Stability of false vacuum in supersymmetric theories with cosmic strings

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We study the stability of supersymmetry breaking vacuum in the presence of cosmic strings arising in the messenger sector. For certain ranges of the couplings, the desired supersymmetry breaking vacua become unstable against decay into phenomenologically unacceptable vacua. This sets constraints on the range of allowed values of the coupling constants appearing in the models and more generally on the chosen dynamics of gauge symmetry breaking.

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I. INTRODUCTION

Supersymmetry (SUSY) is a powerful principle which brings control in quantum field theory. In particular, its nonrenormalization properties [1,2] and its potential for solving the hierarchy problem [3–6] make it appealing as an ingredient of phenomenology, enabling a description in terms of weakly coupled degrees of freedom. In practice however, the implementation of the principle requires breaking supersymmetry in a hidden sector and communicating the effects to observed phenomenology indirectly; for reviews see [7,8]. The issue is further complicated by the fact that breaking supersymmetry is not itself a generic possibility. It was elucidated by [9] that the presence of R parity and its breakdown can play a crucial diagnostic role for understanding generic possibilities of breakdown. A persistent problem of supersymmetry breakdown however is that such vacua may be only local minima and have the danger of relaxing to a true minimum where supersymmetry is restored. This unpleasant possibility is however easy to avoid if the tunneling rate to the true vacuum can be made much longer than the known age of the Universe. New insights into consistency of supersymmetry breakdown in metastable vacua have been obtained recently in [10,11] in the context of supersymmetric QCD. In such models several possibilities exist, e.g., supersymmetry breaking vacuum need not be unique but could be degenerate with several supersymmetry preserving vacua. The issues involved in phenomenological implementation of these ideas have also been elucidated in [10–19]. Applications to cosmology appear, for example, in [20,21]

The question of whether the metastable vacua are cosmologically stable against quantum tunneling for all ranges of couplings received early attention in [22–24] in the context of gauge mediated supersymmetry breaking (see for instance the reviews [25,26]). In these models supersymmetry is dynamically broken in a hidden sector at a scale Λ_s and communicated to the standard model through a ‘‘messenger’’ sector at TeV scale. The messenger sector has suitable interactions with both the hidden and the visible sectors. The true vacuum in some of these models is supersymmetric and color breaking. Supersymmetry is broken and color is preserved only in metastable vacua.

In this paper we advance the point that it is not sufficient to study the stability of supersymmetry breaking vacua in their translationally invariant, i.e., spatially homogeneous avatar. Generically topological defects such as cosmic strings or monopoles may obtain in the course of implementation of any particular scheme. Such defects could arise in the early universe [27]. It is then necessary to take account of their presence when studying stability issues. A class of topological defects can exist which can nucleate the formation of the true vacuum in such a way that the exponential suppression inherent in tunneling phenomenon no longer obtains. Specifically, when a cosmic string is present in a false vacuum, such a local minimum can be rendered unstable against decay to a vacuum of lower energy [28,29]. This process was studied by one of us [30] in the context of phase transitions in grand unified theory models. It was shown that generically, the presence of the cosmic string entailed the consequence that false vacuum would ‘‘roll over’’ smoothly to the true vacuum without recourse to quantum tunneling. In this way, a putative first-order phase transition becomes second order, with important cosmological implications.

The same process is applicable here in the context of gauge mediated supersymmetry breaking models. It is possible to establish the required results numerically where formal methods provide a suggestive answer. We study a model of supersymmetry breaking which contains two classes of vacua, color breaking and color preserving. The color preserving metastable vacuum becomes parametrically unstable in the presence of cosmic strings. This puts constraints on the model of the messenger sector being used and may make some models unviable.

This paper is organized as follows. The next Sec. II provides a brief review of the class of models we have studied here. Section III discusses the vortex (strings) ansatz possible in the relevant vacua and the equations of motion to be solved. The results of the numerical solutions are described in Sec. IV. A semianalytic approach explain-

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ing the numerical results is discussed in Sec. V, followed by concluding remarks in Sec. VI.

