Atiyah-Hitchin space in five-dimensional Einstein-Maxwell theory

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We construct exact solutions to five-dimensional Einstein-Maxwell theory based on Atiyah-Hitchin space. The solutions cannot be written explicitly in a closed form, so their properties are investigated numerically. The five-dimensional metric is regular everywhere except on the location of the original bolt in four-dimensional Atiyah-Hitchin base space. On each time-fixed slice, the metric asymptotically approaches a Euclidean Taub-NUT space.

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I. INTRODUCTION

Atiyah-Hitchin space is part of the set of two monopole solutions of the Bogomol'nyi equation. The moduli space of solutions is of the form

$$\mathbb{R}^3 \otimes \frac{S^1 \otimes \mathcal{M}}{\mathbb{Z}_2}, \tag{1.1}$$

where the factor $\mathbb{R}^3 \otimes S^1$ describes the center of mass of two monopoles and a phase factor that is related to the total electric charge of the system. The interesting part of the moduli space is the four-dimensional manifold $\mathcal{M}$, which has self-dual curvature. The self-duality comes from the hyper-Kähler property of the moduli space. Since $\mathbb{R}^3 \otimes S^1$ is flat and decouples from $\mathcal{M}$, the four-dimensional manifold $\mathcal{M}$ should be hyper-Kähler, which is equivalent to a metric with self-dual curvature in four dimensions. The manifold $\mathcal{M}$ describes the separation of the two monopoles and their relative phase angle (or electric charges). A further aspect concerning $\mathcal{M}$ is that it should be $SO(3)$ invariant, since two monopoles do exist in ordinary flat space; hence the metric on $\mathcal{M}$ can be expressed in terms of three functions of the monopole separation. Self-duality implies that these three functions obey a set of first-order ordinary differential equations. This space has been used recently for the construction of five-dimensional three-charge supergravity solutions that only have a rotational $U(1)$ isometry [1], as well as for the construction of M-brane solutions [2]. Moreover, Atiyah-Hitchin space and its various generalizations were identified with the full quantum moduli space of $\mathcal{N} = 4$ supersymmetric gauge theories in three dimensions [3].

Moreover, in the context of string theory and brane world, investigations of black hole (ring) solutions in higher dimensions have attracted a lot of attention. It is believed that in the strong coupling limit, many horizonless three-charge brane configurations undergo a geometric transition and become smooth horizonless geometries with black hole or black ring charges [4]. These charges come completely from fluxes wrapping on nontrivial cycles. The three-charge black hole (ring) systems are dual to the states of the corresponding conformal field theories, in favor of the idea that nonfundamental black hole (ring) systems effectively arise as a result of many horizonless configurations [5,6]. In 11-dimensional supergravity, there are solutions based on transverse four-dimensional hyper-Kähler metrics (which are equivalent to metrics with self-dual curvatures). The hyper-Kählericity of the transverse metric guarantees (at least partially) to have supersymmetry [7]. There are also many solutions to five-dimensional minimal supergravity. In five dimensions, unlike in four dimensions where the only horizon topology is a two-sphere, we can have different, more interesting, horizon topologies such as black holes with horizon topology of a three-sphere [8], black rings with horizon topology of a two-sphere × circle [9,10], a black saturn (that is, a spherical black hole surrounded by a black ring [11]), and a black lens in which the horizon geometry is a lens space $L(p, q)$ [12]. All allowed horizon topologies have been classified in [13–15]. Recently, it was shown how a uniqueness theorem might be proved for black holes in five dimensions [16,17]. Stationary, asymptotically flat, vacuum black holes with two commuting axial symmetries were shown to be uniquely determined by their mass, angular momentum, and rod structure. Specifically, the rod structure [18] determines the topology of the horizon in five dimensions. In Refs. [19–22], the authors constructed (multi) black hole solutions in the five-dimensional Einstein-Maxwell theory (with and without a cosmological constant) based on four-dimensional Taub-NUT and Eguchi-Hanson spaces. Both spaces have self-dual curvatures and can be put into a Gibbons-Hawking form. Although hyper-Kähler Atiyah-Hitchin space also has self-dual curvature, it cannot be put into a Gibbons-Hawking form.

