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$\mathcal{N} = 4$ supersymmetric Yang-Mills theory on S^3 in the plane wave matrix model at finite temperature

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We investigate the large N reduced model of gauge theory on a curved spacetime through the plane wave matrix model. We formally derive the action of the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory on $\mathbb{R} \times S^3$ from the plane wave matrix model in the large N limit. Furthermore, we evaluate the effective action of the plane wave matrix model up to the two-loop level at finite temperature. We find that the effective action is consistent with the free energy of the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory on S^3 at the high temperature limit where the planar contributions dominate. We conclude that the plane wave matrix model can be used as a large N reduced model to investigate nonperturbative aspects of the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory on $\mathbb{R} \times S^3$.

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I. INTRODUCTION

Matrix models are strong candidates for the nonperturbative formulation of the superstring theory. For example, the BFSS matrix model, which was proposed by Banks, Fischler, Shenker, and Susskind, is the nonperturbative formulation of the M-theory which is the strongly coupled limit of the type-IIA superstring theory [1], and the IKKT matrix model, which was proposed by Ishibashi, Kawai, Kitazawa, and Tsuchiya, is also the nonperturbative formulation of the type-IIB superstring theory [2,3]. Originally, these models were constructed on flat spacetime backgrounds. So, we have problems whether these models can describe curved spacetime, and include symmetries of the general relativity: the diffeomorphism and the local Lorentz invariance.

In 2002, fuzzy homogeneous spaces were constructed using the IKKT matrix model [4]. The homogeneous spaces were constructed as G/H where G is a Lie group and H is a closed subgroup of G. The effective actions of the gauge theory on homogeneous spaces have been investigated for a fuzzy S^2 [5], a fuzzy $S^2 \times S^2$ [6,7], a fuzzy $S^2 \times S^2 \times S^2$ [8], and a fuzzy CP^2 [9]. When a background field is assigned to bosonic matrices in the IKKT matrix model, the stability of this matrix configuration can be examine by investigating the behavior of the effective action under the change of some parameters of the background. Through these investigations, we have found that the IKKT matrix model favors the configurations of four dimensionality. The same conclusion has been obtained by various other approaches [10–16].

Recently, there were interesting developments on the construction of curved spacetime by matrix models. Hanada, Kawai, and Kimura introduced a new interpretation on the IKKT matrix model in which covariant derivatives on any d-dimensional spacetime can be described in terms of d bosonic matrices in the IKKT matrix model [17]. In this interpretation, the Einstein equation follows from the equation of the IKKT matrix model, and symmetries of the diffeomorphism and the local Lorentz transformation are included in the unitary symmetry of the IKKT matrix model.

On the other hand, the formal equivalences between supersymmetric Yang-Mills theories on curved spacetime and a matrix model is shown by Ishiki, Shimasaki, Takayama, and Tsuchiya [18], confirming Lin-Maldacena's gauge/gravity correspondence [19]. They showed the following formal equivalences: the theory around each vacuum of the supersymmetric Yang-Mills on $\mathbb{R} \times S^2$ is equivalent to the theory around a certain vacuum of the plane wave matrix model; the theory around each vacuum of the supersymmetric Yang-Mills on $\mathbb{R} \times S^3$ is equivalent to the theory around a certain vacuum of the supersymmetric Yang-Mills on $\mathbb{R} \times S^2$ with the orbifolding condition imposed [20]. They thus made the connection between the theory around each vacuum of the supersymmetric Yang-Mills on $\mathbb{R} \times S^3$ and the theory around a certain vacuum of the plane wave matrix model with the orbifolding condition imposed. In this identification, S^3 emerges out of a group of the concentric fuzzy spheres. Note that the equivalences shown in [18] are classical, since the equivalences are shown at tree level and the size of matrices are infinite with the orbifolding condition imposed. Recently, they extend the equivalence between the supersymmetric Yang-Mills on $\mathbb{R} \times S^3$ and the plane wave matrix model at quantum level [21]. The equivalence is shown up to the one-loop level and the size of the matrices is finite without the orbifolding conditions. Moreover, they derive the deconfinement phase transition of the supersymmetric Yang-Mills on $S^1 \times S^3$ at weak coupling region from the plane wave matrix model [22].

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In order to elucidate these proposals to construct curved spacetime in matrix models, we investigated the effective action of the deformed IKKT matrix model with a Myers term. Since the classical solution satisfies the commutation relation of the angular momentum, it can be interpreted as the covariant derivatives on S^3 or concentric fuzzy spheres [23]. In both cases, we found that the highly divergent contributions at the tree and one-loop level are sensitive to the UV cutoff. However the two-loop level contributions are universal since they are only logarithmically divergent. We expect that the higher loop contributions are insensitive to the UV cutoff since three-dimensional gauge theory is super renormalizable.

In the large N limit, there is a well-known equivalence between a gauge theory and a matrix model due to Eguchi and Kawai [24]. They proved that a large N gauge theory is equivalent to a matrix model which is dimensionally reduced to zero dimension unless the $U(1)^d$ symmetry is broken, where d represents the dimension of the original gauge theory. However, the $U(1)^d$ symmetry is spontaneously broken in $d > 2$. So, two improved versions of this large N reduced model which preserve the $U(1)^d$ symmetry were proposed. One is the quenched reduced models [25–28] and the other is the twisted reduced models [29–31]. However, in these models the connection is made between matrix models and gauge theories on flat spacetime. In this paper, we investigate the effective action of the plane wave matrix model on a group of concentric fuzzy spheres at finite temperature. We find that the effective action is consistent with the free energy of the $\mathcal{N} = 4$ supersymmetric Yang-Mills on S^3 in the high temperature limit. It is because planar contributions dominate in the high temperature limit. We conclude that the plane wave matrix model can be used as a large N reduced model to investigate nonperturbative aspects of the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory on $\mathbb{R} \times S^3$.

The organization of this paper is as follows. In Sec. II, we formally derive the action of the $\mathcal{N} = 4$ supersymmetric Yang-Mills on S^3 from the plane wave matrix model. In Sec. III, we calculate the effective action of the plane wave matrix model around a group of concentric fuzzy spheres at finite temperature. Section IV is devoted to conclusions and discussions. Some detailed calculations are gathered in the appendix.

II. $\mathcal{N} = 4$ SUPERSYMMETRIC YANG-MILLS ON $\mathbb{R} \times S^3$ AS PLANE WAVE MATRIX MODEL

In this section, we formally derive the action of the supersymmetric Yang-Mills theory on $\mathbb{R} \times S^3$ from the plane wave matrix model in the large N limit.

The authors of [18] observed the following two equivalences between the vacua of different gauge theories and the plane wave matrix model[1]:

(i) The supersymmetric Yang-Mills theory on $\mathbb{R} \times S^2$ is equivalent to the theory around a certain vacuum of the plane wave matrix model.

(ii) The supersymmetric Yang-Mills theory on $\mathbb{R} \times S^3$ is equivalent to the theory around a certain vacuum of the supersymmetric Yang-Mills theory on $\mathbb{R} \times S^2$ with a generalized compactification procedure in the S^1 direction.

From the above equivalences of (i) and (ii), they concluded that S^3 is realized by three matrices. The matrices are as follows:

$$Y_i = -\mu L_i, \tag{2.1}$$

where

$$L_i = \begin{pmatrix} \ddots & & & & & & \\ & L_i^{[j_{s-1}]} & n & & & & \\ & & \ddots & L_i^{[j_{s-1}]} & & & \\ & & & & L_i^{[j_s]} & n & \\ & & & & & \ddots & L_i^{[j_s]} \\ & & & & & & \ddots \end{pmatrix} \oplus \begin{pmatrix} L_i^{[j_{s+1}]} & n & & \\ & \ddots & L_i^{[j_{s+1}]} & \\ & & & \ddots \end{pmatrix}. \tag{2.2}$$

[1]The recent developments are explained in the introduction.

The representation matrix L_i, where $i = 1, 2, 3$, is a reducible representation of $SU(2)$, and obeys the following commutation relation:

$$[L_i, L_j] = \mathrm{i}\epsilon_{ijk}L^k. \tag{2.3}$$

$L_i^{[j_s]}$, where $s = -\infty, \cdots, \infty$, is the $(2j_s + 1) \times (2j_s + 1)$ representation matrix for the spin j_s irreducible representation of $SU(2)$, and obeys the following commutation relation:

$$[L_i^{[j_s]}, L_j^{[j_s]}] = \mathrm{i}\epsilon_{ijk}L^{[j_s]k}. \tag{2.4}$$

Then, the Casimir operator of $L_i^{[j_s]}$ is that

$$L_i^{[j_s]}L^{[j_s]i} = j_s(j_s + 1)\mathbf{1}_{2j_s+1}. \tag{2.5}$$

The matrices (2.2) can be interpreted as n sets of ∞ fuzzy spheres with the radius $\mu\sqrt{j_s(j_s + 1)}$, where all the fuzzy spheres are concentric. In order to make the connection between the supersymmetric Yang-Mills theory on $\mathbb{R} \times S^3$ and the plane wave matrix model, it is necessary to impose the following conditions:

$$\begin{gathered} j_s - j_t = \frac{1}{2}(s - t), \qquad j_s, j_t \to \infty, \\ s, t = -\infty, \cdots, \infty. \end{gathered} \tag{2.6}$$

Let us start with the plane wave matrix model which is defined by the following action:

$$\begin{aligned} S_{\mathrm{PW}} = \frac{1}{g_{\mathrm{PW}}^2}\int \frac{dt}{\mu^2}\,\mathrm{Tr}\Big\{&\frac{1}{2}(D_0X_i)^2 - \frac{1}{2}\Big(\mu X_i - \frac{\mathrm{i}}{2}\epsilon_{ijk}[X^j, X^k]\Big)^2 \\ &+ \frac{1}{2}(D_0X_m)^2 - \frac{\mu^2}{8}X_m^2 + \frac{1}{2}[X_i, X_m]^2 \\ &+ \frac{1}{4}[X_m, X_n]^2 + \frac{\mathrm{i}}{2}\bar{\lambda}\Gamma^0 D_0\lambda + \frac{3\mathrm{i}\mu}{8}\bar{\lambda}\Gamma^{123}\lambda \\ &- \frac{1}{2}\bar{\lambda}\Gamma^i[X_i, \lambda] - \frac{1}{2}\bar{\lambda}\Gamma^m[X_m, \lambda]\Big\}, \end{aligned} \tag{2.7}$$

where X and λ are vector and Majorana-Weyle spinor fields, and both fields are $N \times N$ Hermitian matrices. The vector indices i, j, k, and m, n run over as follows: $i, j, k = 1, 2, 3$ and $m, n = 4, \cdots, 9$. The covariant derivative is given by $D_0\mathcal{O} = \partial_0\mathcal{O} - \mathrm{i}[A_0, \mathcal{O}]$. The radius of S^3: r is fixed to $2/\mu$.

Let us consider such a large N limit as follows:

$$\begin{gathered} X_i(t) \to -\mu\nabla_i + B_i(t, \boldsymbol{x}), \qquad X_m(t) \to X_m(t, \boldsymbol{x}), \\ \lambda(t) \to \lambda(t, \boldsymbol{x}), \end{gathered} \tag{2.8}$$

where ∇_i and B_i are derivatives and space components of gauge fields on S^3 that are defined by Killing vectors (see Ref. [23] for a review on this subject):

$$\nabla_i = K_i^a\partial_a, \qquad B_i(t, \boldsymbol{x}) = K_i^a A_a(t, \boldsymbol{x}), \tag{2.9}$$

where $a = \theta, \phi, \psi$. The nonvanishing components of Killing vectors are given by

$$\begin{aligned} K_1^\theta &= \mu, \qquad K_2^\phi = \frac{\mu}{\sin\theta}, \\ K_2^\psi &= -\frac{\mu\cos\theta}{\sin\theta}, \qquad K_3^\psi = 1. \end{aligned} \tag{2.10}$$

From these Killing vectors, we obtain the metric tensor on S^3 as follows:

$$g^{ab} = K_i^a K_i^b. \tag{2.11}$$

For example, we consider the following term in the action of the plane wave matrix model:

$$\frac{1}{g_{\mathrm{PW}}^2}\int \frac{dt}{\mu^2}\,\mathrm{Tr}\Big\{-\frac{1}{2}\Big(\mu X_i - \frac{\mathrm{i}}{2}\epsilon_{ijk}[X^j, X^k]\Big)^2\Big\}. \tag{2.12}$$

By using (2.8), the part of (2.12) can be rewritten as follows:

$$\begin{aligned} -\frac{1}{2}\Big(&-\mu^2\nabla_i + \mu B_i - \frac{\mathrm{i}}{2}\epsilon_{ijk}(\mu^2[\nabla^j, \nabla^k] \\ &- \mu(\nabla^j B^k - \nabla^k B^j) + [B^j, B^k])\Big)^2. \end{aligned} \tag{2.13}$$

From the commutation relation for the derivatives on S^3:

$$[\nabla_i, \nabla_j] = \mathrm{i}\epsilon_{ijk}\nabla^k, \tag{2.14}$$

we can obtain the following relation:

$$K_i^a\partial_a K_j^b - K_j^a\partial_a K_i^b = \mathrm{i}\epsilon_{ijk}K_k^b. \tag{2.15}$$

Then, we can get the following equation by using (2.14) and (2.15):

$$-\frac{1}{4}g_{ac}g_{bd}F^{ab}F^{cd}, \tag{2.16}$$

where

$$F_{ab} = \partial_a A_b - \partial_b A_a - \mathrm{i}[A_a, A_b]. \tag{2.17}$$

Note that we also rescaled the derivatives on S^3 as follows:

$$\mathrm{i}\,\mu\partial_a \to \partial_a. \tag{2.18}$$

The coupling constant and Tr over matrices have the following correspondence in the large N limit:

$$\frac{2g_{\mathrm{PW}}^2}{\mu\mathrm{N}} \to \frac{g_{\mathrm{SYM}}}{16\pi^2}, \qquad \mathrm{Tr} \to \frac{N}{\mathrm{Vol}(S^3)n}\int d^3x\sqrt{g}\,\mathrm{tr}, \tag{2.19}$$

where

$$\mathrm{Vol}\,(S^3) = 2\pi^2 r^3, \tag{2.20}$$

and tr denotes the trace operation over $SU(n)$ gauge group. Then, we can rewrite (2.12) as follows:

$$\frac{2}{g_{\mathrm{SYM}}^2 n}\int d^4x\sqrt{g}\,\mathrm{tr}\Big\{-\frac{1}{4}g_{ac}g_{bd}F^{ab}F^{cd}\Big\}. \tag{2.21}$$

Similarly, taking the large N limit of the other terms in the action of the plane wave matrix model, we can obtain the action of supersymmetric Yang-Mills theory on $\mathbb{R} \times S^3$ as

follows:

$$S_{\rm SYM} = \frac{2}{g_{\rm SYM}^2 n}\int d^4x\sqrt{g}\,{\rm tr}\Big\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}D_\mu X_m D^\mu X_m - \frac{1}{12}RX_m^2+\frac{\rm i}{2}\bar\lambda\Gamma^\mu D_\mu\lambda-\frac{1}{2}\bar\lambda\Gamma^m[X_m,\lambda] + \frac{1}{4}[X_m,X_n]^2\Big\}, \tag{2.22}$$

where $\mu = 0, 1, 2, 3$, and R is the scalar curvature of S^3.

III. EFFECTIVE ACTION FOR PLANE WAVE MATRIX MODEL

In the preceding section, we have summarized formal arguments for equivalence between the gauge theory on $\mathbb{R}\times S^3$ and a certain vacuum configuration of the plane wave matrix model. However they are formal in the sense that they need to consider the large N limit. Therefore their validity is not automatic especially at the nonperturbative level, since we need to work with finite N. In this section, we work with finite N, namely, finite size matrices. To be precise, we introduce the two cutoffs in the theory with respect to the size and number of the concentric fuzzy spheres. We also put $n = 1$. In such a set up, we investigate the effective action perturbatively to check to what extent formal arguments can be justified.

A. One-loop effective action at zero temperature

In this subsection, we evaluate the one-loop effective action of the plane wave matrix model around S^3 background at zero temperature.

