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Fermion tunneling from higher-dimensional black holes

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Via the semiclassical approximation method, we study the 1/2-spin fermion tunneling from a higher-dimensional black hole. In our work, the Dirac equations are transformed into a simple form, and then we simplify the fermion tunneling research to the study of the Hamilton-Jacobi equation in curved space-time. Finally, we get the fermion tunneling rates and the Hawking temperatures at the event horizon of higher-dimensional black holes. We study fermion tunneling of a higher-dimensional Schwarzschild black hole and a higher-dimensional spherically symmetric quintessence black hole. In fact, this method is also applicable to the study of fermion tunneling from four-dimensional or lower-dimensional black holes, and we will take the rainbow-Finsler black hole as an example in order to make the fact explicit.

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I. INTRODUCTION

Based on Hawking's research on quantum radiation from black holes in 1975 [1,2], allowing for quantum effects, some researchers believe that black holes can produce a kind of thermal radiation. The study of Hawking radiation has become a focus in theoretical physics. Recently, Kraus, Parikh, and Wilczek *et al.* have developed a quantum tunneling theory for researching Hawking radiation of black holes [3–5]. In this theory, the mechanism of Hawking radiation is regarded as the process of tunneling. Then, through the WKB approximation the tunneling rate from the inside of a black hole's horizon to the outside can be determined, $\Gamma \propto \exp(-2\,\mathrm{Im}S)$. (Here S is the classical action of the trajectory.) Via this theory, researchers have studied several black holes [6–11]. In 2008, Kerner and Mann brought forward a method to study the Hawking radiation of 1/2-spin fermions [12,13]. In their research, the wave function of the Dirac equation is decompounded as spin-up and spin-down cases, and then the Dirac equation is simplified by the semiclassical approximation to finally obtain the Hawking temperature and tunneling rate. In this way, the issue of fermion tunneling was successfully solved. Later, Chen *et al.* studied the tunneling radiation from charged dilatonic black holes and de Sitter space-time [14,15]; Li *et al.* researched the tunneling behavior near the horizons of lower-dimensional Bañados-Teitelboim-Zanelli black holes and Kerr black holes [16,17]; fermion tunneling from several nonstatic black holes was researched by us [18–21]; and Jiang *et al.* investigated fermion tunneling from the horizons of five-dimensional black holes [22–24]. However, no one has studied fermion tunneling at the event horizon of higher-dimensional black holes, because the Dirac equations are very complicated in higher-dimensional cases. More so, the incredible difficulty is that the number of the Dirac equation is arbitrary in arbitrary-dimensional space-time.

In this paper, we research fermion tunneling at the event horizons of higher-dimensional black holes. Using the semiclassical approximation method, we first simplify the Dirac equations via separating the variable for the action, and then focus on solving an equation which comes from the condition that there is a nontrivial solution in the Dirac equations. The 1/2-spin fermion tunneling of higher-dimensional black holes could then finally be researched by this method. It should be noted that in our method, the Hamilton-Jacobi equation is obtained, so the research is simplified greatly. In the following section, we research fermion tunneling behavior of higher-dimensional Schwarzschild black holes, and then we study fermion tunneling of higher-dimensional spherically symmetric quintessence black holes. In fact, our method also holds true for fermion tunneling of four-dimensional and lower-dimensional black holes. So, in Sec. IV, we use a rainbow-Finsler black hole as an example to illustrate this point. Finally, we end with some discussions and conclusions.

II. FERMION TUNNELING OF HIGHER-DIMENSIONAL SCHWARZSCHILD BLACK HOLES

We first study fermion tunneling from higher-dimensional Schwarzschild black holes. The metric of a Schwarzschild black hole with extra n dimensions is [25,26]

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega^2_{2+n}, \tag{1}$$

where

$$f(r) = 1 - \left(\frac{r_0}{r}\right)^{n+1}. \tag{2}$$

Obviously, the position of the event horizon is r_0, which should satisfy the equation $f(r_0) = 0$. In this space-time, we can take the higher-dimensional Dirac equation into

