

Lorentzian wormholes in Lovelock gravityM. H. Dehghani^{1,2,*} and Z. Dayyani¹¹*Physics Department and Biruni Observatory, College of Sciences, Shiraz University, Shiraz 71454, Iran*²*Research Institute for Astrophysics and Astronomy of Maragha (RIAAM), Maragha, Iran*

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In this paper, we introduce the n -dimensional Lorentzian wormhole solutions of third order Lovelock gravity. In contrast to Einstein gravity and as in the case of Gauss-Bonnet gravity, we find that the wormhole throat radius r_0 has a lower limit that depends on the Lovelock coefficients, the dimensionality of the spacetime, and the shape function. We study the conditions of having normal matter near the throat, and find that the matter near the throat can be normal for the region $r_0 \leq r \leq r_{\max}$, where r_{\max} depends on the Lovelock coefficients and the shape function. We also find that the third order Lovelock term with negative coupling constant enlarges the radius of the region of normal matter, and conclude that the higher order Lovelock terms with negative coupling constants enlarge the region of normal matter near the throat.

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I. INTRODUCTION

Wormholes are tunnels in the geometry of space and time that connect two separate and distinct regions of spacetimes. Although such objects were long known to be solutions of Einstein equation, a renaissance in the study of wormholes has taken place during 80's motivated by the possibility of quick interstellar travel [1]. Wormhole physics is a specific example of adopting the reverse philosophy of solving the gravitational field equation, by first constructing the spacetime metric, then deducing the stress-energy tensor components. Thus, it was found that these traversable wormholes possess a stress-energy tensor that violates the standard energy conditions (see, e.g., [2,3], or [4] for a more recent review). The literature is rather extensive in candidates for wormhole spacetimes in Einstein gravity, and one may mention several cases, ranging from wormhole solutions in the presence of the cosmological constant [5], wormhole geometries in higher dimensions [6], to geometries in the context of linear and nonlinear electrodynamics [7]. Also, the stability of wormhole solutions has been analyzed by considering specific equations of state [8], or by applying a linearized radial perturbation around a stable solution [9].

One of the main areas in wormhole research is to try to avoid, as much as possible, the violation of the standard energy conditions. For static wormholes of Einstein gravity the null energy condition is violated, and thus, several attempts have been made to somehow overcome this problem. In order to do this, some authors resort to the alternative theories of gravity: the wormhole geometries of Brans-Dicke theory that have been investigated in [10]; of Kaluza-Klein theory in [11]; and of a higher curvature gravity in [12]. In the latter, it was found that the weak energy condition may be respected in the throat vicinity of the wormholes of higher curvature gravity. A special

branch of higher curvature gravity, which respects the assumptions of Einstein—that the left-hand side of the field equations is the most general symmetric conserved tensor containing no more than two derivatives of the metric—is the Lovelock gravity [13]. This theory represents a very interesting scenario to study how the physics of gravity are corrected at short distance due to the presence of higher order curvature terms in the action. Static solutions of second and third order Lovelock gravity have been introduced in [14,15], respectively. For wormholes with small throat radius, the curvature near the throat is very large, and therefore the investigation of the effects of higher curvature terms becomes important. The possibility of obtaining a wormhole solution from the instanton solutions of Lovelock gravity has been studied in [16]. The wormhole solutions of dimensionally continued Lovelock gravity have been introduced in [17], while these kinds of solutions in second order Lovelock gravity and the possibility of obtaining solutions with normal and exotic matter limited to the vicinity of the throat have been explored in [18]. Here, we want to add the third order term of Lovelock theory to the gravitational field equations, and investigate the effects of it on the possibility of having wormhole solutions with normal matter. We also want to explore the effects of higher order Lovelock terms on the region of normal matter near the throat.

The outline of this paper is as follows: We give a brief review of the field equations of third order Lovelock gravity and introduce the wormhole solutions of this theory in Sec. II. In Sec. III, we present the conditions of having normal matter near the throat and exotic matter everywhere. We finish our paper with some concluding remarks.