II. A MODEL FOR MESSENGER SECTOR

In this section, we summarize the models of [31,32] as discussed in [22]. Supersymmetry is broken dynamically in the hidden sector. The messenger sector shares a symmetry G_m with the hidden sector. It is sufficient for G_m to be global, but we consider the possibility of G_m being local, permitting cosmologically viable strings. The case of global G_m is also commented upon. The messenger sector is responsible for transmitting supersymmetry breaking from the hidden sector to the visible sector. This messenger sector consists of the following: a gauge singlet, S, a pair of messenger quarks, q and $\bar{q}$, belonging to the 3 and $\bar{3}$ representation of the $SU(3)_c$ color group, and a pair of chiral superfields, N and P in vectorlike representations of G_m. The possibility of anomalies in the hidden sector forces the introduction of extra superfields E_i to cancel the anomalies. However, in this paper, we consider only "minimal" models which do not include these extra fields. The most general superpotential for the messenger sector is given by [22]

$$W_{\rm mes} = \kappa S\bar{q}q + \frac{\lambda}{3}S^3 + \lambda_1 PNS + W_1(P, N, E_i). \qquad (1)$$

The three coupling constants κ, λ, and λ_1 are all positive by a suitable phase definition of the fields. In the case where G_m is $U(1)$, if e is the $U(1)$ gauge coupling and P, N, and E_i have charges $+1$, -1, and y_i, the effective potential for the charged scalars after integrating out the hidden sector is given by [22]

$$V_{\rm SB} = M_1^2(|P|^2 - |N|^2 + y_i|E_i|^2) + M_2^2(|P|^2 + |N|^2 + y_i^2|E_i|^2) + \cdots \qquad (2)$$

in which the relation between the mass parameters M_i, $i = 1, 2$ and the supersymmetry breaking scale Λ_s is

$$M_i^2 = c_i\Lambda_s^2\bigg(\frac{e^2}{(4\pi)^2}\bigg)^i. \qquad (3)$$

The factors c_i ($i = 1, 2$) are of order unity, with their signs dependent on the content of the dynamical symmetry breaking sector. There are higher-dimensional terms coming from more loops and these are what the ellipsis in the expression for $V_{\rm SB}$ stands for. In addition, there are contributions to the scalar potential coming from the $U(1)$ D-term and various F-terms. In what follows, we will consider the set of minimal models which satisfy $\frac{\partial W_1}{\partial P} = \frac{\partial W_1}{\partial N} = 0$. This set includes models in which there are no E fields, models in which $W_1 = 0$, and models in which the E fields do not couple to P and N. After including the $U(1)$ D-terms and F-terms and setting $W_1 = 0$, we consider an explicit model in which ($M_1 = 0$), and the scalar potential of the messenger sector becomes [22]

$$V_{\rm mes} = \frac{e^2}{2}(|P|^2 - |N|^2)^2 + (M_2^2 + \lambda_1^2|S|^2)(|P|^2 + |N|^2) + \kappa^2|S|^2(|q|^2 + |\bar{q}|^2) + |\kappa\bar{q}q + \lambda S^2 + \lambda_1 PN|^2. \qquad (4)$$

We shall be interested in the case when $M_2^2 < 0$ which signals the breakdown of the $U(1)$. This makes $V_{\rm mes}$ unbounded from below, but in the total potential, higher-order terms in $V_{\rm SB}$ result in a deep global minimum far away in field space in which the visible sector is supersymmetric. For a viable local minimum, one must set $q = \bar{q} = 0$ in the expression for $V_{\rm mes}$ and in order to have $S \neq 0$ simultaneously, we must have

$$\lambda > \lambda_1. \qquad (5)$$

The local minima lie at $q = \bar{q} = 0$ and

$$|P|^2 = |N|^2 = -M_2^2\frac{\lambda}{\lambda_1^3(2 - \lambda_1/\lambda)}, \qquad (6)$$

$$|S|^2 = -M_2^2\frac{1 - \lambda_1/\lambda}{\lambda_1^2(2 - \lambda_1/\lambda)}, \qquad (7)$$

$$\arg(PNS^{*2}) = \pi. \qquad (8)$$

Along with the condition $\lambda > \lambda_1$, stability of the local minima requires the following relations between the couplings [22]:

$$\lambda_1^3 \leq 2\lambda e^2, \qquad (9)$$

$$\lambda_1 \leq \frac{\kappa\lambda}{\kappa + \lambda}. \qquad (10)$$

Condition (10) is also mentioned in [33] wherein it is noted that there is still a possibility of color breaking vacua when (10) holds.