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account, namely,

$$\boldsymbol{\gamma}^{\mu} D_{\mu} \boldsymbol{\Psi} + \frac{m}{\hbar} \boldsymbol{\Psi} = 0, \qquad \mu = t, r, \theta, \varphi, \cdots x^{\eta} \cdots x^{n+4}, \tag{3}$$

where θ and φ are angular coordinates, and $x^{\eta} \cdots x^{n+4}$ are extra-dimensional coordinates,

$$D_{\mu} = \partial_{\mu} + \frac{i}{2} \Gamma^{\alpha}{}_{\mu}{}^{\beta} \Pi_{\alpha\beta}, \tag{4}$$

$$\Pi_{\alpha\beta} = \frac{i}{4} [\boldsymbol{\gamma}^{\alpha}, \boldsymbol{\gamma}^{\beta}], \tag{5}$$

and the gamma matrices satisfy the condition that

$$\{\boldsymbol{\gamma}^{\mu}, \boldsymbol{\gamma}^{\upsilon}\} = 2g^{\mu\upsilon} I. \tag{6}$$

In d-dimensional space-time ($d = n + 4$ in Schwarzschild space-time with extra n dimensions), we choose gamma matrices such as

$$\boldsymbol{\gamma}^{t}_{m\times m} = \frac{1}{\sqrt{f}} \begin{pmatrix} i\boldsymbol{I}_{(m/2)\times(m/2)} & 0 \\ 0 & -i\boldsymbol{I}_{(m/2)\times(m/2)} \end{pmatrix}, \tag{7}$$

$$\boldsymbol{\gamma}^{r}_{m\times m} = \sqrt{f} \hat{\boldsymbol{\gamma}}^{3}_{m\times m} = \sqrt{f} \begin{pmatrix} 0 & \hat{\boldsymbol{\gamma}}^{3}_{(m/2)\times(m/2)} \\ \hat{\boldsymbol{\gamma}}^{3}_{(m/2)\times(m/2)} & 0 \end{pmatrix}, \tag{8}$$

$$\boldsymbol{\gamma}^{\theta}_{m\times m} = \sqrt{g^{\theta\theta}} \hat{\boldsymbol{\gamma}}^{1}_{m\times m} = \sqrt{g^{\theta\theta}} \begin{pmatrix} 0 & \hat{\boldsymbol{\gamma}}^{1}_{(m/2)\times(m/2)} \\ \hat{\boldsymbol{\gamma}}^{1}_{(m/2)\times(m/2)} & 0 \end{pmatrix}, \tag{9}$$

$$\boldsymbol{\gamma}^{\varphi}_{m\times m} = \sqrt{g^{\varphi\varphi}} \hat{\boldsymbol{\gamma}}^{2}_{m\times m} = \sqrt{g^{\varphi\varphi}} \begin{pmatrix} 0 & \hat{\boldsymbol{\gamma}}^{2}_{(m/2)\times(m/2)} \\ \hat{\boldsymbol{\gamma}}^{2}_{(m/2)\times(m/2)} & 0 \end{pmatrix}, \tag{10}$$

$$\cdots\cdots$$

$$\boldsymbol{\gamma}^{\eta}_{m\times m} = \sqrt{g^{\eta\eta}} \hat{\boldsymbol{\gamma}}^{l}_{m\times m} = \sqrt{g^{\eta\eta}} \begin{pmatrix} 0 & \hat{\boldsymbol{\gamma}}^{l}_{(m/2)\times(m/2)} \\ \hat{\boldsymbol{\gamma}}^{l}_{(m/2)\times(m/2)} & 0 \end{pmatrix}, \tag{11}$$

$$\cdots\cdots$$

$$\boldsymbol{\gamma}^{x^{n+4}}_{m\times m} = \sqrt{g^{x^{n+4}x^{n+4}}} \begin{pmatrix} 0 & -i\boldsymbol{I}_{(m/2)\times(m/2)} \\ i\boldsymbol{I}_{(m/2)\times(m/2)} & 0 \end{pmatrix}, \tag{12}$$

