

Generally speaking, the nontrivial aspects of QCD as a relativistic gauge field theoretical system stem from the fact that the non-Abelian gauge symmetry entering its description incorporates an inherent nonlinearity even before interaction terms with matter field agents are introduced. Theoretical schemes aiming at a heads-on analytical confrontation of nonlinear quantum field systems do, of course, exist, possibly the most concrete one being expressed in terms of the infinite battery of Schwinger-Dyson equations. Even in this case, however, the relevant computational procedure for the solution of these (integral) equations is not only methodologically complex but, more importantly, unless totally summed, there is no *a priori* guarantee that they are in the position to capture the full nonperturbative content of any given field theoretical system and/or describe its expected various phases.

A notable field theoretical approach aiming at the study of nonperturbative issues in QCD, such as confinement, chiral symmetry breaking etc., has been proposed by Dosch and Simonov [10–12] and is called the stochastic vacuum model (SVM). By design, the construction of the model takes into consideration the nontrivial structure of the QCD vacuum state [13], while, at the same time, it secures a role for the Stokes' theorem (non-Abelian casting thereof) through which electric-magnetic duality issues can be fully taken into account. The basic building blocks of the SVM scheme are the, so-called, *field-strength correlators*, the definition of which will be given later, while for its computational strategy it employs the so-called field (strength) correlators method, FCM for short. For the reader not familiar with the SVM, we hope that the information provided in this paper will sufficiently illustrate the reasoning behind its definition, as well as its properties and physical content. For a deeper insight into the model, one is referred to the original papers [10–12] and/or the excellent review articles [14,15], which also present a variety of its applications.

The characteristic aspect of the present work is that it adopts, as a basic methodological tool, the worldline casting of gauge field theoretical systems [16] appropriately adjusted to the SVM. Our goal is to confront genuinely nonperturbative issues associated with dynamical processes of physical interest. At the same time—and at a purely theoretical level—we hope that the present effort will offer new insight into further promoting the effectiveness of the SVM as a credible and viable theoretical tool for exploring the nonperturbative content of QCD.

The aim of the first part of this paper (Secs. II and III) is to accomplish the task of carrying out cumbersome computations which reveal fundamental properties of the SVM that are of immediate relevance to our specific purposes, and to establish the consistency of the model with the loop equations [3–6], as well as the Bianchi identity (BI) for QCD. Clearly, such an occurrence strengthens the credibility of the SVM as a theoretical construction which is consistent with QCD as a whole, i.e. in the sense that it goes beyond perturbation theory. Our main theoretical application will be realized in the second part (Secs. IV and V) where we undertake the description of a, theoretically simulated, meson-meson scattering process at the high energy, small momentum transfer, kinematical (Regge) regime. The relevant description of such a dynamical process necessarily ‘‘protrudes’’ into the nonperturbative domain of QCD as represented, in our case, by the SVM.

Our presentation is organized as follows. In Sec. II, we discuss general aspects which prepare the ‘‘fusing’’ of the worldline description of a non-Abelian gauge field theoretical system, such as QCD, with the SVM. We start by focusing our attention on a situation where a matter field entity of spin j interacts with a set of non-Abelian gauge field modes as it propagates along a closed contour in (Euclidean) space-time, hence subjected to a spin-field interaction. The basic dynamical content of the process will be displayed by two alternative formulations. The first focuses on the (closed) Wilson contours traced by the particle entity. The second is based on a shifted field-strength tensor that is integrated over an arbitrary surface bounded by the closed contour. Obviously the (non-Abelian) Stokes' theorem plays a central role in relating the two descriptions, an occurrence of central importance to our purposes, given that the Stokes' theorem enters the SVM scheme in a major way.

We shall subsequently introduce the so-called cluster expansion [14,15,17], which employs the field-strength correlators, the basic dynamical quantities of the overall description. In fact, the cluster expansion is the essence of the stochastic nature of the model and provides the key element for quantifying the stochastic vacuum hypothesis. Once the aforementioned task is accomplished, we shall be in a position to derive, as a first general result, the equation that determines the surface over which the two-point correlator must be integrated.

Section II deals also with fairly demanding calculations within the context of the worldline formalism whose first result is the derivation of an explicit expression for the spin factor. The latter represents the genuinely nonperturbative, spin-field interaction dynamics and necessarily enters [18] the analysis of the meson-meson scattering process. Section III is devoted to the verification of the loop equations and of the Bianchi identity in the framework of the adopted approach. We consider these results to be of importance since they solidify the credibility of the SVM as a construction which reproduces sound, theoretical properties of QCD, and demonstrate the compatibility with its nonperturbative content.

In Sec. IV we present a semiclassical calculation of a simulated meson-meson scattering amplitude. The Wilson loop that carries the dynamics of the process is a helicoidal embedded in a four-dimensional background. The calculation is performed in the framework of the SVM and leads us to a Regge-type behavior for the amplitude valid in the physical region of the scattering. Corrections related to the fluctuations of the aforementioned Wilson contour will be discussed in Sec. V. In the same section the contribution of the spin factor is examined. Some concluding comments will be made in the closing section.

II. WORLDLINE FORMALISM AND FIELD-STRENGTH CORRELATORS

In this section, certain basic features of the field correlator method [15] will be reviewed in the context of the worldline casting of a quantum, gauge field theoretical system interacting with a matter particle mode of a given spin j. More explicitly, the particle entity is taken to propagate along a given, closed, contour (worldline) while interacting with a dynamical set of non-Abelian gauge fields $\mathcal{A}$. According to our introductory discussion, the main objective is to introduce the necessary tools to facilitate nonperturbative, theoretical explorations of QCD in the framework of the SVM.

One of the most important advantages of the worldline formalism is that it allows one to reduce the physical amplitudes to weighted integrals of averaged Wilson loops [16]. In this connection, one can apply powerful techniques, such as the cluster expansion, both at the perturbative (in the sense of series resummation) and at the nonperturbative level. For the latter, the worldline formalism proves to be quite crucial because one can develop methods based on a background gauge fixing strategy [18–22], which enables one to treat the nonperturbative fields as background.

Let us, then, consider a particle entity of spin j propagating from some point and back to the same point while interacting dynamically with a non-Abelian gauge field system $\mathcal{A}$. The basic structure of the quantum mechanical amplitude associated with such a process is written in the worldline formalism (all indices suppressed; Euclidean formalism adopted) as follows [16]:

$$K(L) = \mathrm{Tr}\int_{x(0)=x(1)} \mathcal{D}x(\tau)\exp\left(-\frac{1}{4L}\int_0^1 d\tau \dot{x}^2\right) \times\left\langle P\exp\left(i\int_0^1 d\tau \dot{x}\cdot\mathcal{A} + L\int_0^1 d\tau J\cdot F\right)\right\rangle_{\mathcal{A}}, \tag{1}$$

where it should be noted that the parameter L has dimensions m^{-2} and must be integrated over through a weight factor $\exp(-Lm^2)$ in order to obtain a result with physical content. The matrices $J_{\mu\nu}$ stand for the Lorentz generators, pertaining to the spin of the propagating entity.

Accordingly, the last term represents the spin-field interaction.

The above expression for $K(L)$ can be recast into the form [16]

$$K(L) = \mathrm{Tr} \int_{x(0)=x(1)} \mathcal{D}x(\tau) \exp\left(-\frac{1}{4L} \times \int_0^1 d\tau \dot{x}^2\right) P \exp\left(\frac{i}{2} L \int_0^1 d\tau J \cdot \frac{\delta}{\delta\sigma}\right) \times \left\langle P \exp\left(i \int_0^1 d\tau \dot{x} \cdot \mathcal{A}\right)\right\rangle_{\mathcal{A}}, \quad (2)$$

where

$$\frac{\delta}{\delta\sigma_{\mu\nu}(x(\tau))} = \lim_{\eta\to 0} \int_{-\eta}^{\eta} dhh \frac{\delta^2}{\delta x_\mu(\tau + \frac{h}{2})\delta x_\nu(\tau - \frac{h}{2})} \quad (3)$$

defines a regularized expression for the area derivative [3–6].

Strictly speaking, expression (2) has a well-defined meaning only for smooth [23] loops. On the other hand, when such expressions are used for the purpose of describing physically interesting processes, the contour is forced to pass through points x_i where momentum is imparted by an external agent (field). Such a situation is mathematically realized by inserting a corresponding chain of delta functions $\delta[x(\tau_i) - x]$ in the integral, which produces a loop with cusps, in which case the action of the area derivative operator entering (3) must be understood piecewise, i.e.,

$$P \exp\left(\frac{iL}{2} \int_0^1 d\tau J \cdot \frac{\delta}{\delta\sigma}\right) = \cdots P \exp\left(\frac{iL}{2} \int_{\tau_1}^{\tau_2} d\tau J \cdot \frac{\delta}{\delta\sigma}\right) \times P \exp\left(\frac{iL}{2} \int_0^{\tau_1} d\tau J \cdot \frac{\delta}{\delta\sigma}\right). \quad (4)$$

The first step towards the application of the FCM is taken by employing the non-Abelian Stokes' theorem [24] with the help of which one can write (the symbol P_s stands for surface ordering [14])

$$W[C] \equiv \frac{1}{N_C} \mathrm{Tr}\left\langle P \exp\left(i \oint_C dx \cdot \mathcal{A}\right)\right\rangle_{\mathcal{A}} = \frac{1}{N_C} \mathrm{Tr}\left\langle P_s \exp\left[i \int_{S(C)} dS_{\mu\nu}(z) G_{\mu\nu}(z, x_0)\right]\right\rangle_{\mathcal{A}}. \quad (5)$$

The above expression is valid for any loop C with disc topology, irrespectively of the surface $S(C)$. We also mention that for the area element we adopt the standard expression

$$dS_{\mu\nu} = \frac{1}{2} d^2\xi \sqrt{g} t_{\mu\nu}(\xi), \qquad t_{\mu\nu}(\xi) = \frac{1}{\sqrt{g}} \epsilon^{ab} \partial_a z_\mu \partial_b z_\nu, \qquad a, b = 1, 2; \qquad (\xi^1, \xi^2) = (\tau, s). \quad (6)$$

Finally, in relation (5) we have set [14,15]

$$G_{\mu\nu}(x_0, z) = \phi(x_0, z) F_{\mu\nu}(z) \phi(z, x_0) \quad (7)$$

with

$$\phi(z, x_0) = P \exp\left(i \int_{x_0}^{z} dw \cdot \mathcal{A}\right) \quad (8)$$

a phase factor [14,15] which is a parallel transporter known also, in the SVM nomenclature [17], as a *connector*. The reference point x_0 is chosen arbitrarily on the surface S; the curve that joins the points x_0 and z is also arbitrary.

It can be proved [24] that (5) depends neither on the surface nor on the contour used to define the connector (8), as long as the non-Abelian Bianchi identities are satisfied. In the loop language such a requirement can be cast into the following statement:

The relation

$$\frac{\delta}{\delta z_\lambda(\xi)} \mathrm{Tr}\left\langle P \exp\left[i \int_{S(C)} dS_{\mu\nu}(z) G_{\mu\nu}(z, x_0)\right]\right\rangle_{\mathcal{A}} = 0 \quad (9)$$

is valid independently of the surface choice provided that

$$\epsilon^{\kappa\lambda\mu\nu} \partial_\lambda^{x(\tau)} \frac{\delta}{\delta\sigma_{\mu\nu}(x(\tau))} W[C] = 0, \quad (10)$$

which corresponds to the Bianchi identity for the gauge system [6].