Motivated by these facts, in this article we try to construct solutions to five-dimensional Einstein-Maxwell theory based on Atiyah-Hitchin space. We note that hyper-Kähler Atiyah-Hitchin geometries (unlike Gibbons-Hawking geometries) do not have any triholomorphic $U(1)$ isometry; hence our solutions could be used to study the physical processes that do not respect any triholomor-

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phic $U(1)$ symmetry. We consider the Atiyah-Hitchin space as a base space, and by performing a coordinate transformation, we are able to considerably simplify the structure of the five-dimensional metric. This is the first step toward the construction of more sophisticated solutions (such as black holes or rings) in five-dimensional Einstein-Maxwell theory based on Atiyah-Hitchin space.

The outline of this paper is as follows. In Sec. II, we review briefly Einstein-Maxwell theory, and Atiyah-Hitchin space and its features. In Sec. III, we present our solutions based on two forms for Atiyah-Hitchin space and discuss the asymptotics of the solutions. We conclude in Sec. IV with a summary of our solutions and possible directions for future research.

II. FIVE-DIMENSIONAL EINSTEIN-MAXWELL THEORY AND ATIYAH-HITCHIN SPACE

Five-dimensional Einstein-Maxwell theory is described by the action

$$S = \frac{1}{16\pi}\int d^5x\sqrt{-g}(R - F_{\mu\nu}F^{\mu\nu}), \tag{2.1}$$

where R and $F_{\mu\nu}$ are the five-dimensional Ricci scalar and Maxwell field. The Einstein and Maxwell equations are

$$R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu} = 2F_{\mu\lambda}F^{\nu\lambda} - \tfrac{1}{2}g_{\mu\nu}F^2, \tag{2.2}$$

$$F^{\mu\nu}_{;\nu} = 0, \tag{2.3}$$

respectively. We take the following form for the five-dimensional metric:

$$ds_5^2 = -H(r)^{-2}dt^2 + H(r)ds^2_{\mathrm{AH}}, \tag{2.4}$$

and the only nonvanishing component of the gauge field is

$$A_t = \frac{\eta\sqrt{3}}{2}\frac{1}{H(r)}, \tag{2.5}$$

where $\eta = +1$ or $\eta = -1$. The Atiyah-Hitchin metric ds^2_{AH} is given by the following manifestly $SO(3)$ invariant form [23]:

$$ds^2_{\mathrm{AH}} = f^2(r)dr^2 + a^2(r)\sigma_1^2 + b^2(r)\sigma_2^2 + c^2(r)\sigma_3^2, \tag{2.6}$$

with

$$\sigma_1 = -\sin\psi d\theta + \cos\psi \sin\theta d\phi, \tag{2.7}$$

$$\sigma_2 = \cos\psi d\theta + \sin\psi \sin\theta d\phi, \tag{2.8}$$

$$\sigma_3 = d\psi + \cos\theta d\phi, \tag{2.9}$$

where σ_i are Maurer-Cartan one-forms with the property

$$d\sigma_i = \tfrac{1}{2}\varepsilon_{ijk}\sigma_j \wedge \sigma_k. \tag{2.10}$$

We note that the metric on the $\mathbb{R}^4$ [with a radial coordinate R and Euler angles (θ, ϕ, ψ) on an S^3] could be written in terms of Maurer-Cartan one-forms by

$$ds^2 = dR^2 + \frac{R^2}{4}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2). \tag{2.11}$$