As in the ordinary background field method in quantum field theories, we decompose matrices X and λ into the backgrounds and quantum fluctuations, respectively, as follows:

$$X_i = p_i + x_i, \qquad X_m = p_m + x_m, \qquad \lambda = \chi + \varphi, \tag{3.1}$$

where p_i, p_m, and χ are backgrounds, and x_i, x_m, and φ are quantum fluctuations. Then, we substitute the decomposed matrices (3.1) into the action of the plane wave matrix model, and expand around backgrounds up to the fourth order with respect to quantum fluctuations. The expanded action is expressed as follows:

$$S_{\rm PW} = S_{\rm PW}^{(0)} + S_{\rm PW}^{(1)} + S_{\rm PW}^{(2)} + S_{\rm PW}^{(3)} + S_{\rm PW}^{(4)}, \tag{3.2}$$

where

$$S_{\rm PW}^{(0)} = \frac{1}{g_{\rm PW}^2\mu^2}\int dt\,{\rm Tr}\Big\{\frac{1}{2}(\partial_0 p_i)^2-\frac{1}{2}\mu^2 p_i^2 + \frac{\rm i}{2}\epsilon_{ijk}p^i[p^j,p^k]+\frac{1}{4}[p_i,p_j]^2+\frac{1}{2}(\partial_0 p_m)^2 - \frac{\mu^2}{8}p_m^2+\frac{1}{2}[p_i,p_m]^2+\frac{1}{4}[p_m,p_n]^2+\frac{\rm i}{2}\bar\chi\Gamma^0\partial_0\chi + \frac{3{\rm i}\mu}{8}\bar\chi\Gamma^{123}\chi-\frac{1}{2}\bar\chi\Gamma^i[p_i,\chi]-\frac{1}{2}\bar\chi\Gamma^m[p_m,\chi]\Big\}, \tag{3.3}$$

$$S_{\rm PW}^{(1)} = \frac{1}{g_{\rm PW}^2\mu^2}\int dt\,{\rm Tr}\Big\{-x_k\Big(\partial_0^2 p^k+\mu^2 p^k+\frac{3{\rm i}}{2}\mu\epsilon^{ijk}[p_i,p_j]+[p_i,[p^i,p^k]]+[p_m,[p^m,p^k]]-\frac{1}{2}\{\bar\chi\Gamma^k,\chi\}\Big) - x_n\Big(\partial_0^2 p^n+\frac{\mu^2}{4}p^n+[p_i,[p^i,p^n]]+[p_m,[p^m,p^n]]-\frac{1}{2}\{\bar\chi\Gamma^n,\chi\}\Big)-{\rm i}(\partial_0 p_i)[A_0,p^i]-{\rm i}(\partial_0 p_m)[A_0,p^m] - \frac{1}{2}\bar\chi\Gamma^0[A_0,\chi]+\bar\varphi\Big(\frac{\rm i}{2}\Gamma^0\partial_0\chi+\frac{3{\rm i}\mu}{8}\Gamma^{123}\chi-\frac{1}{2}\Gamma^i[p_i,\chi]-\frac{1}{2}\Gamma^m[p_m,\chi]\Big) + \Big(\frac{\rm i}{2}(\partial_0\bar\chi)\Gamma^0+\frac{3{\rm i}\mu}{8}\bar\chi\Gamma^{123}-\frac{1}{2}[\bar\chi\Gamma^i,p_i]-\frac{1}{2}[\bar\chi\Gamma^m,p_m]\Big)\varphi\Big\}, \tag{3.4}$$

$$S_{\rm PW}^{(2)} = \frac{1}{g_{\rm PW}^2\mu^2}\int dt\,{\rm Tr}\Big\{\frac{1}{2}(\partial_0 x_i)^2-{\rm i}(\partial_0 p_i)[A_0,x^i]-{\rm i}(\partial_0 x_i)[A_0,p^i]-\frac{1}{2}[A_0,p_i]^2-\frac{1}{2}\mu^2 x_i^2-\frac{3{\rm i}}{2}\mu\epsilon^{ijk}x_i[p_k,x_j] + \frac{1}{2}[p_i,x_j]^2-\frac{1}{2}[p_i,x^i]^2+[p_i,p_j][x^i,x^j]+\frac{1}{2}(\partial_0 x_m)^2-{\rm i}(\partial_0 p_m)[A_0,x^m]-{\rm i}(\partial_0 x_m)[A_0,p^m]-\frac{1}{2}[A_0,p_m]^2 - \frac{\mu^2}{8}x_m^2+\frac{1}{2}[p_i,x_m]^2+\frac{1}{2}[p_m,x_i]^2+2[p_i,p_m][x^i,x^m]-[p_i,x^i][p_m,x^m]+\frac{1}{2}[p_m,x_n]^2-\frac{1}{2}[p_m,x^m]^2 + [p_m,p_n][x^m,x^n]+\frac{\rm i}{2}\bar\varphi\Gamma^0\partial_0\varphi+\frac{1}{2}\bar\chi\Gamma^0[A_0,\varphi]+\frac{1}{2}\bar\varphi\Gamma^0[A_0,\chi]+\frac{3{\rm i}\mu}{8}\bar\varphi\Gamma^{123}\varphi-\frac{1}{2}\bar\chi\Gamma^i[x_i,\varphi]-\frac{1}{2}\bar\varphi\Gamma^i[p_i,\varphi] - \frac{1}{2}\bar\varphi\Gamma^i[x_i,\chi]-\frac{1}{2}\bar\chi\Gamma^m[x_m,\varphi]-\frac{1}{2}\bar\varphi\Gamma^m[p_m,\varphi]-\frac{1}{2}\bar\varphi\Gamma^m[x_m,\chi]\Big\}, \tag{3.5}$$

$$S_{\rm PW}^{(3)} = \frac{1}{g_{\rm PW}^2\mu^2}\int dt\,{\rm Tr}\Big\{-{\rm i}(\partial_0 x_i)[A_0, x^i] - [A_0, p_i][A_0, x^i] + \frac{\rm i}{2}\mu\epsilon^{ijk}x_i[x_j, x_k] + [p_i, x_j][x^i, x^j] - {\rm i}(\partial_0 x_m)[A_0, x^m]$$
$$- [A_0, p_m][A_0, x^m] + [p_i, x_m][x^i, x^m] + [p_m, x_i][x^m, x^i] + [p_m, x_n][x^m, x^n] + \frac{1}{2}\bar\varphi\Gamma^0[A_0, \varphi] - \frac{1}{2}\bar\varphi\Gamma^i[x_i, \varphi]$$
$$- \frac{1}{2}\bar\varphi\Gamma^m[x_m, \varphi]\Big\}, \tag{3.6}$$

$$S_{\rm PW}^{(4)} = \frac{1}{g_{\rm PW}^2\mu^2}\int dt\,{\rm Tr}\Big\{-\frac{1}{2}[A_0, x_i]^2 + \frac{1}{4}[x_i, x_j]^2 - \frac{1}{2}[A_0, x_m]^2 + \frac{1}{2}[x_i, x_m]^2 + \frac{1}{4}[x_m, x_n]^2\Big\}. \tag{3.7}$$

Since we need to fix the gauge invariance in the action, we add the gauge fixing and the Faddeev-Popov terms:

$$S_{\rm GF} = \frac{1}{g_{\rm PW}^2\mu^2}\int dt\,{\rm Tr}\Big\{-\frac{1}{2}(\partial_0 A_0 + {\rm i}[p_i, X^i] + {\rm i}[p_m, X^m])^2\Big\}, \tag{3.8}$$

$$S_{\rm FP} = \frac{1}{g_{\rm PW}^2\mu^2}\int dt\,{\rm Tr}\{-b\partial_0 D_0 c - b[p_i, [X^i, c]] - b[p_m, [X^m, c]]\}, \tag{3.9}$$

where c and b are ghost and antighost fields, respectively.

We substitute the matrices Y_i which are the classical solution of the plane wave matrix model for backgrounds as follows:

$$p_i = Y_i = -\mu L_i, \qquad p_m = 0, \qquad \chi = 0, \tag{3.10}$$

where

$$L_i = \begin{pmatrix} L_i^{[j_1]} & & & & \\ & \ddots & & & \\ & & L_i^{[j_s]} & & \\ & & & \ddots & \\ & & & & L_i^{[j_{2\Lambda}]} \end{pmatrix}. \tag{3.11}$$

Here, we introduce a cutoff on s at 2Λ and on the matrix size of $L_i^{[j_s]}$ at $2j_s + 1 = N_0 + s$, and the matrix size N of L_i is finite as follows:

$$N = (2j_1 + 1) + \cdots + (2j_s + 1) + \cdots + (2j_{2\Lambda} + 1). \tag{3.12}$$

Then, we can obtain the following action:

$$\tilde S_{\rm PW} = S_{\rm PW} + S_{\rm GF} + S_{\rm FP}$$
$$= \frac{1}{g_{\rm PW}^2\mu^2}\int dt\sum_{s,t}{\rm Tr}\Big\{-\frac{1}{2}x_i^{(s,t)}(-\delta^{ij}\partial_0^2 + \delta^{ij}\mu^2\mathcal{L}_i^2 + 2\mu^2[\mathcal{L}_i, \mathcal{L}_j] + \mu^2\delta^{ij} - 3{\rm i}\mu^2\epsilon^{ijk}\mathcal{L}_k)x_j^{(t,s)}$$
$$- \frac{1}{2}x_m^{(s,t)}(-\delta^{mn}\partial_0^2 + \delta^{mn}\mu^2\mathcal{L}_i^2 + \frac{\mu^2}{4}\delta^{mn})x_n^{(t,s)} - \frac{1}{2}A_0^{(s,t)}(-\partial_0^2 + \mu^2\mathcal{L}_i^2)A_0^{(t,s)} - b^{(s,t)}(-\partial_0^2 + \mu^2\mathcal{L}_i^2)c^{(t,s)}$$
$$- \frac{1}{2}\bar\varphi^{(s,t)}(-{\rm i}\Gamma^0\partial_0 - \mu\Gamma^i\mathcal{L}_i - \frac{3{\rm i}\mu}{4}\Gamma^{123})\varphi^{(t,s)} - {\rm i}(\partial_0 x_i^{(s,t)})[A_0, x^i]^{(t,s)} + \mu\mathcal{L}_i A_0^{(s,t)}[A_0, x^i]^{(t,s)}$$
$$+ \frac{\rm i}{2}\mu\epsilon^{ijk}x_i^{(s,t)}[x_j, x_k]^{(t,s)} - \mu\mathcal{L}_i x_j^{(s,t)}[x^i, x^j]^{(t,s)} - {\rm i}(\partial_0 x_m^{(s,t)})[A_0, x^m]^{(t,s)} - \mu\mathcal{L}_i x_m^{(s,t)}[x^i, x^m]^{(t,s)}$$
$$+ {\rm i}b^{(s,t)}\partial_0[A_0, c]^{(t,s)} + \mu b^{(s,t)}\mathcal{L}_i[x^i, c]^{(t,s)} + \frac{1}{2}\bar\varphi^{(s,t)}\Gamma^0[A_0, \varphi]^{(t,s)} + \frac{1}{2}\bar\varphi^{(s,t)}\Gamma^i[x_i, \varphi]^{(t,s)} + \frac{1}{2}\bar\varphi^{(s,t)}\Gamma^m[x_m, \varphi]^{(t,s)}$$
$$+ \frac{1}{2}[A_0, x_i]^{(s,t)2} + \frac{1}{4}[x_i, x_j]^{(s,t)2} + \frac{1}{2}[A_0, x_m]^{(s,t)2} + \frac{1}{2}[x_i, x_m]^{(s,t)2} + \frac{1}{4}[x_m, x_n]^{(s,t)2}\Big\}, \tag{3.13}$$

where the suffix (s, t) represents the (s, t) block in the $N \times N$ matrix. We introduce the following operation:

$$\mathcal{L}_i M = [L_i, M], \tag{3.14}$$

which acts on a $(2j_s + 1) \times (2j_t + 1)$ matrix M. Note that the above action is obtained after the Wick rotation for the time components as follows:

$$t \to {\rm i}t, \qquad A_0 \to {\rm i}A_0, \qquad \Gamma^0 \to {\rm i}\Gamma^0. \tag{3.15}$$

The classical action $S_{\rm PW}^{(0)}$ vanishes for the backgrounds we consider here, and the first order action with respect to the quantum fluctuations $S_{\rm PW}^{(1)}$ also vanishes, because the backgrounds satisfy the equations of motion for the plane wave matrix model. We need the quadratic action with

respect to the quantum fluctuations $S^{(2)}_{\rm PW}$ to calculate the one-loop effective action of the plane wave matrix model. To simplify the following calculations, we introduce the notations as follows:

$$p_0 = -\mathrm{i}\partial_0, \qquad p_i = -\mu L_i, \qquad p_m = 0. \tag{3.16}$$

Then, the action is given by

$$\begin{aligned}\tilde{S}^{\text{1-loop}}_{\rm PW} = \frac{2\pi N_0}{g^2_{\rm PW}\mu^2}\sum_{s,t}\sum_{J,M}\Bigg[\sum_l\Bigg\{&-\frac{1}{2}x^{(s,t)}_{ilJM}(\delta^{ij}\mathcal{P}^2_{\rm B} + 2\mathrm{i}\mathcal{F}^{ij}_{\rm B} \\ &+ \mu^2\delta^{ij} + 3\mathrm{i}\mu\epsilon^{ijk}\mathcal{P}_{{\rm B}k})x^{(s,t)\dagger}_{jlJM} \\ &- \frac{1}{2}x^{(s,t)}_{mlJM}\left(\delta^{mn}\mathcal{P}^2_{\rm B} + \frac{\mu^2}{4}\delta^{mn}\right)x^{(s,t)\dagger}_{nlJM} \\ &- \frac{1}{2}A^{(s,t)}_{0lJM}(\mathcal{P}^2_{\rm B})A^{(s,t)\dagger}_{0lJM} - b^{(s,t)}_{lJM}(\mathcal{P}^2_{\rm B})c^{(s,t)\dagger}_{lJM}\Bigg\} \\ &+ \sum_h\left\{-\frac{1}{2}\bar{\varphi}^{(s,t)}_{hJM}\left(\Gamma^\mu\mathcal{P}_{{\rm F}\mu} + \frac{3\mathrm{i}\mu}{4}\Gamma^{123}\right)\varphi^{(s,t)\dagger}_{hJM}\right\}\Bigg],\end{aligned} \tag{3.17}$$

where

$$\begin{gathered}\mathcal{P}_i M = [p_i, M], \qquad \mathcal{F}_{ij}M = [f_{ij}, M], \\ f_{ij} = -\mathrm{i}[p_i, p_j],\end{gathered} \tag{3.18}$$

and the index μ runs from 0 to 3.