where $m = 2^{d/2}$ ($m = 2^{(d-1)/2}$) is the order of the matrix in even- (odd-) dimensional space-time; Eq. (12) is necessary in odd-dimensional space-time, but it is unnecessary in even-dimensional space-time; $\boldsymbol{I}_{(m/2)\times(m/2)}$ is a unit matrix with $\frac{m}{2} \times \frac{m}{2}$ orders; 0 is the zero matrix with $\frac{m}{2} \times \frac{m}{2}$ orders; $\boldsymbol{\gamma}^{\upsilon}_{(m/2)\times(m/2)}$ and $\hat{\boldsymbol{\gamma}}^{\upsilon}_{(m/2)\times(m/2)}$ are the υth gamma matrix with $\frac{m}{2} \times \frac{m}{2}$ orders in curved and flat space-time, respectively. The gamma matrices in flat space-time satisfy $\{\hat{\boldsymbol{\gamma}}^{\upsilon}_{(m/2)\times(m/2)}, \hat{\boldsymbol{\gamma}}^{\mu}_{(m/2)\times(m/2)}\} = 2\delta_{\upsilon\mu} \boldsymbol{I}_{(m/2)\times(m/2)}$, where

$$\delta_{\upsilon\mu} = \begin{cases} 1 & \upsilon = \mu \\ 0 & \upsilon \neq \mu. \end{cases}$$

In particular, the 2×2-order matrices are

$$\hat{\boldsymbol{\gamma}}^{3}_{2\times 2} = \boldsymbol{\sigma}^{3} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{13}$$

$$\hat{\boldsymbol{\gamma}}^{1}_{2\times 2} = \boldsymbol{\sigma}^{1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{14}$$

$$\hat{\boldsymbol{\gamma}}^{2}_{2\times 2} = \boldsymbol{\sigma}^{2} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \tag{15}$$

The spinor wave function $\boldsymbol{\Psi}$ in Eq. (3) can be written as

$$\boldsymbol{\Psi} = \begin{bmatrix} \boldsymbol{A}_{(m/2)\times 1}(t, r, \theta, \varphi, \cdots x^{\mu} \cdots x^{n+4}) \\ \boldsymbol{B}_{(m/2)\times 1}(t, r, \theta, \varphi, \cdots x^{\mu} \cdots x^{n+4}) \end{bmatrix} \times e^{(i/\hbar) S(t,r,\theta,\varphi,\cdots x^{\mu}\cdots x^{n+4})}, \tag{16}$$

where $\boldsymbol{A}_{(m/2)\times 1}(t, r, \theta, \varphi, \cdots x^{\mu} \cdots x^{n+4})$ and $\boldsymbol{B}_{(m/2)\times 1}(t, r, \theta, \varphi, \cdots x^{\mu} \cdots x^{n+4})$ are $\frac{m}{2} \times 1$ function column matrices. Then we substitute Eq. (16) into Eq. (3) near the horizon. After dividing by the exponential terms and multiplying by $\hbar$, the resulting equations to leading order in $\hbar$ are

$$\begin{pmatrix} \boldsymbol{C} & \boldsymbol{D} \\ \boldsymbol{E} & \boldsymbol{F} \end{pmatrix} \begin{pmatrix} \boldsymbol{A}_{(m/2)\times 1} \\ \boldsymbol{B}_{(m/2)\times 1} \end{pmatrix} = 0, \tag{17}$$

where

$$\boldsymbol{C} = \frac{i}{\sqrt{f}} \frac{\partial S}{\partial t} \boldsymbol{I}_{(m/2)\times(m/2)} - im\boldsymbol{I}_{(m/2)\times(m/2)}, \tag{18}$$