The simplest way to prove the non-Abelian Stokes' theorem is to adopt the contour gauge [15]

$$\mathcal{A}_\mu(x) = \int_0^1 ds \partial_s z_\kappa(s, x) \frac{\partial}{\partial x_\mu} z_\lambda(s, x) F_{\kappa\lambda}(z(s, x)), \quad (11)$$

with $\{z_\mu(s, x), s \in [0, 1]\}$ an arbitrary, smooth curve from the reference point x_0 to some point x:

$$z_\mu(0, x) = x_{0\mu}, \qquad z_\mu(1, x) = x_\mu. \quad (12)$$

Indeed, using Eq. (11) one can immediately see that

$$\oint dx_\mu \mathcal{A}_\mu(x) = \int_0^1 d\tau \int_0^1 ds \partial_s z_\mu \partial_\tau z_\nu F_{\mu\nu}(z) = \frac{1}{2} \int_0^1 d\tau \int_0^1 ds \epsilon^{ab} \partial_a z_\mu \partial_b z_\nu F_{\mu\nu}(z) = \frac{1}{2} \int d^2\xi \sqrt{g} t_{\mu\nu}(z) F_{\mu\nu}(z). \quad (13)$$

For the gauge choice (11) the vector potential satisfies the condition $t_\mu(x)\mathcal{A}_\mu(x) = 0$, with $t_\mu(x) = \partial_s z(s, x)|_{s=1}$, which implies that the connector (8) can be considered as the unit matrix in the contour gauge. In any case, the presence of the connectors in (7) guarantees gauge invariance.

The next step towards the application of the FCM is to introduce the so-called cluster expansion [10–12,14,15,17] for the Wilson loop, formally written as

$$W[C] = \frac{1}{N_c}\,\mathrm{Tr}\exp\Bigg(\sum_{n=1}^{\infty}\frac{i^n}{n!}\int_{S(C)} dS_{\mu_n\nu_n}\cdots dS_{\mu_1\nu_1} \times \langle\langle G_{\mu_n\nu_n}(z_n, x_0)\cdots G_{\mu_1\nu_1}(z_1, x_0)\rangle\rangle\Bigg), \tag{14}$$

where the symbol $\langle\langle\cdots\rangle\rangle$ translates as follows:

$$\begin{aligned}\langle\langle O(1)\rangle\rangle &= \langle O(1)\rangle,\\ \langle\langle O(1)O(2)\rangle\rangle &= \langle P_s(O(1)O(2))\rangle - \tfrac{1}{2}(\langle O(1)\rangle\langle O(2)\rangle - \tfrac{1}{2}\langle(O(2)\rangle\langle O(1)\rangle,\\ \langle\langle O(1)O(2)O(3)\rangle\rangle &= \langle P_s(O(1)O(2)O(3))\rangle\\ &\quad - \tfrac{1}{2}(\langle P_s(O(1)O(2))\rangle\langle O(3)\rangle\\ &\quad + \text{cycl. perm.}) + \tfrac{1}{3}(\langle O(1)\rangle\langle O(2)\rangle\\ &\quad \times \langle O(3)\rangle + \text{cycl. perm.}) + \cdots,\end{aligned} \tag{15}$$

and is reminiscent of the cluster expansion in statistical mechanics. It is pointed out that, due to the color neutrality of the vacuum, expectation values of all correlators in (14) are proportional to the unit matrix in color space. This makes color ordering unnecessary.

Equation. (14) quantifies the formulation of the SVM. It turns out [14,15] that the most important contribution to the cluster expansion comes from the two-point correlator:

$$\begin{aligned}\Delta^{(2)}_{\mu\nu,\lambda\rho}(z - z') &= \frac{1}{N_c}\,\mathrm{Tr}\langle G_{\mu\nu}(z, x_0)G_{\lambda\rho}(z', x_0)\rangle_A\\ &= \frac{1}{N_c}\,\mathrm{Tr}\langle F_{\mu\nu}(z)\phi(z, z')F_{\lambda\rho}(z')\phi(z', z)\rangle_A.\end{aligned} \tag{16}$$

The above equation defines the *field-strength correlator*, a quantity which constitutes the basic building block of the model.

Some natural assumptions are incorporated in the above definition. The first has to do with the Lorentz invariance of the vacuum, which is explicitly indicated in the left-hand side of (16) by the fact that the correlator depends on the *distance* between the points z and z'. The second is that, on the other hand, the correlator does not depend on (the gauge parameter) x_0. This is a credible assumption, taking into account the fact that we have been working, from the very beginning, with a gauge invariant amplitude. Accordingly, the basic assumption of the SVM leads to the statement that

$$W[C] \propto \exp\Bigg[-\frac{1}{2}\int_{S(C)} dS_{\mu\nu}(z) \times \int_{S(C)} dS_{\lambda\rho}(z')\Delta^{(2)}_{\mu\nu,\lambda\rho}(z - z')\Bigg], \tag{17}$$

where the surface element enters through the use of the (non-Abelian) Stokes' theorem.

Two important points should now be made. First, the last expression is supposed to be valid in a certain limit. Explicitly, it is assumed [10–12,14,15] that the vacuum fluctuations establish a *correlation length* T_g beyond which correlations decay very fast. If Δ is an order of magnitude estimation for the two-point correlator, relation (17) is considered as an asymptotic approximation which is valid in the limit $T_g^2\sqrt{\Delta} \to 0$. The second is that, while expression (14) does not depend on the particular surface one uses for the application of the non-Abelian Stokes' theorem, approximation (17) does. Thus, the stochasticity assumption transforms relation (9) to an equation which determines the dominant surface in the cluster expansion. To quantify this statement we write

$$A[C] = \frac{1}{2}\int_{S(C)} dS_{\mu\nu}(z)\int_{S(C)} dS_{\lambda\rho}(z')\Delta^{(2)}_{\mu\nu,\lambda\rho}(z - z'), \tag{18}$$

from which it is easily determined that

$$\frac{\delta A}{\delta z_\sigma(\tilde{\xi})} = -\frac{1}{2}\sqrt{g(\tilde{z})}t_{\mu\nu}(\tilde{z})\int_{S(C)} dS_{\lambda\rho}(z')[\tilde{\partial}_\mu\Delta^{(2)}_{\sigma\nu,\lambda\rho}(\tilde{z} - z') + \tilde{\partial}_\nu\Delta^{(2)}_{\mu\sigma\lambda\rho}(\tilde{z} - z') + \tilde{\partial}_\sigma\Delta^{(2)}_{\nu\mu,\lambda\rho}(\tilde{z} - z')], \tag{19}$$

where we have written $\tilde{\partial}_\mu \equiv \frac{\partial}{\partial\tilde{z}_\mu}$. The calculation of the derivative of the correlator (16) needs to take into account [15] the presence of the connectors $\phi(z, x_0)$. In Appendix A we show that

$$\frac{\partial}{\partial z_\mu}\phi(z, x_0) = i\mathcal{A}_\mu(z)\phi(z, x_0) - iI_\mu(z, x_0), \tag{20}$$

with

$$I_\mu(z, x_0) = \int_0^1 dt(\partial_t\omega_\kappa)\frac{\partial\omega_\lambda}{\partial z_\mu}\phi(z, \omega)F_{\kappa\lambda}(\omega)\phi(\omega, x_0),$$

$$\omega = \omega(t, z), \qquad \omega(1, z) = z, \qquad \omega(0, z) = x_0. \tag{21}$$

Given the above equations, we conclude that

$$\partial_\mu G_{\alpha\beta}(z, x_0) = \phi(x_0, z)D_\mu F_{\alpha\beta}(z)\phi(z, x_0) + i[I_\mu(z, x_0), G_{\alpha\beta}(z, x_0)] \tag{22}$$

and consequently write

$$\partial_\mu\tilde{G}_{\mu\nu}(z, x_0) = \phi(x_0, z)D_\mu\tilde{F}_{\mu\nu}(z)\phi(z, x_0) + i[I_\mu(z, x_0), \tilde{G}_{\mu\nu}(z, x_0)], \tag{23}$$

where we have set $\tilde{G}_{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}G_{\alpha\beta}$.

The above analysis establishes the following relation [15] among the derivatives of the correlator:

$$\begin{aligned}\frac{1}{2}\epsilon^{\sigma\kappa\mu\nu}\partial_\sigma\Delta^{(2)}_{\mu\nu,\lambda\rho} &\equiv \partial_\sigma\tilde{\Delta}^{(2)}_{\sigma\kappa,\lambda\rho}\\ &= \frac{1}{N_c}\,\mathrm{Tr}\langle D_\sigma\tilde{F}_{\sigma\kappa}(z)F_{\lambda\rho}(z')\phi(z', z)\rangle_{\mathcal{A}}\\ &\quad - \Delta_{\kappa\lambda\rho}(z, z'),\end{aligned} \tag{24}$$

where

$$\Delta_{\kappa\lambda\rho}(z,z') = \frac{1}{N_c}\,\mathrm{Tr}\langle \tilde{F}_{\sigma\kappa}(z)I_\sigma(z,z')F_{\lambda\rho}(z')\phi(z,z') - \tilde{F}_{\sigma\kappa}(z)\phi(z,z')F_{\lambda\rho}(z')I_\sigma(z',z)\rangle_{\mathcal{A}}. \quad (25)$$

Thus, if one assumes the validity of the Bianchi identities $D_\mu \tilde{F}_{\mu\nu} = 0$, one concludes that

$$\partial_\mu \Delta^{(2)}_{\sigma\nu,\lambda\rho} + \partial_\nu \Delta^{(2)}_{\mu\sigma,\lambda\rho} + \partial_\sigma \Delta^{(2)}_{\nu\mu,\lambda\rho} = \epsilon^{\mu\kappa\sigma\nu}\Delta_{\kappa\lambda\rho}. \quad (26)$$

Accordingly, Eq. (9) can be represented as follows:

$$\frac{\delta A}{\delta z_\sigma(\tilde{\xi})} = \frac{1}{2}\sqrt{g(\tilde{z})}t_{\mu\nu}\int_{S(C)} dS_{\lambda\rho}(z)\epsilon^{\sigma\kappa\mu\nu}\Delta_{\kappa\lambda\rho}(\tilde{z}-z) = 0. \quad (27)$$

It is worth noting that $\Delta_{\kappa\lambda\rho}$ is a three-point correlation function, and it would be *identically zero* if we were considering an Abelian gauge theory so that the above relation becomes, really, an identity, telling us nothing about the particular surface involved in Stokes' theorem, an expected result given that relation (17) is exact in the framework of QED. An extensive discussion of the physical content of the correlator $\Delta_{\kappa\lambda\rho}$ can be found in [14,15]. According to the analysis presented in the aforementioned references, confinement in QCD occurs due to the nonzero value of the (non-Abelian) correlator $\Delta_{\kappa\lambda\rho}$. Equation (27) indicates that this correlator also defines the relevant surface on which the two-point correlator "lives."

Now, it has been demonstrated [10–12,14,15] that, in the asymptotic limit $|\tilde{z}-z| \gg T_g$, Eq. (27) determines the surface S bounded by the contour C as the minimal one. To demonstrate this we employ the following general Lorentz structure representation [14,15] for the two-point correlation function:

$$\Delta^{(2)}_{\mu\nu,\lambda\rho}(\bar{z}) = (\delta_{\mu\lambda}\delta_{\nu\rho} - \delta_{\mu\rho}\delta_{\nu\lambda})D(\bar{z}) + \left\{\frac{1}{2}\frac{\partial}{\partial \bar{z}_\mu}[(\bar{z}_\lambda\delta_{\nu\rho} - \bar{z}_\rho\delta_{\nu\lambda})D_1(\bar{z})] - (\mu \leftrightarrow \nu)\right\}, \quad (28)$$

where we have set $\bar{z} = z - z'$.

