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Reconstructing baryon oscillations: A Lagrangian theory perspective

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Recently Eisenstein and collaborators introduced a method to ''reconstruct'' the linear power spectrum from a nonlinearly evolved galaxy distribution in order to improve precision in measurements of baryon acoustic oscillations. We reformulate this method within the Lagrangian picture of structure formation, to better understand what such a method does, and what the resulting power spectra are. We show that reconstruction does *not* reproduce the linear density field, at second order. We however show that it does reduce the damping of the oscillations due to nonlinear structure formation, explaining the improvements seen in simulations. Our results suggest that the reconstructed power spectrum is potentially better modeled as the sum of three different power spectra, each dominating over different wavelength ranges and with different nonlinear damping terms. Finally, we also show that reconstruction reduces the mode-coupling term in the power spectrum, explaining why miscalibrations of the acoustic scale are reduced when one considers the reconstructed power spectrum.

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I. INTRODUCTION

The baryon acoustic oscillation (BAO) method [1] is an integral part of current and next-generation dark energy experiments. Oscillations in the baryon-photon fluid, frozen into the matter distribution at decoupling, provide a standard ruler to constrain the expansion of the Universe. These sound waves imprint an almost harmonic series of peaks in the power spectrum $P(k)$, corresponding to a feature in the correlation function $\xi(r)$ at $\sim$100 Mpc, with width $\sim$10% due to Silk damping (see [2,3] for a detailed description of the physics, and [4] for a comparison of Fourier and configuration space pictures). While the early Universe physics is linear and well understood, the low redshift observations are complicated by the nonlinear evolution of matter (not to mention galaxy bias and redshift space distortions [5], but we will defer these to future work) which erases the oscillations on small scales and shifts the peaks [4,6–8]

$$P_{\rm obs}(k) = e^{-k^2\Sigma^2/2}P_{\rm lin}(k) + P_{\rm mc}(k) + \cdots \tag{1}$$

by coupling individual k-modes which are at early times independent. The exponential damping of the linear power spectrum (or equivalently the smoothing of the correlation function) reduces the contrast of the feature and thereby the precision with which the size of ruler may be measured. Neglect or incorrect modeling of the ''mode-coupling'' term $P_{\rm mc}$ may bias the resulting distance measurements.

In [4] it was pointed out that much of the modification to the power spectrum comes from large-scale modes, bulk flows, and supercluster formation, in principle enabling their effects to be corrected. Eisenstein *et al.* [9] introduced a method for removing the nonlinear degradation of the acoustic signature, sharpening the feature in configuration space or restoring/correcting the higher k oscillations in Fourier space. Given the ambitious nature of future experiments, there has been considerable interest [9–11] in ''reconstruction'' schemes which remove the effects of nonlinearities, reducing the damping and mode-coupling terms above.

Since the method proposed in [9] is an inherently nonlinear mapping of the observed density field, it is difficult to intuitively understand. It is however easily formulated within the Lagrangian picture of structure formation, where the fundamental quantity is the displacement of particles from their initial positions (contrasted with the Eulerian picture where one tracks the evolution of the density field at a fixed location). Motivated by recent developments in Lagrangian perturbation theory (LPT) [7,12], we discuss reconstruction within the context of LPT, both to elucidate how it works and to expose possible shortcomings. Although we use the method of [9] for specificity, the lessons learned have broader validity.

We proceed as follows: Sec. II introduces the essential aspects of both LPT as well as reconstruction. We then compute the reconstructed density field to second order, and demonstrate that there are corrections to the linear density at this order. Section III then explains why the BAO feature is enhanced in the reconstructed power spectrum. We conclude in Sec. IV, highlighting potential avenues for improvements.