By using the above action $\tilde{S}^{\text{1-loop}}_{\rm PW}$, we can calculate the one-loop effective action of the plane wave matrix model on S^3 as follows:

$$W = -\log\int dx_i dx_m dA_0 dbdcd\varphi \mathrm{e}^{-\tilde{S}^{\text{1-loop}}_{\rm PW}}. \tag{3.19}$$

First, we evaluate the bosonic parts of the effective action as follows:

$$\begin{aligned}W_{\rm B} = \sum_{s,t}\sum_l\Bigg\{&\frac{1}{2}\,\mathrm{Tr}\log(\delta^{ij}\mathcal{P}^2_{\rm B} + 2\mathrm{i}\mathcal{F}^{ij}_{\rm B} + \mu^2\delta^{ij} \\ &+ 3\mathrm{i}\mu\epsilon^{ijk}\mathcal{P}_{{\rm B}k}) + \frac{1}{2}\,\mathrm{Tr}\log\left(\delta^{mn}\mathcal{P}^2_{\rm B} + \frac{\mu^2}{4}\delta^{mn}\right) \\ &+ \frac{1}{2}\,\mathrm{Tr}\log(\mathcal{P}^2_{\rm B}) - \mathrm{Tr}\log(\mathcal{P}^2_{\rm B})\Bigg\}.\end{aligned} \tag{3.20}$$

We expand the bosonic parts (3.20) of the effective action into the inverse power series of $\mathcal{P}^2_{\rm B} = (\omega^2_l + \mu^2 J(J+1))$. In this way, we obtain the leading term of the bosonic parts of the one-loop effective action as follows:

$$\begin{aligned}W_{\rm B} \sim \sum_{s,t}\sum_l\Bigg\{&4\,\mathrm{Tr}\log(\mathcal{P}^2_{\rm B}) + \frac{9\mu^2}{4}\,\mathrm{Tr}\frac{1}{\mathcal{P}^2_{\rm B}} \\ &- \frac{\mu^2}{2}\,\mathrm{Tr}\mathcal{P}^2_{{\rm B}i}\left(\frac{1}{\mathcal{P}^2_{\rm B}}\right)^2\Bigg\},\end{aligned} \tag{3.21}$$

where $\mathcal{P}^2_{{\rm B}i} = \mu^2 J(J+1)$. Then, we evaluate the fermionic parts of the effective action as follows:

$$\begin{aligned}W_{\rm F} = -\sum_{s,t}\sum_h\frac{1}{4}\,\mathrm{Tr}\log\Bigg(&\mathcal{P}^2_{\rm F} + \frac{\mathrm{i}}{2}\Gamma^{\mu\nu}\mathcal{F}_{{\rm F}\mu\nu} \\ &- \frac{3\mathrm{i}}{4}\Gamma^{\mu\nu}\mathcal{F}_{{\rm F}\mu\nu} + \frac{9\mu^2}{16}\Bigg).\end{aligned} \tag{3.22}$$

Similarly, we expand the fermionic parts (3.22) of the effective action into the power series of $\mathcal{P}^2_{\rm F} = (\omega^2_h + \mu^2 J(J+1))$. So, we obtain the leading term of the fermionic parts of the one-loop effective action as follows:

$$\begin{aligned}W_{\rm F} \sim \sum_{s,t}\sum_h\Bigg\{&-4\,\mathrm{Tr}\log(\mathcal{P}^2_{\rm F}) - \frac{9\mu^2}{4}\,\mathrm{Tr}\frac{1}{\mathcal{P}^2_{\rm F}} \\ &+ \frac{\mu^2}{2}\,\mathrm{Tr}\mathcal{P}^2_{{\rm F}i}\left(\frac{1}{\mathcal{P}^2_{\rm F}}\right)^2\Bigg\},\end{aligned} \tag{3.23}$$

where $\mathcal{P}^2_{{\rm F}i} = \mu^2 J(J+1)$. Therefore, we find that the one-loop effective action of the plane wave matrix model on S^3 vanishes to the next leading order in this expansion:

$$W = W_{\rm B} + W_{\rm F} \sim 0. \tag{3.24}$$

In fact, it vanishes exactly due to supersymmetry [32].

B. Free energy of $\mathcal{N} = 4$ supersymmetric Yang-Mills on S^3

In this subsection, we calculate the effective action of the plane wave matrix model on S^3 at finite temperature up to the two-loop level. In order to study the plane wave matrix model on S^3 at finite temperature, we compactify the Eucliedean time direction with a periodicity $\beta = 1/T$, where T is a temperature. Thus, we impose the constraint of periodicity for the bosonic, ghost and antighost fields as follows:

$$\begin{aligned}&x^{(s,t)}_i(0) = x^{(s,t)}_i(\beta), \qquad x^{(s,t)}_m(0) = x^{(s,t)}_m(\beta), \\ &A^{(s,t)}_0(0) = A^{(s,t)}_0(\beta), \qquad c^{(s,t)}(0) = c^{(s,t)}(\beta), \\ &b^{(s,t)}(0) = b^{(s,t)}(\beta).\end{aligned} \tag{3.25}$$

So, we can obtain the conditions for frequencies in (3.20) as follows:

$$\omega_l = 2\pi l T, \tag{3.26}$$

where l is the integer. On the other hand, we impose the constraint of antiperiodicity for the fermion fields as follows:

$$\varphi^{(s,t)}(0) = -\varphi^{(s,t)}(\beta), \tag{3.27}$$

and hence

$$\omega_h = 2\pi h T, \tag{3.28}$$

where h is the half-integers.

Therefore, we can obtain the one-loop effective action of the plane wave matrix model on S^3 at finite temperature as follows:

$$\hat{W}^{\text{1-loop}} = \sum_{s,t}\Big\{\sum_{l} 4\,\mathrm{Tr}\log(\mathcal{P}_{\mathrm{B}}^2) - \sum_{h} 4\,\mathrm{Tr}\log(\mathcal{P}_{\mathrm{F}}^2) + \sum_{l}\frac{9\mu^2}{4}\,\mathrm{Tr}\frac{1}{\mathcal{P}_{\mathrm{B}}^2} - \sum_{h}\frac{9\mu^2}{4}\,\mathrm{Tr}\frac{1}{\mathcal{P}_{\mathrm{F}}^2} - \sum_{l}\frac{\mu^2}{2}\,\mathrm{Tr}\mathcal{P}_{\mathrm{B}i}^2\Big(\frac{1}{\mathcal{P}_{\mathrm{B}}^2}\Big)^2 + \sum_{h}\frac{\mu^2}{2}\,\mathrm{Tr}\mathcal{P}_{\mathrm{F}i}^2\Big(\frac{1}{\mathcal{P}_{\mathrm{F}}^2}\Big)^2\Big\}. \tag{3.29}$$

However, since the supersymmetry is broken at finite temperature, the contributions from the bosons and fermions do not cancel each other. For example, we consider the leading terms of the one-loop effective action as follows:

$$\hat{W}_{(0)}^{\text{1-loop}} = \sum_{s,t}\Big\{\sum_{l} 4\,\mathrm{Tr}\log((2\pi lT)^2 + \mu^2 J(J+1)) - \sum_{h} 4\,\mathrm{Tr}\log((2\pi hT)^2 + \mu^2 J(J+1))\Big\}. \tag{3.30}$$

It is easy to calculate the sums over l and h by using the following formulas:

$$\sum_{l=-\infty}^{\infty}\log(l^2\pi^2 + z^2) - \sum_{l=1}^{\infty} 2\log(l^2\pi^2) = 2\,\mathrm{logsinh}z, \tag{3.31}$$

$$\sum_{h=-\infty}^{\infty}\log(h^2\pi^2 + z^2) - \sum_{h=1/2}^{\infty} 2\log(h^2\pi^2) = 2\,\mathrm{logcosh}z, \tag{3.32}$$

where z is the complex number. We may discard the infinite constants which do not depend on physical parameters. Thus, we can obtain the leading terms of the one-loop effective action as follows:

$$\hat{W}_{(0)}^{\text{1-loop}} = \sum_{s,t}\Big\{8\,\mathrm{Tr}\,\mathrm{logsinh}\Big(\frac{\mu\sqrt{J(J+1)}}{2T}\Big) - 8\,\mathrm{Tr}\,\mathrm{logcosh}\Big(\frac{\mu\sqrt{J(J+1)}}{2T}\Big)\Big\} = \sum_{s,t}\sum_{J,M} 8\log\Big(\frac{\exp(\mu\sqrt{J(J+1)}/T) - 1}{\exp(\mu\sqrt{J(J+1)}/T) + 1}\Big). \tag{3.33}$$

In analogy with the large N reduced model on a flat background, we rewrite the summations in (3.33) as the summations of degrees of freedom on S^3 $(J, M, \tilde{M})$ and an extra degree of freedom (s) as follows:

$$\hat{W}_{(0)}^{\text{1-loop}} = \sum_{s}\sum_{J,M,\tilde{M}} 8\log\Big(\frac{\exp(\mu\sqrt{J(J+1)}/T) - 1}{\exp(\mu\sqrt{J(J+1)}/T) + 1}\Big), \tag{3.34}$$

where $\tilde{M} = \frac{1}{2}(s - t)$. We have introduced a cutoff such that $s < 2\Lambda$, so that the maximal value of J and $\tilde{M}$ are N_0 and Λ, respectively. Then we separate the summation over J into two parts at the value Λ as follows:

$$\sum_{s=1}^{2\Lambda}\sum_{J=0}^{\Lambda}\sum_{M=-J}^{J}\sum_{\tilde{M}=-J}^{J} 8\log\Big(\frac{\exp(\mu\sqrt{J(J+1)}/T) - 1}{\exp(\mu\sqrt{J(J+1)}/T) + 1}\Big) + \sum_{s=1}^{2\Lambda}\sum_{J=\Lambda+1/2}^{N_0}\sum_{M=-J}^{J}\sum_{\tilde{M}=-\Lambda}^{\Lambda} 8\log\Big(\frac{\exp(\mu\sqrt{J(J+1)}/T) - 1}{\exp(\mu\sqrt{J(J+1)}/T) + 1}\Big). \tag{3.35}$$

The second term in the above expression can be safely neglected since we assume that

$$T \ll \Lambda. \tag{3.36}$$

If we further divide this effective action by the overall factor $\sum_s$, it agrees with that of the supersymmetric Yang-Mills theory on S^3. In this sense, the plane wave matrix model is a large N reduced model of the supersymmetric Yang-Mills theory on S^3.

In this way, we can obtain that

$$\hat{W}_{(0)}^{\text{1-loop}} = \sum_{J=0}^{\infty}\sum_{M=-J}^{J}\sum_{\tilde{M}=-J}^{J} 8\log\Big(\frac{\exp(\mu\sqrt{J(J+1)}/T) - 1}{\exp(\mu\sqrt{J(J+1)}/T) + 1}\Big) = \sum_{J=0}^{\infty} 8\log\Big(\frac{\exp(\mu\sqrt{J(J+1)}/T) - 1}{\exp(\mu\sqrt{J(J+1)}/T) + 1}\Big)(2J+1)^2 = \sum_{k=0}^{\infty} 8\log\Big(\frac{\exp(\sqrt{k(k+2)}/rT) - 1}{\exp(\sqrt{k(k+2)}/rT) + 1}\Big)(k+1)^2, \tag{3.37}$$

where we set $k = 2J$. Here we take the high temperature limit such that the temperature is much larger than the inverse radius of S^3:

$$T \gg \frac{1}{r}. \tag{3.38}$$

Thus, this limit represent a flat space limit. The summation over k can be well approximated by the integrals over

$$x = \frac{\sqrt{k(k+2)}}{rT}. \tag{3.39}$$

We can obtain the following equation:

$$\int_0^\infty dx 8 \log\left(\frac{e^x - 1}{e^x + 1}\right)\left(r^3 T^3 x^2 + \frac{1}{2} rT\right). \tag{3.40}$$

This integral is evaluated as

$$\hat{W}_{(0)}^{\text{1-loop}} = -\frac{\pi^4}{3} r^3 T^3 - \pi^2 rT + \mathcal{O}\left(\frac{1}{T}\right). \tag{3.41}$$

Similarly, we evaluate the subleading terms of the one-loop effective action:

$$\hat{W}_{(1)}^{\text{1-loop}} = \sum_{s,t}\Bigg\{\sum_l \frac{9\mu^2}{4} \operatorname{Tr} \frac{1}{(2\pi lT)^2 + \mu^2 J(J+1)} - \sum_h \frac{9\mu^2}{4} \operatorname{Tr} \frac{1}{(2\pi hT)^2 + \mu^2 J(J+1)} - \sum_l \frac{\mu^4}{2} \operatorname{Tr} J(J+1)\left(\frac{1}{(2\pi lT)^2 + \mu^2 J(J+1)}\right)^2 + \sum_h \frac{\mu^4}{2} \operatorname{Tr} J(J+1)\left(\frac{1}{(2\pi hT)^2 + \mu^2 J(J+1)}\right)^2\Bigg\}. \tag{3.42}$$

By taking the high temperature limit such that the temperature is much larger than the inverse radius of S^3, it can be evaluated as follows:

$$\hat{W}_{(1)}^{\text{1-loop}} = \frac{3\pi^2}{2} rT + \mathcal{O}\left(\frac{1}{T}\right). \tag{3.43}$$

In order to examine to what extent a plane wave matrix model can explore the planar sector of super Yang-Mills theory on S^3, we further calculate the two-loop effective action of the plane wave matrix model at finite temperature. We describe the detailed calculations of the two-loop effective action in the appendix. The main conclusion is that the equivalence is valid in the high temperature limit as the contributions from the nonplanar diagrams can be neglected in comparison to those from the planar diagrams in such a limit. The two-loop effective action is given by

$$\hat{W}^{\text{2-loop}} = 2\pi^4 \frac{g_{\text{PW}}^2 \mu^2}{N_0} r^6 T^3. \tag{3.44}$$

Here, recalling the following relation between the coupling constants in Sec. II:

$$\lim_{N_0 \to \infty} \frac{2 g_{\text{PW}}^2}{\mu N_0} = \frac{g_{\text{SYM}}^2}{16\pi^2}, \tag{3.45}$$

we obtain the following equation:

$$\hat{W}^{\text{2-loop}} = \frac{\pi^2}{2} g_{\text{SYM}}^2 r^3 T^3. \tag{3.46}$$

We summarize the effective action of the plane wave matrix model at a finite temperature up to the two-loop level:

$$\hat{W}/\text{Vol}(S^3) = -\frac{\pi^2}{6} T^3 + \frac{1}{4} g_{\text{SYM}}^2 T^3 + \frac{1}{4r^2} T + \mathcal{O}\left(\frac{1}{T}\right), \tag{3.47}$$

where we have divided the effective action by the volume of S^3. This effective action is equal to β times the free energy density of the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory on S^3 [33–37].

IV. CONCLUSIONS AND DISCUSSIONS

In this paper, we have investigated the properties of the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory on S^3 at finite temperature by using the plane wave matrix model.

We have formally derived the action of the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory on $\mathbb{R} \times S^3$ from the action of the plane wave matrix model by taking the large N limit. Furthermore, we have calculated the effective action of the plane wave matrix model around the S^3 configuration at the two-loop level. We have found that the effective action of the plane wave matrix model agrees with the free energy of the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory on S^3 at the two-loop level in the high temperature limit. Therefore, we can conclude that the nonperturbative properties of the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory on S^3 at finite temperature can be explored by the plane wave matrix model. Our results serve as a nontrivial check that the plane wave matrix model can be regarded as a large N reduced model of the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory on $\mathbb{R} \times S^3$. However, the nonplanar contributions at the two-loop level differ from those on S^3. They are rather of S^2 type since the propagators carry vanishing $\tilde{M}$ in these contributions. They can be neglected only in the high temperature limit. In this sense, a construction of a large N reduced model on a curved manifold (S^3 in this case) is successful only in a flat manifold limit.

It is interesting to investigate nonperturbative properties of the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory on $S^1 \times S^3$ in connection to AdS/CFT correspondence. This corre-

spondence states that the large N $\mathcal{N} = 4$ supersymmetric Yang-Mills theory on $\mathbb{R} \times S^3$ at strong coupling region is solved in terms of the type-IIB supergravity on $AdS_5 \times S^5$. We have shown that the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory on $S^1 \times S^3$ at weak coupling region is consistent with the plane wave matrix model at quantum level. We hope to evaluate the behavior of the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory on $S^1 \times S^3$ at strong coupling region by using the plane wave matrix model.