II. STATIC WORMHOLE SOLUTIONS

We first give a brief review of the field equations of third order Lovelock gravity and then we consider the static wormhole solutions of the theory. The most fundamental

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assumption in standard general relativity is the requirement that the field equations be generally covariant and contain at most second order derivative of the metric. Based on this principle, the most general classical theory of gravitation in n dimensions is the Lovelock gravity. The Lovelock equation up to third order terms without the cosmological constant term may be written as [19]

$$G_{\mu\nu}^{(1)} + \sum_{p=2}^3 \alpha'_p \left(H_{\mu\nu}^{(p)} - \frac{1}{2} g_{\mu\nu} \mathcal{L}^{(p)} \right) = \kappa_n^2 T_{\mu\nu}, \quad (1)$$

where α'_p 's are Lovelock coefficients, $T_{\mu\nu}$ is the energy-momentum tensor, $G_{\mu\nu}^{(1)}$ is just the Einstein tensor, $\mathcal{L}^{(2)} = R_{\mu\nu\gamma\delta} R^{\mu\nu\gamma\delta} - 4R_{\mu\nu} R^{\mu\nu} + R^2$ is the Gauss-Bonnet Lagrangian,

$$\begin{aligned} \mathcal{L}^{(3)} = & 2R^{\mu\nu\sigma\kappa} R_{\sigma\kappa\rho\tau} R^{\rho\tau}_{\mu\nu} + 8R^{\mu\nu}_{\sigma\rho} R^{\sigma\kappa}_{\nu\tau} R^{\rho\tau}_{\mu\kappa} \\ & + 24R^{\mu\nu\sigma\kappa} R_{\sigma\kappa\nu\rho} R^{\rho}_{\mu} + 3RR^{\mu\nu\sigma\kappa} R_{\sigma\kappa\mu\nu} \\ & + 24R^{\mu\nu\sigma\kappa} R_{\sigma\mu} R_{\kappa\nu} + 16R^{\mu\nu} R_{\nu\sigma} R^{\sigma}_{\mu} \\ & - 12RR^{\mu\nu} R_{\mu\nu} + R^3 \end{aligned} \quad (2)$$

is the third order Lovelock Lagrangian, and $H_{\mu\nu}^{(2)}$ and $H_{\mu\nu}^{(3)}$ are

$$\begin{aligned} H_{\mu\nu}^{(2)} = & 2(R_{\mu\sigma\kappa\tau} R_{\nu}^{\sigma\kappa\tau} - 2R_{\mu\rho\nu\sigma} R^{\rho\sigma} \\ & - 2R_{\mu\sigma} R^{\sigma}_{\nu} + RR_{\mu\nu}), \end{aligned} \quad (3)$$