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II. RECONSTRUCTION AND THE DENSITY FIELD

The Lagrangian description of structure formation [13–15] relates the current, or Eulerian, position of a mass element, $\mathbf{x}$, to its initial, or Lagrangian, position, $\mathbf{q}$, through a displacement vector field $\mathbf{\Psi}(\mathbf{q})$,

$$\mathbf{x} = \mathbf{q} + \mathbf{\Psi}(\mathbf{q}). \tag{2}$$

The displacements can be related to overdensities by [16]

$$\delta(\mathbf{x}) = \int d^3q\,\delta^{(D)}(\mathbf{x} - \mathbf{q} - \mathbf{\Psi}) - 1, \tag{3}$$

where $\delta^{(D)}$ is the 3D Dirac δ function, or in Fourier space by

$$\delta(\mathbf{k}) = \int d^3q\,e^{-i\mathbf{k}\cdot\mathbf{q}}(e^{-i\mathbf{k}\cdot\mathbf{\Psi}(\mathbf{q})} - 1). \tag{4}$$

The displacements evolve according to

$$\frac{d^2\mathbf{\Psi}}{dt^2} + 2H\frac{d\mathbf{\Psi}}{dt} = -\nabla_x\phi[\mathbf{q} + \mathbf{\Psi}(\mathbf{q})], \tag{5}$$

where ϕ is the gravitational potential. Analogous to Eulerian perturbation theory, LPT expands the displacement in powers of the linear density field, δ_l,

$$\mathbf{\Psi} = \mathbf{\Psi}^{(1)} + \mathbf{\Psi}^{(2)} + \cdots, \tag{6}$$

where [17]

$$\mathbf{\Psi}^{(n)}(\mathbf{k}) = \frac{i}{n!}\int\prod_{i=1}^{n}\left[\frac{d^3k_i}{(2\pi)^3}\right](2\pi)^3\delta^{(D)}\left(\sum_i\mathbf{k}_i - \mathbf{k}\right) \times \mathbf{L}^{(n)}(\mathbf{k}_1,\ldots,\mathbf{k}_n,\mathbf{k})\delta_l(\mathbf{k}_1)\cdots\delta_l(\mathbf{k}_n) \tag{7}$$

and the $\mathbf{L}^{(n)}$ have closed form expressions, generated by recurrence relations. Specifically,

$$\mathbf{L}^{(1)} = \frac{\mathbf{k}}{k^2} \tag{8}$$

is the well-known Zel'dovich displacement, e.g., [18], which is 1st order LPT. Expanding the exponential in Eq. (4) we obtain a perturbative series for the overdensity, $\delta = \delta^{(1)} + \delta^{(2)} + \cdots$ where, e.g.,

$$\delta^{(2)} = \int d^3q\,e^{-i\mathbf{k}\cdot\mathbf{q}}\left[-i\mathbf{k}\mathbf{\Psi}^{(2)} - \frac{(\mathbf{k}\cdot\mathbf{\Psi}^{(1)})^2}{2}\right] \tag{9}$$

or in terms of the $\mathbf{L}^{(n)}$'s

$$\delta^{(2)} = \frac{1}{2}\int\frac{d^3k_1d^3k_2}{(2\pi)^3}\delta^{(D)}(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k})\delta_l(\mathbf{k}_1)\delta_l(\mathbf{k}_2) \times[\mathbf{k}\cdot\mathbf{L}^{(2)}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}) + \mathbf{k}\cdot\mathbf{L}^{(1)}(\mathbf{k}_1)\mathbf{k}\cdot\mathbf{L}^{(1)}(\mathbf{k}_2)] \tag{10}$$

is second order in the linear density field δ_l.

The prescription of [9] can be cast into this framework as follows:

(i) Smooth the density field to filter out high k nonlinearities. In Fourier space, this is equivalent to multiplying by a function $\mathcal{S}(k)$ which monotonically decreases from unity at low k to zero at high k,

$$\delta(\mathbf{k}) \rightarrow \mathcal{S}(k)\delta(\mathbf{k}). \tag{11}$$

(ii) Compute the negative Zel'dovich displacement from the smoothed density field

$$\mathbf{s}(\mathbf{k}) \equiv -i\frac{\mathbf{k}}{k^2}\mathcal{S}(k)\delta(\mathbf{k}). \tag{12}$$

(iii) Shift the original particles by $\mathbf{s}$ and compute the "displaced" density field,

$$\delta_d(\mathbf{k}) = \int d^3q\,e^{-i\mathbf{k}\cdot\mathbf{q}}(e^{-i\mathbf{k}\cdot[\mathbf{\Psi}(\mathbf{q})+\mathbf{s}(\mathbf{q})]} - 1). \tag{13}$$

Note that if the original density field were linear, and $\mathcal{S} = 1$, this would undo their displacements exactly, moving them back to their original positions and giving $\delta_d = 0$.