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APPENDIX A: TWO-LOOP EFFECTIVE ACTION OF PLANE WAVE MATRIX MODEL

In this appendix, we calculate the two-loop effective action of the plane wave matrix model on S^3 at finite temperature. The effective action $\hat{W}$ is evaluated as follows:

$$\hat{W} = -\log \int dx_i dx_m dA_0 dc db d\varphi e^{-\tilde{S}_{PW}} = \hat{W}^{1\text{-loop}} + \hat{W}^{2\text{-loop}}, \tag{A1}$$

where

$$\hat{W}^{2\text{-loop}} = -\log\left(\frac{\int dx_i dx_m dA_0 dc db d\varphi e^{-\tilde{S}_{PW}^{2\text{-loop}}} e^{-\tilde{S}_{PW}^{1\text{-loop}}}}{\int dx_i dx_m dA_0 dc db d\varphi e^{-\tilde{S}_{PW}^{1\text{-loop}}}}\right) \equiv \langle e^{-\tilde{S}_{PW}^{2\text{-loop}}} \rangle_{1PI}, \tag{A2}$$

and

$$\begin{aligned}\tilde{S}_{PW}^{2\text{-loop}} = \frac{1}{g_{PW}^2 \mu^2} \int_0^\beta dt \sum_{s,t} \mathrm{Tr}\Big\{ & i(\partial_0 x_i^{(s,t)})[A_0, x^i]^{(t,s)} - \mu(\mathcal{L}_i A_0^{(s,t)})[A_0, x^i]^{(t,s)} + \frac{i}{2}\mu \epsilon^{ijk} x_i^{(s,t)}[x_j, x_k]^{(t,s)} + \mu(\mathcal{L}_i x_j^{(s,t)})[x^i, x^j]^{(t,s)} \\ & + i(\partial_0 x_m^{(s,t)})[A_0, x^m]^{(t,s)} + \mu(\mathcal{L}_i x_m^{(s,t)})[x^i, x^m]^{(t,s)} + i(\partial_0 b^{(s,t)})[A_0, c]^{(t,s)} + \mu(\mathcal{L}_i b^{(s,t)})[x^i, c]^{(t,s)} \\ & - \frac{1}{2}\bar{\varphi}^{(s,t)}\Gamma^0[A_0, \varphi]^{(t,s)} - \frac{1}{2}\bar{\varphi}^{(s,t)}\Gamma^i[x_i, \varphi]^{(t,s)} - \frac{1}{2}\bar{\varphi}^{(s,t)}\Gamma^m[x_m, \varphi]^{(t,s)} - \frac{1}{2}[A_0, x_i]^{(s,t)2} - \frac{1}{4}[x_i, x_j]^{(s,t)2} \\ & - \frac{1}{2}[A_0, x_m]^{(s,t)2} - \frac{1}{2}[x_i, x_m]^{(s,t)2} - \frac{1}{4}[x_m, x_n]^{(s,t)2}\Big\}. \end{aligned} \tag{A3}$$

We define $\langle \cdots \rangle_{1PI}$ as a summation over only 1-particle-irreducible (1PI) diagrams. To simplify the following calculations, we combine the action $\tilde{S}_{PW}^{2\text{-loop}}$ as follows:

$$\begin{aligned}\tilde{S}_{PW}^{2\text{-loop}} = \frac{1}{g_{PW}^2 \mu^2} \int_0^\beta dt \sum_{s,t} \mathrm{Tr}\Big\{ & -(\mathcal{P}_I x_J^{(s,t)})[x^I, x^J]^{(t,s)} \\ & + \frac{i}{2}\mu \epsilon^{ijk} x_i^{(s,t)}[x_j, x_k]^{(t,s)} - (\mathcal{P}_I b^{(s,t)})[x^I, c]^{(t,s)} \\ & - \frac{1}{2}\bar{\varphi}^{(s,t)}\Gamma^I[x_I, \varphi]^{(t,s)} - \frac{1}{4}[x_I, x_J]^{(s,t)2}\Big\}, \end{aligned} \tag{A4}$$

where the index $I = 0, \cdots, 9$, and we set that

$$p_0 = -i\partial_0, \qquad p_i = -\mu L_i, \qquad p_m = 0, \qquad x_0 = A_0. \tag{A5}$$

Now, there are five 1PI diagrams to evaluate which are illustrated in Fig. 1. The diagrams (a), (b), and (c) represent the contributions from gauge fields, and (c) involves the Myers type interaction. The diagrams (d) and (e) represent the contributions from ghost and fermion fields, respectively.

1. Propagators

a. Bosonic propagators

First of all, we derive the bosonic propagators of the plane wave matrix model. From the quadratic terms for the gauge fields $x_i^{(s,t)}$ in the action (3.13), we can read out the propagators of gauge boson modes $x_{iJM}^{(s,t)}$ as follows:

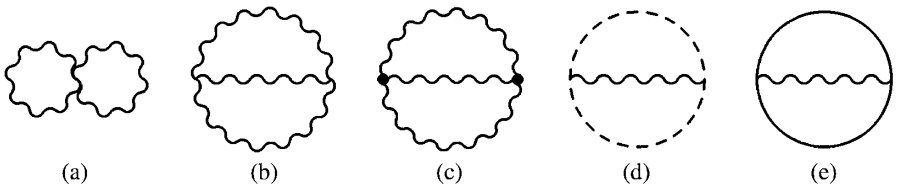

FIG. 1. Feynman diagrams of two-loop corrections to the effective action [6].

$$\frac{1}{g_{\mathrm{PW}}^2\mu^2}\int_0^\beta dt\sum_{s,t}\mathrm{Tr}\Big\{-\frac{1}{2}x_i^{(s,t)}(-\delta^{ij}\partial_0^2+\delta^{ij}\mu^2\mathcal{L}_k^2+\mu^2\delta^{ij}-\mathrm{i}\mu^2\epsilon^{ijk}\mathcal{L}_k)x_j^{(t,s)}\Big\}$$
$$=\frac{1}{g_{\mathrm{PW}}^2\mu^2}\int_0^\beta dt\sum_{s,t}\mathrm{Tr}\Big\{-\frac{1}{2}\sum_{l_1}\sum_{J_1,M_1}\mathrm{e}^{\mathrm{i}\omega_{l_1}t}x_{il_1J_1M_1}^{(s,t)}\otimes\hat{Y}_{J_1M_1}^{(j_s,j_t)}(\delta^{ij}\omega_{l_2}^2+\delta^{ij}\mu^2J_2(J_2+1)+\mu^2\delta^{ij}-\mathrm{i}\mu^2\epsilon^{ijk}\mathcal{L}_k)$$
$$\times\sum_{l_2}\sum_{J_2,M_2}\mathrm{e}^{\mathrm{i}\omega_{l_2}t}x_{jl_2J_2M_2}^{(t,s)}\otimes\hat{Y}_{J_2M_2}^{(j_t,j_s)}\Big\}$$
$$=\frac{1}{2}\sum_{s,t}\sum_{l_1,l_2}\sum_{J_1,M_1}\sum_{J_2,M_2}x_{il_1J_1M_1}^{(s,t)}(-1)^{M_1-(j_s-j_t)}\frac{-\beta N_0}{g_{\mathrm{PW}}^2\mu^2}\delta_{l_1-l_2}\delta_{J_1J_2}\delta_{M_1-M_2}(\delta^{ij}\omega_{l_2}^2+\delta^{ij}\mu^2J_2(J_2+1)+\mu^2\delta^{ij}$$
$$-\mathrm{i}\mu^2\epsilon^{ijk}\mathcal{L}_k)x_{jl_2J_2M_2}^{(t,s)}. \tag{A6}$$

Note that the quantum fluctuations are expanded by a plane wave and a fuzzy spherical harmonics[2] as follows:

$$x_i^{(s,t)}(t)=\sum_{l=-\infty}^{\infty}\sum_{J=0}^{\infty}\sum_{M=-J}^{J}\mathrm{e}^{\mathrm{i}\omega_l t}x_{ilJM}^{(s,t)}\otimes\hat{Y}_{JM}^{(j_s,j_t)}. \tag{A7}$$

Then, we can obtain the propagator of gauge boson modes as follows:

$$\langle x_{il_1J_1M_1}^{(j_s,j_t)}x_{jl_2J_2M_2}^{(j_t,j_s)}\rangle=\frac{g_{\mathrm{PW}}^2\mu^2}{\beta N_0}\frac{(-1)^{M_1-(j_s-j_t)}}{\delta^{ij}\omega_{l_2}^2+\delta^{ij}\mu^2J_2(J_2+1)+\mu^2\delta^{ij}-\mathrm{i}\mu^2\epsilon^{ijk}\mathcal{L}_k}\delta_{l_1-l_2}\delta_{J_1J_2}\delta_{M_1-M_2}. \tag{A8}$$

We expand this propagator into the power series of $(\omega_l^2+\mu^2J(J+1))=\mathcal{P}_{\mathrm{B}}^2$ as follows:

$$\frac{1}{\delta^{ij}\omega_{l_2}^2+\delta^{ij}\mu^2J_2(J_2+1)+\mu^2\delta^{ij}-\mathrm{i}\mu^2\epsilon^{ijk}\mathcal{L}_k}=\frac{1}{\mathcal{P}_{\mathrm{B}}^2}\delta^{ij}-\mathrm{i}\mu\Big(\frac{1}{\mathcal{P}_{\mathrm{B}}^2}\Big)^2\epsilon^{ijk}\mathcal{P}_{\mathrm{B}k}-\mu^2\Big(\frac{1}{\mathcal{P}_{\mathrm{B}}^2}\Big)^3\mathcal{P}_{\mathrm{B}}^i\mathcal{P}_{\mathrm{B}}^j+\mathcal{O}\Big(\frac{1}{\mathcal{P}_{\mathrm{B}}^5}\Big). \tag{A9}$$

Therefore, the propagator of gauge boson fields are given by

$$\langle x_i^{(s,t)}(t_1)x_j^{(t,s)}(t_2)\rangle=\sum_{l_1,l_2}\sum_{J_1,M_1}\sum_{J_2,M_2}\mathrm{e}^{\mathrm{i}\omega_{l_1}t_1}\mathrm{e}^{\mathrm{i}\omega_{l_2}t_2}\langle x_{il_1J_1M_1}^{(s,t)}x_{jl_2J_2M_2}^{(t,s)}\rangle\otimes\hat{Y}_{J_1M_1}^{(j_s,j_t)}\hat{Y}_{J_2M_2}^{(j_t,j_s)}$$
$$\sim\frac{g_{\mathrm{PW}}^2\mu^2}{\beta N_0}\sum_{l=-\infty}^{\infty}\sum_{J=0}^{\infty}\sum_{M=-J}^{J}\Big\{\frac{1}{\mathcal{P}_{\mathrm{B}}^2}\delta_{ij}-\mathrm{i}\mu\Big(\frac{1}{\mathcal{P}_{\mathrm{B}}^2}\Big)^2\epsilon_{ijk}\mathcal{P}_{\mathrm{B}}^k-\mu^2\Big(\frac{1}{\mathcal{P}_{\mathrm{B}}^2}\Big)^3\mathcal{P}_{\mathrm{B}i}\mathcal{P}_{\mathrm{B}j}\Big\}$$
$$\times\mathrm{e}^{\mathrm{i}\omega_l(t_1-t_2)}(-1)^{M-(j_s-j_t)}\hat{Y}_{JM}^{(j_s,j_t)}\hat{Y}_{J-M}^{(j_t,j_s)}, \tag{A10}$$

where we note that $\omega_{-l}=-2\pi lT=-\omega_l$.

In the same way, we can read off the other propagators of bosonic fields as follows:

$$\langle x_m^{(s,t)}(t_1)x_n^{(t,s)}(t_2)\rangle\sim\frac{g_{\mathrm{PW}}^2\mu^2}{\beta N_0}\sum_{l=-\infty}^{\infty}\sum_{J=0}^{\infty}\sum_{M=-J}^{J}\Big\{\frac{1}{\mathcal{P}_{\mathrm{B}}^2}\delta_{mn}-\frac{\mu^2}{4}\Big(\frac{1}{\mathcal{P}_{\mathrm{B}}^2}\Big)^2\delta_{mn}\Big\}\mathrm{e}^{\mathrm{i}\omega_l(t_1-t_2)}(-1)^{M-(j_s-j_t)}\hat{Y}_{JM}^{(j_s,j_t)}\hat{Y}_{J-M}^{(j_t,j_s)}, \tag{A11}$$

$$\langle A_0^{(s,t)}(t_1)A_0^{(t,s)}(t_2)\rangle=\frac{g_{\mathrm{PW}}^2\mu^2}{\beta N_0}\sum_{l=-\infty}^{\infty}\sum_{J=0}^{\infty}\sum_{M=-J}^{J}\frac{1}{\mathcal{P}_{\mathrm{B}}^2}\mathrm{e}^{\mathrm{i}\omega_l(t_1-t_2)}(-1)^{M-(j_s-j_t)}\hat{Y}_{JM}^{(j_s,j_t)}\hat{Y}_{J-M}^{(j_t,j_s)}, \tag{A12}$$

$$\langle c^{(s,t)}(t_1)b^{(t,s)}(t_2)\rangle=\frac{g_{\mathrm{PW}}^2\mu^2}{\beta N_0}\sum_{l=-\infty}^{\infty}\sum_{J=0}^{\infty}\sum_{M=-J}^{J}\frac{1}{\mathcal{P}_{\mathrm{B}}^2}\mathrm{e}^{\mathrm{i}\omega_l(t_1-t_2)}(-1)^{M-(j_s-j_t)}\hat{Y}_{JM}^{(j_s,j_t)}\hat{Y}_{J-M}^{(j_t,j_s)}, \tag{A13}$$

where we expand the other bosonic fields by the plane wave and the fuzzy spherical harmonics as follows:

[2]The fuzzy spherical harmonics is based on the work [18].

$$
\begin{aligned}
x_m^{(s,t)}(t) &= \sum_{l=-\infty}^{\infty}\sum_{J=0}^{\infty}\sum_{M=-J}^{J} \mathrm{e}^{\mathrm{i}\omega_l t} x_{mlJM}^{(s,t)} \otimes \hat{Y}_{JM}^{(j_s,j_t)}, \\
A_0^{(s,t)}(t) &= \sum_{l=-\infty}^{\infty}\sum_{J=0}^{\infty}\sum_{M=-J}^{J} \mathrm{e}^{\mathrm{i}\omega_l t} A_{0lJM}^{(s,t)} \otimes \hat{Y}_{JM}^{(j_s,j_t)}, \\
c^{(s,t)}(t) &= \sum_{l=-\infty}^{\infty}\sum_{J=0}^{\infty}\sum_{M=-J}^{J} \mathrm{e}^{\mathrm{i}\omega_l t} c_{lJM}^{(s,t)} \otimes \hat{Y}_{JM}^{(j_s,j_t)}, \\
b^{(s,t)}(t) &= \sum_{l=-\infty}^{\infty}\sum_{J=0}^{\infty}\sum_{M=-J}^{J} \mathrm{e}^{\mathrm{i}\omega_l t} b_{lJM}^{(s,t)} \otimes \hat{Y}_{JM}^{(j_s,j_t)}.
\end{aligned}
\tag{A14}
$$

Moreover, we can get the following bosonic propagator from the above propagators:

$$
\begin{aligned}
&\langle x_I^{(s,t)}(t_1) x_J^{(t,s)}(t_2)\rangle \\
&\sim \frac{g_{\mathrm{PW}}^2\mu^2}{\beta N_0}\sum_{l=-\infty}^{\infty}\sum_{J=0}^{\infty}\sum_{M=-J}^{J}\Bigg\{\frac{1}{\mathcal{P}_{\mathrm{B}}^2}\delta_{IJ} - \mathrm{i}\mu\left(\frac{1}{\mathcal{P}_{\mathrm{B}}^2}\right)^2 f_{IJK}\mathcal{P}_{\mathrm{B}}^K \\
&+ \mu^2\left(\frac{1}{\mathcal{P}_{\mathrm{B}}^2}\right)^3 (G_{IJ}\mathcal{P}_{\mathrm{B}i}^2 - H_{IJ}) - \mu^2\left(\frac{1}{\mathcal{P}_{\mathrm{B}}^2}\right)^2 G_{IJ} \\
&- \frac{\mu^2}{4}\left(\frac{1}{\mathcal{P}_{\mathrm{B}}^2}\right)^2 I_{IJ}\Bigg\}\mathrm{e}^{\mathrm{i}\omega_l(t_1-t_2)}(-1)^{M-(j_s-j_t)}\hat{Y}_{JM}^{(j_s,j_t)}\hat{Y}_{JM}^{(j_t,j_s)},
\end{aligned}
\tag{A15}
$$

where

$$
\begin{aligned}
f_{ijk} &= \epsilon_{ijk}, \quad \text{other } f_{IJK} = 0, \\
G_{ij} &= \delta_{ij}, \quad \text{other } G_{IJ} = 0, \\
H_{ij} &= \mathcal{P}_{\mathrm{B}i}\mathcal{P}_{\mathrm{B}j}, \quad \text{other } H_{IJ} = 0, \\
I_{mn} &= \delta_{mn}, \quad \text{other } I_{IJ} = 0.
\end{aligned}
\tag{A16}
$$

b. Fermion propagator

Next, we derive the fermion propagator to read out the quadratic term of φ in the action (3.13) as follows:

$$
\begin{aligned}
&\frac{1}{g_{\mathrm{PW}}^2\mu^2}\int_0^\beta dt \sum_{s,t}\mathrm{Tr}\left\{-\frac{1}{2}\bar{\varphi}^{(s,t)}\left(-\mathrm{i}\Gamma^0\partial_0 - \mu\Gamma^i\mathcal{L}_i - \frac{3\mathrm{i}\mu}{4}\Gamma^{123}\right)\varphi^{(t,s)}\right\} \\
&= \frac{1}{g_{\mathrm{PW}}^2\mu^2}\int_0^\beta dt \sum_{s,t}\mathrm{Tr}\left\{-\frac{1}{2}\sum_{h_1}\sum_{J_1,M_1}\mathrm{e}^{-\mathrm{i}\omega_{h_1}t}\bar{\varphi}_{h_1J_1M_1}^{(s,t)} \otimes \hat{Y}_{J_1M_1}^{(j_s,j_t)\dagger}\left(-\mathrm{i}\Gamma^0\partial_0 - \mu\Gamma^i\mathcal{L}_i - \frac{3\mathrm{i}\mu}{4}\Gamma^{123}\right)\sum_{h_2}\sum_{J_2,M_2}\mathrm{e}^{\mathrm{i}\omega_{h_2}t}\varphi_{h_2J_2M_2}^{(t,s)} \otimes \hat{Y}_{J_2M_2}^{(j_t,j_s)}\right\} \\
&= \frac{1}{2}\sum_{s,t}\sum_{h_1,h_2}\sum_{J_1,M_1}\sum_{J_2,M_2}\bar{\varphi}_{h_1J_1M_1}^{(s,t)}\frac{-\beta N_0}{g_{\mathrm{PW}}^2\mu^2}\left(\Gamma^0\omega_{h_2} - \mu\Gamma^i\mathcal{L}_i - \frac{3\mathrm{i}\mu}{4}\Gamma^{123}\right)\delta_{h_1h_2}\delta_{J_1J_2}\delta_{M_1M_2}\varphi_{h_2J_2M_2}^{(t,s)},
\end{aligned}
\tag{A17}
$$

where we expanded the quantum fluctuations by a plane wave and fuzzy spherical harmonics as follows:

$$
\varphi^{(s,t)}(t) = \sum_{h=-\infty}^{\infty}\sum_{J=0}^{\infty}\sum_{M=-J}^{J}\mathrm{e}^{\mathrm{i}\omega_h t}\varphi_{hJM}^{(s,t)} \otimes \hat{Y}_{JM}^{(j_s,j_t)}.
\tag{A18}
$$