We also note that $\sigma_1^2 + \sigma_2^2$ is the standard metric of the round unit radius S^2 and $4(\sigma_1^2 + \sigma_2^2 + \sigma_3^2)$ gives the same for S^3. The metric (2.6) satisfies Einstein's equations provided that

$$a' = f\frac{(b-c)^2 - a^2}{2bc}, \tag{2.12}$$

$$b' = f\frac{(c-a)^2 - b^2}{2ca}, \tag{2.13}$$

$$c' = f\frac{(a-b)^2 - c^2}{2ab}. \tag{2.14}$$

Choosing $f(r) = -\frac{b(r)}{r}$ the explicit expressions for the metric functions a, b, and c are given by

$$a(r) = \sqrt{\frac{r\Upsilon\sin(\gamma)\{\frac{1-\cos(\gamma)}{2}r - \sin(\gamma)\Upsilon\}}{\Upsilon\sin(\gamma) + r\cos^2(\frac{\gamma}{2})}}, \tag{2.15}$$

$$b(r) = \sqrt{\frac{\{\Upsilon\sin(\gamma) - \frac{1-\cos\gamma}{2}r\}r\{-\Upsilon\sin(\gamma) - \frac{1+\cos\gamma}{2}r\}}{\Upsilon\sin(\gamma)}}, \tag{2.16}$$

$$c(r) = -\sqrt{\frac{r\Upsilon\sin(\gamma)\{\frac{1+\cos(\gamma)}{2}r + \sin(\gamma)\Upsilon\}}{-\Upsilon\sin(\gamma) + \frac{1-\cos\gamma}{2}r}}, \tag{2.17}$$

where

$$\Upsilon = \frac{2nE\{\sin(\frac{\gamma}{2})\}}{\sin(\gamma)} - \frac{nK\{\sin(\frac{\gamma}{2})\}\cos(\frac{\gamma}{2})}{\sin(\frac{\gamma}{2})} \tag{2.18}$$

and

$$K\left(\sin\left(\frac{\gamma}{2}\right)\right) = \frac{r}{2n}. \tag{2.19}$$

In the above equations, K and E are the elliptic integrals,

$$K(k) = \int_0^1 \frac{dt}{\sqrt{1-t^2}\sqrt{1-k^2t^2}} = \int_0^{\pi/2}\frac{d\theta}{\sqrt{1-k^2\cos^2\theta}}, \tag{2.20}$$

$$E(k) = \int_0^1 \frac{\sqrt{1-k^2t^2}dt}{\sqrt{1-t^2}} = \int_0^{\pi/2}\sqrt{1-k^2\cos^2\theta}d\theta, \tag{2.21}$$

and the coordinate r ranges over the interval $[n\pi, \infty)$, which corresponds to $\gamma \in [0, \pi)$. The positive number n is a constant number with unit of length that is related to the NUT charge of the metric at infinity obtained from the Atiyah-Hitchin metric (2.6).

In fact, as $r \longrightarrow \infty$, the metric (2.6) reduces to

$$ds^2_{\rm AH} \longrightarrow \left(1 - \frac{2n}{r}\right)(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2) + 4n^2 \left(1 - \frac{2n}{r}\right)^{-1} (d\psi + \cos\theta d\phi)^2, \tag{2.22}$$

which is the well-known Euclidean Taub-NUT metric with a negative NUT charge $N = -n$. The metric (2.22) is obtained from consideration of the limiting behaviors of the functions a, b, and c at large monopole separation, which are given by

$$a(r) = r\left(1 - \frac{2n}{r}\right)^{1/2} + O(e^{-r/n}), \tag{2.23}$$

$$b(r) = r\left(1 - \frac{2n}{r}\right)^{1/2} + O(e^{-r/n}), \tag{2.24}$$

$$c(r) = -2n\left(1 - \frac{2n}{r}\right)^{-1/2} + O(e^{-r/n}). \tag{2.25}$$

In the other extreme limit where $\epsilon = r - n\pi \longrightarrow 0$, from Eqs. (2.15), (2.16), and (2.17), we find the following behaviors for the metric functions $a(r)$, $b(r)$, and $c(r)$:

$$a(r) = 2\epsilon + O(\epsilon^2), \tag{2.26}$$

$$b(r) = n\pi + \frac{\epsilon}{2} + O(\epsilon^2), \tag{2.27}$$

$$c(r) = -n\pi + \frac{\epsilon}{2} + O(\epsilon^2). \tag{2.28}$$

Equation (2.26) clearly shows a bolt singularity as $\epsilon \longrightarrow 0$. Actually, by using the $SO(3)$ invariance of the metric, we can write the metric element (2.6) near the bolt location as

$$ds^2 = d\epsilon^2 + 4\epsilon^2(d\tilde{\psi} + \cos\tilde{\theta} d\tilde{\phi})^2 + \pi^2 n^2 (d\tilde{\theta} + \sin^2\tilde{\theta} d\tilde{\phi}), \tag{2.29}$$

where $\tilde{\psi}$, $\tilde{\theta}$, and $\tilde{\phi}$ are a new set of Euler angles related to ψ, θ, ϕ by

$$\mathcal{R}_1(\tilde{\psi})\mathcal{R}_3(\tilde{\theta})\mathcal{R}_1(\tilde{\phi}) = \mathcal{R}_3(\psi)\mathcal{R}_2(\theta)\mathcal{R}_3(\phi), \tag{2.30}$$

in which $R_i(\alpha)$ represents a rotation by α about the ith axis. We note that the last term in (2.29) is the induced metric on the two-dimensional bolt.

III. EINSTEIN-MAXWELL SOLUTIONS OVER ATIYAH-HITCHIN BASE SPACE

To find the five-dimensional metric function $H(r)$, we consider equations of motion (2.2) and (2.3). The metric (2.4) [along with the gauge field (2.5)] is a solution to the Einstein-Maxwell equations provided that $H(r)$ is a solution to the differential equation,

$$ra(r)c(r)\frac{d^2H(r)}{dr^2} + (a(r)b(r) + a(r)c(r) + b(r)c(r) - b(r)^2)\frac{dH(r)}{dr} = 0. \tag{3.1}$$

So, we find

$$H(r) = H_0 + H_1 \int dr \times e^{\int \{[b(r)^2 - a(r)b(r) - a(r)c(r) - b(r)c(r)]/ra(r)c(r)\}dr}, \tag{3.2}$$

where H_0 and H_1 are two constants of integration. Although the r dependences of the metric functions a, b, c are given explicitly in Eqs. (2.15), (2.16), and (2.17), it is unlikely to find an analytic expression for $H(r)$ given by (3.2). As $r \longrightarrow \infty$, the metric function (3.2) goes to

$$H(r) = H_0 - \frac{H_1}{r}. \tag{3.3}$$

On the other hand, near the bolt the metric function $H(r)$ has a logarithmic divergence as

$$H(r) \simeq \frac{H_1}{4n^2\pi^2}\ln(\epsilon) + H_0 + O(\epsilon), \tag{3.4}$$

where $\epsilon = r - n\pi$. This type of divergence in the metric function has been observed previously in the metric function of an M2-brane in transverse Atiyah-Hitchin space [23].

We could not find a closed analytic expression for the metric function given in (3.2). To overcome this problem, we choose $f(\xi)$ in (2.6) to be $16a(\xi)b(\xi)c(\xi)$, and so the Atiyah-Hitchin metric reads

$$ds^2_{\rm AH} = 16a^2(\xi)b^2(\xi)c^2(\xi)d\xi^2 + a^2(\xi)\sigma_1^2 + b^2(\xi)\sigma_2^2 + c^2(\xi)\sigma_3^2, \tag{3.5}$$

where the functions $a(\xi)$, $b(\xi)$, and $c(\xi)$ satisfy Eqs. (2.12), (2.13), and (2.14) with $f(\xi) = 4a(\xi)b(\xi)c(\xi)$, and $'$ means $\frac{d}{d\xi}$. By introducing the new functions $\psi_1(\xi)$, $\psi_2(\xi)$, and $\psi_3(\xi)$ such that