(iv) Shift a spatially uniform grid of particles by $\mathbf{s}$ to form the "shifted" density field,

$$\delta_s(\mathbf{k}) = \int d^3q\,e^{-i\mathbf{k}\cdot\mathbf{q}}(e^{-i\mathbf{k}\cdot\mathbf{s}(\mathbf{q})} - 1). \tag{14}$$

Again, assuming linear theory would imply $\delta_s(\mathbf{k}) = -\delta(\mathbf{k})$.

(v) The reconstructed density field is defined as $\delta_{\rm recon} \equiv \delta_d - \delta_s$

$$\delta_{\rm recon}(\mathbf{k}) = \int d^3q\,e^{-i\mathbf{k}\cdot\mathbf{q}}e^{-i\mathbf{k}\cdot\mathbf{s}(\mathbf{q})}(e^{-i\mathbf{k}\cdot\mathbf{\Psi}(\mathbf{q})} - 1) \tag{15}$$

with power spectrum $P_{\rm recon}(k) \propto \langle|\delta^2_{\rm recon}|\rangle$.

Note that $\mathcal{S} \propto \mathbf{s} \rightarrow 0$ is equivalent to no reconstruction, which is helpful in interpreting some of the expressions below.

When applied to simulations this process yields an enhanced BAO feature [8–11] with a reduced "shift" in the peak. Our focus here is to understand what this procedure is doing within an analytic framework.

In the spirit of LPT we can expand the reconstructed density field in a perturbative series

$$\delta_{\rm recon} = \delta^{(1)}_{\rm recon} + \delta^{(2)}_{\rm recon} + \cdots. \tag{16}$$

As anticipated above, the reconstructed field equals the linear density field to lowest order. Working to the next order, we find

$$\delta^{(2)}_{\rm recon} = \delta^{(2)} - \frac{1}{2}\int \frac{d^3k_1 d^3k_2}{(2\pi)^3}\delta^{(D)}(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k})\delta_l(\mathbf{k}_1) \times \delta_l(\mathbf{k}_2)\mathbf{k}\cdot\mathbf{L}^{(1)}(\mathbf{k}_1)\mathbf{k}\cdot\mathbf{L}^{(1)}(\mathbf{k}_2)[\mathcal{S}(\mathbf{k}_1) + \mathcal{S}(\mathbf{k}_2)]. \tag{17}$$

We observe that the second-order term in the reconstructed density field does not vanish. While $\delta^{(2)}$ contains $\mathbf{L}^{(2)}$, the correction only involves $\mathbf{L}^{(1)}$ and so cannot fully cancel the nonlinearity. This is a general feature—the corrections to $\delta^{(n)}$ only involve terms $\mathbf{L}^{(i<n)}$—and follows from the fact that we only worked to first order when shifting objects. We note in passing that one might be able to construct higher order reconstruction schemes such that $\delta_l^{(n>1)}$ contributions to the reconstructed density vanish, but that is beyond our scope here.

To recap: the reconstruction algorithm above generates a density field with second-order corrections, *not* the linear density field. The next section explains why simulations saw an improvement when using reconstruction, by considering the reconstructed power spectrum.

III. THE POWER SPECTRUM

A. A toy model

To best highlight the effects of reconstruction on the power spectrum, we start with a toy model that captures both the physics and the algebraic structure of the full gravitational perturbation problem. This toy model is particularly useful for identifying the effect of reconstruction on the nonlinear damping of the linear power spectrum in Eq. (1). Section III B describes the correspondence between the toy model and the full gravitational instability problem, extending the analysis of the effect of reconstruction to the mode-coupling terms as well.

Consider a model, inspired by the peak-background split, where $\mathbf{\Psi}$ can be split into low (L) and high (H) frequency pieces,

$$\mathbf{\Psi} = \mathbf{\Psi}_L + \mathbf{\Psi}_H, \tag{18}$$

with $\mathbf{\Psi}_L$ the Zel'dovich displacement based on a linear density field δ_l,

$$\mathbf{\Psi}_L(\mathbf{k}) = i\frac{\mathbf{k}}{k^2}\delta_l(\mathbf{k}). \tag{19}$$

For simplicity we assume that $\mathbf{\Psi}_H$ is also Gaussian and is uncorrelated with $\mathbf{\Psi}_L$. The intuitive picture behind this model is that $\mathbf{\Psi}_L$ encodes the linear density field, while $\mathbf{\Psi}_H$ encodes the nonlinearities; importantly, the baryon oscillations only exist in $\mathbf{\Psi}_L$ and not in $\mathbf{\Psi}_H$.