The scalar potential of the messenger sector given in Eq. (4) contains two important types of vacua. One of these, which will be denoted as $|V_1\rangle$ later in the paper, is the supersymmetry breaking local minimum described above. This vacuum has $\langle S\rangle \neq 0$ and $\langle q\rangle = \langle\bar{q}\rangle = 0$, which means that SUSY is broken while the color gauge group is unbroken. The other vacuum, which shall be denoted $|V_2\rangle$, also has $\langle S\rangle \neq 0$ but the fields q and $\bar{q}$ get nonzero vacuum expectation values (VEVs). This means that $|V_2\rangle$ is a phenomenologically undesirable minimum in which SUSY is still broken but the color gauge group is also broken. If cosmic strings are supported, these vacua are modified to what will be denoted as $|V_1^{(\rm string)}\rangle$ and $|V_2^{(\rm string)}\rangle$, respectively. It will be shown in a later section that $|V_1^{(\rm string)}\rangle$ becomes parametrically unstable toward decay into $|V_2^{(\rm string)}\rangle$. The next section describes the available string solutions.

III. THE STRING ANSATZ

With the scalar potential for the messenger sector as given in (4), the full Lagrangian for all fields ($P, N, S, q, \bar{q}$) can be written down. The fact that string solutions are cylindrically symmetric makes it convenient to use cylindrical coordinates. We ignore the z-dependence and look for time-independent solutions. In a local minimum, $|P| = |N|$ from (6) and $\langle S \rangle$ is as given in (7). The ansatz functions for the scalar fields are a simple generalization of the Abrikosov-Nielsen-Olesen string [34,35] (see also [36])

$$P = \eta f(r) e^{i\theta}, \tag{11}$$

$$N = \eta f(r) e^{-i\theta}, \tag{12}$$

$$S = \eta g(r) e^{i\pi/2}, \tag{13}$$

$$q = \bar{q} = \eta h(r) e^{i\pi/2}. \tag{14}$$

The value of η is given by the right-hand side of Eq. (6) Note that the winding number for the P and N fields has been chosen equal and opposite. Single valuedness requires that the phase completes a circuit of 2π at infinity. This requires the phase to be an integer multiple of cylindrical angle θ. Along with the choice of phase for S as in (13), this winding ensures the condition (8). This is the expected minimal energy configuration. The phases for q, and $\bar{q}$ satisfy the expectation that their product remains negative. At the core of the vortex, the fields P and N must vanish, and hence $f(0) = 0$. At infinity, P and N approach their vacuum expectation values which are denoted by η, and hence $f(\infty) = 1$. Similarly, $g(r)$ approaches the value given in (7) at infinity but can be nonzero in the core. The value of $h(r)$ can be nonzero in the core also, but its value at infinity is of importance in determining whether the resulting vacuum is phenomenologically viable or not. This is because the vacuum with h and hence q and $\bar{q}$ nonzero is color breaking.

A vortex solution in P and N also includes a coupling of the fields to a gauge field $A(r)$. The gauge field behavior is described by a function $a(r)$ defined through

$$A_\theta(r) = \frac{1}{er} a(r), \tag{15}$$

$$A_0 = A_r = A_z = 0, \tag{16}$$

where for continuity of $A_\theta(r)$, we have $a(0) = 0$, and e is the unit of the Abelian G_m charge. At infinity, $A_\theta(r)$ is pure gauge and goes as $1/r$ and hence $a(\infty) = 1$. The Lagrangian for the system can be written as

$$L = \tfrac{1}{2}(\partial_\mu S)^2 + \tfrac{1}{2}(\partial_\mu q)^2 + \tfrac{1}{2}(\partial_\mu \bar{q})^2 + \tfrac{1}{2}|(\partial_\mu + ieA_\mu)P|^2 + \tfrac{1}{2}|(\partial_\mu + ieA_\mu)N|^2 - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - V_{\mathrm{mes}}, \tag{17}$$

where V_{mes} is as given in (4). After substituting Eqs. (11)–(15) into the Lagrangian, the equations of motion can be written in terms of $f(r)$, $a(r)$, $g(r)$, and $h(r)$. They are a set of coupled second-order and nonlinear ordinary differential equations given by

$$\frac{d^2 f}{dr^2} + \frac{1}{r}\frac{df}{dr} - \frac{1}{r^2} f(a-1)^2 - \left[\frac{M_2^2}{\eta^2} - \lambda_1(\lambda - \lambda_1)g^2 - \lambda_1 \kappa h^2\right] f - \lambda_1^2 f^3 = 0, \tag{18}$$

$$\frac{d^2 a}{dr^2} - \frac{1}{r}\frac{da}{dr} - 2e^2 f^2 (a-1) = 0, \tag{19}$$

$$\frac{d^2 g}{dr^2} + \frac{1}{r}\frac{dg}{dr} - [2\lambda_1(\lambda_1 - \lambda)f^2 + 2\kappa(\kappa + \lambda)h^2]g - 2\lambda^2 g^3 = 0, \tag{20}$$

$$\frac{d^2 h}{dr^2} + \frac{1}{r}\frac{dh}{dr} - [\kappa(\kappa + \lambda)g^2 - \kappa\lambda_1 f^2]h - k^2 h^3 = 0. \tag{21}$$