$$a^2(\xi) = \frac{\psi_2\psi_3}{4\psi_1}, \tag{3.6}$$

$$b^2(\xi) = \frac{\psi_3\psi_1}{4\psi_2}, \tag{3.7}$$

$$c^2(\xi) = \frac{\psi_1\psi_2}{4\psi_3}, \tag{3.8}$$

the set of equations (2.12), (2.13), and (2.14) with $f(\xi) = 4a(\xi)b(\xi)c(\xi)$ reduces to a Darboux-Halpern system,

$$\frac{d}{d\xi}(\psi_1 + \psi_2) + 2\psi_1\psi_2 = 0, \tag{3.9}$$

$$\frac{d}{d\xi}(\psi_2 + \psi_3) + 2\psi_2\psi_3 = 0, \tag{3.10}$$

$$\frac{d}{d\xi}(\psi_3 + \psi_1) + 2\psi_3\psi_1 = 0. \tag{3.11}$$

We can find the solutions to the above equations as

$$\psi_1 = -\frac{1}{2}\left(\frac{d}{d\vartheta}\mu^2 + \frac{\mu^2}{\sin\vartheta}\right), \tag{3.12}$$

$$\psi_2 = -\frac{1}{2}\left(\frac{d}{d\vartheta}\mu^2 - \frac{\mu^2\cos\vartheta}{\sin\vartheta}\right), \tag{3.13}$$

$$\psi_3 = -\frac{1}{2}\left(\frac{d}{d\vartheta}\mu^2 - \frac{\mu^2}{\sin\vartheta}\right), \tag{3.14}$$

where

$$\mu(\vartheta) = \frac{1}{\pi}\sqrt{\sin\vartheta}K\left(\sin\frac{\vartheta}{2}\right). \tag{3.15}$$

The new coordinate ϑ is related to the coordinate ξ by

$$\xi = -\int_{\vartheta}^{\pi}\frac{d\vartheta}{\mu^2(\vartheta)}. \tag{3.16}$$

Figure 1 shows the result of the numerical integration of the relation between the two coordinates ξ and ϑ. The coordinate ϑ takes values over $[0, \pi]$ if the coordinate ξ is chosen to take values on $(-\infty, 0]$. In Fig. 2, the function $\mu(\vartheta)$ is plotted and shows an increasing behavior from $\vartheta = 0$ to $\vartheta_0 = 2.281\,318$. At $\vartheta = \vartheta_0$, the function μ reaches the maximum value 0.643 243 and then decreases to zero at $\vartheta = \pi$. Hence, in the range of $0 < \vartheta < \pi$, μ is positive, and so the change of variables, given in (3.16), is completely well defined. As one can see from Fig. 3, the functions ψ_1, ψ_2 are always negative and ψ_3 is always positive. Hence, Eqs. (3.6), (3.7), and (3.8) show that the metric functions are always positive. In Fig. 4, the behaviors of the functions a, b, and c versus ϑ are plotted.

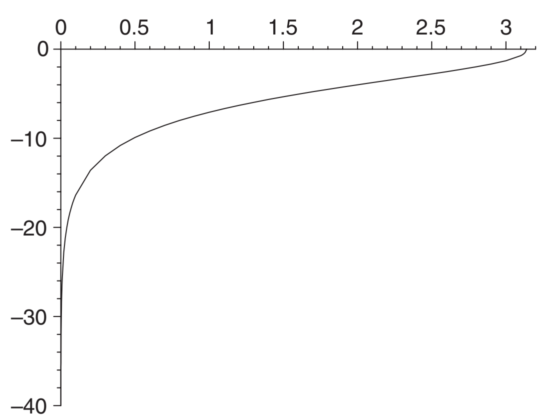

FIG. 1. The coordinate $0 \leq \vartheta \leq \pi$ versus the coordinate $-\infty < \xi \leq 0$.