Thus, we obtain the propagator of fermion modes as follows:

$$
\langle \varphi_{h_1J_1M_1}^{(s,t)}\bar{\varphi}_{h_2J_2M_2}^{(t,s)}\rangle = \frac{g_{\mathrm{PW}}^2\mu^2}{\beta N_0}\frac{1}{\Gamma^0\omega_{h_2} - \mu\Gamma^i\mathcal{L}_i - \frac{3\mathrm{i}\mu}{4}\Gamma^{123}}\delta_{h_1h_2}\delta_{J_1J_2}\delta_{M_1M_2}.
\tag{A19}
$$

Here, we expand this propagator into power series of $(\omega_h^2 + \mu^2 J(J+1)) = \mathcal{P}_{\mathrm{F}}^2$ as follows:

$$
\begin{aligned}
\frac{1}{\Gamma^0\omega_{h_2} - \mu\Gamma^i\mathcal{L}_i - \frac{3\mathrm{i}\mu}{4}\Gamma^{123}} &= \frac{1}{\mathcal{P}_{\mathrm{F}}^2}\Gamma^I\mathcal{P}_{\mathrm{F}I} + \frac{3\mathrm{i}\mu}{4}\frac{1}{\mathcal{P}_{\mathrm{F}}^2}\Gamma^{123} - \frac{\mathrm{i}}{2}\left(\frac{1}{\mathcal{P}_{\mathrm{F}}^2}\right)^2\Gamma^{IJ}\mathcal{F}_{\mathrm{F}IJ}\Gamma^K\mathcal{P}_{\mathrm{F}K} + \frac{3\mu}{8}\left(\frac{1}{\mathcal{P}_{\mathrm{F}}^2}\right)^2\Gamma^{IJ}\mathcal{F}_{\mathrm{F}IJ}\Gamma^{123} \\
&\quad + \frac{9\mu^2}{16}\left(\frac{1}{\mathcal{P}_{\mathrm{F}}^2}\right)^2\Gamma^K\mathcal{P}_{\mathrm{F}K} - \frac{1}{4}\left(\frac{1}{\mathcal{P}_{\mathrm{F}}^2}\right)^3\Gamma^{IJ}\mathcal{F}_{\mathrm{F}IJ}\Gamma^{MN}\mathcal{F}_{\mathrm{F}MN}\Gamma^K\mathcal{P}_{\mathrm{F}K} + \mathcal{O}\left(\frac{1}{\mathcal{P}_{\mathrm{F}}^4}\right).
\end{aligned}
\tag{A20}
$$

The third term of (A20) is that

$$-\frac{\mathrm{i}}{2}\left(\frac{1}{\mathcal{P}_{\mathrm{F}}^2}\right)^2\Gamma^{IJ}\mathcal{F}_{\mathrm{F}IJ}\Gamma^K\mathcal{P}_{\mathrm{F}K}=-\frac{\mathrm{i}}{2}\left(\frac{1}{\mathcal{P}_{\mathrm{F}}^2}\right)^2(-\mu f_{IJK}\Gamma^{IJM}\mathcal{P}_{\mathrm{F}}^K\mathcal{P}_{\mathrm{F}M}-2\mathrm{i}\mu^2\Gamma^I\mathcal{P}_{\mathrm{F}I})$$
$$=\frac{\mathrm{i}\mu}{2}\left(\frac{1}{\mathcal{P}_{\mathrm{F}}^2}\right)^2 f_{IJK}\Gamma^{IJM}\mathcal{P}_{\mathrm{F}}^K\mathcal{P}_{\mathrm{F}M}-\mu^2\left(\frac{1}{\mathcal{P}_{\mathrm{F}}^2}\right)^2\Gamma^I\mathcal{P}_{\mathrm{F}I}, \tag{A21}$$

where we used the multiplication law of the gamma matrices as follows:

$$\Gamma^{IJ}\Gamma^K=\Gamma^{IJK}-\delta^{IK}\Gamma^J+\delta^{JK}\Gamma^I. \tag{A22}$$

Moreover, the last term of (A20) is that

$$-\frac{1}{4}\left(\frac{1}{\mathcal{P}_{\mathrm{F}}^2}\right)^3\Gamma^{IJ}\mathcal{F}_{\mathrm{F}IJ}\Gamma^{MN}\mathcal{F}_{\mathrm{F}MN}\Gamma^K\mathcal{P}_{\mathrm{F}K}=-\frac{1}{4}\left(\frac{1}{\mathcal{P}_{\mathrm{F}}^2}\right)^3(-4\mu^2\Gamma^I\mathcal{P}_{\mathrm{F}I}\mathcal{P}_{\mathrm{F}}^2+\cdots)=\mu^2\left(\frac{1}{\mathcal{P}_{\mathrm{F}}^2}\right)^2\Gamma^I\mathcal{P}_{\mathrm{F}I}+\cdots, \tag{A23}$$

where we also used the multiplication law of the gamma matrices as follows:

$$\begin{aligned}\Gamma^{IJ}\Gamma^{MN}\Gamma^K=&\Gamma^{IJMNK}-\delta^{IM}\Gamma^{JNK}+\delta^{IN}\Gamma^{JMK}-\delta^{IK}\Gamma^{JMN}+\delta^{JM}\Gamma^{INK}-\delta^{JN}\Gamma^{IMK}+\delta^{JK}\Gamma^{IMN}-\delta^{MK}\Gamma^{IJN}+\delta^{NK}\Gamma^{IJM}\\&-\delta^{IM}\delta^{JN}\Gamma^K+\delta^{IM}\delta^{JK}\Gamma^N-\delta^{IN}\delta^{JK}\Gamma^M+\delta^{IN}\delta^{JN}\Gamma^K-\delta^{IK}\delta^{JN}\Gamma^M+\delta^{IK}\delta^{JM}\Gamma^N-\delta^{MK}\delta^{JN}\Gamma^I\\&+\delta^{MK}\delta^{IN}\Gamma^J-\delta^{NK}\delta^{IM}\Gamma^J+\delta^{NK}\delta^{JM}\Gamma^I.\end{aligned} \tag{A24}$$

Thus, we can obtain the following equation:

$$\begin{aligned}\frac{1}{\Gamma^I\mathcal{P}_{\mathrm{F}I}-\frac{3\mathrm{i}\mu}{4}\Gamma^{123}}=&\frac{1}{\mathcal{P}_{\mathrm{F}}^2}\Gamma^I\mathcal{P}_{\mathrm{F}I}+\frac{3\mathrm{i}\mu}{4}\frac{1}{\mathcal{P}_{\mathrm{F}}^2}\Gamma^{123}+\frac{\mathrm{i}\mu}{2}\left(\frac{1}{\mathcal{P}_{\mathrm{F}}^2}\right)^2 f_{IJK}\Gamma^{IJM}\mathcal{P}_{\mathrm{F}}^K\mathcal{P}_{\mathrm{F}M}-\frac{3\mu^2}{8}\left(\frac{1}{\mathcal{P}_{\mathrm{F}}^2}\right)^2 f_{IJK}\Gamma^{IJ}\mathcal{P}_{\mathrm{F}}^K\Gamma^{123}\\&+\frac{9\mu^2}{16}\left(\frac{1}{\mathcal{P}_{\mathrm{F}}^2}\right)^2\Gamma^I\mathcal{P}_{\mathrm{F}I}+\mathcal{O}\left(\frac{1}{\mathcal{P}_{\mathrm{F}}^4}\right).\end{aligned} \tag{A25}$$

Therefore, we obtain the fermion propagator of the plane wave matrix model as follows:

$$\begin{aligned}\langle\varphi^{(s,t)}(t_1)\bar{\varphi}^{(t,s)}(t_2)\rangle=&\sum_{h_1,h_2}\sum_{J_1,M_1}\sum_{J_2,M_2}\mathrm{e}^{\mathrm{i}\omega_{h_1}t_1}\mathrm{e}^{-\mathrm{i}\omega_{h_2}t_2}\langle\varphi^{(s,t)}_{h_1J_1M_1}\bar{\varphi}^{(t,s)}_{h_2J_2M_2}\rangle\otimes\hat{Y}^{(j_s,j_t)}_{J_1M_1}\hat{Y}^{(j_t,j_s)\dagger}_{J_2M_2}\\\sim&\frac{g_{\mathrm{PW}}^2\mu^2}{\beta N_0}\sum_{h=-\infty}^{\infty}\sum_{J=0}^{\infty}\sum_{M=-J}^{J}\left\{\frac{1}{\mathcal{P}_{\mathrm{F}}^2}\Gamma^I\mathcal{P}_{\mathrm{F}I}+\frac{3\mathrm{i}\mu}{4}\frac{1}{\mathcal{P}_{\mathrm{F}}^2}\Gamma^{123}+\frac{\mathrm{i}\mu}{2}\left(\frac{1}{\mathcal{P}_{\mathrm{F}}^2}\right)^2 f_{IJK}\Gamma^{IJM}\mathcal{P}_{\mathrm{F}}^K\mathcal{P}_{\mathrm{F}M}\right.\\&\left.-\frac{3\mu^2}{8}\left(\frac{1}{\mathcal{P}_{\mathrm{F}}^2}\right)^2 f_{IJK}\Gamma^{IJ}\mathcal{P}_{\mathrm{F}}^K\Gamma^{123}+\frac{9\mu^2}{16}\left(\frac{1}{\mathcal{P}_{\mathrm{F}}^2}\right)^2\Gamma^I\mathcal{P}_{\mathrm{F}I}\right\}\mathrm{e}^{\mathrm{i}\omega_h(t_1-t_2)}(-1)^{M-(j_s-j_t)}\hat{Y}^{(j_s,j_t)}_{JM}\hat{Y}^{(j_t,j_s)}_{J-M}.\end{aligned} \tag{A26}$$

2. Feynman diagrams

a. Feynman diagram involving four-point gauge boson vertex (a)

We evaluate the 1PI diagram involving a four-point gauge boson vertex. The four-point gauge boson vertex is given by

$$V_4=\frac{1}{g_{\mathrm{PW}}^2\mu^2}\int_0^\beta dt\sum_{s,t}\mathrm{Tr}\left\{-\frac{1}{4}[x_I,x_J]^{(s,t)2}\right\}. \tag{A27}$$

It gives rise to the following contribution:

$$\langle-V_4\rangle_{1\mathrm{PI}}=\frac{1}{g_{\mathrm{PW}}^2\mu^2}\int_0^\beta dt\sum_{s,t}\mathrm{Tr}\left\{\frac{1}{4}[x_I,x_J]^{(s,t)2}\right\}=\frac{1}{g_{\mathrm{PW}}^2\mu^2}\int_0^\beta dt\sum_{s,t,u,v}\mathrm{Tr}\left\{\frac{1}{2}(x_I^{(s,t)}x_J^{(t,u)}x^{(u,v)I}x^{(v,s)J}-x_I^{(s,t)}x^{(t,u)I}x_J^{(u,v)}x^{(v,s)J})\right\}. \tag{A28}$$

We can calculate $\langle-V_4\rangle_{1\mathrm{PI}}$ by performing the Wick contraction.

$$\begin{aligned}\langle-V_4\rangle_{1\mathrm{PI}}=&\frac{1}{g_{\mathrm{PW}}^2\mu^2}\int_0^\beta dt\sum_{s,t,u,v}\mathrm{Tr}\left\{\frac{1}{2}(\langle x_I^{(s,t)}x_J^{(t,u)}\rangle\langle x^{(u,v)I}x^{(v,s)J}\rangle+\langle x_J^{(s,t)}x^{(t,u)I}\rangle\langle x^{(u,v)J}x_I^{(v,s)}\rangle-\langle x_I^{(s,t)}x^{(t,u)I}\rangle\langle x_J^{(u,v)}x^{(v,s)J}\rangle\right.\\&\left.-\langle x^{(s,t)I}x_J^{(t,u)}\rangle\langle x^{(u,v)J}x_I^{(v,s)}\rangle)\right\}.\end{aligned} \tag{A29}$$

The leading contribution for the diagram involving four-point gauge boson vertex is given by

$$\begin{aligned}\langle -V_4\rangle_{1\mathrm{PI}} &= \frac{g_{\mathrm{PW}}^2\mu^2}{2\beta N_0}\int_0^\beta dt \sum_{s,t,u,v}\mathrm{Tr}\\ &\times\Big\{\sum_{l_1}\sum_{J_1,M_1}\frac{1}{\mathcal{P}_{\mathrm{B}}^2}\delta_{IJ}e^{i\omega_{l_1}(t_1-t_2)}(-1)^{M_1-(j_s-j_t)}\hat{Y}^{(j_s,j_t)}_{J_1M_1}\hat{Y}^{(j_t,j_u)}_{J_1-M_1}\sum_{l_2}\sum_{J_2,M_2}\frac{1}{\mathcal{P}_{\mathrm{B}}^2}\delta^{IJ}e^{i\omega_{l_2}(t_1-t_2)}(-1)^{M_2-(j_u-j_v)}\hat{Y}^{(j_u,j_v)}_{J_2M_2}\hat{Y}^{(j_v,j_s)}_{J_2-M_2}\\ &+\sum_{l_1}\sum_{J_1,M_1}\frac{1}{\mathcal{P}_{\mathrm{B}}^2}\delta^I_Je^{i\omega_{l_1}(t_1-t_2)}(-1)^{M_1-(j_s-j_t)}\hat{Y}^{(j_s,j_t)}_{J_1M_1}\hat{Y}^{(j_t,j_u)}_{J_1-M_1}\sum_{l_2}\sum_{J_2,M_2}\frac{1}{\mathcal{P}_{\mathrm{B}}^2}\delta_{IJ}e^{i\omega_{l_2}(t_1-t_2)}(-1)^{M_2-(j_u-j_v)}\hat{Y}^{(j_u,j_v)}_{J_2M_2}\hat{Y}^{(j_v,j_s)}_{J_2-M_2}\\ &-\sum_{l_1}\sum_{J_1,M_1}\frac{1}{\mathcal{P}_{\mathrm{B}}^2}\delta^I_Je^{i\omega_{l_1}(t_1-t_2)}(-1)^{M_1-(j_s-j_t)}\hat{Y}^{(j_s,j_t)}_{J_1M_1}\hat{Y}^{(j_t,j_u)}_{J_1-M_1}\sum_{l_2}\sum_{J_2,M_2}\frac{1}{\mathcal{P}_{\mathrm{B}}^2}\delta^J_Ie^{i\omega_{l_2}(t_1-t_2)}(-1)^{M_2-(j_u-j_v)}\hat{Y}^{(j_u,j_v)}_{J_2M_2}\hat{Y}^{(j_v,j_s)}_{J_2-M_2}\\ &-\sum_{l_1}\sum_{J_1,M_1}\frac{1}{\mathcal{P}_{\mathrm{B}}^2}\delta^I_Je^{i\omega_{l_1}(t_1-t_2)}(-1)^{M_1-(j_s-j_t)}\hat{Y}^{(j_s,j_t)}_{J_1M_1}\hat{Y}^{(j_t,j_u)}_{J_1-M_1}\sum_{l_2}\sum_{J_2,M_2}\frac{1}{\mathcal{P}_{\mathrm{B}}^2}\delta^J_Ie^{i\omega_{l_2}(t_1-t_2)}(-1)^{M_2-(j_u-j_v)}\hat{Y}^{(j_u,j_v)}_{J_2M_2}\hat{Y}^{(j_v,j_s)}_{J_2-M_2}\Big\}\\ &=\frac{1}{2}\sum_{s,t,u,v}\sum_{p,q}\mathrm{Tr}\sum_{l_1,l_2}\sum_{J_1,M_1}\sum_{J_2,M_2}\left(\frac{1}{\beta^2N_0}\right)^2\frac{(-1)^{M_1+M_2-(j_s-j_t)-(j_u-j_v)}}{J_1(J_1+1)J_2(J_2+1)}\hat{Y}^{(j_s,j_t)}_{J_1M_1}\hat{Y}^{(j_t,j_p)}_{J_1-M_1}\hat{Y}^{(j_u,j_v)}_{J_2M_2}\hat{Y}^{(j_v,j_q)}_{J_2-M_2}\delta_{pu}\delta_{qs}. \end{aligned}\tag{A30}$$