It is easy to see that

$$\tfrac{1}{2}\epsilon^{\mu\kappa\sigma\nu}\partial_\sigma \Delta^{(2)}_{\mu\nu,\lambda\rho} = \epsilon^{\kappa\lambda\rho\sigma}\partial_\sigma D = \Delta_{\kappa\lambda\rho}. \quad (29)$$

With the help of the above relation, Eq. (27) can be cast into the form

$$\frac{\delta}{\delta z_\sigma(\tilde{\xi})}\int_{S(C)} dS_{\mu\nu}(z)\int_{S(C)} dS_{\mu\nu}(z')D(z-z') = 0. \quad (30)$$

The functions D and D_1 have been measured in lattice calculations [25] and have been found to decrease very fast as $|z-z'|^2 \to \infty$. In the considered region it was found that both of them are of the form $f(\frac{|z-z'|^2}{T_g^2})$ and that they go exponentially fast to zero for $|z-z'| > T_g$. In this region, we write

$$z(\xi') \simeq z(\xi) + (\xi' - \xi)^a \partial_a z(\xi), \quad |z-z'|^2 \simeq (\xi'-\xi)^a(\xi'-\xi)^b g_{ab}. \quad (31)$$

In the considered limit and taking into account that $t_{\mu\nu}t_{\mu\nu} = 2$, we find

$$\frac{\delta}{\delta z_\sigma(\tilde{\xi})}[\sigma S + \mathcal{O}(T_g^4\Delta)] = 0, \quad (32)$$

where the string tension

$$\sigma \equiv \frac{T_g^2}{2}\int d^2w D(w^2) \quad (33)$$

has been introduced and where we have also written

$$S \equiv \int d^2w\sqrt{g(w)} \quad (34)$$

for the area of the surface bounded by the Wilson curve. In the last equations we used the dimensionless parameter $w_\mu = \frac{1}{T_g}(z-z')_\mu$, and we have written $g_{ab}(w) = \partial_a w_\mu \partial_b w_\mu$ as the induced metric. Accordingly, it follows that, in the limit $T_g \to 0$, the surface on which the two-point correlation dominates is the minimal one.

Now we turn our attention to the *spin factor*, whose role is to incorporate the spin-field interaction in the framework of the worldline formalism. We mention that in the present work we shall be dealing with massive fermions. It should be noted, at the same time, that the spin factor can also be extended [21,22] for the case of massless bosons of spin 1. Accordingly, the calculations to be presented in this section can easily be extended to bosonic fields.

We start by inserting into the SVM formula for the Wilson loop, cf. Eq. (17), the worldline integral expression (1), the basic goal being that of calculating the spin-field interaction with the help of the area derivative operator defined in (3). Once this is accomplished the stage will be in place for performing specific calculations of physical interest.

We start by introducing the spin factor by formally casting Eq. (1) into the form

$$K(L) = \mathrm{Tr}\int_{x(0)=x(1)} \mathcal{D}x(\tau)\exp\left(-\frac{1}{4L}\int_0^1 d\tau \dot{x}^2\right) \times \Phi^{(j)}[C]e^{-A[C]}, \quad (35)$$

where the quantity

$$\Phi^{(j)}[C] \equiv e^{A[C]}P\exp\left(\frac{i}{2}\int_0^1 d\tau J\cdot\frac{\delta}{\delta\sigma}\right)e^{-A[C]} \quad (36)$$

defines the spin factor characterizing a particle entity propagating on the Wilson curve $A[C]$. Its calculation is not trivial and requires a number of steps, the first of which is to determine the action of the area derivative operator on A.

The first objective of the computation of the spin factor is to study the change in $A[C]$ induced by an infinitesimal variation of the boundary. The relevant problem is formulated as follows:

$$\frac{\delta A}{\delta x_\mu(\tau_1)} = \int_0^1 ds[a(\tau_1,s)\dot{z}_\alpha(\tau_1,s)]' \int_{S(C)} dS_{\gamma\delta}(z')\Delta^{(2)}_{\alpha\mu,\gamma\delta} + \int_0^1 ds a(\tau_1,s)\dot{z}_\alpha(\tau_1,s) z'_\beta(\tau_1,s)\int_{S(C)} dS_{\gamma\delta}(z') \times (\partial_\mu \Delta^{(2)}_{\alpha\beta,\gamma\delta} + \partial_\alpha \Delta^{(2)}_{\beta\mu,\gamma\delta}), \quad (37)$$

where the dot denotes a (partial) derivation with respect to τ, while the prime shows a derivation with respect to s. Moreover, the correlators depend on the distance $|z(\tau_1,s) - z(\tau',s')|$, and we have written $\frac{\delta z_\alpha(\xi)}{\delta x_\mu(\tau_1)} = \delta_{\alpha\mu}\delta(\tau-\tau_1)\alpha(\tau_1,s)$.

Using Eq. (26), Eq. (37) is recast into the form

$$\begin{aligned}\frac{\delta A}{\delta x_\mu(\tau_1)} &= \int_0^1 ds[a(\tau_1,s)\dot{z}_\alpha(\tau_1,s)]' \int_{S(C)} dS_{\gamma\delta}(z')\Delta^{(2)}_{\mu\alpha,\gamma\delta} - \int_0^1 ds a(\tau_1,s)\dot{z}_\alpha(\tau_1,s)z'_\beta(\tau_1,s) \\ &\quad\times \int_{S(C)} dS_{\gamma\delta}(z')(\partial_\beta \Delta^{(2)}_{\mu\alpha,\gamma\delta} - \epsilon^{\mu\nu\alpha\beta}\Delta_{\gamma\nu\delta}) \\ &= \dot{x}_\alpha(\tau_1)\int_{S(C)} dS_{\gamma\delta}(z')\Delta^{(2)}_{\alpha\mu,\gamma\delta}[x(\tau_1) - z(\tau',s')] + \int_0^1 ds a(\tau_1,s)\dot{z}_\alpha(\tau_1,s)z'_\beta(\tau_1,s) \\ &\quad\times \int_{S(C)} dS_{\gamma\delta}(z')\epsilon^{\mu\nu\alpha\beta}\Delta_{\nu\gamma\delta}[z(\tau_1,s) - z(\tau',s')]. \end{aligned} \quad (38)$$

The last term in the above relation is zero on account of condition (27):

$$\dot{z}_\alpha(\tau_1,s)z'_\beta(\tau_1,s)\int_{S(C)} dS_{\gamma\delta}(z')\epsilon^{\mu\nu\alpha\beta}\Delta_{\nu\gamma\delta}[z(\tau_1,s)-z(\tau',s')] = \frac{1}{2}\sqrt{g(z)}t_{\alpha\beta}(z)\int_{S(C)} dS_{\gamma\delta}(z')\epsilon^{\mu\nu\alpha\beta}\Delta_{\nu\gamma\delta}(z-z'). \quad (39)$$

We have consequently determined that

$$\frac{\delta A}{\delta x_\mu(\tau)} = \dot{x}_\alpha(\tau)\int_{S(C)} dS_{\gamma\delta}(z')\Delta^{(2)}_{\alpha\mu,\gamma\delta}[x(\tau) - z(\tau',s')]. \quad (40)$$

In order to find the area derivative [cf. Eq. (3)], we need to calculate the second functional derivative of A, at the points $x(\tau_1) = x(\tau + \frac{h}{2})$ and $x(\tau_2) = (\tau - \frac{h}{2})$. From the definition of the area derivative we also surmise that only terms $\sim\delta'(h)$ are relevant. Accordingly, it is straightforward to surmise that

$$\frac{\delta A}{\delta\sigma_{\mu\nu}(x(\tau))} = \int_{S(C)} dS_{\gamma\delta}(z')\Delta^{(2)}_{\mu\nu,\gamma\delta}[x(\tau) - z(\tau',s')]. \quad (41)$$

It is also easy to determine that

$$\frac{\delta}{\delta\sigma_{\mu_2\nu_2}(x(\tau_2))}\frac{\delta}{\delta\sigma_{\mu_1\nu_1}(x(\tau_1))}A = \Delta^{(2)}_{\mu_2\nu_2,\mu_1\nu_1}[x(\tau_2) - x(\tau_1)], \quad (42)$$

while all higher derivatives give null contributions.

On the basis of the above analysis we determine

$$\begin{aligned}\Phi^{(j)}[C] &= 1 - \frac{iL}{2}\int_0^1 d\tau_1 \int_{S(C)} dS\cdot\Delta^{(2)}(z-x_1)\cdot J + \left(\frac{iL}{2}\right)^2 \int_0^1 d\tau_2 \int_0^{\tau_2} d\tau_1 \Big[-J\cdot\Delta^{(2)}(x_2-x_1)\cdot J \\ &\quad + \int_{S(C)} dS\cdot\Delta^{(2)}(z-x_2)\cdot J\int_{S(C)} dS\cdot\Delta^{(2)}(z-x_1)\cdot J\Big] + \left(\frac{iL}{2}\right)^3 \int_0^1 d\tau_3\int_0^{\tau_3} d\tau_2\int_0^{\tau_2} d\tau_1 \Big[J\cdot\Delta^{(2)}(x_2-x_1)\cdot J \\ &\quad \times \int_{S(C)} dS\cdot\Delta^{(2)}(z-x_3)\cdot J + J\cdot\Delta^{(2)}(x_3-x_2)\cdot J\int_{S(C)} dS\cdot\Delta^{(2)}(z-x_1)\cdot J - \int_{S(C)} dS\cdot\Delta^{(2)}(z-x_3)\cdot J \\ &\quad \times \int_{S(C)} dS\cdot\Delta^{(2)}(z-x_2)\cdot J\int_{S(C)} dS\cdot\Delta^{(2)}(z-x_1)\cdot J + \cdots\Big]. \end{aligned} \quad (43)$$

In the above expression we have omitted terms in which the field-strength correlator $\Delta^{(2)}$ depends on distances between two nonsuccessive points. For example, the last line of the above equation does not include the correlator $\Delta^{(2)}(x(\tau_3) - x(\tau_1))$. The reason is that such a correlator [14,15,17,25] is assumed to behave as $f(\frac{|x_3-x_1|^2}{T_g^2}) \simeq f(\dot{x}^2\frac{(\tau_3-\tau_1)^2}{T_g^2})$, which, in turn, means that its contribution is suppressed by powers of $T_g^2\sqrt{\Delta}$. Accordingly, we obtain the following expression for the spin factor:

$$\Phi^{(j)}[C] = P\exp\Big[-\frac{iL}{2}\int_0^1 d\tau\int_{S(C)} dS\cdot\Delta^{(2)}(z-x)\cdot J + \frac{L^2}{4}\int_0^1 d\tau_2\int_0^{\tau_2} d\tau_1 J\cdot\Delta^{(2)}(x_2-x_1)\cdot J\Big]. \quad (44)$$

We have arrived at a result of considerable interest for our purposes, which we would like to analyze further. Let us start from the second term in the exponent, which is of special interest [26] and represents the interaction of the quark color-magnetic moment with the non-Abelian background. In particular, let us refer to the representation (28) of the two-point correlator, which we rewrite in the form

$$\Delta^{(2)}_{\mu\nu,\lambda\rho}(\bar{x}) = (\delta_{\mu\lambda}\delta_{\nu\rho} - \delta_{\mu\rho}\delta_{\nu\lambda})(D + D_1) + (\bar{x}_\mu\bar{x}_\lambda\delta_{\nu\rho} - \bar{x}_\mu\bar{x}_\rho\delta_{\nu\lambda} - (\mu \leftrightarrow \nu))D_1', \quad (45)$$

where we have denoted $D_1' = \frac{\partial}{\partial \bar{x}^2} D_1(\bar{x})$.