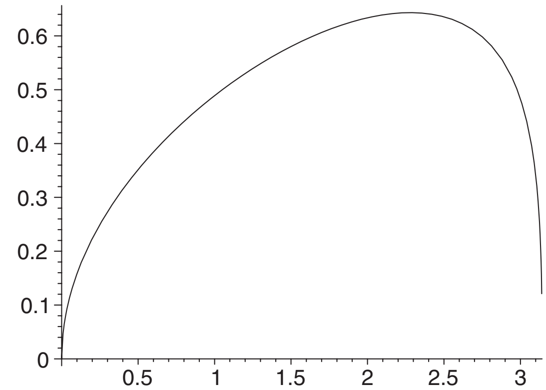

FIG. 2. The function μ versus ϑ.

The five-dimensional metric and gauge field are given by

$$ds^2 = -\frac{dt^2}{(\alpha\xi + \beta)^2} + (\alpha\xi + \beta)\{16a^2(\xi)b^2(\xi)c^2(\xi)d\xi^2 + a^2(\xi)\sigma_1^2 + b^2(\xi)\sigma_2^2 + c^2(\xi)\sigma_3^2\}, \tag{3.17}$$

and

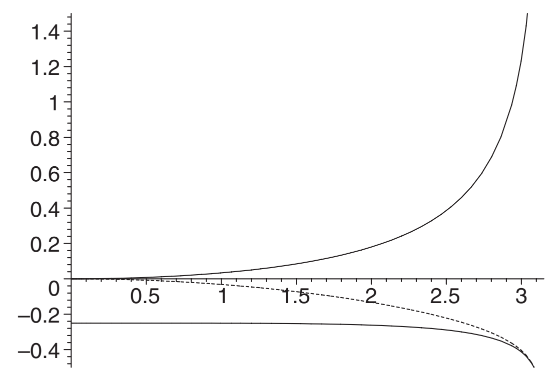

FIG. 3. The functions ψ_1 (lower solid line), ψ_2 (dashed line), and ψ_3 (upper solid line) plotted as functions of ϑ.

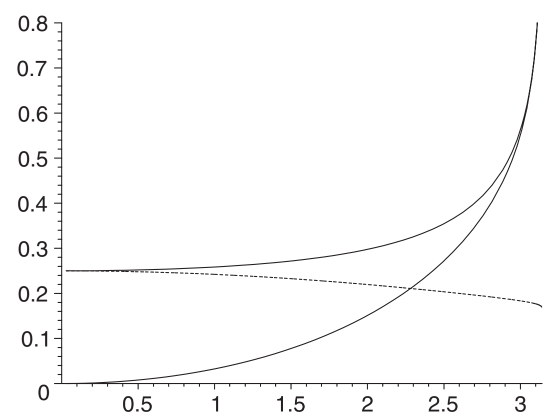

FIG. 4. The functions a (lower solid line), b (upper solid line), and c (dashed line) plotted as functions of ϑ.

$$A_t = \frac{\eta\sqrt{3}}{2(\alpha\xi + \beta)}. \tag{3.18}$$

To avoid any singularity at a finite ξ, the sign of α must be opposite to the sign of β. Moreover, to get a regular positive-definite metric, we should choose $\alpha < 0$ and $\beta > 0$. The geometry of a solution on a $t =$ constant hypersurface is quite simple. At $\xi = 0$, which corresponds to $\vartheta = \pi$, the metric functions given in Eqs. (3.6), (3.7), and (3.8) reduce to

$$a = b \simeq \frac{-1}{2\pi}\ln\epsilon + \frac{\ln 8}{2\pi}, \tag{3.19}$$

$$c \simeq \frac{1}{2\pi}, \tag{3.20}$$

where $\epsilon = \pi - \vartheta$. Hence, we get

$$ds^2|_{t=\text{const}} \sim \frac{-\alpha}{4\zeta}\{d\zeta^2 + \zeta^2(d\theta^2 + \sin^2\theta d\phi^2) + (d\psi + \cos\theta d\phi)^2\}, \tag{3.21}$$