Here, we have inserted the complete set as follows:

$$\frac{1}{N_0}\mathrm{Tr}\sum_{J_3M_3}(-1)^{M_3-(j_p-j_q)}\hat{Y}^{(j_p,j_u)}_{J_3M_3}\hat{Y}^{(j_q,j_s)}_{J_3-M_3}=\delta_{pu}\delta_{qs}.\tag{A31}$$

Therefore, we can get the leading term

$$\langle -V_4\rangle_{1\mathrm{PI}}\sim -45\frac{g_{\mathrm{PW}}^2\mu^2}{\beta N_0}\sum_{l_1,l_2}\sum_{s,t,u}\sum_{123}\hat{\Psi}^\dagger_{123}\frac{1}{\mathcal{P}_{\mathrm{B}}^2\mathcal{Q}_{\mathrm{B}}^2}\hat{\Psi}_{123},\tag{A32}$$

where $\mathcal{P}_{\mathrm{B}}$, $\mathcal{Q}_{\mathrm{B}}$, and $\mathcal{R}_{\mathrm{B}}$ are defined as follows:

$$\begin{aligned}\mathcal{P}_I\hat{Y}^{(j_s,j_t)}_{J_1M_1}\equiv[p_I,\hat{Y}^{(j_s,j_t)}_{J_1M_1}],\qquad \mathcal{Q}_I\hat{Y}^{(j_s,j_t)}_{J_2M_2}\equiv[p_I,\hat{Y}^{(j_s,j_t)}_{J_2M_2}],\\ \mathcal{R}_I\hat{Y}^{(j_s,j_t)}_{J_3M_3}\equiv[p_I,\hat{Y}^{(j_s,j_t)}_{J_3M_3}].\end{aligned}\tag{A33}$$

We have introduced the following wave function:

$$\hat{\Psi}_{123}\equiv\frac{1}{N_0}\mathrm{Tr}\hat{Y}^{(j_s,j_t)}_{J_1M_1}\hat{Y}^{(j_t,j_u)}_{J_2M_2}\hat{Y}^{(j_u,j_s)}_{J_3M_3},\tag{A34}$$

and

$$\sum_{123}\equiv\sum_{J_1=0}^{\infty}\sum_{M_1=-J_1}^{J_1}\sum_{J_2=0}^{\infty}\sum_{M_2=-J_2}^{J_2}\sum_{J_3=0}^{\infty}\sum_{M_3=-J_3}^{J_3}.\tag{A35}$$

b. Feynman diagram involving three-point gauge boson vertex (b)

We evaluate the 1PI diagram involving three-point gauge boson vertices. The three-point gauge boson vertex is expressed as follows:

$$V_3=\frac{1}{g_{\mathrm{PW}}^2\mu^2}\int_0^\beta dt\sum_{s,t}\mathrm{Tr}\{-(\mathcal{P}_Ix_J^{(s,t)})[x^I,x^J]^{(t,s)}\}.\tag{A36}$$

We can express the contribution corresponding to the diagram (b) as follows:

$$\begin{aligned}\left\langle\frac{1}{2}V_3V_3\right\rangle_{1\mathrm{PI}}&=\frac{1}{2}\left[\frac{1}{g_{\mathrm{PW}}^2\mu^2}\int_0^\beta dt\sum_{s,t}\mathrm{Tr}\{(\mathcal{P}_Ix_J^{(s,t)})[x^I,x^J]^{(t,s)}\}\right]^2\\ &=\frac{1}{2}\left[\frac{1}{g_{\mathrm{PW}}^2\mu^2}\int_0^\beta dt\sum_{s,t,u}\mathrm{Tr}\{(\mathcal{P}_Ix_J^{(s,t)})x^{(t,u)I}x^{(u,s)J}-(\mathcal{P}_Ix_J^{(s,t)})x^{(t,u)J}x^{(u,s)I}\}\right]^2\\ &=\frac{1}{2g_{\mathrm{PW}}^4\mu^4}\int_0^\beta dt_1dt_2\sum_{s,t,u}[\mathrm{Tr}\{(\mathcal{P}_Ix_J^{(s,t)})x^{(t,u)I}x^{(u,s)J}\}\mathrm{Tr}\{(\mathcal{P}_Mx_N^{(s,t)})x^{(t,u)M}x^{(u,s)N}\}\\ &-\mathrm{Tr}\{(\mathcal{P}_Ix_J^{(s,t)})x^{(t,u)I}x^{(u,s)J}\}\mathrm{Tr}\{(\mathcal{P}_Mx_N^{(s,t)})x^{(t,u)N}x^{(u,s)M}\}-\mathrm{Tr}\{(\mathcal{P}_Ix_J^{(s,t)})x^{(t,u)J}x^{(u,s)I}\}\mathrm{Tr}\{(\mathcal{P}_Mx_N^{(s,t)})x^{(t,u)M}x^{(u,s)N}\}\\ &+\mathrm{Tr}\{(\mathcal{P}_Ix_J^{(s,t)})x^{(t,u)J}x^{(u,s)I}\}\mathrm{Tr}\{(\mathcal{P}_Mx_N^{(s,t)})x^{(t,u)N}x^{(u,s)M}\}].\end{aligned}\tag{A37}$$

For example, we calculate the first term of (A37) by applying Wick's theorem.

$$\frac{1}{2g_{\mathrm{PW}}^4\mu^4}\int_0^\beta dt_1dt_2\sum_{s,t,u}[\mathrm{Tr}\{(\mathcal{P}_I x_J^{(s,t)})x^{(t,u)I}x^{(u,s)J}\}\,\mathrm{Tr}\{(\mathcal{P}_M x_N^{(s,t)})x^{(t,u)M}x^{(u,s)N}\}]$$
$$=\frac{1}{2g_{\mathrm{PW}}^4\mu^4}\int_0^\beta dt_1dt_2\sum_{s,t,u}\mathrm{TrTr}\{\langle(\mathcal{P}_I x_J^{(s,t)}(t_1))x^{(u,s)N}(t_2)\rangle\langle x^{(t,u)I}(t_1)x^{(t,u)M}(t_2)\rangle\langle x^{(u,s)J}(t_1)(\mathcal{P}_M x_N^{(s,t)}(t_2))\rangle$$
$$+\langle(\mathcal{P}_I x_J^{(s,t)}(t_1))(\mathcal{P}_M x_N^{(u,s)}(t_2))\rangle\langle x^{(t,u)I}(t_1)x^{(t,u)N}(t_2)\rangle\langle x^{(u,s)J}(t_1)x^{(s,t)M}(t_2)\rangle$$
$$+\langle(\mathcal{P}_I x_J^{(s,t)}(t_1))x^{(u,s)M}(t_2)\rangle\langle x^{(t,u)I}(t_1)(\mathcal{P}_M x_N^{(t,u)}(t_2))\rangle\langle x^{(u,s)J}(t_1)x^{(s,t)N}(t_2)\rangle\}. \tag{A38}$$

Here, we evaluate the particular contraction which is the first term of the above equation as follows:

$$\frac{g_{\mathrm{PW}}^2\mu^2}{\beta N_0}\int_0^\beta dt_1dt_2\sum_{s,t,u}\mathrm{TrTr}\Big\{\sum_{l_1}\sum_{J_1,M_1}\frac{1}{\mathcal{P}_{\mathrm{B}}^2}\delta_J{}^N e^{i\omega_{l_1}(t_1-t_2)}(-1)^{M_1-(j_s-j_t)}(\mathcal{P}_{\mathrm{B}I}\hat{Y}_{J_1M_1}^{(j_s,j_t)})\hat{Y}_{J_1-M_1}^{(j_u,j_s)}$$
$$\times\sum_{l_2}\sum_{J_2,M_2}\frac{1}{\mathcal{Q}_{\mathrm{B}}^2}\delta^{IM}e^{i\omega_{l_2}(t_1-t_2)}(-1)^{M_2-(j_t-j_u)}\hat{Y}_{J_2M_2}^{(j_t,j_u)}\hat{Y}_{J_2-M_2}^{(j_t,j_u)}\sum_{l_3}\sum_{J_3,M_3}\frac{1}{\mathcal{R}_{\mathrm{B}}^2}\delta^J{}_N e^{i\omega_{l_3}(t_1-t_2)}(-1)^{M_3-(j_u-j_s)}\hat{Y}_{J_3M_3}^{(j_u,j_s)}(\mathcal{R}_{\mathrm{B}M}\hat{Y}_{J_3-M_3}^{(j_s,j_t)})$$
$$+\sum_{l_1}\sum_{J_1,M_1}\frac{1}{\mathcal{P}_{\mathrm{B}}^2}\delta_{JN}e^{i\omega_{l_1}(t_1-t_2)}(-1)^{M_1-(j_s-j_t)}(\mathcal{P}_{\mathrm{B}I}\hat{Y}_{J_1M_1}^{(j_s,j_t)})(\mathcal{P}_{\mathrm{B}M}\hat{Y}_{J_1-M_1}^{(j_u,j_s)})\sum_{l_2}\sum_{J_2,M_2}\frac{1}{\mathcal{Q}_{\mathrm{B}}^2}\delta^{IN}e^{i\omega_{l_2}(t_1-t_2)}(-1)^{M_2-(j_t-j_u)}\hat{Y}_{J_2M_2}^{(j_t,j_u)}\hat{Y}_{J_2-M_2}^{(j_t,j_u)}$$
$$\times\sum_{l_3}\sum_{J_3,M_3}\frac{1}{\mathcal{R}_{\mathrm{B}}^2}\delta^{JM}e^{i\omega_{l_3}(t_1-t_2)}(-1)^{M_3-(j_u-j_s)}\hat{Y}_{J_3M_3}^{(j_u,j_s)}\hat{Y}_{J_3-M_3}^{(j_s,j_t)}+\sum_{l_1}\sum_{J_1,M_1}\frac{1}{\mathcal{P}_{\mathrm{B}}^2}\delta_J{}^M e^{i\omega_{l_1}(t_1-t_2)}(-1)^{M_1-(j_s-j_t)}(\mathcal{P}_{\mathrm{B}I}\hat{Y}_{J_1M_1}^{(j_s,j_t)})\hat{Y}_{J_1-M_1}^{(j_u,j_s)}$$
$$\times\sum_{l_2}\sum_{J_2,M_2}\frac{1}{\mathcal{Q}_{\mathrm{B}}^2}\delta^I{}_N e^{i\omega_{l_2}(t_1-t_2)}(-1)^{M_2-(j_t-j_u)}\hat{Y}_{J_2M_2}^{(j_t,j_u)}(\mathcal{P}_{\mathrm{B}M}\hat{Y}_{J_2-M_2}^{(j_t,j_u)})\sum_{l_3}\sum_{J_3,M_3}\frac{1}{\mathcal{R}_{\mathrm{B}}^2}\delta^{JN}e^{i\omega_{l_3}(t_1-t_2)}(-1)^{M_3-(j_u-j_s)}\hat{Y}_{J_3M_3}^{(j_u,j_s)}\hat{Y}_{J_3-M_3}^{(j_s,j_t)}\Big\}$$
$$=\frac{1}{2}\frac{g_{\mathrm{PW}}^2\mu^2}{\beta N_0}\sum_{s,t,u}\sum_{l_1,l_2}\sum_{J_1,M_1}\sum_{J_2,M_2}\sum_{J_3,M_3}\frac{1}{N_0}\mathrm{Tr}\frac{1}{N_0}\mathrm{Tr}\Big\{\frac{-10\mathcal{P}_{\mathrm{B}}\cdot\mathcal{R}_{\mathrm{B}}}{\mathcal{P}_{\mathrm{B}}^2\mathcal{Q}_{\mathrm{B}}^2\mathcal{R}_{\mathrm{B}}^2}(-1)^{M_1-(j_s-j_t)}\hat{Y}_{J_1M_1}^{(j_s,j_t)}\hat{Y}_{J_1-M_1}^{(j_u,j_s)}$$
$$+\frac{-\mathcal{P}_{\mathrm{B}}^2}{\mathcal{P}_{\mathrm{B}}^2\mathcal{Q}_{\mathrm{B}}^2\mathcal{R}_{\mathrm{B}}^2}\delta^{IM}(-1)^{M_2-(j_t-j_u)}\hat{Y}_{J_2M_2}^{(j_t,j_u)}\hat{Y}_{J_2-M_2}^{(j_t,j_u)}+\frac{-\mathcal{P}_{\mathrm{B}}\cdot\mathcal{Q}_{\mathrm{B}}}{\mathcal{P}_{\mathrm{B}}^2\mathcal{Q}_{\mathrm{B}}^2\mathcal{R}_{\mathrm{B}}^2}(-1)^{M_3-(j_u-j_s)}\hat{Y}_{J_3M_3}^{(j_u,j_s)}\hat{Y}_{J_3-M_3}^{(j_s,j_t)}\Big\}, \tag{A39}$$

where we used the following relation:

$$\sum_{M=-J}^{J}(-1)^{M-(j_s-j_t)}\hat{Y}_{JM}^{(j_s,j_t)}\mathcal{P}_I\hat{Y}_{J-M}^{(j_t,j_s)}$$
$$=-\sum_{M=-J}^{J}(-1)^{M-(j_s-j_t)}(\mathcal{P}_I\hat{Y}_{JM}^{(j_s,j_t)})\hat{Y}_{J-M}^{(j_t,j_s)}. \tag{A40}$$

We can obtain the following compact equation to calculate the other contractions:

$$\left\langle\frac{1}{2}V_3V_3\right\rangle_{1\mathrm{PI}}\sim\frac{9}{2}\frac{g_{\mathrm{PW}}^2\mu^2}{\beta N_0}\sum_{s,t,u}\sum_{l_1,l_2}\sum_{123}\hat{\Psi}_{123}^\dagger\times\frac{2\mathcal{P}_{\mathrm{B}}^2-\mathcal{P}_{\mathrm{B}}\cdot\mathcal{Q}_{\mathrm{B}}-\mathcal{P}_{\mathrm{B}}\cdot\mathcal{R}_{\mathrm{B}}}{\mathcal{P}_{\mathrm{B}}^2\mathcal{Q}_{\mathrm{B}}^2\mathcal{R}_{\mathrm{B}}^2}\hat{\Psi}_{123}. \tag{A41}$$

Moreover, we use the following relation:

$$\mathcal{P}_{\mathrm{B}}\cdot\mathcal{Q}_{\mathrm{B}}\hat{\Psi}_{123}=\frac{1}{2}(\mathcal{R}_{\mathrm{B}}^2-\mathcal{P}_{\mathrm{B}}^2-\mathcal{Q}_{\mathrm{B}}^2)\hat{\Psi}_{123}, \tag{A42}$$

and the momentum conserved relation:

$$\mathcal{P}_{\mathrm{B}}+\mathcal{Q}_{\mathrm{B}}+\mathcal{R}_{\mathrm{B}}=0. \tag{A43}$$