The fields P and N in which we have set up string configurations have a $U(1)$ symmetry. As is well known, existence of cosmic string solutions requires that the space of equivalent vacua be necessarily multiply connected. Generically in a larger gauge group such as $SO(N)$, Spin(N), or $SU(N)$, cosmic strings are in principle possible for P and N in any representation other than the vector or the fundamental. Thus, the conclusions of this paper have relevance to more general cases.

IV. TESTING THE VACUA

A. Homogeneous vacuum

We begin with calculating the field values at infinity which can be obtained easily from Eqs. (18)–(21). At $r = \infty$, all the derivative terms vanish due to translational invariance and so do the terms containing $(1/r^2)$. The equations get reduced to the following simple set of simultaneous equations:

$$\left[\frac{M_2^2}{\eta^2} - \lambda_1(\lambda - \lambda_1)g^2 - \lambda_1 \kappa h^2\right] f + \lambda_1^2 f^3 = 0, \tag{22}$$

$$2e^2 f^2 (a-1) = 0, \tag{23}$$

$$[2\lambda_1(\lambda_1 - \lambda)f^2 + 2\kappa(\kappa + \lambda)h^2]g + 2\lambda^2 g^3 = 0, \tag{24}$$

$$[\kappa(\kappa + \lambda)g^2 - \kappa\lambda_1 f^2]h + k^2 h^3 = 0. \tag{25}$$

One must note that values for three couplings ($\kappa, \lambda, \lambda_1$) must be given for each unique set of vacuum solutions. The set of vacuum solutions consists of four solutions. One is the trivial solution in which all fields vanish. Another is one in which f and a are nonzero but SUSY breaking is not being communicated since g and h are both zero.

The other two translation invariant minima shall be denoted $|V_1\rangle$ and $|V_2\rangle$. Their field content is as follows:

$$|V_1\rangle\colon\ f = a = 1, \quad g \neq 0, \quad h = 0, \tag{26}$$

$$|V_2\rangle\colon\ f \lesssim 1, \quad a = 1, \quad g \neq 0, \quad h \neq 0. \tag{27}$$

$|V_1\rangle$ is the desired supersymmetry breaking and color preserving local minimum described in Sec. II. The value of g is exactly as required by Eq. (7), and it is a function of the couplings. $|V_2\rangle$ is an undesirable minimum in which h and hence q and $\bar{q}$ get VEVs. Furthermore, what is interesting is that for all ranges of couplings, its energy is always lower than that of $|V_1\rangle$.

$$\langle V_2|V_{\rm mes}|V_2\rangle < \langle V_1|V_{\rm mes}|V_1\rangle. \tag{28}$$

The expression for $V_{\rm mes}$ in terms of (f, g, h) can be written as

$$V_{\rm mes} = \eta^4\left[2f^2\left(\lambda_1^2 g^2 - \frac{|M_2|^2}{\eta^2}\right) + 2\kappa^2 g^2 h^2 + (\lambda_1 f^2 - \lambda g^2 - \kappa h^2)^2\right]. \tag{29}$$

It is a simple exercise to check that for each set of solutions, $V_{\rm mes}$ is always smaller for $|V_2\rangle$ compared to $|V_1\rangle$.

B. Vacuum with gauge string

A numerical strategy exists for solving Eqs. (11)–(15). This consists of using relaxation techniques, after discretizing the equations of motion on a grid converting them to a set of coupled polynomial equations. In conformity with physical expectations the initial guess is one with $f = a = 0$ in the core and with g and h arbitrary. At infinity, the trial is chosen to be either settling into $|V_1\rangle$ or $|V_2\rangle$. The exact values of (f, a, g, h) in $|V_1\rangle$ and $|V_2\rangle$ can be found by first solving Eqs. (22)–(25). We denote by $|V_1^{(\rm string)}\rangle$ the static state of the system in which the cosmic string configuration approaches $|V_1\rangle$ (up to a θ-dependent phase) as $|\vec{r}| \to \infty$. Similarly, $|V_2^{(\rm string)}\rangle$ denotes a static string solution which asymptotes to $|V_2\rangle$. An example of the solutions in a case where both $|V_1^{(\rm string)}\rangle$ and $|V_2^{(\rm string)}\rangle$ are stable is shown in Figs. 1–4.