Using Eq. (4) the power spectrum is

$$P(k) = \int d^3q e^{-i\mathbf{k}\cdot\mathbf{q}}(\langle e^{-ik_i\Delta\Psi_i(\mathbf{q})}\rangle - 1), \tag{20}$$

where $\mathbf{q} = \mathbf{q}_1 - \mathbf{q}_2$, and $\Delta\mathbf{\Psi} = \mathbf{\Psi}(\mathbf{q}_1) - \mathbf{\Psi}(\mathbf{q}_2)$. For Gaussian $\mathbf{\Psi}$

$$\langle e^{-i\mathbf{k}\cdot\Delta\mathbf{\Psi}(q)}\rangle = \exp[-\tfrac{1}{2}k_ik_j\langle\Delta\Psi_i(\mathbf{q})\Delta\Psi_j(\mathbf{q})\rangle] \tag{21}$$

with

$$k_ik_j\langle\Delta\Psi_i(\mathbf{q})\Delta\Psi_j(\mathbf{q})\rangle = 2k_i^2\langle\Psi_i^2(\mathbf{0})\rangle - 2k_ik_j\xi_{ij}(\mathbf{q}), \tag{22}$$

where $\xi_{ij}(\mathbf{q}) \equiv \langle\Psi_i(\mathbf{q}_1)\Psi_j(\mathbf{q}_2)\rangle$ is the displacement correlation function and we have used translational invariance for the correlation function at zero lag. To lowest order the zero-lag correlation function is $\xi_{ij}(\mathbf{0}) = \delta_{ij}\Sigma^2/2$, with Σ^2 the mean-squared Zel'dovich displacement of particles,

$$\Sigma_L^2 = \frac{1}{3\pi^2}\int dp P_L(p) \tag{23}$$

with a similar expression for Σ_H. Note that the relation of the damping to the Zel'dovich displacement follows naturally from the LPT formalism and shows the similarity of the treatments in Refs. [4,6,7,12].

Given our assumption of uncorrelated low and high frequency pieces, we have $\Sigma^2 = \Sigma_L^2 + \Sigma_H^2$. However Fig. 1 demonstrates that the dominant contribution comes from relatively large ($k < 0.3h$ Mpc^{-1}) scales. If we estimate Σ_H by substituting the nonlinear power spectrum in the equation above, we find that the dominant contribution comes from linear motions, even at $z = 0$. For simplicity, we will therefore assume $\Sigma^2 \simeq \Sigma_L^2$ in what follows.

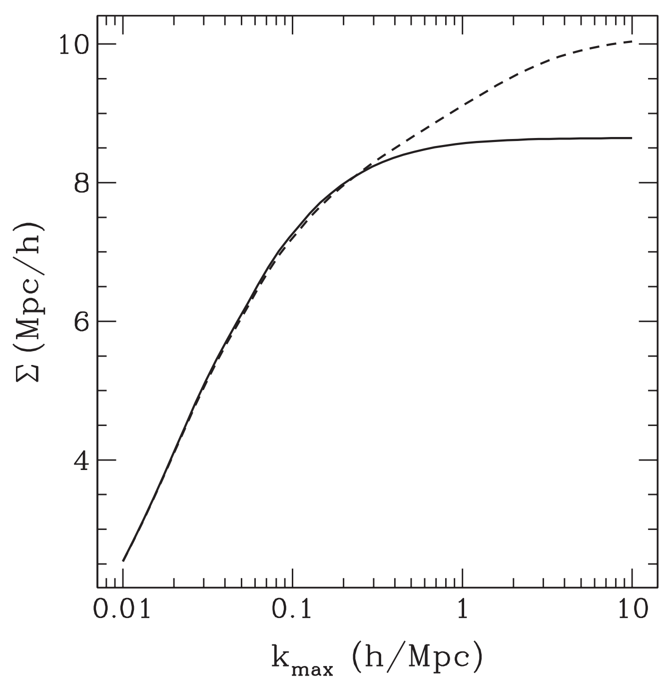

FIG. 1. The damping scale at $z = 0$ as a function of the maximum wave number for the linear (solid line) and nonlinear (dashed line) power spectra. Note that the dominant contribution to the damping scale comes from linear motions.