Therefore, we can simplify as follows:

$$\left\langle\frac{1}{2}V_3V_3\right\rangle_{1\mathrm{PI}}\sim\frac{27}{2}\frac{g_{\mathrm{PW}}^2\mu^2}{\beta N_0}\sum_{s,t,u}\sum_{l_1,l_2}\sum_{123}\hat{\Psi}_{123}^\dagger\frac{1}{\mathcal{P}_{\mathrm{B}}^2\mathcal{Q}_{\mathrm{B}}^2}\hat{\Psi}_{123}. \tag{A44}$$

c. Feynman diagram involving the ghost interactions (d)

We evaluate the 1PI diagram involving the ghost interactions. The ghost vertex is expressed as follows:

$$V_{\mathrm{gh}}=\frac{1}{g_{\mathrm{PW}}^2\mu^2}\int_0^\beta dt\sum_{s,t}\mathrm{Tr}\{-(\mathcal{P}_I b^{(s,t)})[x^I,c]^{(t,s)}\}. \tag{A45}$$

We can express the contribution corresponding to the diagram (d) as follows:

$$\begin{aligned}\left\langle \frac{1}{2} V_{\rm gh} V_{\rm gh} \right\rangle_{\rm 1PI} &= \frac{1}{2}\left[\frac{1}{g_{\rm PW}^2 \mu^2} \int_0^\beta dt \sum_{s,t} {\rm Tr}\{(\mathcal{P}_I b^{(s,t)})[x^I, c]^{(t,s)}\} \right]^2 \\ &= \frac{1}{2}\left[\frac{1}{g_{\rm PW}^2 \mu^2} \int_0^\beta dt \sum_{s,t,u} {\rm Tr}\{(\mathcal{P}_I b^{(s,t)}) x^{(t,u)I} c^{(u,s)} - (\mathcal{P}_I b^{(s,t)}) c^{(t,u)} x^{(u,s)I}\} \right]^2 \\ &= \frac{1}{2 g_{\rm PW}^4 \mu^4} \int_0^\beta dt_1 dt_2 \sum_{s,t,u} \Big[{\rm Tr}\{(\mathcal{P}_I b^{(s,t)}) x^{(t,u)I} c^{(u,s)}\} {\rm Tr}\{(\mathcal{P}_J b^{(s,t)}) x^{(t,u)J} c^{(u,s)}\} \\ &\quad - {\rm Tr}\{(\mathcal{P}_I b^{(s,t)}) x^{(t,u)I} c^{(u,s)}\} {\rm Tr}\{(\mathcal{P}_J b^{(s,t)}) c^{(t,u)} x^{(u,s)J}\} - {\rm Tr}\{(\mathcal{P}_I b^{(s,t)}) c^{(t,u)} x^{(u,s)I}\} {\rm Tr}\{(\mathcal{P}_J b^{(s,t)}) x^{(t,u)J} c^{(u,s)}\} \\ &\quad + {\rm Tr}\{(\mathcal{P}_I b^{(s,t)}) c^{(t,u)} x^{(u,s)I}\} {\rm Tr}\{(\mathcal{P}_J b^{(s,t)}) c^{(t,u)} x^{(u,s)J}\} \Big]. \end{aligned} \tag{A46}$$

For example, we calculate the first term of (A46) by applying Wick's theorem.

$$\begin{aligned}&\frac{1}{2 g_{\rm PW}^4 \mu^4} \int_0^\beta dt_1 dt_2 \sum_{s,t,u} [{\rm Tr}\{(\mathcal{P}_I b^{(s,t)}) x^{(t,u)I} c^{(u,s)}\} {\rm Tr}\{(\mathcal{P}_J b^{(s,t)}) x^{(t,u)J} c^{(u,s)}\}] \\ &= \frac{1}{2 g_{\rm PW}^4 \mu^4} \int_0^\beta dt_1 dt_2 \sum_{s,t,u} {\rm TrTr}\{\langle ((\mathcal{P}_I b^{(s,t)}(t_1)) c^{(u,s)}(t_2)\rangle \langle x^{(t,u)I}(t_1) x^{(t,u)J}(t_2)\rangle \langle c^{(u,s)}(t_1)(\mathcal{P}_J b^{(s,t)}(t_2)))\rangle\}. \\ &= \frac{1}{2 g_{\rm PW}^4 \mu^4} \int_0^\beta dt_1 dt_2 \sum_{s,t,u} {\rm TrTr}\Big\{ \frac{g_{\rm PW}^2 \mu^2}{\beta N_0} \sum_{l_1} \sum_{J_1,M_1} \frac{1}{\mathcal{P}_{\rm B}^2} e^{i\omega_{l_1}(t_1-t_2)} (-1)^{M_1-(j_s-j_t)} (\mathcal{P}_{{\rm B}I} \hat{Y}^{(j_s,j_t)}_{J_1 M_1}) \hat{Y}^{(j_u,j_s)}_{J_1-M_1} \frac{g_{\rm PW}^2 \mu^2}{\beta N_0} \\ &\quad \times \sum_{l_2} \sum_{J_2,M_2} \frac{1}{\mathcal{Q}_{\rm B}^2} \delta^{IJ} e^{i\omega_{l_2}(t_1-t_2)} (-1)^{M_2-(j_t-j_u)} \hat{Y}^{(j_t,j_u)}_{J_2 M_2} \hat{Y}^{(j_t,j_u)}_{J_2-M_2} \frac{g_{\rm PW}^2 \mu^2}{\beta N_0} \sum_{l_3} \sum_{J_3,M_3} \frac{1}{\mathcal{R}_{\rm B}^2} e^{i\omega_{l_3}(t_1-t_2)} (-1)^{M_3-(j_u-j_s)} \hat{Y}^{(j_u,j_s)}_{J_3 M_3} (\mathcal{R}_{{\rm B}M} \hat{Y}^{(j_s,j_t)}_{J_3-M_3}) \Big\} \\ &= -\frac{1}{2} \frac{g_{\rm PW}^2 \mu^2}{\beta N_0} \sum_{s,t,u} \sum_{l_1,l_2} \sum_{J_1,M_1} \sum_{J_2,M_2} \sum_{J_3,M_3} \frac{1}{N_0} {\rm Tr} \frac{1}{N_0} {\rm Tr}\Big\{ \frac{1}{\mathcal{P}_{\rm B}^2} (-1)^{M_1-(j_s-j_t)} (\mathcal{P}_{{\rm B}I} \hat{Y}^{(j_s,j_t)}_{J_1 M_1}) \hat{Y}^{(j_u,j_s)}_{J_1-M_1} \frac{1}{\mathcal{Q}_{\rm B}^2} \delta^{IJ} (-1)^{M_2-(j_t-j_u)} \hat{Y}^{(j_t,j_u)}_{J_2 M_2} \hat{Y}^{(j_t,j_u)}_{J_2-M_2} \\ &\quad \times \frac{1}{\mathcal{R}_{\rm B}^2} (-1)^{M_3-(j_u-j_s)} (\mathcal{R}_{{\rm B}J} \hat{Y}^{(j_u,j_s)}_{J_3 M_3}) \hat{Y}^{(j_s,j_t)}_{J_3-M_3} \Big\}, \end{aligned} \tag{A47}$$

where we also used the relation (A40). We obtain the following equation to evaluate the other contractions:

$$\left\langle \frac{1}{2} V_{\rm gh} V_{\rm gh} \right\rangle_{\rm 1PI} \sim \frac{1}{2} \frac{g_{\rm PW}^2 \mu^2}{\beta N_0} \sum_{s,t,u} \sum_{l_1,l_2} \sum_{123} \hat{\Psi}^\dagger_{123} \frac{\mathcal{P}_{\rm B} \cdot \mathcal{Q}_{\rm B} + \mathcal{P}_{\rm B} \cdot \mathcal{R}_{\rm B}}{\mathcal{P}_{\rm B}^2 \mathcal{Q}_{\rm B}^2 \mathcal{R}_{\rm B}^2} \hat{\Psi}_{123} = \frac{g_{\rm PW}^2 \mu^2}{\beta N_0} \sum_{s,t,u} \sum_{l_1,l_2} \sum_{123} \hat{\Psi}^\dagger_{123} \frac{\mathcal{P}_{\rm B} \cdot \mathcal{Q}_{\rm B}}{\mathcal{P}_{\rm B}^2 \mathcal{Q}_{\rm B}^2 \mathcal{R}_{\rm B}^2} \hat{\Psi}_{123}. \tag{A48}$$

Moreover, we obtain the following simplified equation to use the relation (A42) and the conservation law of momenta (A43):

$$\left\langle \frac{1}{2} V_{\rm gh} V_{\rm gh} \right\rangle_{\rm 1PI} \sim -\frac{1}{2} \frac{g_{\rm PW}^2 \mu^2}{\beta N_0} \sum_{s,t,u} \sum_{l_1,l_2} \sum_{123} \hat{\Psi}^\dagger_{123} \frac{1}{\mathcal{P}_{\rm B}^2 \mathcal{Q}_{\rm B}^2} \hat{\Psi}_{123}. \tag{A49}$$

d. Feynman diagram involving the fermion interaction (e)

Finally, we evaluate the diagram involving fermion interactions. The fermion vertex is expressed as follows:

$$V_{\rm F} = \frac{1}{g_{\rm PW}^2 \mu^2} \int_0^\beta dt \sum_{s,t} {\rm Tr}\Big\{ -\frac{1}{2} \bar{\varphi}^{(s,t)} \Gamma^I [x_I, \varphi]^{(t,s)} \Big\}. \tag{A50}$$

The 1PI diagram involving fermion vertices is calculated as follows:

$$\begin{aligned}\left\langle \frac{1}{2} V_{\rm F} V_{\rm F} \right\rangle_{\rm 1PI} &= \frac{1}{2}\left[\frac{1}{g_{\rm PW}^2 \mu^2} \int_0^\beta dt \sum_{s,t} {\rm Tr}\Big\{ \frac{1}{2} \bar{\varphi}^{(s,t)} \Gamma^I [x_I, \varphi]^{(t,s)} \Big\} \right]^2 \\ &= \frac{1}{2}\left[\frac{1}{g_{\rm PW}^2 \mu^2} \int_0^\beta dt \sum_{s,t,u} {\rm Tr}\{ \bar{\varphi}^{(s,t)} \Gamma^I x_I^{(t,u)} \varphi^{(u,s)} \} \right]^2. \end{aligned} \tag{A51}$$

We can perform the Wick contractions:

$$\left\langle \frac{1}{2} V_{\rm F} V_{\rm F} \right\rangle_{\rm 1PI} \sim \frac{1}{2} \frac{1}{g_{\rm PW}^4 \mu^4} \int_0^\beta dt_1 dt_2 \sum_{s,t,u} {\rm TrTr}$$
$$\times \frac{g_{\rm PW}^2 \mu^2}{\beta N_0} \sum_{h_1} \sum_{J_1, M_1} \left(-\frac{1}{\mathcal{P}_{\rm F}^2} \Gamma^I \mathcal{P}_{{\rm F}I} - \frac{3{\rm i}\mu}{4} \frac{1}{\mathcal{P}_{\rm F}^2} \Gamma^{123} - \frac{{\rm i}\mu}{2} \left(\frac{1}{\mathcal{P}_{\rm F}^2} \right)^2 f_{IJK} \Gamma^{IJM} \mathcal{P}_{\rm F}^K \mathcal{P}_{{\rm F}M} \right)$$
$$\times \Gamma^A (-1)^{M_1 - (j_s - j_t)} {\rm e}^{{\rm i}\omega_{h_1}(t_1 - t_2)} \hat{Y}^{(j_s, j_t)}_{J_1 M_1} \hat{Y}^{(j_u, j_s)}_{J_1 - M_1} \frac{g_{\rm PW}^2 \mu^2}{\beta N_0} \sum_{l_2} \sum_{J_2, M_2} \frac{1}{\mathcal{P}_{\rm B}^2} \delta_{AB} (-1)^{M_2 - (j_t - j_u)} {\rm e}^{{\rm i}\omega_{l_2}(t_1 - t_2)} \hat{Y}^{(j_t, j_u)}_{J_2 M_2} \hat{Y}^{(j_u, j_t)}_{J_2 - M_2}$$
$$\times \frac{g_{\rm PW}^2 \mu^2}{\beta N_0} \sum_{h_3} \sum_{J_3, M_3} \left(\frac{1}{\mathcal{P}_{\rm F}^2} \Gamma^P \mathcal{P}_{{\rm F}P} + \frac{3{\rm i}\mu}{4} \frac{1}{\mathcal{P}_{\rm F}^2} \Gamma^{123} + \frac{{\rm i}\mu}{2} \left(\frac{1}{\mathcal{P}_{\rm F}^2} \right)^2 f_{PQR} \Gamma^{PQS} \mathcal{P}_{\rm F}^R \mathcal{P}_{{\rm F}S} \right)$$
$$\times \Gamma^B (-1)^{M_3 - (j_u - j_s)} {\rm e}^{{\rm i}\omega_{h_3}(t_1 - t_2)} \hat{Y}^{(j_u, j_s)}_{J_3 M_3} \hat{Y}^{(j_s, j_t)}_{J_3 - M_3}. \tag{A52}$$

We can evaluate the traces of products of gamma matrices. We obtain the following result

$$\left\langle \frac{1}{2} V_{\rm F} V_{\rm F} \right\rangle_{\rm 1PI} \sim \sum_{123} \hat{\Psi}^\dagger_{123} \left(-32 \frac{g_{\rm PW}^2 \mu^2}{\beta N_0} \sum_{h_1, h_2} \frac{1}{\mathcal{P}_{\rm F}^2 \mathcal{Q}_{\rm F}^2} + 64 \frac{g_{\rm PW}^2 \mu^2}{\beta N_0} \sum_{l_1, h_2} \frac{1}{\mathcal{P}_{\rm B}^2 \mathcal{Q}_{\rm F}^2} \right) \hat{\Psi}_{123}. \tag{A53}$$

3. Two-loop effective action

a. Bosonic two-loop effective action

We calculate the bosonic two-loop effective action of the plane wave matrix model as follows:

$$\hat{W}_{\rm B}^{\text{2-loop}} = \left\langle -V_4 + \frac{1}{2} V_3 V_3 + \frac{1}{2} V_{\rm gh} V_{\rm gh} \right\rangle_{\rm 1PI} = -32 \frac{g_{\rm PW}^2 \mu^2}{\beta N_0} \sum_{l_1, l_2} \sum_{s,t,u} \sum_{123} \hat{\Psi}^\dagger_{123} \frac{1}{\mathcal{P}_{\rm B}^2 \mathcal{Q}_{\rm B}^2} \hat{\Psi}_{123}$$
$$= -32 \frac{g_{\rm PW}^2 \mu^2}{\beta N_0^3} \sum_{l_1, l_2} \sum_{s,t,u} \sum_{J_1, M_1} \sum_{J_2, M_2} \sum_{J_3, M_3} {\rm Tr} \hat{Y}^{(j_s, j_t)\dagger}_{J_1 M_1} \hat{Y}^{(j_t, j_u)\dagger}_{J_2 M_2} \hat{Y}^{(j_u, j_s)\dagger}_{J_3 M_3} \frac{1}{(\omega_{l_1}^2 + \mu^2 J_1(J_1+1))(\omega_{l_2}^2 + \mu^2 J_2(J_2+1))}$$
$$\times {\rm Tr} \hat{Y}^{(j_s, j_t)}_{J_1 M_1} \hat{Y}^{(j_t, j_u)}_{J_2 M_2} \hat{Y}^{(j_u, j_s)}_{J_3 M_3}. \tag{A54}$$