It may happen that for some values of the couplings, a solution with the expected field values at infinity cannot be obtained. When this happens, that solution with its asymptotic vacuum values is not tenable. We have explored the stability of $|V_1^{(\rm string)}\rangle$ and $|V_2^{(\rm string)}\rangle$ over a range of the couplings. For the entire range of couplings studied, the string solution $|V_2^{(\rm string)}\rangle$ is always obtained. However, no stable static solution $|V_1^{(\rm string)}\rangle$ can be obtained for certain ranges which are indicated in Table I. Note specifically that for $\kappa = 1$ and $\lambda = 1.1$, values of $\lambda_1 < 0.1$, which are of phenomenological interest, are immediately ruled out. It is for these ranges of the couplings that the corresponding metastable vacua are no longer local minima due to the presence of cosmic strings.

In the context of the early universe, the parameters in the effective action are as a rule temperature dependent. The above results then imply that we may start with a phase wherein a state of the type $|V_1^{(\rm string)}\rangle$ may be a local minimum but with reduction of temperature, the string solution may not exist in the sense argued above. How this comes about in the present example is taken up at the end of Sec. V. The subsequent dynamics can be simulated by restoring time dependence in Eqs. (18)–(21). It was shown in [30] that the string configuration rendered parametrically unstable undergoes a real time evolution into a possible stable string configuration with a modified vacuum at infinity. This can be referred to as a roll-over process, a

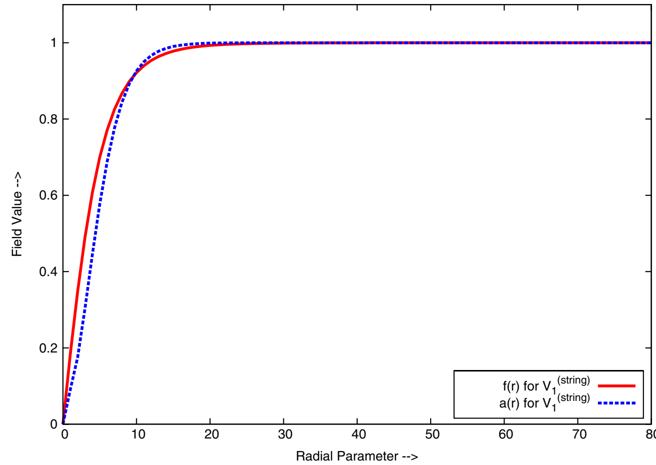

FIG. 1 (color online). $f(r)$ and $a(r)$ for $|V_1^{(\rm string)}\rangle$ with boundary conditions as in Eq. (26), and $\kappa = 1.3$, $\lambda = 1.45$, and $\lambda_1 = 0.65$. Here, $f_\infty = 1$ and $a_\infty = 1$.

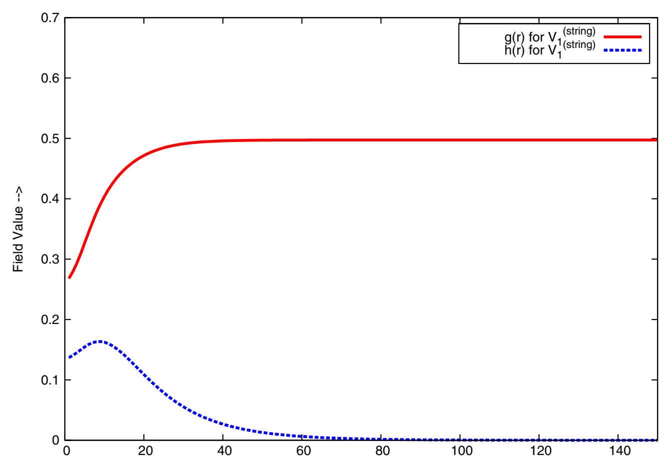

FIG. 2 (color online). $g(r)$ and $h(r)$ for $|V_1^{(\rm string)}\rangle$ with boundary conditions as in Eq. (26), and $\kappa = 1.3$, $\lambda = 1.45$, and $\lambda_1 = 0.65$. Here, $g_\infty = 0.497\,317$ and $h_\infty = 0$.