which is conformally the Euclidean Taub-NUT metric [2] or the fibration of a unit circle (parametrized with ψ) over R^3. Here the coordinate ζ is related to ϵ by $\zeta = -\ln\epsilon$ and to ξ by $\zeta = -\frac{\pi^2}{\xi}$, respectively. The Ricci scalar of the space-time (3.17) approaches $f(\alpha, \beta)\xi^4(1 + O(\xi))$ as $\xi \to 0$, and the Kretschman invariant approaches $g(\alpha, \beta)\xi^6(1 + O(\xi))$, where f, g are functions of α and β. On the other extreme level where $\xi \to -\infty$, which corresponds to $\vartheta \to 0$, the metric functions behave as

$$a \simeq \frac{\vartheta^2}{768}(24 + \vartheta^2 + O(\vartheta^4)), \tag{3.22}$$

$$b \simeq \frac{1}{4}\left(1 + \frac{\vartheta^2}{32} + O(\vartheta^4)\right), \tag{3.23}$$

$$c \simeq \frac{1}{4}\left(1 - \frac{\vartheta^2}{32} + O(\vartheta^4)\right). \tag{3.24}$$

In this limit, the elliptic integral in Eq. (3.15) approaches

$$K\left(\sin\frac{\vartheta}{2}\right) \simeq \frac{\pi}{2}\left(1 + \frac{1}{16}\vartheta^2 + O(\vartheta^4)\right). \tag{3.25}$$

Hence from Eqs. (3.15) and (3.16), we get

$$\xi \simeq 4\ln\vartheta, \tag{3.26}$$

and we find the metric

$$ds^2|_{t=\text{cons}} \sim 4\alpha\ln\vartheta\left\{\left(\frac{\vartheta}{32}\right)^2 d\vartheta^2 + \left(\frac{\vartheta^2}{32}\right)^2\sigma_1^2 + \frac{1}{16}(\sigma_2^2 + \sigma_3^2)\right\}. \tag{3.27}$$

By changing to the coordinate $\varrho = \frac{\vartheta^2}{32}$, the metric (3.27) changes to

$$ds^2|_{t=\text{cons}} \sim d\varrho^2 + 4\varrho^2\sigma_1^2 + \tfrac{1}{4}(\sigma_2^2 + \sigma_3^2) \tag{3.28}$$

(up to a conformal factor) which clearly shows a bolt at $\vartheta = 0$ of fixed radius 1/2. The Ricci scalar and Kretschman invariant of the metric (3.17) near the bolt behave as $\frac{(\ln\vartheta)^3}{\vartheta^4}$ and $\frac{1}{(\ln\vartheta)^6\vartheta^8}$, respectively. One way to avoid the bolt region is to consider positive values for both α and β. In this case, the range of ξ is limited to $\xi_0 \leq \xi \leq 0$, where $\xi_0 = -\frac{\beta}{\alpha}$. There is still a curvature singularity at $\xi = \xi_0$ of the order of $\frac{1}{\varepsilon^3}$ where $\varepsilon = \xi - \xi_0$, but it is less divergent than the singularity on the bolt. This latter singularity is a simple result of our symmetric metric function $H(r)$ in the ansatz (2.4) that could be removed by considering some nonsymmetric metric functions. We leave this case along with some other open issues for a future article.

IV. CONCLUDING REMARKS

The main result of this article is the metric (3.17) along with the gauge field (3.18) which are exact solutions to the five-dimensional Einstein-Maxwell equations. To our knowledge, these solutions are the first known solutions of five-dimensional Einstein-Maxwell theory based on non-triholomorphic base space; hence, they could be used to study the physical processes that do not have any triholomorphic symmetry. The simplicity of these solutions (simple analytic metric functions) is a result of taking the base Atiyah-Hitchin metric in the form of (3.5); otherwise the metric function (3.2) cannot be obtained in a simple analytic form. The metric function and the gauge field are regular everywhere in space-time. The metric is regular everywhere except on the location of the original bolt in four-dimensional Atiyah-Hitchin space. Similar results have been observed previously in higher-dimensional (super)gravity solutions based on transverse self-dual hyper-Kähler manifolds [2,24,25].