$$\begin{aligned} H_{\mu\nu}^{(3)} = & -3(4R^{\tau\rho\sigma\kappa} R_{\sigma\kappa\lambda\rho} R^{\lambda}_{\nu\tau\mu} - 8R^{\tau\rho}_{\lambda\sigma} R^{\sigma\kappa}_{\tau\mu} R^{\lambda}_{\nu\rho\kappa} \\ & + 2R_{\nu}^{\tau\sigma\kappa} R_{\sigma\kappa\lambda\rho} R^{\lambda\rho}_{\tau\mu} - R^{\tau\rho\sigma\kappa} R_{\sigma\kappa\tau\rho} R_{\nu\mu} \\ & + 8R^{\tau}_{\nu\sigma\rho} R^{\sigma\kappa}_{\tau\mu} R^{\rho}_{\kappa} + 8R^{\sigma}_{\nu\tau\kappa} R^{\tau\rho}_{\sigma\mu} R^{\kappa}_{\rho} \\ & + 4R_{\nu}^{\tau\sigma\kappa} R_{\sigma\kappa\mu\rho} R^{\rho}_{\tau} - 4R_{\nu}^{\tau\sigma\kappa} R_{\sigma\kappa\tau\rho} R^{\rho}_{\mu} \\ & + 4R^{\tau\rho\sigma\kappa} R_{\sigma\kappa\tau\mu} R_{\nu\rho} + 2RR_{\nu}^{\kappa\tau\rho} R_{\tau\rho\kappa\mu} \\ & + 8R^{\tau}_{\nu\mu\rho} R^{\rho}_{\sigma} R^{\sigma}_{\tau} - 8R^{\sigma}_{\nu\tau\rho} R^{\tau}_{\sigma} R^{\rho}_{\mu} \\ & - 8R^{\tau\rho}_{\sigma\mu} R^{\sigma}_{\tau} R_{\nu\rho} - 4RR^{\tau}_{\nu\mu\rho} R^{\rho}_{\tau} + 4R^{\tau\rho} R_{\rho\tau} R_{\nu\mu} \\ & - 8R^{\tau}_{\nu} R_{\tau\rho} R^{\rho}_{\mu} + 4RR_{\nu\rho} R^{\rho}_{\mu} - R^2 R_{\mu\nu}), \end{aligned} \quad (4)$$

respectively.

As in the paper of Morris and Thorne [1], we adopt the reverse philosophy in solving the third order Lovelock field equation, namely, we first consider an interesting and exotic spacetime metric, then find the matter source responsible for the respective geometry. The generalized metric of Morris and Thorne in n dimensions may be written as

$$\begin{aligned} ds^2 = & -e^{2\phi(r)} dt^2 + \left(1 - \frac{b(r)}{r}\right)^{-1} dr^2 + r^2 d\theta_1^2 \\ & + \sum_{i=2}^{n-2} \prod_{j=1}^{i-1} \sin^2 \theta_j d\theta_i^2, \end{aligned} \quad (5)$$

where $\phi(r)$ and $b(r)$ are the redshift function and shape function, respectively. Although the metric coefficient g_{rr} becomes divergent at the throat of the wormhole $r = r_0$, where $b(r_0) = r_0$, the proper radial distance

$$l(r) = \int_{r_0}^r \frac{dr}{\sqrt{1 - b/r}}$$

is required to be finite everywhere. The metric (5) represents a traversable wormhole provided the function $\phi(r)$ is finite everywhere and the shape function $b(r)$ satisfies the following two conditions:

$$1) \quad b(r) \leq r, \quad (6)$$

$$2) \quad rb' < b, \quad (7)$$

where the prime denotes the derivative with respect to r . The first condition is due to the fact that the proper radial distance should be real and finite for $r > r_0$, and the second condition comes from the flaring-out condition [1].

The mathematical analysis and the physical interpretation will be simplified using a set of orthonormal basis vectors

$$\begin{aligned} \mathbf{e}_{\hat{t}} &= e^{-\phi} \frac{\partial}{\partial t}, & \mathbf{e}_{\hat{r}} &= \left(1 - \frac{b(r)}{r}\right)^{1/2} \frac{\partial}{\partial r}, \\ \mathbf{e}_{\hat{1}} &= r^{-1} \frac{\partial}{\partial \theta_1}, & \mathbf{e}_{\hat{i}} &= \left(r \prod_{j=1}^{i-1} \sin \theta_j\right)^{-1} \frac{\partial}{\partial \theta_i}. \end{aligned}$$