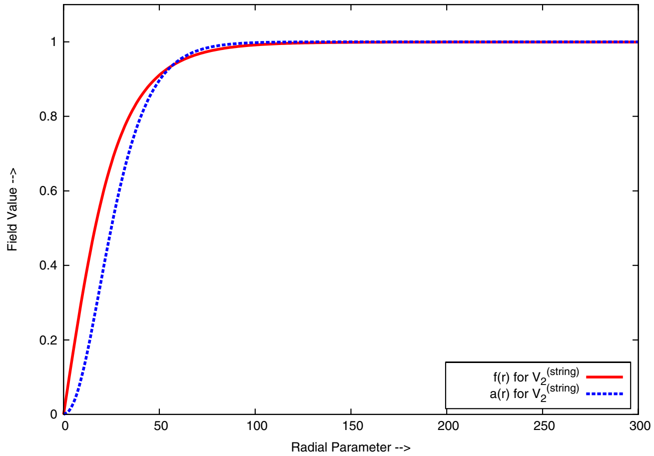

FIG. 3 (color online). $f(r)$ and $a(r)$ for $|V_2^{(\mathrm{string})}\rangle$ with boundary conditions as in Eq. (27), and $\kappa = 1.3$, $\lambda = 1.45$, and $\lambda_1 = 0.65$. Here, $f_\infty = 0.999\,405$ and $a_\infty = 1$.

prompt, semiclassical evolution rather than tunneling by spontaneous formation of bubbles.

C. Global cosmic strings

Our demonstration though specific to the messenger group of this example may have relevance to a more general setting where some sector of the theory possesses global symmetries which are broken in the desirable vacuum. In this case, the strings arising are global cosmic strings. In our example this corresponds to setting $a(r)$ identically to zero. We have looked for solutions without the $a(r)$ field for some ranges of the couplings. The results are shown in Table II. It is found that this condition rules out $|V_1^{(\mathrm{string})}\rangle$ for a larger range of couplings as compared to the case in which $a(r)$ is present. The results for $|V_2^{(\mathrm{string})}\rangle$ remain unchanged.

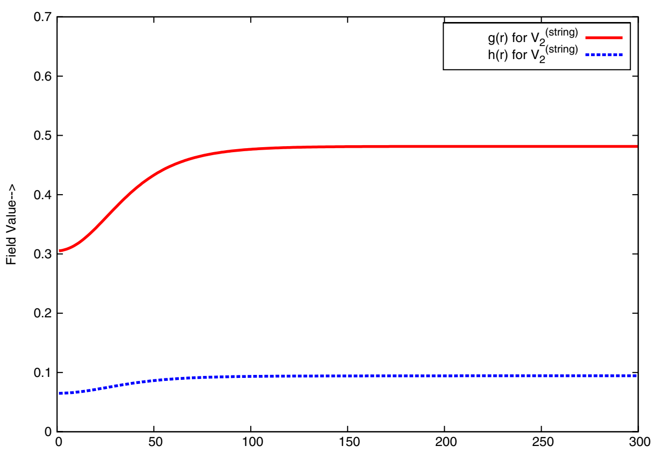

FIG. 4 (color online). $g(r)$ and $h(r)$ for $|V_2^{(\mathrm{string})}\rangle$ with boundary conditions as in Eq. (27), and $\kappa = 1.3$, $\lambda = 1.45$, and $\lambda_1 = 0.65$. Here, $g_\infty = 0.481\,526$ and $h_\infty = 0.094\,434$.

TABLE I. Testing the existence of local minima viable in the presence of a cosmic string. Check marks indicate when solutions are found and crosses indicate when solutions cannot be obtained. Vacua of type 1, Eq. (26), become parametrically disallowed due to the presence of a cosmic string.