Using expression (45) we find

$$J_{\mu\nu}\Delta^{(2)}_{\mu\nu,\lambda\rho}(\bar{x})J_{\lambda\rho} = 2J^2(D + D_1) + 4(J \cdot \bar{x})^2 D_1'. \quad (46)$$

For the case at hand, we have that $J_{\mu\nu} = \frac{i}{4}[\gamma_\mu, \gamma_\nu]$, so we can easily determine that

$$J_{\mu\nu}\Delta^{(2)}_{\mu\nu,\lambda\rho}(\bar{x})J_{\lambda\rho} = 6(D + D_1) + 3\bar{x}^2 D_1'. \quad (47)$$

Expression (47) is positive definite and is associated [27] with the ghost tachyonic pole which appears in the fermionic, or the gluonic, propagator—an issue we shall not discuss further in this paper.

Now, with the help of the result (41), the first term in the exponential (44) can be recast into the form

$$\int_0^1 d\tau \int_{S(C)} dS \cdot \Delta^{(2)}(z - x) \cdot J = \int_0^1 d\tau J_{\mu\nu} \frac{\delta A[C]}{\delta \sigma_{\mu\nu}(x(\tau))}. \quad (48)$$

It is convenient, for the applications we have in mind, to rewrite the above relation, by referring to the variation of $A[C]$ as it has been computed in Eq. (40):

$$\frac{\delta A[C]}{\delta x_\mu(\tau)} \equiv g_\mu[x(\tau)]. \quad (49)$$

The above function is reparametrization invariant; thus $\dot{x}_\mu g_\mu = 0$.

Taking into account the result displayed in Eq. (40), we can write

$$\dot{x}_\mu \frac{\delta A[C]}{\delta \sigma_{\mu\nu}(x(\tau))} = g_\nu[x(\tau)]. \quad (50)$$

An obvious solution to the above equation is

$$\frac{\delta A[C]}{\delta \sigma_{\mu\nu}(x)} = \frac{1}{\dot{x}^2}[\dot{x}_\mu g_\nu(x) - \dot{x}_\nu g_\mu(x)]. \quad (51)$$

It can also be shown [28] that the above equation is the only possible solution. The proof follows, basically, dimensional arguments: Making the changes $x \rightarrow \lambda x$ and $\tau \rightarrow \lambda\tau$, the area derivative of $A[C]$ scales like $\frac{1}{\lambda^2}$. The same scaling behavior goes for the function g, as can be easily concluded from Eq. (40). Thus, the term appearing in (51) has the right scaling properties. Any other term must be an antisymmetric combination $K_{\mu\nu}$ which scales as $\frac{1}{\lambda^2}$ and is perpendicular to the velocity, i.e., $\dot{x}_\mu K_{\mu\nu} = 0$. However, such a combination expressible in terms of the boundary $x(\tau)$ cannot be found. It thereby follows that the first term in the exponential which defines the spin factor reads

$$\int_0^1 d\tau \int_{S(C)} dS \cdot \Delta^{(2)}(z - x) \cdot J = \int_0^1 d\tau \frac{\dot{x}_\mu g_\nu - \dot{x}_\nu g_\mu}{\dot{x}^2} J_{\mu\nu}. \quad (52)$$

III. LOOP EQUATIONS AND THE BIANCHI IDENTITY

In this section we shall proceed to assess the capacity of the SVM to expedite nonperturbative investigations in QCD by examining whether Eq. (17), as formulated within the framework of the stochastic approach, satisfies the Polyakov/Makeenko-Migdal equations formulated in loop space. The latter constitute the most credible proposal for achieving a nonperturbative casting of the theory, equivalently, one which provides a solid basis for conducting nonperturbative investigations within its framework.[1] In addition, we shall explicitly demonstrate the validity of the Bianchi identity. We claim that these properties *must* be satisfied if one is to assert that the expression for the Wilson loop, as given in Eq. (17), is to constitute a credible approximation to the full theory.

Now, the loop equation for a contour without self-intersections can be stated [6] as follows:

$$\partial_\mu^{x(\tau)} \frac{\delta}{\delta \sigma_{\mu\nu}(x(\tau))} W[C] = \lim_{\epsilon \rightarrow 0} \int_{\tau-\epsilon}^{\tau+\epsilon} d\tilde{\tau} \frac{\delta}{\delta x_\mu(\tilde{\tau})} \frac{\delta}{\delta \sigma_{\mu\nu}(x(\tau))} \times W[C] = 0. \quad (53)$$

Its verification constitutes the first, as well as the simplest, test the SVM must pass. To this end, let us insert Eq. (28) into Eq. (41), whereupon, using the fact that the boundary is a closed contour, we determine

$$\frac{\delta}{\delta \sigma_{\mu\nu}(x(\tau))} A = \int d^2\xi' \epsilon^{ab} \partial_a z_\mu(\xi') \partial_b z_\nu(\xi') D[z(\xi') - x(\tau)] + \frac{1}{2}\int_0^1 d\tau' [\dot{x}_\mu(\tau')(x_\nu(\tau') - x_\nu(\tau)) - (\mu \leftrightarrow \nu)] D_1[x(\tau') - x(\tau)]. \quad (54)$$

Our next step is to take the functional derivative of the above equation. Our task becomes relatively easy as we notice, from Eq. (53), that we only need those terms which contain the delta function $\delta(\tilde{\tau} - \tau)$. Accordingly, we obtain

[1] One might consider this approach as the continuum space casting of lattice gauge theories, which defines the "going" standard for such investigations.

$$\frac{\delta}{\delta x_\mu(\tilde{\tau})}\frac{\delta A[C]}{\delta\sigma_{\mu\nu}(x(\tau))} = \int d^2\xi' \epsilon^{ab}\partial_a z_\mu(\xi')\partial_b z_\nu(\xi')\frac{\delta}{\delta x_\mu(\tilde{\tau})}D(z(\xi') - x(\tau)) + \frac{1}{2}\int_0^1 d\tau'[\dot{x}_\mu(\tau')(x_\nu(\tau') - x_\nu(\tau)) - (\mu\leftrightarrow\nu)]\frac{\delta}{\delta x_\mu(\tilde{\tau})}D_1(x(\tau') - x(\tau)) + \frac{3}{2}\delta(\tau - \tilde{\tau})\int_0^1 d\tau'\dot{x}_\nu(\tau')D_1(x' - x). \quad (55)$$

We now write

$$\frac{\delta}{\delta x_\mu(\tilde{\tau})}D(z(\xi') - x(\tau)) = \delta(\tau - \tilde{\tau})\frac{\partial D(z - x)}{\partial x_\mu} = -\delta(\tau - \tilde{\tau})\frac{\partial D(z - x)}{\partial z_\mu} \quad (56)$$

and similarly for the derivative of D_1. Thus

$$\partial_\mu^{x(\tau)}\frac{\delta A[C]}{\delta\sigma_{\mu\nu}(x(\tau))} = -\int d^2\xi'\epsilon^{ab}\partial_a z_\mu(\xi')\partial_b z_\nu(\xi') \times\frac{\partial}{\partial z_\mu}D(z - x) - \frac{1}{2}\int_0^1 d\tau'[\dot{x}_\mu(\tau') \times(x_\nu(\tau') - x_\nu(\tau)) - (\mu\leftrightarrow\nu)] \times\frac{\partial}{\partial x'_\mu}D_1(x' - x) + \frac{3}{2}\oint d\bar{x}_\nu D_1(\bar{x}). \quad (57)$$

Since the boundary is a (closed) loop, we conclude that the first term takes the form

$$\int d^2\xi'\epsilon^{ab}\partial_a z_\mu(\xi')\partial_b z_\nu(\xi')\frac{\partial}{\partial z_\mu}D(z - x) = \int d^2\xi'\epsilon^{ab}\partial_a D(z - x)\partial_b z_\nu(\xi') = -\int_0^1 d\tau'\dot{x}_\nu(\tau')D(x' - x) = -\oint_C d\bar{x}_\nu D(\bar{x}). \quad (58)$$

With the same reasoning we have for the second term

$$\int_0^1 d\tau'[\dot{x}_\mu(\tau')(x_\nu(\tau') - x_\nu(\tau)) - (\mu\leftrightarrow\nu)]\frac{\partial}{\partial x'_\mu}D_1(x' - x) = -\int_0^1 d\tau'\dot{x}_\nu(\tau')(x_\mu(\tau') - (x_\mu(\tau))\frac{\partial}{\partial x'_\mu}D_1(x' - x) = -2\oint_C d\bar{x}_\nu \mid\bar{x}\mid^2\frac{\partial}{\partial\mid\bar{x}\mid^2}D_1(\bar{x}). \quad (59)$$

Accordingly,

$$\partial_\mu^{x(\tau)}\frac{\delta A[C]}{\delta\sigma_{\mu\nu}(x(\tau))} = -\oint_C d\bar{x}_\nu\Big[D(\bar{x}) + \frac{3}{2}D_1(\bar{x}) + \mid\bar{x}\mid^2\frac{\partial}{\partial\mid\bar{x}\mid^2}D_1(\bar{x})\Big]. \quad (60)$$

Given that the functions D and D_1 depend only on the distance $\mid\bar{x}\mid$, the right-hand side (rhs) of the above equation vanishes and the loop equation is satisfied. In fact, what *is known* is that the minimal area satisfies the Makeenko-Migdal equation asymptotically. The above result, on the other hand, can be considered as new in the sense that it is an *exact result* in the framework of the stochastic vacuum hypothesis.

Our next concern is to confirm the BI in the framework of the SVM. The reason we are interested in such a confirmation is based on the fact that the zigzag, or backtracking, symmetry characterizes the Wilson loop functional: It is invariant under reparametrizations of the form $x(\tau)\to x(\alpha(\tau))$, even if $\alpha' < 0$. It can be shown [24] that a Stokes-type functional, of which the Wilson loop is a prime example, which respects the aforementioned symmetry also satisfies Eq. (10), and can be written in the form

$$\epsilon^{\kappa\lambda\mu\nu}\partial_\lambda^{x(\tau)}\frac{\delta}{\delta\sigma_{\mu\nu}(x(\tau))}W[C] = \frac{1}{N_c}\epsilon^{\kappa\lambda\mu\nu}\Big\langle\nabla_\lambda F_{\mu\nu}\exp\Big(i\oint_C dx\cdot\mathcal{A}\Big)\Big\rangle_{\mathcal{A}} = 0. \quad (61)$$

The above relation forms the bridge between the zigzag symmetry and the BI in the framework of a gauge field theory.