Using the orthonormal basis (8), the components of energy-momentum tensor $T_{\hat{\mu}\hat{\nu}}$ carry a simple physical interpretation, i.e.,

$$T_{\hat{t}\hat{t}} = \rho, \quad T_{\hat{r}\hat{r}} = -\tau, \quad T_{\hat{i}\hat{i}} = p,$$

in which $\rho(r)$ is the energy density, $\tau(r)$ is the radial tension, and $p(r)$ is the pressure measured in the tangential directions orthogonal to the radial direction. The radial tension $\tau(r) = -p_r(r)$, where $p_r(r)$ is the radial pressure. Using a unit system with $\kappa_n^2 = 1$, and defining $\alpha_2 \equiv (n-3)(n-4)\alpha'_2$ and $\alpha_3 \equiv (n-3)\dots(n-6)\alpha'_3$ for simplicity, the nonvanishing components of field Eq. (1) reduce to

$$\begin{aligned} \rho(r) = & \frac{(n-2)}{2r^2} \left\{ -\left(1 + \frac{2\alpha_2 b}{r^3} + \frac{3\alpha_3 b^2}{r^6}\right) \frac{(b-rb')}{r} \right. \\ & \left. + \frac{b}{r} \left[(n-3) + (n-5) \frac{\alpha_2 b}{r^3} + (n-7) \frac{\alpha_3 b^2}{r^6} \right] \right\}, \end{aligned} \quad (8)$$

$$\begin{aligned} \tau(r) = & \frac{(n-2)}{2r} \left\{ -2\left(1 - \frac{b}{r}\right) \left(1 + \frac{2\alpha_2 b}{r^3} + \frac{3\alpha_3 b^2}{r^6}\right) \phi' \right. \\ & \left. + \frac{b}{r^2} \left[(n-3) + (n-5) \frac{\alpha_2 b}{r^3} + (n-7) \frac{\alpha_3 b^2}{r^6} \right] \right\}, \end{aligned} \quad (9)$$

$$\begin{aligned}
 p(r) = & \left(1 - \frac{b}{r}\right) \left(1 + \frac{2\alpha_2 b}{r^3} + \frac{3\alpha_3 b^2}{r^6}\right) \left[\phi'' + \phi'^2 + \frac{(b - rb')\phi'}{2r(r-b)}\right] + \left(1 - \frac{b}{r}\right) \left(\frac{\phi'}{r} + \frac{b - b'r}{2r^2(r-b)}\right) \left[(n-3) + (n-5)\frac{2\alpha_2 b}{r^3}\right. \\
 & + (n-7)\frac{3\alpha_3 b^2}{r^6} \left. \right] - \frac{b}{2r^3} \left[(n-3)(n-4) + (n-5)(n-6)\frac{\alpha_2 b}{r^3} + (n-7)(n-8)\frac{\alpha_3 b^2}{r^6}\right] \\
 & - \frac{2\phi'}{r^4} \left(1 - \frac{b}{r}\right) (b - b'r) \left(\alpha_2 + 3\alpha_3 \frac{b}{r^3}\right). \tag{10}
 \end{aligned}$$

III. EXOTICITY OF THE MATTER

To gain some insight into the matter threading the wormhole, one should consider the sign of ρ , $\rho - \tau$, and $\rho + p$. If the values of these functions are nonnegative, the weak energy condition ($T_{\mu\nu}u^\mu u^\nu \geq 0$, where u^μ is the timelike velocity of the observer) is satisfied, and therefore the matter is normal. In the case of negative ρ , $\rho - \tau$ or $\rho + p$, the weak energy condition is violated and the matter is exotic. We consider a specific class of particularly simple solutions corresponding to the choice of $\phi(r) = \text{const}$, which can be set equal to zero without loss of generality. In this case, $\rho - \tau$ and $\rho + p$ reduce to

$$\rho - \tau = -\frac{(n-2)}{2r^3} (b - rb') \left(1 + \frac{2\alpha_2 b}{r^3} + \frac{3\alpha_3 b^2}{r^6}\right), \tag{11}$$

$$\begin{aligned}
 \rho + p = & -\frac{(b - rb')}{2r^3} \left(1 + \frac{6\alpha_2 b}{r^3} + \frac{15\alpha_3 b^2}{r^6}\right) \\
 & + \frac{b}{r^3} \left\{ (n-3) + (n-5)\frac{2\alpha_2 b}{r^3} + (n-7)\frac{3\alpha_3 b^2}{r^6} \right\}. \tag{12}
 \end{aligned}$$

A. Positivity of ρ and $\rho + p$

Here, we investigate the conditions of positivity of ρ and $\rho + p$ for different choices of shape function $b(r)$.