$$\begin{aligned} \boldsymbol{D} = {} & \sqrt{f} \frac{\partial S}{\partial r} \hat{\boldsymbol{\gamma}}^{3}_{(m/2)\times(m/2)} + \sqrt{g^{\theta\theta}} \frac{\partial S}{\partial \theta} \hat{\boldsymbol{\gamma}}^{1}_{(m/2)\times(m/2)} \\ & + \sqrt{g^{\varphi\varphi}} \frac{\partial S}{\partial \varphi} \hat{\boldsymbol{\gamma}}^{2}_{(m/2)\times(m/2)} \cdots \\ & + \sqrt{g^{\eta\eta}} \frac{\partial S}{\partial x^{\eta}} \hat{\boldsymbol{\gamma}}^{l}_{(m/2)\times(m/2)} + \cdots \\ & - i\sqrt{g^{x^{n+4}x^{n+4}}} \frac{\partial S}{\partial x^{n+4}} \boldsymbol{I}_{(m/2)\times(m/2)}, \end{aligned} \tag{19}$$

$$\begin{aligned} \boldsymbol{E} = &\sqrt{f}\frac{\partial S}{\partial r}\hat{\boldsymbol{\gamma}}^3_{(m/2)\times(m/2)} + \sqrt{g^{\theta\theta}}\frac{\partial S}{\partial\theta}\hat{\boldsymbol{\gamma}}^1_{(m/2)\times(m/2)} \\ &+ \sqrt{g^{\varphi\varphi}}\frac{\partial S}{\partial\varphi}\hat{\boldsymbol{\gamma}}^2_{(m/2)\times(m/2)}\cdots \\ &+ \sqrt{g^{\eta\eta}}\frac{\partial S}{\partial x^\eta}\hat{\boldsymbol{\gamma}}^l_{(m/2)\times(m/2)} + \cdots \\ &+ i\sqrt{g^{x^{n+4}x^{n+4}}}\frac{\partial S}{\partial x^{n+4}}\boldsymbol{I}_{(m/2)\times(m/2)}, \end{aligned} \tag{20}$$

$$\boldsymbol{F} = -\frac{i}{\sqrt{f}}\frac{\partial S}{\partial t}\boldsymbol{I}_{(m/2)\times(m/2)} - im\boldsymbol{I}_{(m/2)\times(m/2)}. \tag{21}$$

If we assume that Eq. (17) has a nontrivial solution, it is expected to comply with

$$\begin{vmatrix} \boldsymbol{C} & \boldsymbol{D} \\ \boldsymbol{E} & \boldsymbol{F} \end{vmatrix} = 0. \tag{22}$$

Taking into account the anticommutation relation of gamma matrices in flat space-time, we can get the following equation:

$$\begin{aligned} &-\frac{1}{f}\left(\frac{\partial S}{\partial t}\right)^2 + f\left(\frac{\partial S}{\partial r}\right)^2 + g^{\theta\theta}\left(\frac{\partial S}{\partial\theta}\right)^2 + g^{\varphi\varphi}\left(\frac{\partial S}{\partial\varphi}\right)^2 + \cdots \\ &+ g^{\eta\eta}\left(\frac{\partial S}{\partial x^\eta}\right)^2 + \cdots + g^{x^{n+4}x^{n+4}}\left(\frac{\partial S}{\partial x^{n+4}}\right)^2 + m^2 = 0. \end{aligned} \tag{23}$$

Obviously, it is a fermion semiclassical dynamic equation with mass m in the higher-dimensional curved space-time defined by Eq. (1). Up to now, research has been greatly simplified. Next we resolve Eq. (23). In Eq. (23), we can separate the variable for the action as

$$S = -\omega t + R(r) + Y(\theta, \varphi, \cdots, x^\nu, \cdots), \tag{24}$$

and then Eq. (23) can be broken up as

$$-\frac{1}{f}\omega^2 + f\left(\frac{\partial R}{\partial r}\right)^2 + m^2 = \frac{\lambda}{r^2}, \tag{25}$$

$$\sum_{i=\theta,\varphi,\cdots x^\nu\cdots} G^{ii}\left(\frac{\partial Y}{\partial x^i}\right)^2 + \lambda = 0, \tag{26}$$

where λ is a constant, and

$$G^{ii}(\theta, \varphi, \cdots, x^\nu\cdots) = r^2 g^{ii}. \tag{27}$$

Equation (25) is the radial equation, and Eq. (26) is the nonradial equation (where g^{ii} are the inverter metric components of angular and extra-dimensional coordinates. Allowing for the formula of this metric, G^{ii} is not dependent on either time or radial coordinates). However, we are not concerned with Eq. (26) [of course, Eq. (26) must be true], because fermion tunneling at the event horizon of black holes is radial. From Eq. (25), we can get

$$\frac{dR(r)}{dr} = \pm\frac{\sqrt{\omega^2 r^2 + f(-m^2 r^2 + \lambda)}}{fr}. \tag{28}$$