The nonlinear power spectrum is then given by

$$P(k) = e^{-k^2\Sigma_L^2/2} \int d^3q e^{-ik_iq_i} e^{k_ik_j\xi_{ij}(q)}. \quad (24)$$

Following [7] we leave the zero-lag piece exponentiated, but expand the exponential inside the integral. The first term of the expansion gives P_L. This procedure can be viewed as a resummation of terms in the standard perturbative expansion which leads to a power spectrum of the form in Eq. (1),

$$P_{\rm obs}(k) = e^{-k^2\Sigma_L^2/2} P_L(k) + P_{\rm mc}(k) + \cdots \quad (25)$$

with $P_{\rm lin} = P_L(k)$. Note that $P_{\rm mc}(k)$ contains terms $\mathcal{O}(\mathbf{\Psi}_H^2)$ representing the high frequency part of the power spectrum and terms $\mathcal{O}(\mathbf{\Psi}_L^4)$ corresponding to second-order (in P_L) corrections. We will consider these terms in the next section.

The above can be extended to compute the reconstructed power spectrum for this model. Since $\mathbf{\Psi}_H$ has no low frequency piece by construction, we assume that the inferred shift, $\mathbf{s}(\mathbf{k})$, is simply given by

$$\mathbf{s}(\mathbf{k}) = -\mathcal{S}(k)\mathbf{\Psi}_L(\mathbf{k}) + O(\mathbf{\Psi}_L^2). \quad (26)$$

The fields δ_d and δ_s of Sec. II are then generated to first order by $(1-\mathcal{S})\mathbf{\Psi}_L + \mathbf{\Psi}_H$ and $-\mathcal{S}\mathbf{\Psi}_L$, respectively. Since the reconstructed density field is the difference of the two fields, there are three terms (two autospectra, P_{ss}, P_{dd} and one cross spectrum P_{sd}) that make up the reconstructed power spectrum: $P_{\rm recon} = P_{ss} + P_{dd} - 2P_{sd}$. The autopower spectra are exactly analogous to the nonlinear power spectra, except for the damping terms,

$$P_{ss}(k) = e^{-k^2\Sigma_{ss}^2/2}\mathcal{S}^2(k)P_L(k) + \cdots \quad (27)$$

and

$$P_{dd}(k) = e^{-k^2\Sigma_{dd}^2/2}[1-\mathcal{S}(k)]^2 P_L(k) + \cdots \quad (28)$$

where we have dropped higher order terms. The Gaussian damping is modified to

$$\Sigma_{ss}^2 = \frac{1}{3\pi^2}\int dp \mathcal{S}^2(p)P_L(p) \quad (29)$$

with an analogous expression for Σ_{dd} with $\mathcal{S}^2 \to (1-\mathcal{S})^2$. The cross power spectrum is

$$P_{sd}(k) = -e^{-k^2\Sigma_{sd}^2/2}\mathcal{S}(k)[1-\mathcal{S}(k)]P_L(k) + \cdots \quad (30)$$

where

$$\Sigma_{sd}^2 = \tfrac{1}{2}(\Sigma_{ss}^2 + \Sigma_{dd}^2), \quad (31)$$

and the negative sign comes from the fact that the random field was shifted by the negative Zel'dovich term. Putting the pieces together, we find that the damping term becomes

$$D(k) \equiv e^{-k^2\Sigma^2/2} \to \mathcal{S}^2(k)e^{-k^2\Sigma_{ss}^2/2} + [1-\mathcal{S}(k)]^2 e^{-k^2\Sigma_{dd}^2/2} + 2\mathcal{S}(k)[1-\mathcal{S}(k)]e^{-k^2\Sigma_{sd}^2/2}. \quad (32)$$

Before proceeding, it is useful to choose an explicit form for the smoothing; the standard choice is a Gaussian,

$$\mathcal{S}(k) = \exp\left(-\frac{k^2R^2}{4}\right). \quad (33)$$

Figure 2 plots the various damping scales as a function of the smoothing scale. As expected, for nonzero smoothing, both Σ_{ss} and Σ_{dd} (and therefore Σ_{sd} as well) are less than the nonlinear damping scale. This is the crux of the reconstruction method—that the P_L contribution to the reconstructed power spectrum is less damped than in the nonlinear power spectrum. This holds even when taking into account that there are additional terms depending upon $\mathcal{S}(k)$ in $D(k)$ as we now show.