1. Power law shape function

First, we consider the positivity of ρ and $\rho + p$ for the power law shape function $b = r_0^m / r^{m-1}$ with positive m . The positivity of m comes from the conditions (6) and (7). The functions ρ and $\rho + p$ for the power law shape function are positive for $r > r_0$ provided $r_0 > r_c$, where r_c is the largest positive real root of the following equations:

$$\begin{aligned}
 (n-3-m)r_c^4 + (n-5-2m)\alpha_2 r_c^2 \\
 + (n-7-3m)\alpha_3 = 0, \\
 (2n-6-m)r_c^4 + 2\alpha_2(2n-10-3m)r_c^2 \\
 + 3\alpha_3(2n-14-5m) = 0. \tag{13}
 \end{aligned}$$

Of course if Eqs. (13) have no real root, then there is no lower limit for r_0 and ρ and $\rho + p$ are positive everywhere.

2. Logarithmic shape function

Next, we investigate the positivity of ρ and $\rho + p$ for logarithmic shape function, $b(r) = r \ln r_0 / \ln r$. In this case the conditions (6) and (7) include $r_0 > 1$. The functions ρ and $\rho + p$ are positive for $r > r_0$ provided $r_0 \geq r_c$, where r_c is the largest real root of the following equations:

$$\begin{aligned}
 [(n-3)r_c^4 + \alpha_2(n-5)r_c^2 + \alpha_3(n-7)] \ln r_c \\
 - (r_c^4 + 2\alpha_2 r_c^2 + 3\alpha_3) = 0, \\
 2[(n-3)r_c^4 + 2\alpha_2(n-5)r_c^2 + 3\alpha_3(n-7)] \ln r_c \\
 - (r_c^4 + 6\alpha_2 r_c^2 + 15\alpha_3) = 0. \tag{14}
 \end{aligned}$$

If $r_c > 1$, then ρ and $\rho + p$ are positive for $r > r_0 \geq r_c$, but in the case that Eqs. (14) have no real positive root or their real roots are less than 1, then the lower limit for r_0 is just 1, and ρ and $\rho + p$ are positive for $r \geq r_0 > 1$

3. Hyperbolic solution

Finally, we consider the positivity of density ρ and $\rho + p$ for the hyperbolic shape function, $b(r) = r_0 \tanh(r) / \tanh(r_0)$ with $r_0 > 0$, which satisfies the conditions (6) and (7). The functions ρ and $\rho + p$ will be positive provided $r_0 > r_c$, where r_c is the largest real root of the following equations:

$$\begin{aligned}
 (n-4)r_c^4 + \alpha_2(n-7)r_c^2 + \alpha_3(n-10) \\
 + \frac{r_c^5 + 2\alpha_2 r_c^3 + 3\alpha_3 r_c}{\sinh r_c \cosh r_c} = 0, \\
 (2n-7)r_c^4 + 2\alpha_2(2n-13)r_c^2 + 3\alpha_3(2n-19) \\
 + \frac{r_c^5 + 6\alpha_2 r_c^3 + 15\alpha_3 r_c}{\sinh r_c \cosh r_c} = 0. \tag{15}
 \end{aligned}$$

Again for the case that Eqs. (15) have no real root, the functions ρ and $\rho + p$ are positive everywhere.