Near the event horizon, we expand f as

$$f(r) = f'(r_0)(r - r_0) + f''(r_0)(r - r_0)^2/2 + \cdots. \tag{29}$$

Choosing the leading term of f, Eq. (28) can be rewritten as

$$\begin{aligned} R_\pm(r) &= \pm\int\frac{\sqrt{\omega^2 + f'(r_0)(r - r_0)(-m^2 + \lambda/r^2)}}{f'(r_0)(r - r_0)}dr \\ &= \pm\frac{i\pi\omega}{f'(r_0)}, \end{aligned} \tag{30}$$

where R_+ is the radial outgoing solution, and R_- is the radial incoming solution. The total imaginary part of the action is

$$\mathrm{Im}\, S = \mathrm{Im}R = \mathrm{Im}R_+(r) - \mathrm{Im}R_-(r). \tag{31}$$

Therefore, the tunneling rate is

$$\Gamma = \exp(-2\,\mathrm{Im}S) = \exp\left(\frac{-4\pi\omega}{f'(r_0)}\right), \tag{32}$$

where ImS is the imaginary part of the action, so the Hawking temperature is

$$T_0 = \frac{f'(r_0)}{4\pi} = \frac{n+1}{4\pi r_0}. \tag{33}$$

In fact, Eq. (23) is none other than the Hamilton-Jacobi equation in higher-dimensional Schwarzschild space-time. That is to say, we can also get the Hamilton-Jacobi equations of higher-dimensional space-time using the semiclassical approximate fermion tunneling theory. This shows that the Hamilton-Jacobi equation is an elementary equation which can describe a quantum particle's behavior in semiclassical curved space-times. Namely, in the semiclassical approximation theory, the Hamilton-Jacobi equation can describe the property of fermions, as well as scalar particles.

III. FERMION TUNNELING OF HIGHER-DIMENSIONAL SPHERICALLY SYMMETRIC QUINTESSENCE BLACK HOLES

Quintessence is one of the candidates for dark energy. In the study of quintessential black holes, researchers have worked on higher-dimensional spherically symmetric black holes surrounded by quintessence. The metric is given by [27]

$$\begin{aligned} ds^2 = &-\left(1 - \frac{2M}{r^{n-3}} - \frac{c}{r^{(n-1)\omega_q + n-3}}\right)dt^2 \\ &+ \left(1 - \frac{2M}{r^{n-3}} - \frac{c}{r^{(n-1)\omega_q + n-3}}\right)^{-1}dr^2 + r^2 d\Omega^2_{n-2}, \end{aligned} \tag{34}$$

where c is a normalized constant, the ratio of the pressure and energy density of quintessence is

$$\omega_q = p_q/\rho_q, \tag{35}$$

and

$$f(r) = 1 - \frac{2M}{r^{n-3}} - \frac{c}{r^{(d-1)\omega_q+n-3}}. \tag{36}$$

The event horizon of the black hole satisfies the equation $f(r_0) = 0$. Similarly to what we have done in Sec. II, we can get the Hamilton-Jacobi equation of higher-dimensional quintessential black holes. After separating the variables, we have

$$-\frac{1}{f}\omega^2 + f\left(\frac{\partial R}{\partial r}\right)^2 + m^2 = \frac{\lambda}{r^2}, \tag{37}$$

$$\sum_{i=\theta,\varphi,\cdots x^v\cdots} G^{ii}\left(\frac{\partial Y}{\partial x^i}\right)^2 + \lambda = 0, \tag{38}$$

where

$$G^{ii}(\theta, \varphi, \cdots, x^v \cdots) = r^2 g^{ii}, \tag{39}$$

in which Eq. (37) is the radial equation and Eq. (38) is the nonradial equation. From Eq. (37), we can get

$$\frac{dR(r)}{dr} = \pm\frac{\sqrt{\omega^2 r^2 + f(-m^2r^2 + \lambda)}}{fr}. \tag{40}$$