As it is now obvious, the area $S = \int d^2w\sqrt{g}$ does not respect the zigzag symmetry, and in this sense $e^{-\sigma S}$ is not a good representative for $W[C]$. On the other hand, the action $A[C]$ in the stochastic approximation *is* invariant under zigzag parametrizations, but due to the truncation e^{-A} it is not obvious that it is a Stokes-type functional. We are thereby obliged to confirm that the BI is explicitly satisfied and that, consequently, the stochastic approximation produces Stokes-type functionals. To this end we first observe, using Eq. (41), that

$$\epsilon^{\kappa\lambda\mu\nu}\partial_\lambda^{x(\tau)}\frac{\delta}{\delta\sigma_{\mu\nu}(x(\tau))}e^{-A[C]} = \Big(\epsilon^{\kappa\lambda\mu\nu}\int_{S(C)}dS_{\alpha\beta}(z)\partial_\lambda\Delta^{(2)}_{\mu\nu,\alpha\beta}(z - x)\Big)e^{-A[C]}. \quad (62)$$

Using Eq. (29) we find that

$$\epsilon^{\kappa\lambda\mu\nu}\int_{S(C)}dS_{\alpha\beta}(z)\partial_\lambda\Delta^{(2)}_{\mu\nu,\alpha\beta}(z - x) = 2X_\kappa, \quad (63)$$

where we have set

$$X_\kappa = \int_{S(C)}dS_{\alpha\beta}\Delta_{\kappa\alpha\beta}(z - x). \quad (64)$$

Now, we have established that the surface over which we integrate the correlators is determined by Eq. (27). Multiplying that equation by $\epsilon^{\sigma\kappa'\mu'\nu'}$ we determine

$$t_{\mu\nu}X_\kappa + t_{\nu\kappa}X_\mu + t_{\kappa\mu}X_\nu = 0. \quad (65)$$

The above equations form a homogeneous system whose only solution is $X_\kappa = 0$, a result which, together with Eqs. (61) and (62), confirms the validity of the BI within the framework of the SVM.

IV. MESON-MESON SCATTERING

In addition to confinement, which constitutes a profoundly nonperturbative problem, there do exist specific *dynamical* processes, whose theoretical confrontation also calls for nonperturbative methods of analysis. One such situation arises in connection with the theoretical description of high energy scattering amplitudes for which the soft sector of the theory is involved. From the experimental point of view, one such case arises in connection with Regge kinematics, directly entering the theoretical description of, among others, diffractive and low-x physics processes. In this section we shall study a simulated case of a meson-meson scattering process whose quark-based description is of the general form

$$(1\bar{1}) + (2\bar{2}) \to (3\bar{3}) + (4\bar{4}).$$

We adopt a standard picture, which has already been employed in the QCD literature (see, for example, [29–31]), according to which quark 1 from the first meson and antiquark $\bar{2}$ from the second meson are very heavy, in comparison to the incoming total energy, and hence their worldlines from the gluon field action are considered to remain intact and can be described in the framework of the eikonal approximation. The light pairs $\bar{1}$, 2 and $\bar{3}$, 4, on the other hand, are annihilated and produced in the t channel, where the eikonal approximation is not valid and a full treatment is needed for their description. In the worldline framework the process is schematically pictured in spacetime by the straight eikonal lines $(1 \to 3)$ and $(\bar{2} \to \bar{4})$, describing an intact quark and antiquark, and by the curves $(\bar{1} \to 2)$ and $(\bar{3} \to 4)$ which correspond, respectively, to the annihilated and produced quark-antiquark pairs. The structure of the field theoretical amplitude can be written as follows (see Fig. 1):

$$G(x_4, x_3, x_2, x_1) = \langle iS_F(x_4, x_3 \mid \mathcal{A}) iS_F(x_3, x_1 \mid \mathcal{A}) iS_F(x_1, x_2 \mid \mathcal{A}) iS_F(x_2, x_4 \mid \mathcal{A})\rangle_{\mathcal{A}}. \tag{66}$$

In the above expression iS_F is the full fermionic propagator which, in the framework of the worldline formalism, assumes the form [16]

$$iS_F(y, x \mid \mathcal{A}) = \int_0^\infty dL e^{-Lm^2} \int_{\substack{x(0)=x \\ x(L)=y}} Dx(\tau) e^{-(1/4)\int_0^L d\tau \dot{x}^2} \times \left[m - \frac{\gamma \cdot \dot{x}(L)}{2}\right] \Phi^{(1/2)}(L, 0) P \times \exp\left(i \int_0^L d\tau \dot{x} \cdot \mathcal{A}\right), \tag{67}$$

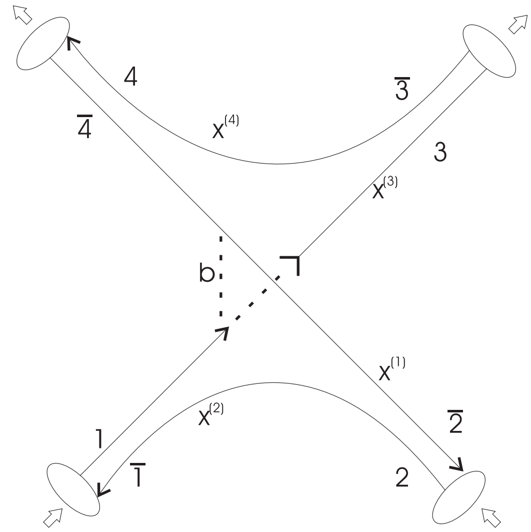

FIG. 1. The helicoidal geometry of the Wilson loop in Euclidean space.

where $\Phi^{(j)}$ is the so-called spin factor for the matter particles entering the system. For us, it means that $j = \frac{1}{2}$.

Inserting the above formula into Eq. (66) we find

$$G(x_4, x_3, x_2, x_1) = \prod_{i=1}^{4} \int_0^\infty d\tau_i \theta(\tau_i - \tau_{i-1}) e^{-(\tau_i - \tau_{i-1})m_i^2} \times \int_{\substack{x(0)=x_4 \\ x(\tau_4)=x_4}} \int Dx(\tau) \delta[x(\tau_3) - x_3] \times \delta[x(\tau_2) - x_1]\delta[x(\tau_1) - x_2] \times \exp\left[-\frac{1}{4}\int_0^{\tau_4} d\tau \dot{x}^2(\tau)\right](\text{spin structure}) \times \left\langle P \exp\left(i \oint_C dx \cdot \mathcal{A}\right)\right\rangle_{\mathcal{A}}, \tag{68}$$

where the term *spin structure* corresponds to the following expression:

$$(\text{spin structure}) = \prod_{i=4}^{1}\left[m_i - \frac{1}{2}\gamma \cdot \dot{x}(\tau_i)\right]\Phi^{(1/2)}(\tau_i, \tau_{i-1}) \quad (\tau_0 \equiv 0). \tag{69}$$

In principle, the Wilson loop appearing in Eq. (68) incorporates the dynamics (perturbative, as well as nonperturbative) of the process. In the framework of the SVM it assumes the form

$$\left\langle P \exp\left(i \oint_C dx \cdot \mathcal{A}\right)\right\rangle_{\mathcal{A}} = \exp\left[-\frac{1}{2}\int_{S(C)} dS_{\mu\nu}(z) \int_{S(C)} dS_{\lambda\rho}(z') \Delta^{(2)}_{\mu\nu,\lambda\rho}(z - z')\right] \equiv e^{-A[C]}. \tag{70}$$

In the present Section we are going to calculate the amplitude (68) using the above expression, which gives the structure of the Wilson loop in the framework of the SVM. The particular method to be adopted is a kind of a "semiclassical" approximation based on a combined minimization of the action $A[C]$—see Eq. (39)—with respect to the surface $S[C]$, and of the surface $S[C]$ with respect to the boundary C. The reasoning behind this procedure is that, according to Eq. (68), in order to obtain the full amplitude it does not suffice to determine the minimal surface bounded by a given specific contour; one needs to proceed even further and sum over all possible boundaries with a weight of the form

$$S[x] = \frac{1}{4}\int_0^{\tau_4} d\tau \dot{x}^2 + A[C]. \tag{71}$$

The above-described approximation will allow us to determine the dominant contribution to the worldline integral (68) in the stochastic limit $T_g^2\sqrt{\Delta} \ll 1$.

The variation $g_\mu[x(\tau)]$ of $A[C]$ under changes of the boundary is given by Eq. (39). Accordingly, the correlator contributions become stationary for the "classical" trajectory

$$g_\mu[x_{\rm cl}] = 0. \tag{72}$$

Using the expansion for the correlator according to Eq. (28), it is easy to see that

$$g_\mu[x(\tau)] = 2\dot{x}_\alpha(\tau)R_{\alpha\mu}[x(\tau)] - \tfrac{1}{2}\dot{x}_\alpha(\tau)Q_{\alpha\mu}[x(\tau)], \tag{73}$$

with

$$R_{\alpha\mu}[x(\tau)] = \int_{S(C)} dS_{\alpha\mu}(z')D[x(\tau) - z(s',\tau')] \tag{74}$$

and

$$Q_{\alpha\mu}[x(\tau)] = \int d\tau'[\dot{x}_\mu(\tau')(x_\alpha(\tau') - x_\alpha(\tau)) - (\mu \leftrightarrow \alpha)]D_1[x(\tau) - x(\tau')]. \tag{75}$$

It is worth noting that the above expressions are reparametrization invariant. Also, in the last relation the integration covers the whole range of the τ variable. Following Refs. [30,31] the minimal surface bounded by two infinite rods at a relative angle θ has (in four-dimensional Euclidean space) the shape of a (three-dimensional) helicoid, which is the only surface that can be spanned by straight lines [32]. In the process considered the eikonal lines $1 \to 3$, $\bar{2} \to \bar{4}$ play the role of the "rods," while the angle θ is connected, via analytic continuation [33], to the logarithm of their total energy s.

Given the above specifications, consider the following, helpful parametrization of the boundary C: For $0 < \tau < \tau_1$ we have a straight line segment, $x^{(1)}$, going from the point x_4 to the point x_2. Moreover, introducing the *length* $2T = | x_4 - x_2 |$ for convenience and reparametrizing according to $\tau \to \frac{2T}{\tau_1}\tau - T$, we write

$$x_\mu^{(1)} = (\tau, 0, 0, 0), \qquad -T < \tau < T, \tag{76}$$

with $x_\mu^{(1)}(-T) = x_4$, $x_\mu^{(1)}(T) = x_2$.

The second eikonal line $x^{(3)}(\tau)$, $\tau_2 < \tau < \tau_3$, goes from the point x_1 to the point x_3 at a relative angle θ with respect to $x^{(1)}$, while a distance b (impact parameter) separates the two linear contours in the transverse direction. Introducing the distance $2T_1 =| x_3 - x_1 |$ and reparametrizing according to

$$\tau \to T_1\left(\frac{2}{\tau_3 - \tau_2}\tau - \frac{\tau_3 + \tau_2}{\tau_3 - \tau_2}\right), \tag{77}$$

we write

$$x_\mu^{(3)}(\tau) = (-\tau\cos\theta, -\tau\sin\theta, b, 0), \qquad -T_1 < \tau < T_1,$$

with $x_\mu^{(3)}(-T_1) = x_1$, $x_\mu^{(3)}(T_1) = x_3$.