κ	λ_1	λ	$\lvert V_1^{(\mathrm{string})}\rangle$	$\lvert V_2^{(\mathrm{string})}\rangle$
2.4	1.2	≥ 2.41	$\times$	$\surd$
2	1	≥ 2.01	$\times$	$\surd$
2	≤ 0.75	2.5	$\times$	$\surd$
1.7	0.85	≥ 1.71	$\times$	$\surd$
1.3	0.65	≤ 1.42	$\times$	$\surd$
1.3	0.65	≥ 1.43	$\surd$	$\surd$
1.3	≤ 0.65	1.5	$\surd$	$\surd$
1	0.5	≤ 1.07	$\times$	$\surd$
1	0.5	≥ 1.1	$\surd$	$\surd$
1	≥ 0.3	1.1	$\surd$	$\surd$
1	≤ 0.15	1.1	$\times$	$\surd$
0.6	0.3	≤ 0.68	$\times$	$\surd$

TABLE II. Sample results for global strings, i.e., no $a(r)$ field present. These are to be compared to rows 5–9 of Table I.

κ	λ_1	λ	$\lvert V_1^{(\mathrm{string})}\rangle$	$\lvert V_2^{(\mathrm{string})}\rangle$
1.3	0.65	≥ 1.3	$\times$	$\surd$
1	0.5	≤ 1.19	$\times$	$\surd$
1	0.5	≥ 1.20	$\surd$	$\surd$

A global cosmic string network is not desirable for stable cosmology [27]. This is because the energy per unit length of a global string is divergent and can dominate the energy density of the Universe. However the divergence of the energy with distance from the string is logarithmic and a transient network of such strings cannot be ruled out. We expect such a network to eventually relax to a homogeneous vacuum. Even a transient network, however, may be capable of destabilizing a particular local minimum according to the discussion of this section.

V. STABILITY ANALYSIS OF $|V_1^{(\mathrm{string})}\rangle$

In the previous section, the stability of $|V_1^{(\mathrm{string})}\rangle$ has been studied numerically. As illustrated in Table I, there are certain domains in the parameter space of κ, λ_1, and λ in which $|V_1^{(\mathrm{string})}\rangle$ is not admissible. The nonexistence of such solutions was then understood as an unavailability of $|V_1^{(\mathrm{string})}\rangle$ for those parameter values. This numerical result can be confirmed by a semianalytic treatment following an approach discussed in [28].

A preliminary analysis can be made for the translation invariant vacuum to study the effect of changing λ. Referring to Eq. (29), the effective squared mass m_{eff}^2 for the h field is given by

$$m_{\mathrm{eff}}^2 = 2g^2(\kappa^2 + \kappa\lambda) - 2\kappa\lambda_1 f^2. \qquad (30)$$

Now if κ and λ_1 are held fixed, when λ is large enough, this quantity is positive and $h = 0$ is admissible as a local minimum. As λ reduces, there comes a point when $m^2_{\rm eff}$ becomes negative. This corresponds to the regions of instability shown in Table I. For example, when $\kappa = 1.3$ and $\lambda_1 = 0.65$, reducing λ below 1.43 drives its effective squared mass negative making $h = 0$ unstable.

We now proceed along the lines of [28] to analyze the linear stability of $|V_1^{(\rm string)}\rangle$ by adding a small time-dependent term to the time-independent numerical solution. If small oscillations around this solution have only real frequencies, the configuration is considered stable. For nonavailability of $|V_1^{(\rm string)}\rangle$, at least one of the modes of oscillation must possess an imaginary frequency. The stability analysis for $|V_1^{(\rm string)}\rangle$ is greatly simplified by the following observation. The fields f and g do not differ significantly in states $|V_1^{(\rm string)}\rangle$ and $|V_2^{(\rm string)}\rangle$ as seen from Figs. 1–4. Only h differs significantly and the possible time dependence of the background fields f and g can be ignored. The equation of motion for h with time dependence restored is

$$-\frac{d^2h}{dt^2} + \frac{d^2h}{dr^2} + \frac{1}{r}\frac{dh}{dr} - [\kappa(\kappa+\lambda)g^2 - \kappa\lambda_1 f^2]h - k^2h^3 = 0, \tag{31}$$

and we now write

$$h(r, t) = \tilde{h}(r) + p(r)e^{i\omega t}. \tag{32}$$

Here, $\tilde{h}$ is the time-independent solution obtained for the h field and $p \ll \tilde{h}$. Similarly, the time-independent fields f and g in Eq. (31) are written as $\tilde{f}$ and $\tilde{g}$. Substituting Eq. (32) in (31) and linearizing the equation for $p(r)$, we get

$$\omega^2 p = -\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}\right]p + [3\kappa^2\tilde{h}^2 + \kappa(\kappa+\lambda)\tilde{g}^2 - \kappa\lambda_1\tilde{f}^2]p. \tag{33}$$