B. Positivity of $\rho - \tau$

Now, we investigate the conditions of the positivity of $\rho - \tau$. Since $b - rb' > 0$, as one may see from Eq. (6), the positivity of $\rho - \tau$ reduces to

$$1 + \frac{2\alpha_2 b}{r^3} + \frac{3\alpha_3 b^2}{r^6} < 0. \tag{16}$$

One may note that when the Lovelock coefficients are positive, the condition (16) does not satisfy. For the cases that either of α_2 and α_3 or both of them are negative, the

condition (16) is satisfied in the vicinity of the throat for power law, logarithmic, and hyperbolic shape function provided that the throat radius is chosen in the range $r_- < r_0 < r_+$, where

$$\begin{aligned} r_- &= (-\alpha_2 - \sqrt{\alpha_2^2 - 3\alpha_3})^{1/2}, \\ r_+ &= (-\alpha_2 + \sqrt{\alpha_2^2 - 3\alpha_3})^{1/2}. \end{aligned} \quad (17)$$

For the choices of Lovelock coefficients where r_+ is not real, then the condition (16) does not hold. For the cases where r_- is not real, then there is no lower limit for the throat radius that satisfies the condition (16). Even for the cases where r_+ exists and r_0 is chosen in the range $r_- < r_0 < r_+$, the condition (16) will be satisfied in the region $r_{\min} < r < r_{\max}$, where r_{\min} and r_{\max} are the positive real roots of the following equation:

$$r^6 + 2\alpha_2 r^3 b(r) + 3\alpha_3 b^2(r) = 0. \quad (18)$$

For negative α_3 , Eq. (18) has only one real root and the condition (16) is satisfied in the range $0 \leq r < r_{\max}$. It is worth noting that the value of r_{\max} depends on the Lovelock coefficients and the shape function. The value of r_{\max} for the power law shape function is

$$r_{\max} = \left(\frac{r_+}{r_0}\right)^{2/(m+2)} r_0, \quad (19)$$

which means that one cannot have a wormhole with normal matter everywhere. It is worth noting that $r_+ > r_0$, and therefore $r_{\max} > r_0$, as it should be.

C. Normal and exotic matter

Now, we are ready to give some comments on the exoticity or normality of the matter. First, we investigate the condition of having normal matter near the throat. There exist two constraints on the value of r_0 for the power law, logarithmic, and hyperbolic shape functions, while for the logarithmic shape function r_0 should also be larger than 1. The first constraint comes from the positivity of ρ and $\rho + p$, which state that r_0 should be larger or equal to r_c , where r_c is the largest real root of Eqs. (13)–(15) for power law, logarithmic, and hyperbolic shape functions, respectively. Of course, if there exists no real root for these equations, then there is no lower limit for r_0 . The second constraint, which comes from the condition (16), states that r_+ should be real. For positive Lovelock coefficients, there exists no real value for r_+ , and therefore we consider the cases where either of α_2 and α_3 or both of them are negative. The condition (16) is satisfied near the throat for the following two cases:

- (1) $\alpha_2 < 0$ and $0 < \alpha_3 \leq \alpha_2^2/3$.
- (2) $\alpha_3 < 0$.

The root r_- is real for the first case, while it is not real for the second case. In these two cases, one has normal matter in the vicinity of the throat provided $r_c < r_+$, and r_0 is

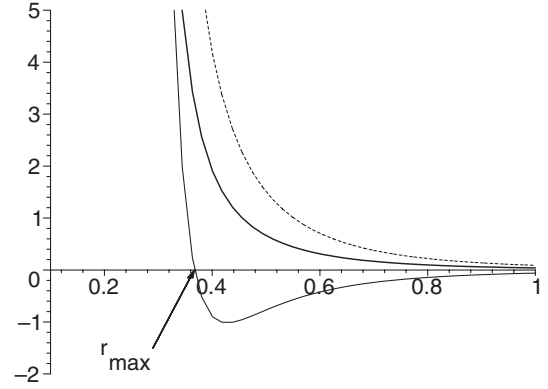


FIG. 1. $\rho - \tau$ (solid line), $\rho + p$ (bold line) and ρ (dotted line) vs r for power law shape function with $n = 8$, $m = 2$, $r_0 = .1$, $\alpha_2 = -.5$, and $\alpha_3 = -.5$.