Before considering Eq. (32) for arbitrary choices of smoothing scales, we consider the special case where $\Sigma_{ss} = \Sigma_{dd} = \Sigma_{sd}$; for the Gaussian smoothing above, this corresponds to a smoothing scale $R \sim 30h^{-1}$ Mpc. The damping of P_L in the reconstructed power spectrum simplifies considerably; the reconstructed power spectrum has the form,

$$P_{\rm recon}(k) = e^{-k^2\Sigma_{ss}^2/2}P_L(k) + \cdots. \quad (34)$$

Note that this is identical to the form of the nonlinear power spectrum Eq. (25) except that $\Sigma_{ss} < \Sigma$, reducing the damping.

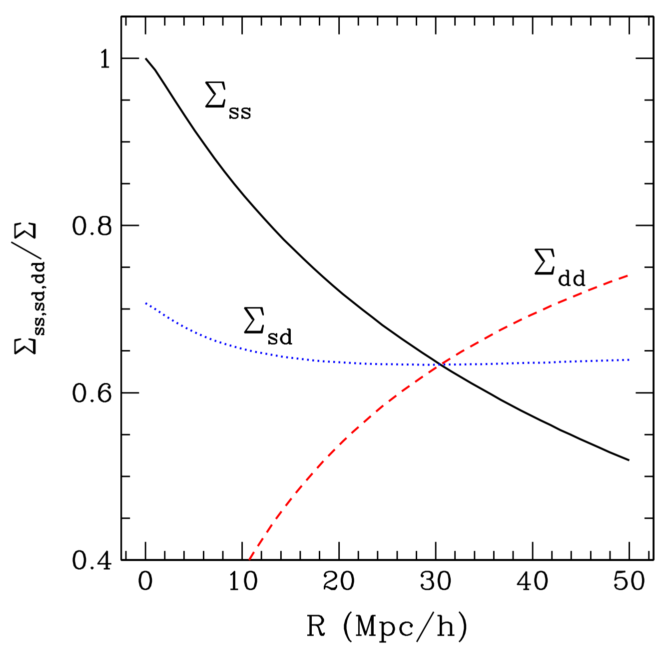

FIG. 2 (color online). The ratio of Σ_{ss}, Σ_{sd}, and Σ_{dd} to Σ, as a function of the Gaussian smoothing scale, R. Note that for no smoothing, $\Sigma_{ss} = \Sigma$ and $\Sigma_{dd} = 0$, while for infinite smoothing, $\Sigma_{dd} = \Sigma$ with $\Sigma_{ss} = 0$.

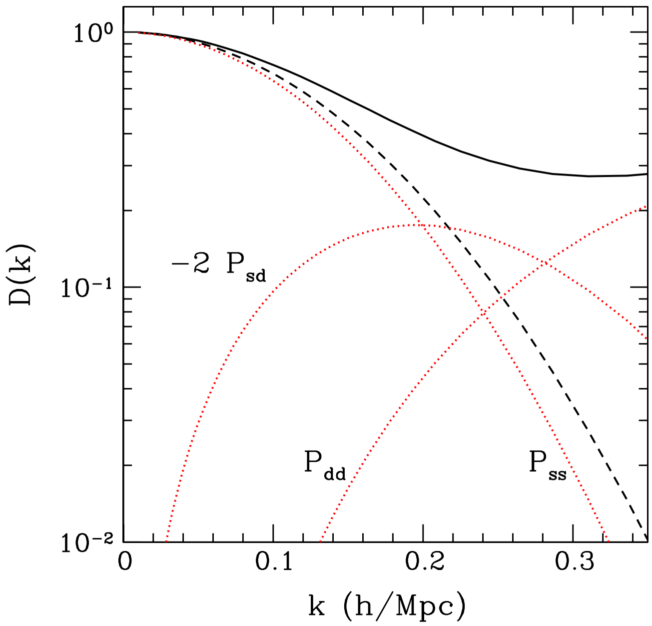

FIG. 3 (color online). The damping of the linear power spectrum for the nonlinear power spectrum (dashed line), and the reconstructed power spectrum [Eq. (32), solid line, assuming a smoothing scale $R = 5h^{-1}$ Mpc]. The dotted lines decompose the reconstructed damping into the leading contributions from its P_{ss}, P_{sd}, and P_{dd} components These curves have been calculated assuming $z = 0$.