Equation (33) has the form of a one-dimensional Schrödinger equation with a potential $u(r)$ given by

$$u(r) = 3\kappa^2\tilde{h}(r)^2 + \kappa(\kappa+\lambda)\tilde{g}(r)^2 - \kappa\lambda_1\tilde{f}(r)^2. \tag{34}$$

Looking for imaginary modes of the frequency ω now reduces to finding negative-energy bound states for this potential. Figure 5 depicts the behavior of the potential for values of couplings which lie in both the stable and unstable regions for $|V_1^{(\rm string)}\rangle$. Referring to Table I, when $\kappa = 1.3$ and $\lambda_1 = 0.65$, $|V_1^{(\rm string)}\rangle$ is stable for $\lambda = 1.5$. This is in agreement with the fact that the minimum of the potential for $\lambda = 1.5$ has positive energy. As λ reduces toward $\lambda = 1.43$, notice that the minimum of the energy starts reducing until it starts becoming negative near $\lambda = 1.43$. This once again confirms the numerical result that vacua of type $|V_1^{(\rm string)}\rangle$ become unavailable for values of $\lambda < 1.43$. We have similarly computed the potential $u(r)$ for critical values of κ and λ, and found similar solutions confirming the results stated in Table I.

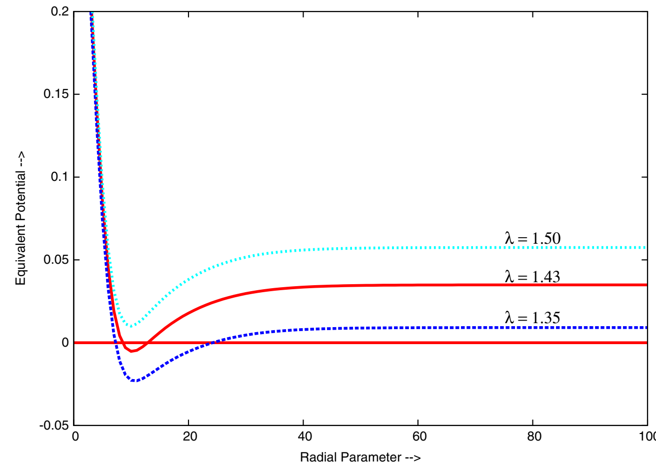

FIG. 5 (color online). The equivalent Schrödinger potential of Eq. (33) with $\kappa = 1.3$, $\lambda_1 = 0.65$, and different values of λ. A positive energy minimum results in a stable solution. When a negative-energy bound state is possible, vacua of type $|V_1^{(\rm string)}\rangle$ become unavailable.

Returning to our comment on the early universe setting at the end of Sec. IV B, the field f (the VEV of P and N) has negative effective mass squared, $M_2^2 < 0$. High temperature corrections from the gauge sector add a term of the form AT^2 with $A > 0$ to this effective mass squared [37,38], which would drive the temperature dependent value f_T to zero at high temperature, restoring the gauge symmetry. Below the symmetry breaking critical temperature f_T becomes nonzero. As long as this value remains small, the effective mass squared for the h in Eq. (30) remains positive. This makes $|V_1\rangle$ a valid local minimum and admits a vortex sector $|V_1^{(\rm string)}\rangle$. At an even lower temperature f_T grows sufficiently large, and the roll-over transition sets in.

VI. CONCLUSION

One of the main requirements in the minimal models of gauge mediated supersymmetry breaking is that the messenger squarks do not develop VEVs [22]. In this paper, we have shown an example in which cosmic strings are present in the metastable vacua. These configurations can roll over classically into undesirable configurations which asymptote into vacua in which $\langle q\rangle = \langle\bar{q}\rangle \neq 0$. This happens for certain ranges of the couplings κ, λ, and λ_1, including the phenomenologically interesting region in which $\lambda_1 < 0.1$. For a general messenger group G_m stable cosmic strings would exist depending on the choice of representation of the fields P and N. Furthermore, we have evidence that if G_m is global the instability issue becomes more severe. In all such models, it should be possible to obtain new constraints on the coupling constants from detailed simulations of the cosmic strings. Such studies provide tighter constraints on the parameters as compared to those from quantum tunneling effects.

In principle, these techniques are applicable to the SUSY breaking sector itself provided the model supports solitonic solutions. This aspect of the problem will be dealt with in future work.

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