chosen in the range $r_0 \leq r_+ < r_c$, where r_+ is the largest value of r_c and r_- . The above discussion shows that there are some constraints on the Lovelock coefficients and the parameters of shape function. In spite of these constraints,

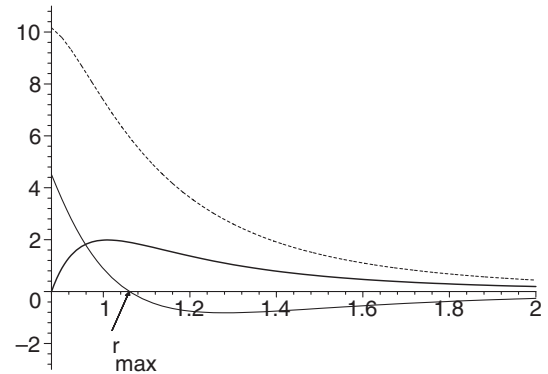


FIG. 2. $\rho - \tau$ (solid line), $\rho + p$ (bold line) and ρ (dotted line) vs r for power law shape function with $n = 8$, $m = 2$, $r_0 = r_c = .88$, $\alpha_2 = -1$, and $\alpha_3 = .2$.

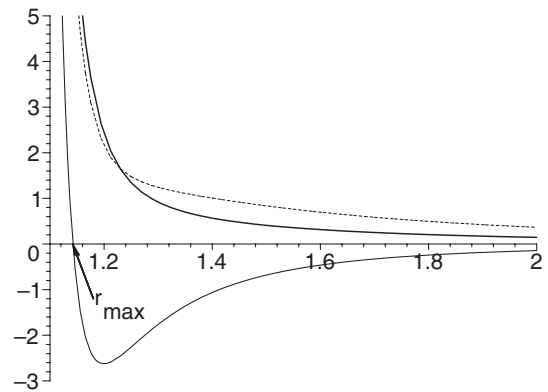


FIG. 3. $\rho - \tau$ (solid line), $\rho + p$ (bold line) and ρ (dotted line) vs r for logarithmic shape function with $n = 8$, $r_0 = 1.1$, $\alpha_2 = -.5$, and $\alpha_3 = -.5$.

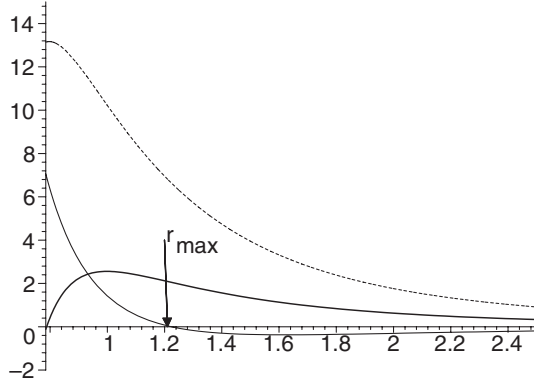


FIG. 4. $\rho - \tau$ (solid line), $\rho + p$ (bold line) and ρ (dotted line) and vs r for hyperbolic shape function with $n = 8$, $r_0 = r_c = .785$, $\alpha_2 = -.5$, and $\alpha_3 = -.5$.

one can choose the parameters suitable to havng normal matter near the throat. Even if the conditions of having normal matter near the throat are satisfied, there exists an upper limit for the radius of region of normal matter given in Eq. (18). Figures 1–4 are the diagrams of ρ , $\rho + p$, and $\rho - \tau$ versus r for various shape functions. In Figs. 2 and 4, the parameters have been chosen such that r_c is real, while r_c is not real in Figs. 1 and 3 and therefore r_0 has no lower limit. Note that for logarithmic shape function r_0 is larger than 1. All of these figures show that one is able to choose suitable values for the metric parameters in order to have normal matter near the throat. Also, it is worth mentioning that the radius of normal matter increases as α_3 becomes more negative, as one may note from Eq. (18) or Fig. 5. That is, the third order Lovelock term with negative coupling constant enlarges the region of normal matter.