We conclude with a few comments about possible directions for future work. In our solutions, we have considered the simplest dependence of the five-dimensional metric function and gauge field on the coordinates (i.e. dependence only on the radial coordinate). We can seek other solutions for which the functions appearing in the metric depend on more coordinates. It is quite possible that in these solutions, the singularity in the location of the bolt can be converted to a regular hypersurface(s) in five-dimensional space-time, and we obtain Atiyah-Hitchin black hole solutions. The other possibility is to include the cosmological constant in the theory, which may lift the singularity behind some regular hypersurface(s). Moreover, the solutions could be used to study (A)dS/CFT correspondence, where Atiyah-Hitchin space is part of the bulk space-time. The other open issue is the study of the thermodynamics of solutions constructed in this paper.

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[1] I. Bena, N. Bobev, and N. P. Warner, J. High Energy Phys. 08 (2007) 004.
[2] A. M. Ghezelbash and R. B. Mann, J. High Energy Phys. 10 (2004) 012.
[3] N. Seiberg and E. Witten, Nucl. Phys. **B426**, 19 (1994); **B431**, 484 (1994).
[4] I. Bena and N. P. Warner, Phys. Rev. D **74**, 066001 (2006).
[5] I. Bena and N. P. Warner, Lect. Notes Phys. **755**, 1 (2008).
[6] S. D. Mathur, Classical Quantum Gravity **23**, R115 (2006).
[7] J. P. Gauntlett, J. B. Gutowski, C. M. Hull, S. Pakis, and H. S. Reall, Classical Quantum Gravity **20**, 4587 (2003).
[8] R. C. Myers and M. J. Perry, Ann. Phys. (N.Y.) **172**, 304 (1986).
[9] R. Emparan and H. S. Reall, Phys. Rev. Lett. **88**, 101101 (2002).
[10] H. Elvang, R. Emparan, D. Mateos, and H. S. Reall, Phys. Rev. Lett. **93**, 211302 (2004).
[11] H. Elvang and P. Figueras, J. High Energy Phys. 05 (2007) 050.
[12] Y. Chen and E. Teo, Phys. Rev. D **78**, 064062 (2008).
[13] M. Cai and G. J. Galloway, Classical Quantum Gravity **18**, 2707 (2001).
[14] C. Helfgott, Y. Oz, and Y. Yanay, J. High Energy Phys. 02 (2006) 025.
[15] G. J. Galloway and R. Schoen, Commun. Math. Phys. **266**, 571 (2006).
[16] S. Hollands and S. Yazadjiev, Commun. Math. Phys. **283**, 749 (2008).
[17] S. Hollands and S. Yazadjiev, Classical Quantum Gravity **25**, 095010 (2008).
[18] T. Harmark, Phys. Rev. D **70**, 124002 (2004).
[19] H. Ishihara, M. Kimura, K. Matsuno, and S. Tomizawa, Phys. Rev. D **74**, 047501 (2006).
[20] H. Ishihara, M. Kimura, and S. Tomizawa, Classical Quantum Gravity **23**, L89 (2006).
[21] D. Ida, H. Ishihara, M. Kimura, K. Matsuno, Y. Morisawa, and S. Tomizawa, Classical Quantum Gravity **24**, 3141 (2007).
[22] C. Yoo, H. Ishihara, M. Kimura, K. Matsuno, and S. Tomizawa, Classical Quantum Gravity **25**, 095017 (2008).
[23] G. W. Gibbons and N. S. Manton, Nucl. Phys. **B274**, 183 (1986).
[24] S. A. Cherkis and A. Hashimoto, J. High Energy Phys. 11 (2002) 036.
[25] A. M. Ghezelbash, Phys. Rev. D **77**, 026006 (2008); **74**, 126004 (2006).