Near the event horizon, we choose the leading term when we expand f, and Eq. (40) is

$$\begin{aligned} R_\pm(r) &= \pm\int\frac{\sqrt{\omega^2 + f'(r_0)(r-r_0)(-m^2+\lambda/r^2)}}{f'(r_0)(r-r_0)}dr \\ &= \pm\frac{i\pi\omega}{f'(r_0)}, \end{aligned} \tag{41}$$

where R_+ is the radial outgoing solution, while R_- is the radial incoming solution. Now we can obtain the tunneling rate

$$\Gamma = \frac{\exp(-2\,\mathrm{Im}R_+)}{\exp(-2\,\mathrm{Im}R_-)} = \exp\left(\frac{-4\pi\omega}{f'(r_0)}\right), \tag{42}$$

and the Hawking temperature

$$T_0 = \frac{f'(r_0)}{4\pi}. \tag{43}$$

In this section, we proved the validity of our method again. Similarly, using this method, we can research fermion tunneling from other higher-dimensional black holes. We should emphasize that the method can apply not only to research in higher-dimensional space-times, but also to research of fermion tunneling from four-dimensional or lower-dimensional black holes. Next, we will take the spherically symmetric rainbow black hole as an example to illustrate this fact.

IV. FERMION TUNNELING OF STATIC SPHERICALLY SYMMETRIC BLACK HOLES IN THE GRAVITY RAINBOW THEORY

Recently, some researchers have used the gravity rainbow theory to study the quantum field theory [28,29]. Researchers think the energy-momentum relation in special relativity should be modified as

$$l_1^2E^2 - l_2^2\vec{p}\bullet\vec{p} = m^2. \tag{44}$$

In the above formula, we have defined the light velocity as $c = 1$. l_1 and l_2 are correction terms related to the energy of probe particles. When the energy is far less than Planck energy, l_1 and l_2 are equivalent to 1. Generalizing the theory in general relativity, the Einstein field equation is rewritten as

$$G_{\mu v}(E, \lambda') = 8\pi G(E, \lambda')T_{\mu v}(E, \lambda') + g_{\mu v}\Lambda(E, \lambda'), \tag{45}$$

where λ' is a parameter of order the Planck length, and the metric of a rainbow Schwarzschild black hole can be expressed as

$$ds^2 = -\frac{f(r)}{l_1^2}dt^2 + \frac{1}{f(r)l_2^2}dr^2 + \frac{r^2}{l_2^2}(d\theta^2 + \sin^2\theta d\varphi^2) \tag{46}$$

and

$$f(r) = 1 - \frac{2M}{r} \tag{47}$$

in which $2M = r_0$ is the black hole's event horizon. The influence of the correction terms l_1 and l_2 can be found in the research of high energy physics. Because four-dimensional space-time is even dimensional, the gamma matrices in the Dirac equation can be chosen as

$$\boldsymbol{\gamma}^t = \frac{l_1}{\sqrt{f}}\begin{pmatrix} i\boldsymbol{I}_{2\times2} & 0 \\ 0 & -i\boldsymbol{I}_{2\times2}\end{pmatrix}, \tag{48}$$

$$\boldsymbol{\gamma}^r = \sqrt{f}l_2\begin{pmatrix} 0 & \boldsymbol{\sigma}^3 \\ \boldsymbol{\sigma}^3 & 0\end{pmatrix}, \tag{49}$$

$$\boldsymbol{\gamma}^\theta = \frac{l_2}{r}\begin{pmatrix} 0 & \boldsymbol{\sigma}^1 \\ \boldsymbol{\sigma}^1 & 0\end{pmatrix}, \tag{50}$$

$$\boldsymbol{\gamma}^\varphi = \frac{l_2}{r\sin\theta}\begin{pmatrix} 0 & \boldsymbol{\sigma}^2 \\ \boldsymbol{\sigma}^2 & 0\end{pmatrix}. \tag{51}$$