In the following we shall assume, just for convenience, that

$$2T =| x_4 - x_2 | \sim | x_3 - x_1 |= 2T_1.$$

For $\tau_1 < \tau < \tau_2$, we have a helical curve $x_\mu^{(2)}(\tau)$, which joins the points $x_2 = x_\mu^{(2)}(\tau_1)$ and $x_1 = x_\mu^{(2)}(\tau_2)$, representing the exchanged light quarks. Now, performing the change $\sigma = \frac{b}{\tau_2 - \tau_1}(\tau - \tau_1)$, we write

$$x_\mu^{(2)}(\sigma) = \left(\phi(\sigma)\cos\frac{\theta\sigma}{b}, \phi(\sigma)\sin\frac{\theta\sigma}{b}, \sigma, 0\right), \quad 0 < \sigma < b \tag{78}$$

The continuity of the boundary requires

$$x_\mu^{(1)}(T) = x_\mu^{(2)}(0) = x_2 \quad \text{and} \quad x_\mu^{(2)}(b) = x_\mu^{(3)}(-T) = x_1,$$

or

$$\phi(0) = \phi(b) = T. \tag{79}$$

The final helical curve is $x^{(4)}(\tau)$, which, for $\tau_3 < \tau < \tau_4$, joins the points $x_3 = x^{(4)}(\tau_3)$ and $x_4 = x^{(4)}(\tau_4)$. Making one more, final reparametrization, namely, $\sigma = \frac{b}{\tau_4 - \tau_3}(\tau - \tau_3)$, we write

$$x_\mu^{(4)}(\sigma) = \left(-\phi(\sigma)\cos\frac{\theta\sigma}{b}, -\phi(\sigma)\sin\frac{\theta\sigma}{b}, \sigma, 0\right), \quad 0 < \sigma < b. \tag{80}$$

Once again, Eq. (79) takes care of the continuity of the boundary. Now, the minimal surface is bounded by the (four) curves specified by Eqs. (76)–(80) and can be spanned by straight lines parametrized as follows:

$$z_\mu(\xi) = \frac{T-\tau}{2T}x_\mu^{(4)}(\sigma) + \frac{T+\tau}{2T}x_\mu^{(2)}(\sigma) = \left(\frac{\tau}{T}\phi(\sigma)\cos\frac{\theta\sigma}{b}, \frac{\tau}{T}\phi(\sigma)\sin\frac{\theta\sigma}{b}, \sigma, 0\right). \tag{81}$$

It can be easily proved that the surface defined by the above equation is minimal, irrespectively of the function ϕ:

$$\partial_\tau\left[\frac{(\dot{z}\cdot z')z'_\mu - z'^2\dot{z}_\mu}{\sqrt{g}}\right] + \partial_\sigma\left[\frac{(\dot{z}\cdot z')\dot{z}_\mu - \dot{z}^2 z'_\mu}{\sqrt{g}}\right] = 0. \tag{82}$$

One observes that the minimization of the surface is not enough for the complete specification of the parametrization of the helicoid. Accordingly, we go back to Eq. (72), which determines the boundary that dominates the path integration (68). A first observation is that, due to the antisymmetric nature of $R_{\alpha\mu}$ and $Q_{\alpha\mu}$, the function g_μ vanishes when $x_\mu(\tau)$ represents a straight line. Thus Eq. (72) is trivially satisfied for the eikonal sector of the boundary. Nontrivial contributions are coming only from the helices $x_\mu^{(2)}$ and $x_\mu^{(4)}$. One can simplify Eq. (73) by computing the leading behavior of the functions $R_{\alpha\mu}$ and $Q_{\alpha\mu}$ using the fact that the functions D and D_1, as defined in the SVM scheme—and measured in lattice calculations [25]—decay exponentially fast for distances which are large in comparison with the correlation length T_g. In this connection and upon writing

$$x(\sigma') = x(\sigma) + (\sigma' - \sigma)\dot{x}(\sigma) + \tfrac{1}{2}(\sigma' - \sigma)^2\ddot{x}(\sigma) + \cdots,$$

we find, for the second term in Eq. (73),

$$\begin{aligned}\dot{x}_\alpha Q_{\alpha\mu} &= \frac{1}{2}[(\dot{x}^2)\ddot{x}_\mu - (\dot{x}\cdot\ddot{x})\dot{x}_\mu]\int_0^b d\sigma'(\sigma'-\sigma)^2 D_1\\ &\quad\times\left[\dot{x}^2\frac{(\sigma'-\sigma)^2}{T_g^2}\right] + \cdots\\ &= \frac{1}{|\,\dot{x}\,|}\left(\ddot{x} - \frac{\dot{x}\cdot\ddot{x}}{\dot{x}^2}\right)\frac{1}{T_g\alpha_1} + \cdots,\end{aligned} \tag{83}$$

where[2]

$$\frac{1}{\alpha_1} \equiv T_g^4\int_0^\infty dw w^2 D_1(w^2).$$

In the last expression we have used, as in Eqs. (33) and (34), the dimensionless parameter $w = |\,z\,|/T_g$. It must be noted that the coefficient $1/\alpha_1$ is a small number. Taking into account that T_g is of the order of 0.1 fm and the string tension [see Eq. (33)] $\sigma \simeq 0.18\ \mathrm{GeV}^2$, it is readily seen that $1/\alpha_1 = O(\sigma T_g^2)$.

Noting that

$$z_\mu(\sigma, \tau = T) = x_\mu^{(2)}(\sigma), \qquad z_\mu(\sigma, \tau = -T) = x_\mu^{(4)}(\sigma),$$
$$\partial_\tau z_\mu(\sigma,\tau) = \dot{z}_\mu(\sigma,\tau) = \frac{1}{2T}[x_\mu^{(2)}(\sigma) - x_\mu^{(4)}(\sigma)], \tag{84}$$

the leading behavior of the first term of the rhs of (73) can be easily determined. One finds

$$\begin{aligned}\dot{x}_\alpha R_{\alpha\mu} &= \frac{1}{2}\dot{x}^2\left(\dot{z}_\mu - \frac{(\dot{x}\cdot\dot{z})}{\dot{x}^2}\dot{x}_\mu\right)\int_{-T}^{T} d\tau'\\ &\quad\times\int_0^b d\sigma' D\left[\dot{x}^2\frac{(\sigma'-\sigma)^2}{T_g^2}\right] + \cdots\\ &= 2T\,|\,\dot{x}\,|\left(\dot{z}_\mu - \frac{(\dot{x}\cdot\dot{z})}{\dot{x}^2}\dot{x}_\mu\right)\frac{\mu^2}{T_g} + \cdots,\end{aligned} \tag{85}$$

where we have introduced the parameter

$$\mu^2 \equiv T_g^2\int_0^\infty dw D(w^2) \sim O(\sigma). \tag{86}$$

Thus, the function g takes, to leading order, the form

$$\begin{aligned}g_\mu &= \frac{1}{|\,\dot{x}\,|\,T_g}\Bigg[4T\mu^2\dot{x}^2\left(\dot{z}_\mu - \frac{\dot{x}\cdot\dot{z}}{\dot{x}^2}\dot{x}_\mu\right)\\ &\quad - \frac{1}{2\alpha_1}\left(\ddot{x}_\mu - \frac{\dot{x}\cdot\ddot{x}}{\dot{x}^2}\dot{x}_\mu\right)\Bigg].\end{aligned} \tag{87}$$

Now, we recall from its definition that the g function provides a measure of the change of $A[C]$ when the Wilson contour is altered as a result of some interaction which reshapes its geometrical profile. In this sense, it contains important information concerning the dynamics of the problem under study. The structure of the g function, as it appears in the above equation, is quite general and exhibits its dependence not only on the boundary, but on the minimal surface as well. It is worth noting that this fact is strictly associated with the non-Abelian nature of the theory since the function D—and consequently μ^2—disappears [14,15] in an Abelian gauge theory.

Taking into account that for the helicoid parametrization the velocity $\dot{x}$ has three nonzero components, while $\ddot{x}$ and $\dot{z}$ have only two, we conclude that Eq. (72) can be satisfied only if

$$4T\mu^2\dot{x}^2\dot{z}_\mu - \frac{1}{2\alpha_1}\ddot{x}_\mu = 0. \tag{88}$$

Inserting the helical parametrization into Eq. (88), one easily finds that the function ϕ must be a constant. Now, taking into account Eq. (79), we determine this constant to be the length T. It is then very easy to see that this result leads to the conclusions

$$\dot{x}\cdot\dot{z} = 0, \qquad \dot{x}\cdot\ddot{x} = 0 \tag{89}$$

and

$$\dot{x}^2 = -\frac{1}{8\mu^2\alpha_1}\frac{\theta^2}{b^2} = 1 + \frac{T^2\theta^2}{b^2}. \tag{90}$$

This equation cannot be satisfied in Euclidean space. In Minkowski space the angle θ becomes imaginary, $\theta \to -i\chi \simeq -i\ln(\frac{s}{m^2})$ [s is the total energy of the "heavy" quarks that form the rods and $m(\simeq m_1 \simeq m_3)$ their mass], and Eq. (90) has a positive definite solution:

[2]We have omitted terms suppressed by powers of T_g^2.

$$\frac{T^2\chi^2}{b^2} = 1 - \frac{1}{8\mu^2\alpha_1}\frac{\chi^2}{b^2}. \quad (91)$$

In the last equation we have analytically continued only the angle between the rods and not the parameter T. Our reasoning is based on the one presented in [31]: Eqs. (79) and (90) define a contour of the steepest descent for the path integration. For such a contour the parameter T is determined to be imaginary (the formulation is still Euclidean and θ is real). After performing the T integration the only parameter remaining for analytic continuation is the angle between the rods. Equation (91) indicates that, equivalently, one can first analytically continue the angle variable to imaginary values, leaving the T parameter real. In any case it is obvious that the impact parameter must grow with the incoming energy—$T_g b \sim \ln s$—a conclusion which is in agreement with the landmark result of Cheng and Wu [34].

The preceding analysis obviously repeats itself for the two helical curves $x^{(2)}$ and $x^{(4)}$ and has led us to a specific parametrization for the Wilson loop, which plays the dominant role in the path integration in Eq. (68). We are now in a position to determine the leading contribution to the action (71):

$$S_{\rm cl} = \frac{1}{4}\int_0^{\tau_4} d\tau \dot{x}^2_{cl}(\tau) + A[C]_{\rm cl}. \quad (92)$$

Our first step is to expand the second term of the integrand in powers of $T_g^2\sqrt{\Delta}$. The first term of such an expansion is the familiar Nambu-Goto string. The next term, which reveals the rich structure of the SVM, is the so-called "rigidity term," representing the extrinsic curvature of a surface embedded in a four-dimensional [35] background:

$$A[C] = \sigma\int d^2\xi\sqrt{g} + \frac{1}{\alpha_0}\int d^2\xi\sqrt{g}g^{ab}\partial_a t_{\mu\nu}\partial_b t_{\mu\nu} + \cdots, \quad (93)$$

where σ is the string tension as defined in Eq. (32).

The coefficient of the rigidity term reads

$$\frac{1}{\alpha_0} \equiv \frac{1}{32}T_g^4\int d^2w w^2(2D_1(w^2) - D(w^2)) \sim O(\sigma T_g^2). \quad (94)$$

Terms proportional to T_g^6 entering the expansion in Eq. (93) will be considered negligible in our analysis. We have also omitted the term $\int d^2\xi\sqrt{g}R$, since in two dimensions the curvature is a total derivative. Using the helicoid parametrization (81), with $\phi = T$, the Nambu-Goto term in Eq. (93) takes the form

$$\int d^2\xi\sqrt{g} = \int_{-T}^{T} d\tau\int_0^b ds\sqrt{1 + \frac{\tau^2\theta^2}{b^2}} = bT\left[\sqrt{1+p^2} + \frac{1}{p}\ln\left(\sqrt{1+p^2} + p\right)\right], \quad (95)$$

where $p = \frac{T\theta}{b}$.