Second, we consider the conditions where the matter is exotic for $r \geq r_0$ with positive ρ and $\rho + p$. These functions are positive for $r > r_0$ provided $r_0 > r_c$, where r_c is the largest real root of Eqs. (13)–(15) for power law, logarithmic, and hyperbolic solutions, respectively. Of course, there is no lower limit on r_0 , when these equations

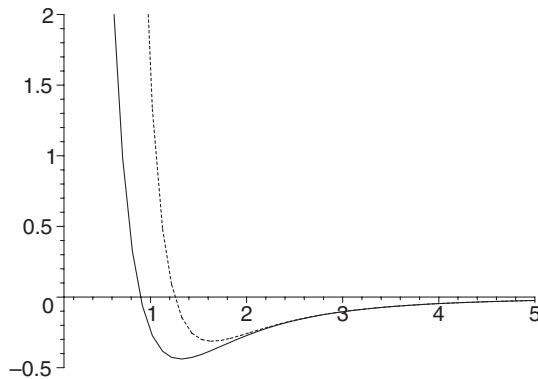


FIG. 5. $\rho - \tau$ vs r for hyperbolic shape function with $n = 8$, $r_0 = .1$, $\alpha_2 = -.5$, $\alpha_3 = 0$ (solid line) and $\alpha_3 = -1$ (dotted line).

have no real root. On the other hand, the condition (16) does not hold for $r_0 > r_+$. Thus, if both of r_c and r_+ are real, and one choose $r_0 \geq r_+$, where r_+ is the largest value of r_c and r_+ , then the matter is exotic with positive ρ and $\rho + p$ in the range $r_0 \leq r < \infty$. If none of r_c and r_+ are real, then there is no lower limit for r_0 , and one can have wormhole with exotic matter everywhere.

IV. CLOSING REMARKS

For wormholes with small throat radius, the curvature near the throat is very large, and therefore higher order curvature corrections are invited to the investigation of the wormholes. Thus, we presented the wormhole solutions of third order Lovelock gravity. Here, it is worth comparing the distinguishing features of wormholes of third order Lovelock gravity with those of Gauss-Bonnet and Einstein gravities. While the positivity of ρ and $\rho + p$ does not impose any lower limit on r_0 in Einstein gravity, there may exist a lower limit on the throat radius in Lovelock gravity, which is the largest real root of Eqs. (13)–(15) for power law, logarithmic, and hyperbolic shape functions, respectively. Although the existence of normal matter near the throat is a common feature of the wormholes of Gauss-Bonnet and third order Lovelock gravity, but the radius of the region with normal matter near the throat of third order Lovelock wormholes with negative α_3 is larger than that of Gauss-Bonnet wormholes. That is, the third order Lovelock term with negative α_3 enlarges the radius of the region of normal matter. Thus, one may conclude that inviting higher order Lovelock term with negative coupling constants into the gravitational field equation, enlarges the region of normal matter near the throat. For n th order Lovelock gravity with a suitable definition of α_p in terms of Lovelock coefficients, the condition (16) may be generalized to

$$1 + \sum_{p=2}^{[n-1]/2} p\alpha_p \left(\frac{b}{r^3}\right)^{p-1} < 0,$$

which can be satisfied only up to a radius $r_{\max} < \infty$. One may conclude from the above equation that as more Lovelock terms with negative Lovelock coefficients contribute to the field equation, the value of r_{\max} increases, but one cannot have wormhole in Lovelock gravity with normal matter everywhere for the metric (5) with $\phi(r) = 0$. The case of arbitrary $\phi(r)$ needs further investigation.

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