Similar to what we have done before, we can get the Hamilton-Jacobi equation in the space-time

$$\begin{aligned} &-\frac{l_1^2}{f}\left(\frac{\partial S}{\partial t}\right)^2 + fl_2^2\left(\frac{\partial S}{\partial r}\right)^2 + \frac{l_2^2}{r^2}\left(\frac{\partial S}{\partial \theta}\right)^2 \\ &\quad + \frac{l_2^2}{r^2\sin^2\theta}\left(\frac{\partial S}{\partial \varphi}\right)^2 + m^2 = 0. \end{aligned} \tag{52}$$

Separating the variables for the action, we have

$$S = -\omega t + R(r) + Y(\theta, \varphi), \tag{53}$$

so we can decompose Eq. (52) into a radial equation and an angular equation,

$$-\frac{l_1^2}{f}\omega^2 + f l_2^2\left(\frac{\partial R}{\partial r}\right)^2 + m^2 = \frac{\lambda}{r^2}, \tag{54}$$

$$l_2^2\left(\frac{\partial Y}{\partial \theta}\right)^2 + \frac{l_2^2}{\sin^2\theta}\left(\frac{\partial Y}{\partial \varphi}\right)^2 + \lambda = 0. \tag{55}$$

From Eq. (54), we obtain

$$\frac{dR(r)}{dr} = \pm\frac{\sqrt{\omega^2 r^2 l_1^2 + f(-m^2 r^2 + \lambda)}}{f r l_2}. \tag{56}$$

Near the event horizon, $f(r_0) = 0$, so Eq. (56) becomes

$$R_{\pm}(r) = \pm\frac{l_1}{l_2}\frac{i\pi\omega}{f'(r_0)}, \tag{57}$$

where R_+ is the outgoing solution and R_- is the incoming solution. Then we can get the tunneling rate

$$\Gamma = \frac{\exp(-2\,\mathrm{Im}R_+)}{\exp(-2\,\mathrm{Im}R_-)} = \exp\left(\frac{l_1}{l_2}\frac{-4\pi\omega}{f'(r_0)}\right), \tag{58}$$

and the Hawking temperature

$$T_0 = \frac{l_2}{l_1}\frac{f'(r_0)}{4\pi} = \frac{l_2}{l_1}\frac{1}{4\pi r_0}. \tag{59}$$

The conclusions arrived upon here are meaningful. We can see that the tunneling rate and the Hawking temperature of four-dimensional rainbow black holes depend on the correction terms, which means that the tunneling rate and the Hawking temperature at the horizon in rainbow space-time depend on the energy of probe particles. Namely, they depend on the mathematic tangent vector. Therefore, Girelli *et al.* thought the rainbow metric was exactly the Finsler metric [30,31]. The conclusion about the rainbow-Finsler theory needs to be proven through more experiments and observations in the future. Similarly, we can also study fermion tunneling from the horizons of other four-dimensional and lower-dimensional black holes.

V. CONCLUSIONS

In this paper, we have researched fermion tunneling from higher-dimensional Schwarzschild black holes, higher-dimensional spherically symmetric quintessence black holes, and rainbow-Finsler black holes. In the course of our research, we made full use of Eq. (23) to simplify the problem of solving a semiclassical approximate Dirac equation in higher-dimensional space-time. By solving a semiclassical differential equation, we can finally obtain the Hamilton-Jacobi equation in the corresponding space-times. It is shown that the Dirac equation is self-consistent in semiclassical approximation theory, and the method we used in this paper is correct. Although we achieved significant conclusions in our work, the space-time background of Hawking radiation studied in this paper is unchangeable. Taking the unfixed background space-time into account, the research could help resolve the information-loss paradox of black holes. Work on these fields is currently in progress.

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