To proceed further we analytically continue to Minkowski space where we can use Eq. (91) to determine

$$bT\sqrt{1+p^2} \to bT\sqrt{1 - \frac{T^2\chi^2}{b^2}} \simeq b\left(1 - \frac{1}{8\alpha_1\mu^2}\frac{\chi^2}{b^2}\right)^{1/2}\frac{1}{\sqrt{8\alpha_1\mu^2}} \simeq \frac{b}{\sqrt{8\alpha_1\mu^2}} + \mathcal{O}(T_g^3) \quad (96)$$

and

$$\frac{bT}{p}\ln\left(\sqrt{1+p^2} + p\right) \to \frac{bT}{-iT\chi/b}\ln\left[\sqrt{1 - \frac{T^2\chi^2}{b^2}} - i\frac{T\chi}{b}\right] \simeq \frac{\pi b^2}{2\chi} - \frac{b}{\sqrt{8\alpha_1\mu^2}} + \mathcal{O}(T_g^3). \quad (97)$$

Thus

$$\sigma\int d^2\xi\sqrt{g} \to \frac{\sigma\pi b^2}{2\chi}. \quad (98)$$

In the same framework, the contribution of the rigidity term takes the form

$$\int d^2\xi\sqrt{g}g^{ab}\partial_a t_{\mu\nu}\partial_b t_{\mu\nu} = \int_{-T}^{T} d\tau\int_0^b ds\frac{1}{\sqrt{1+\frac{\theta^2\tau^2}{b^2}}}\left(\frac{\theta^2}{b^2} + \frac{1}{2}\frac{\theta^4}{b^4}\tau^2\right) = \theta\left[\frac{3}{2}\ln(\sqrt{1+p^2} + p) + \frac{1}{2}p\sqrt{1+p^2}\right]. \quad (99)$$

It follows that in Minkowski space we have

$$\frac{1}{\alpha_0}\int d^2\xi\sqrt{g}g^{ab}\partial_a t_{\mu\nu}\partial_b t_{\mu\nu} \to -\frac{3\pi}{4\alpha_0}\chi. \quad (100)$$

For the full estimation of the classical action [cf. Eq. (92)], one should also take into account the presence of the classical kinetic term. Nontrivial contributions come from the helical curves $x^{(2)}(\bar{1} \to 2)$ and $x^{(4)}(\bar{3} \to 4)$:

$$\frac{b}{4(\tau_2 - \tau_1)}\int_0^b ds(\dot{x}^{(2)})^2 + \frac{b}{4(\tau_4 - \tau_3)}\int_0^b ds(\dot{x}^{(4)})^2 = \frac{b^2\dot{x}^2}{4(\tau_2 - \tau_1)} + \frac{b^2\dot{x}^2}{4(\tau_4 - \tau_3)}. \quad (101)$$

Now we have to take into account that both $\tau_2 - \tau_1$ and $\tau_4 - \tau_3$ must be integrated with weights $e^{-(\tau_2-\tau_1)m_0^2}$ and $e^{-(\tau_4-\tau_3)m_0^2}$, respectively. These integrals, as it turns out, are dominated by the values $\tau_2 - \tau_1 = \tau_4 - \tau_3 = \frac{b|\dot{x}|}{2m_0}$, leading to a final kinetic contribution of the form

$$2m_0 b \mid \dot{x} \mid = 2\frac{m_0}{\sqrt{8\alpha_1\mu^2}}\chi. \tag{102}$$

Here, $m_0(\simeq m_2 \simeq m_4)$ is the (current) mass of the light quarks; thus the result expressed by (102) can be considered negligible.

From the above analysis we conclude that

$$S_{\rm cl} \approx \frac{\sigma\pi b^2}{2\chi} - \frac{3\pi}{4\alpha_0}\chi. \tag{103}$$

Putting aside, for now, the possible corrections to $A[C]$ which arise from fluctuations of the boundary as well as the spin-factor contribution, let us consider the result (103) as a whole, except for terms ~mass. To obtain the final expression for the scattering amplitude one must integrate over the impact parameter:

$$\int d^2b \exp\left(i\vec{q}\cdot\vec{b} - \frac{\sigma\pi}{2\chi}b^2\right) \propto \exp\left(-\frac{1}{2\pi\sigma}q^2\chi\right). \tag{104}$$

Combining (103) and (104) we find, for the scattering amplitude, a Regge behavior of the form $s^{\alpha_R'(0)t+\alpha_R(0)}$ with

$$\alpha_R'(0) = \frac{1}{2\pi\sigma} \quad \text{and} \quad \alpha_R(0) = \frac{3\pi}{4\alpha_0}. \tag{105}$$

The form of the slope $a_R'(0)$, indicated in the last equation, is the same as the one obtained in Ref. [32]. In that work the authors have applied, in the framework of the SVM, a different method based on the path-integral-Hamiltonian duality, and consequently, their result is confined in the region $t > 0$. In this sense our result extends the same value $a_R'(0) = 1/2\pi\sigma \simeq 0.9$ GeV^{-2} for all the values of square momentum transfer.

It is well known [26,32] that the intercept $a_R(0)$ receives a significant contribution from nonperturbative corrections to quark self-energy. We shall comment on this interesting issue in the next section. The result indicated in Eq. (105) does not take into account the aforementioned corrections; thus it is very sensitive to different lattice data or parametrizations. For example, using Ref. [25], the coefficient $1/a_0$ of the rigidity term is negative and one needs [32] the large (and negative) quark self-energy corrections to restore the phenomenological value of the intercept. On the other hand, adopting a certain [17,36] parametrization for the functions D and D_1 (see Appendix B) one obtains for the string tension the value $\sigma = 0.175$ GeV^2 and for the coefficient of the rigidity term the value $1/a_0 = 0.276$. With these numbers we obtain for the Reggeon slope the value $a_R'(0) = 0.91$ GeV^{-2} and for the Reggeon intercept the value $a_R(0) = 0.65$ in good agreement with the phenomenological values $\alpha_R'(0) = 0.93$ GeV^{-2} and $\alpha_R(0) = 0.55$ [37].

V. BOUNDARY FLUCTUATIONS AND THE ROLE OF THE SPIN FACTOR

As repeatedly mentioned in our narration, corrections to the amplitude (68), beyond semiclassical ones, are expected to arise from fluctuations of the boundary of the surface on which the two-point correlator lives. Fluctuations of the surface itself can be taken into account by higher order correlators. This, in fact, is the big difference which distinguishes the SVM approach from Nambu-Goto-type approaches.

We begin our related considerations by expanding the action (71) around the helicoid classical solution:

$$S = S_{\rm cl} - \frac{1}{2}\int_0^{\tau_4} d\tau y(\tau)\ddot{x}^{\rm cl}(\tau) + \frac{1}{2}\int_0^{\tau_4} d\tau \int_0^{\tau_4} d\tilde{\tau} y_\alpha \tau) \times \left[-\frac{1}{2}\delta_{\alpha\beta}\frac{\partial^2}{\partial\tau^2}\delta(\tau-\tilde{\tau}) + \frac{\delta^2 A[C]}{\delta x_\alpha^{\rm cl}(\tau)\delta x_\beta(\tilde{\tau})}\right] y_\beta(\tilde{\tau}) + \cdots, \tag{106}$$

where $y = x - x^{\rm cl}$.

Using the results of Secs. II and III one can easily determine that

$$\frac{\delta^2 A[C]}{\delta x_\alpha(\tau)\delta x_\beta(\tilde{\tau})} = \dot{x}_\mu(\tau)\dot{x}_\nu(\tilde{\tau})\Delta^{(2)}_{\mu\alpha,\nu\beta}[x(\tau) - x(\tilde{\tau})] - \frac{\partial}{\partial\tau}\delta(\tau-\tilde{\tau})\int_{S(C)} dS_{\lambda\rho}(z')\Delta^{(2)}_{\alpha\beta,\lambda\rho} \times [z(\xi' - x(\tau)] + \dot{x}_\alpha(\tau)\int ds\alpha(\tilde{\tau}, s) \times \dot{z}_\lambda(\tilde{\tau}, s)z'_\rho(\tilde{\tau}, s)\epsilon^{\kappa\nu\lambda\rho}\Delta_{\kappa\alpha\mu} \times [z(\tilde{\tau}, s) - x(\tau)], \tag{107}$$

where we have written

$$\frac{\delta z_\mu(\tau, s)}{\delta x_\nu} = \delta_{\mu\nu}\delta(\tau - \tilde{\tau})a(\tilde{\tau}, s).$$

The second term on the rhs of Eq. (107) is simply the area derivative which, as we have seen in Sec. III, has the general form $\frac{\delta A[C]}{\delta\sigma_{\alpha\beta}} \sim g_\alpha\dot{x}_\beta - g_\beta\dot{x}_\alpha$. Thus, for the classical solution $g[x^{\rm cl}]$ it gives zero contribution. It is, furthermore, easy to verify that the third term in (107) also disappears for $x = x^{\rm cl}$. We, therefore, conclude that

$$\frac{\delta^2 A[C]}{\delta x_\alpha^{\rm cl}(\tau)\delta x_\beta^{\rm cl}(\tilde{\tau})} = \dot{x}_\mu^{\rm cl}(\tau)\dot{x}_\nu^{\rm cl}(\tilde{\tau})\Delta^{(2)}_{\mu\alpha\nu\beta}[x^{\rm cl}(\tau) - x(\tilde{\tau})]. \tag{108}$$

Inserting Eq. (107) into Eq. (108) and taking into account that the dominant contribution to the two-point correlator comes from the region $\tau \approx \tilde{\tau}$, we find

$$S \approx S_{\rm cl} + \int_0^b d\sigma y_\alpha(\sigma)\Big[-\frac{1}{2}\frac{m_0}{|\dot{x}|}\delta_{\alpha\beta}\frac{\partial^2}{\partial\sigma^2} + \frac{\lambda^2}{T_g}\omega_{\alpha\beta}(\sigma)\Big] y_\beta(\sigma). \quad (109)$$

Let it be remarked that to arrive at the above relation we have adopted the expansion of the two-point correlator indicated in Eq. (28). We have also used the helicoid parametrization, observing, at the same time, that the eikonal lines give null contribution. One further realizes that the contributions of the two helical curves to the linear term in (106) cancel each other, since $\ddot{x}^{(2)}_\mu(s) = -\ddot{x}^{(4)}_\mu(s)$ and $\tau_2 - \tau_1 \simeq \tau_4 - \tau_3 \sim \frac{b|\dot{x}|}{2m_0}$.

The nontrivial contribution of the helical curves is incorporated in the term

$$\omega_{\alpha\beta} = \delta_{\alpha\beta} - \frac{1}{2\dot{x}^2}(\dot{x}^{(2)}_\alpha \dot{x}^{(2)}_\beta + \dot{x}^{(4)}_\alpha \dot{x}^{(4)}_\beta), \quad (110)$$

the origin of which is the second functional derivative; cf. (106). The mass parameter λ^2 in (109) has the same source and is defined as

$$\lambda^2 \equiv |\dot{x}| T_g^2 \int_0^\infty dw \Big(D(w^2) + D_1(w^2) + \frac{d}{dw^2}D_1(w^2)\Big). \quad (111)$$

The differential operator entering Eq. (109) has no zero eigenvalues since the classical solution is, in fact, the one that annihilates the g function. Accordingly, the calculation of the path integral over $y = x - x^{\rm cl}$ does not require any particular regularization. A straightforward calculation shows that

$$\det\omega_{\alpha\beta} = \frac{1}{\dot{x}^2}\Big(1 - \frac{1}{\dot{x}^2}\Big) = \frac{T^2\theta^2/b^2}{1+\theta^2/b^2}. \quad (112)$$

Thus the matrix $\omega_{\alpha\beta}$ can be diagonalized and the y integral can be easily performed. However, in the limit $m_0 \to 0$ it can be immediately seen that the integration over the boundary fluctuations gives prefactors which are powers of the logarithm of the incoming energy, and as far as the Regge behavior is concerned, they cannot change the behavior that was determined in the previous section.