Note that on the analogy with the large N reduced model on a flat background. So we obtain that

$$\hat{W}_{\rm B}^{\text{2-loop}} = -32 \frac{g_{\rm PW}^2 \mu^2}{\beta N_0} \sum_{l_1, l_2} \sum_{r=1}^{\infty} \sum_{J_1=0}^{\infty} \sum_{M_1=-J_1}^{J_1} \sum_{\tilde{M}_1=-J_1}^{J_1} \sum_{J_2=0}^{\infty} \sum_{M_2=-J_2}^{J_2} \sum_{\tilde{M}_2=-J_2}^{J_2} \sum_{J_3=0}^{\infty} \sum_{M_3=-J_3}^{J_3} \sum_{\tilde{M}_3=-J_3}^{J_3}$$
$$\times \frac{(2J_1+1)(2J_2+1)(2J_3+1)}{(\omega_{l_1}^2 + \mu^2 J_1(J_1+1))(\omega_{l_2}^2 + \mu^2 J_2(J_2+1))} \begin{pmatrix} J_1 & J_2 & J_3 \\ M_1 & M_2 & M_3 \end{pmatrix}^2 \begin{pmatrix} J_1 & J_2 & J_3 \\ \tilde{M}_1 & \tilde{M}_2 & \tilde{M}_3 \end{pmatrix}^2, \tag{A55}$$

where we define that $p/2 = \tilde{M}_1$, $q/2 = \tilde{M}_2$, and $(-p-q)/2 = \tilde{M}_3$, and use the following relation:

$$\frac{1}{N_0} {\rm Tr} \hat{Y}^{(j_s, j_t)}_{J_1 M_1} \hat{Y}^{(j_t, j_u)}_{J_2 M_2} \hat{Y}^{(j_u, j_s)}_{J_3 M_3} \underset{N_0 \to \infty}{\to} (-1)^{2J_2 - 2J_3 - \tilde{M}_1} \sqrt{(2J_1+1)(2J_2+1)(2J_3+1)} \begin{pmatrix} J_1 & J_2 & J_3 \\ M_1 & M_2 & M_3 \end{pmatrix} \begin{pmatrix} J_1 & J_2 & J_3 \\ \tilde{M}_1 & \tilde{M}_2 & \tilde{M}_3 \end{pmatrix}, \tag{A56}$$

where $(\cdots)$ represents the 3-j symbol of Wigner [38,39]. We have a cutoff such that $r < 2\Lambda$, so the maximal value of J and $\tilde{M}$ are N_0 and Λ, respectively. Then, we separate the sums over J into two parts at Λ. After dividing the overall factor $\sum_r$, we can obtain that

$$-32 \frac{g_{\rm PW}^2 \mu^2}{\beta N_0} \sum_{l_1, l_2} \sum_{J_1=0}^{\infty} \sum_{M_1=-J_1}^{J_1} \sum_{\tilde{M}_1=-J_1}^{J_1} \sum_{J_2=0}^{\infty} \sum_{M_2=-J_2}^{J_2} \sum_{\tilde{M}_2=-J_2}^{J_2} \sum_{J_3=0}^{\infty} \sum_{M_3=-J_3}^{J_3} \sum_{\tilde{M}_3=-J_3}^{J_3} \frac{(2J_1+1)(2J_2+1)(2J_3+1)}{(\omega_{l_1}^2 + \mu^2 J_1(J_1+1))(\omega_{l_2}^2 + \mu^2 J_2(J_2+1))}$$
$$\times \begin{pmatrix} J_1 & J_2 & J_3 \\ M_1 & M_2 & M_3 \end{pmatrix}^2 \begin{pmatrix} J_1 & J_2 & J_3 \\ \tilde{M}_1 & \tilde{M}_2 & \tilde{M}_3 \end{pmatrix}^2 = -32 \frac{g_{\rm PW}^2 \mu^2}{\beta N_0} \sum_{l_1, l_2} \sum_{J_1=0}^{\infty} \sum_{J_2=0}^{\infty} \sum_{J_3=0}^{\infty} \frac{(2J_1+1)(2J_2+1)(2J_3+1)}{(\omega_{l_1}^2 + \mu^2 J_1(J_1+1))(\omega_{l_2}^2 + \mu^2 J_2(J_2+1))}. \tag{A57}$$

Note that we consider the following cutoff scale: $T \ll \Lambda$. We thus obtain that

$$\hat{W}_{\mathrm{B}}^{2\text{-loop}} = -32\frac{g_{\mathrm{PW}}^2\mu^2}{\beta N_0}\sum_{l_1,l_2}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\frac{(k_1+1)(k_2+1)(k_3+1)}{(\omega_{l_1}^2+\frac{\mu^2}{4}k_1(k_1+2))(\omega_{l_2}^2+\frac{\mu^2}{4}k_2(k_2+2))}, \tag{A58}$$

where we set that $k_1 = 2J_1$, $k_2 = 2J_2$, and $k_3 = 2J_3$. The summations over k_1, k_2, and k_3 can be approximated by the integrals over

$$x_1 = \frac{\sqrt{k_1(k_1+2)}}{rT}, \qquad x_2 = \frac{\sqrt{k_2(k_2+2)}}{rT}, \qquad x_3 = \frac{\sqrt{k_3(k_3+2)}}{rT}. \tag{A59}$$

In a high temperature limit, we obtain the following equation:

$$\begin{aligned}
&-32\frac{g_{\mathrm{PW}}^2\mu^2}{N_0}\sum_{l_1,l_2}\int_0^{\infty}dx_1dx_2dx_3r^2T^2x_1r^2T^2x_2r^2T^2x_3 \\
&\quad\times\frac{1}{((2\pi l_1T)^2+x_1^2T^2)((2\pi l_2T)^2+x_2^2T^2)} \\
&=-32r^6T^2\frac{g_{\mathrm{PW}}^2\mu^2}{N_0}\sum_{l_1,l_2}\int_0^{\infty}dx_1dx_2dx_3 \\
&\quad\times\frac{x_1x_2x_3}{((2\pi l_1)^2+x_1^2)((2\pi l_2)^2+x_2^2)}.
\end{aligned} \tag{A60}$$

We want to evaluate the sum of the following form:

$$\sum_{l_1=-\infty}^{\infty}\sum_{l_2=-\infty}^{\infty}\frac{x_1}{(2\pi l_1)^2+x_1^2}\frac{x_2}{(2\pi l_2)^2+x_2^2}. \tag{A61}$$

Since the function $\frac{1}{2}\coth(\frac{z}{2})$ has poles at $z = 2\pi l\mathrm{i}$ and is everywhere else bounded and analytic, we may express Eq. (A61) as a contour integral as follows:

$$\begin{aligned}
&\frac{1}{2\pi\mathrm{i}}\oint dz_1\frac{-x_1}{z_1^2-x_1^2}\frac{1}{2}\coth\left(\frac{z_1}{2}\right)\frac{1}{2\pi\mathrm{i}}\oint dz_2\frac{-x_2}{z_2^2-x_2^2}\frac{1}{2} \\
&\quad\times\coth\left(\frac{z_2}{2}\right)=\frac{1}{2}\coth\left(\frac{x_1}{2}\right)\cdot\frac{1}{2}\coth\left(\frac{x_2}{2}\right).
\end{aligned} \tag{A62}$$

Then, with a suitable rearrangement of the exponentials in the hyperbolic cotangent, we obtain that

$$\frac{1}{4}+\frac{1}{2}\frac{1}{\mathrm{e}^{x_1}-1}+\frac{1}{2}\frac{1}{\mathrm{e}^{x_2}-1}+\frac{1}{\mathrm{e}^{x_1}-1}\frac{1}{\mathrm{e}^{x_2}-1}. \tag{A63}$$

Therefore, we can get the bosonic two-loop effective action as follows:

$$\begin{aligned}
\hat{W}_{\mathrm{B}}^{2\text{-loop}} &= -32r^6T^2\frac{g_{\mathrm{PW}}^2\mu^2}{N_0}\int_0^{\infty}dx_1dx_2dx_3x_3\left(\frac{1}{4}+\frac{1}{2}\right. \\
&\quad\left.\times\frac{1}{\mathrm{e}^{x_1}-1}+\frac{1}{2}\frac{1}{\mathrm{e}^{x_2}-1}+\frac{1}{\mathrm{e}^{x_1}-1}\frac{1}{\mathrm{e}^{x_2}-1}\right).
\end{aligned} \tag{A64}$$

b. Fermionic two-loop effective action

We calculate the fermionc two-loop effective action of the plane wave matrix model as follows:

$$\begin{aligned}
\hat{W}_{\mathrm{F}}^{2\text{-loop}} &= \left\langle\frac{1}{2}V_{\mathrm{F}}V_{\mathrm{F}}\right\rangle_{1\mathrm{PI}} \\
&\sim\sum_{123}\hat{\Psi}_{123}^{\dagger}\left(-32\frac{g_{\mathrm{PW}}^2\mu^2}{\beta N_0}\sum_{h_1,h_2}\frac{1}{\mathcal{P}_{\mathrm{F}}^2\mathcal{Q}_{\mathrm{F}}^2}\right. \\
&\quad\left.+64\frac{g_{\mathrm{PW}}^2\mu^2}{\beta N_0}\sum_{l_1,h_2}\frac{1}{\mathcal{P}_{\mathrm{B}}^2\mathcal{Q}_{\mathrm{F}}^2}\right)\hat{\Psi}_{123}.
\end{aligned} \tag{A65}$$

First, we calculate the first term of the fermionic two-loop effective action as follows:

$$\begin{aligned}
\hat{W}_{\mathrm{F}(0)}^{2\text{-loop}} &= -32\frac{g_{\mathrm{PW}}^2\mu^2}{\beta N_0}\sum_{h_1,h_2}\sum_{s,t,u}\sum_{123}\hat{\Psi}_{123}^{\dagger}\frac{1}{\mathcal{P}_{\mathrm{F}}^2\mathcal{Q}_{\mathrm{F}}^2}\hat{\Psi}_{123} \\
&=-32\frac{g_{\mathrm{PW}}^2\mu^2}{\beta N_0^3}\sum_{h_1,h_2}\sum_{s,t,u}\sum_{J_1,M_1}\sum_{J_2,M_2}\sum_{J_3,M_3} \\
&\quad\times\mathrm{Tr}\hat{Y}_{J_1M_1}^{(j_s,j_t)\dagger}\hat{Y}_{J_2M_2}^{(j_t,j_u)\dagger}\hat{Y}_{J_3M_3}^{(j_u,j_s)\dagger} \\
&\quad\times\frac{1}{(\omega_{h_1}^2+\mu^2J_1(J_1+1))(\omega_{h_2}^2+\mu^2J_2(J_2+1))} \\
&\quad\times\mathrm{Tr}\hat{Y}_{J_1M_1}^{(j_s,j_t)}\hat{Y}_{J_2M_2}^{(j_t,j_u)}\hat{Y}_{J_3M_3}^{(j_u,j_s)}.
\end{aligned} \tag{A66}$$

In the same way as the bosonic two-loop effective action, we obtain that

$$\begin{aligned}
&-32r^6T^2\frac{g_{\mathrm{PW}}^2\mu^2}{\beta N_0}\sum_{h_1,h_2}\int_0^{\infty}dx_1dx_2dx_3 \\
&\quad\times\frac{x_1x_2x_3}{((2\pi h_1)^2+x_1^2)((2\pi h_2)^2+x_2^2)}.
\end{aligned} \tag{A67}$$

Similarly, we evaluate the sum of the following form:

$$\sum_{h_1=-\infty}^{\infty}\sum_{h_2=-\infty}^{\infty}\frac{x_1}{(2\pi h_1)^2+x_1^2}\frac{x_2}{(2\pi h_2)^2+x_2^2}. \tag{A68}$$

Since the function $\frac{1}{2}\tanh(\frac{z}{2})$ has poles at $z = 2\pi h\mathrm{i}$ and is everywhere else bounded and analytic, we may express Eq. (A68) as a contour integral as follows:

$$\begin{aligned}
&\frac{1}{2\pi\mathrm{i}}\oint dz_1\frac{-x_1}{z_1^2-x_1^2}\frac{1}{2}\tanh\left(\frac{z_1}{2}\right)\frac{1}{2\pi\mathrm{i}}\oint dz_2\frac{-x_2}{z_2^2-x_2^2}\frac{1}{2} \\
&\quad\times\tanh\left(\frac{z_2}{2}\right)=\frac{1}{2}\tanh\left(\frac{x_1}{2}\right)\cdot\frac{1}{2}\tanh\left(\frac{x_2}{2}\right).
\end{aligned} \tag{A69}$$

Then, with a suitable rearrangement of the exponentials in the hyperbolic tangent, we obtain that

$$\frac{1}{4}-\frac{1}{2}\frac{1}{e^{x_1}+1}-\frac{1}{2}\frac{1}{e^{x_2}+1}+\frac{1}{e^{x_1}+1}\frac{1}{e^{x_2}+1}. \tag{A70}$$

Then, we calculate the second term of the fermionic two-loop effective action as follows:

$$\begin{aligned}\hat{W}^{\text{2-loop}}_{\text{F(1)}} &= \frac{64g^2_{\text{PW}}\mu^2}{\beta N_0}\sum_{l_1,h_2}\int_0^\infty dx_1dx_2dx_3r^2T^2x_1r^2T^2x_2r^2T^2x_3\frac{1}{((2\pi l_1T)^2+x_1^2T^2)((2\pi h_2T)^2+x_2^2T^2)}\\ &= \frac{64g^2_{\text{PW}}\mu^2}{N_0}\sum_{l_1,h_2}\int_0^\infty dx_1dx_2dx_3r^6T^2\frac{x_1x_2x_3}{((2\pi l_1)^2+x_1^2)((2\pi h_2)^2+x_2^2)}.\end{aligned} \tag{A71}$$

Similarly, we calculate the sum of the following form:

$$\begin{aligned}\sum_{l_1,h_2}\frac{x_1}{(2\pi l_1)^2+x_1^2}\frac{x_2}{(2\pi h_2)^2+x_2^2} &= \frac{1}{2\pi i}\oint dz_1\frac{-x_1}{z_1^2-x_1^2}\frac{1}{2}\coth\left(\frac{z_1}{2}\right)\frac{1}{2\pi i}\oint dz_2\frac{-x_2}{z_2^2-x_2^2}\frac{1}{2}\tanh\left(\frac{z_2}{2}\right)\\ &= \frac{1}{2}\coth\left(\frac{x_1}{2}\right)\cdot\frac{1}{2}\tanh\left(\frac{x_2}{2}\right)=\frac{1}{4}\left(1+\frac{2}{e^{x_1}-1}\right)\left(1-\frac{2}{e^{x_2}+1}\right)\\ &= \frac{1}{4}+\frac{1}{2}\frac{1}{e^{x_1}-1}-\frac{1}{2}\frac{1}{e^{x_2}+1}-\frac{1}{e^{x_1}-1}\frac{1}{e^{x_2}+1}.\end{aligned} \tag{A72}$$

Therefore, we can get the fermionic two-loop effective action as follows:

$$\begin{aligned}\hat{W}^{\text{2-loop}}_{\text{F}} &= -\frac{32g^2_{\text{PW}}\mu^2}{\beta N_0}r^6T^2\int_0^\infty dx_1dx_2dx_3x_3\left(\frac{1}{4}-\frac{1}{2}\frac{1}{e^{x_1}+1}-\frac{1}{2}\frac{1}{e^{x_2}+1}+\frac{1}{e^{x_1}+1}\frac{1}{e^{x_2}+1}\right)\\ &\quad+\frac{64g^2_{\text{PW}}\mu^2}{\beta N_0}r^6T^2\int_0^\infty dx_1dx_2dx_3x_3\left(\frac{1}{4}+\frac{1}{2}\frac{1}{e^{x_1}-1}-\frac{1}{2}\frac{1}{e^{x_2}+1}-\frac{1}{e^{x_1}-1}\frac{1}{e^{x_2}+1}\right).\end{aligned} \tag{A73}$$

c. All contributions of two-loop effective action

We summarize the two-loop effective action of the plane wave matrix model at finite temperature as follows:

$$\begin{aligned}\hat{W}^{\text{2-loop}} &= \hat{W}^{\text{2-loop}}_{\text{B}}+\hat{W}^{\text{2-loop}}_{\text{F}}\\ &= -\frac{32g^2_{\text{PW}}\mu^2}{\beta N_0}r^6T^2\int_0^\infty dx_1dx_2\int_{|x_1-x_2|}^{x_1+x_2}dx_3x_3\left(\frac{1}{e^{x_1}-1}\frac{1}{e^{x_2}-1}+\frac{1}{e^{x_1}+1}\frac{1}{e^{x_2}+1}+2\frac{1}{e^{x_1}-1}\frac{1}{e^{x_2}+1}\right)\\ &= -\frac{64g^2_{\text{PW}}\mu^2}{\beta N_0}r^6T^2\int_0^\infty dx_1dx_2\left(\frac{x_1}{e^{x_1}-1}\frac{x_2}{e^{x_2}-1}+\frac{x_1}{e^{x_1}+1}\frac{x_2}{e^{x_2}+1}+2\frac{x_1}{e^{x_1}-1}\frac{x_2}{e^{x_2}+1}\right)=-2\pi^4\frac{g^2_{\text{PW}}\mu^2}{N_0}r^6T^3.\end{aligned} \tag{A74}$$

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