The next task is to take up the issue of the spin-field dynamics contribution to the scattering amplitude. As seen in Sec. III a spin factor is associated with each segment of the worldline path. This factor receives contributions from two sources. The first one is

$$\int d\tau \int_{S(C)} dS \cdot \Delta^{(2)}(z-x)\cdot J = \int d\tau \frac{\dot{x}_\mu g_\nu - \dot{x}_\nu g_\mu}{\dot{x}^2}\frac{i}{4}[\gamma_\mu, \gamma_\nu] \quad (113)$$

and is obviously zero for the classical trajectory (72).

The other term has the form

$$P = \frac{1}{8}\int d\tau \int d\tau' J_{\mu\nu}\Delta^{(2)}_{\mu\nu,\lambda\rho}(x-x')J_{\lambda\rho} = \frac{3}{4}\int d\tau\int d\tau'(D + D_1) + \frac{3}{8}\int d\tau \int d\tau'(x-x')^2 D_1'. \quad (114)$$

In the stochastic limit, within which we are working, the integrals in the above equation give an appreciable contribution only for $|x(\tau) - x(\tau')| \approx |\dot{x}||\tau - \tau'| \ll T_g$. More concretely, consider the contribution to (114) from the helical curve $(\bar{1} \to 2)$. A straightforward calculation shows that the analytically continued result is

$$P = -(t_2 - t_1)^2 \frac{M^4}{\chi}, \quad (115)$$

where we have written $\tau = it$ for the time variable and denoted

$$M^4 = \Big(8\frac{\int_0^\infty dw D(w^2)}{\int_0^\infty dw w^2 D_1(w^2)}\Big)^{1/2}\int_0^\infty dw\Big(D(w^2) + D_1(w^2) + \frac{1}{2}\frac{d}{dw^2}D_1(w^2)\Big). \quad (116)$$

As has been mentioned in Sec. III and discussed in [27], contribution (115) has an interesting role as far as the form of the fermionic propagator is concerned. As it has been shown in [26,32] this nonperturbative "paramagnetic" contribution corrects the self-energy of a bound light quark and, consequently, the Regge intercept without changing the slope. We shall not discuss this interesting issue here, leaving it for a forthcoming study. In this paper we bypass the problem using the parametrization [17,36].

The remaining spin structure is summarized in the chain

$$I = \prod_{i=4}^{1} m_i \Big[1 - \frac{1}{2m_i}\gamma\cdot\dot{x}^{(i)}(\tau_i)\Big], \quad (117)$$

which must be sandwiched between the external spinor wave functions representing the incoming and outgoing quarks (in the simple picture wherein the meson wave function is just the product of free spinors). The nontrivial dynamics of the process is now incorporated into the fact that the vectors $x^{(i)}_\mu$, $i = 1, 2, 3, 4$, forming the boundary of the helicoids, are three-dimensional vectors with $|\dot{x}^{(i)}|^2 =$ const. For $i = 1, 3$, the factor in (117) becomes the operator $1 - \frac{\gamma\cdot p^{(i)}}{|p^{(i)}|}$.

For $i = 2, 4$, the matrices

$$I_2 = 1 - \frac{1}{2m}\frac{b}{\tau_2 - \tau_1}\gamma\cdot\dot{x}^{(2)}(b) \to 1 - \frac{\gamma\cdot\dot{x}^{(2)}(b)}{|\dot{x}^{(2)}|} \quad (118)$$

and

$$I_4 = 1 - \frac{1}{2m}\frac{b}{\tau_4 - \tau_3}\gamma \cdot \dot{x}^{(4)}(b) \to 1 - \frac{\gamma \cdot \dot{x}^{(4)}(b)}{|\dot{x}^{(4)}|} \quad (119)$$

are also representations of projection operators. As shown in [31] the matrices (118) and (119) are the direct product of two 2×2 matrices, each of which are, by themselves, projection operators. Given these observations it becomes a matter of simple algebra to find that the standard kinematics is reproduced.

VI. CONCLUDING REMARKS

The central objective of this paper was to assess the merits of the stochastic vacuum model of Dosch and Simonov as a credible representative of QCD. From a methodological standpoint we employed the path-integral approach for the casting of the theory, a practice that has been proved an ideal tool for the exploration of its nonperturbative aspects on which the present study is focused. In the first part of the paper—and at a purely theoretical level—we verified both the loop equations and the Bianchi identity through the SVM, an occurrence which further solidifies the credibility of the model. We have also derived an explicit expression for the spin factor that represents the nonperturbative spin-field dynamics and necessarily enters the analysis of physical processes. In the second part of the paper we assessed the effectiveness of the SVM, always in its path-integral casting, to confront a dynamical problem where a nonperturbative treatment is essentially important. More explicitly, we appropriately modeled a meson-meson scattering process in the Regge kinematical regime. In a semiclassical approximation and always working in the framework of the stochastic vacuum model, we found a Regge-type behavior for the scattering amplitude with linear Regge trajectories. The specific methodology we followed is entirely based on the capability of the SVM to represent the nonperturbative content of QCD, and, perhaps, it traces a way for analytically calculating Regge trajectories in the physical region of scattering, i.e., square momentum transfer $t < 0$.

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APPENDIX A

We give here the proof of relation (20), which appears in the text and whose role is significant for the derivation of the equation that determines the surface on which the two-point connector lives. We begin by writing the expression for the connector in Eq. (8):

$$\phi(z, x_0) = P\exp\left(i\int_{x_0}^{z} dw \cdot \mathcal{A}\right) = P\exp\left[i\int_0^1 d\tau \dot{w}(\tau)\cdot\mathcal{A}(w(\tau))\right], \quad (A1)$$

where $w_\mu(0) = x_{0\mu}$, $w_\mu(1) = z_\mu$. Taking the functional derivative of (A1) we find

$$\frac{\delta\phi}{\delta w_\mu(\tau')} = i\int_0^1 d\tau P\exp\left(i\int_\tau^1 d\tau\dot{w}\cdot\mathcal{A}(w)\right)[\partial_\tau\delta(\tau-\tau') + \delta(\tau-\tau')\dot{w}_\nu\partial_\mu\mathcal{A}_\nu(w)]P \times \exp\left(i\int_0^\tau d\tau\dot{w}\cdot\mathcal{A}(w)\right) \quad (A2)$$

or

$$\frac{\delta\phi}{\delta w_\mu(\tau')} = i\delta(1-\tau')\mathcal{A}_\mu(z)P\exp\left[i\int_{x_0}^z dw\cdot\mathcal{A}(w)\right] - i\delta(\tau')P\exp\left[i\int_{x_0}^z dw\cdot\mathcal{A}(w)\right]\mathcal{A}_\mu(x_0) + ig\dot{w}_\nu(\tau')P\exp\left(i\int_{w'}^z dw\cdot\mathcal{A}(w)\right) \times F_{\mu\nu}(w(\tau'))P\exp\left[i\int_{x_0}^z dw\cdot\mathcal{A}(w)\right]. \quad (A3)$$

Accordingly, it follows that the variation of the connector reads

$$\delta\phi = \mathcal{A}_\mu(z)\delta z_\mu\phi(z,x_0) - i\phi(z,x_0)\mathcal{A}_\mu(x_0)\delta x_{0_\mu} + \int_0^1 d\tau\dot{w}_\nu(\tau)\phi(z,w(\tau))F_{\mu\nu}(w(\tau))\phi(w(\tau),x_0) \times \delta w_\mu(\tau). \quad (A4)$$

Keeping everything but the end point constant, one immediately deduces that

$$\frac{\partial\phi}{\partial z_\mu} = A_\mu(z)\phi(z,x_0) - \int_0^1 d\tau\dot{w}_\kappa(\tau)\phi(z,w(\tau)) \times F_{\kappa\lambda}(w(\tau))\phi(w(\tau),x_0)\frac{\partial w_\lambda}{\partial z_\mu}. \quad (A5)$$

APPENDIX B

In this appendix we present a parametrization of the functions D and D_1 used extensively in the present paper. This parametrization is supported by lattice data and is extensively discussed in Refs. [17,36].

The exact relations defining the functions are

$$D = \frac{\pi^2(N_C^2-1)}{2N_C}\frac{G_2}{24}\kappa D_N, \qquad D_1 = \frac{\pi^2(N_C^2-1)}{2N_C}\frac{G_2}{24}(1-\kappa)D_{1,N}, \quad (B1)$$

where D_N and $D_{1,N}$ are functions which determine the structure of the two-point correlators, as defined in [17]. The factor G_2 is defined as follows:

$$G_2 \equiv \langle 0 \mid \frac{g^2}{4\pi^2} F^{\alpha}_{\mu\nu}(0) F^{\alpha}_{\mu\nu}(0) \mid 0 \rangle = \frac{2N_C}{4\pi^4} \Delta^{(2)}_{\mu\nu,\mu\nu}(0). \tag{B2}$$

For the above correlator we shall adopt the value given in Ref [8], namely, $G_2 = (0.496)^4$ GeV^4. The value of the numerical quantity κ in (B1) is estimated in the same reference to be 0.74. The ansatz for the function D_N is [17]

$$D_N(z) = \frac{27}{64}\frac{1}{a^2}\int d^4k e^{ik\cdot z}\frac{k^2}{[k^2 + (\frac{3\pi}{8a})^2]^4}, \tag{B3}$$

where

$$a \equiv \int_0^\infty dz D_N(z). \tag{B4}$$

A simple calculation shows that

$$D_N(z) = wK_1(w) - \frac{1}{4}w^2K_0(w), \qquad w = \frac{3\pi}{8a}\mid z \mid, \tag{B5}$$

with K_ν denoting a Bessel function. The correlation length T_g can be deduced from Eq. (B5):

$$T_g = \frac{8a}{3\pi}. \tag{B6}$$

The estimated value of a is

$$a \approx 0.35 \text{ fm} \quad \text{or} \quad T_g \approx 0.297 \text{ fm}. \tag{B7}$$

With the help of ansatz (B3) and using (B7), one can determine the string tension:

$$\sigma = \frac{1}{2}T_g^2 \int d^2 w D(w) = \frac{1}{2}T_g^2 \frac{\pi^2(N_C^2 - 1)}{2N_C}\frac{G_2}{24}\kappa \int d^2w\left[wK_1(w) - \frac{1}{4}w^2K_0(w)\right] \tag{B8}$$

or

$$\sigma = \frac{1}{2}T_g^2\frac{\pi^2(2N_C^2 - 1)}{2N_C}\frac{G_2}{24}\kappa 2\pi = a^2 G_2 \kappa\pi\frac{32}{81} \approx 0.175 \text{ GeV}^2. \tag{B9}$$

The ansatz for the function $D_{1,N}$ is deduced from the equation [17,36]

$$\left(4 + z_\mu \frac{\partial}{\partial z_\mu}\right)D_{1,N}(z) = 4D_N(z) \tag{B10}$$

or

$$D_{1,N}(z) = \frac{1}{z^4}\int_0^z dw[4w^4K_1(w) - w^5K_0(w)]. \tag{B11}$$

The coefficient of the rigidity term entering Eq. (94) can now be calculated:

$$\frac{1}{\alpha_0} = \frac{1}{32}T_g^4 \int d^2 w w^2[2D_1(w) - D(w)] = \frac{1}{32}T_g^4\frac{\pi^2(N_C^2 - 1)}{2N_C}\frac{G_2}{24}\int d^2ww^2[2(1-\kappa)D_{1,N}(w) - \kappa D_N(w)] = \frac{1}{32}T_g^4\frac{\pi^2(N_C^2 - 1)}{2N_C}\frac{G_2}{24}2(1-\kappa)32\pi \approx 0.276. \tag{B12}$$

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