Figure 3 shows the damping, $D(k)$, for smoothing scale $R = 5h^{-1}$ Mpc, as an example of its general form for an arbitrary choice of smoothing scale. Given R, the factors involving $\mathcal{S}(k)$ determine the range of wave numbers for which each of the three power spectra dominate. For large R (see below), P_{dd} dominates over the wave numbers important for baryon oscillations $[0.07 < k(h/\mathrm{Mpc}) < 0.35]$ but as we argue below, this limit is not optimal. As we decrease R, we might have expected that P_{ss} would have dominated; however, decreasing R quickly increases Σ_{ss} to close to the nonlinear damping scale (Fig. 2), limiting the importance of P_{ss}. Indeed, in Fig. 3, we see that P_{ss} has the linear power spectrum more strongly damped than the nonlinear power spectrum. The dominant term at small R is therefore P_{sd}; Fig. 2 shows that Σ_{sd} is $\sim 0.6\Sigma$ and is only weakly dependent on R. This suggests that such a reconstruction method can reduce the damping of the linear power spectrum by a factor ~ 2.

The above discussion argues that the smoothing scale determines the wave number where P_{dd} becomes dominant. The obvious question is whether the above analysis suggests a value for the smoothing scale. We argue that the natural choice is $R \sim \Sigma$, the nonlinear (damping) scale. To see why, we start by observing that the terms we ignored in P_{dd} are $\mathcal{O}(\mathbf{\Psi}_H^2)$, whereas for P_{ss} and P_{dd} they involve higher powers of the displacement. This is just the statement that the small-scale displacements have their largest effect on P_{dd} which is not surprising, given that P_{dd} is based on the original density field. We would ideally want to reduce these terms, which argues for making R as small as possible. However, from Eq. (25), we see that the linear field is damped on scales smaller than Σ. Smoothing on scales much smaller would then violate our assumption that $\mathbf{s}(\mathbf{k})$ is derived from the linear density field, which leads to choosing $R \sim \Sigma$ as might have intuitively been expected.

The above discussion explains how reconstruction reduces the damping of the acoustic oscillations (or equivalently, how it sharpens the peak in the correlation function). We now turn to its effect on the mode-coupling terms, by considering the reconstructed power spectrum within LPT.

B. Lagrangian perturbation theory

Many of the features of reconstruction in the last section carry across to the gravitational instability problem within LPT. We will closely follow the LPT formalism developed in [7,12] in which the broadening of the peak and the mode-coupling terms appear naturally.

For the unreconstructed power spectrum the derivation leading to Eq. (20) still holds. However now we must use the cumulant expansion theorem

$$\langle e^{-iX}\rangle = \exp\left[\sum_{N=1}\frac{(-i)^N}{N!}\langle X^N\rangle_c\right] \tag{35}$$

(where the $\langle X^N\rangle_c$ are the connected moments) to compute the expectation value of the exponential. In the toy model only the $N = 2$ term survived, for the full problem higher orders contribute as well. Expanding $(\mathbf{k}\cdot\Delta\mathbf{\Psi})^N$ using the binomial theorem we have two types of terms: those where the $\mathbf{\Psi}$ are all evaluated at the same point (which we can take to be the origin) and those with $j\ \mathbf{q}_1$'s and $N - j\ \mathbf{q}_2$'s. As in the toy model, and following [7], we leave the first set of terms exponentiated while expanding the second set of terms in powers of $\mathbf{\Psi}$. If we keep only the lowest order terms in the exponential we regain the form of Eq. (1) with Σ given by the rms Zel'dovich displacement

$$P(k) = e^{-k^2\Sigma^2/2}\left\{P_L(k)\left[1 + \int d^3k_1 P_L(k_1)G(\mathbf{k},\mathbf{k}_1)\right] + \int d^3k_1 d^3k_2 P_L(\mathbf{k}_1)P_L(\mathbf{k}_2)F^2(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}) + \cdots\right\} \tag{36}$$

where F and G can be expressed in terms of $\mathbf{L}^{(1)}$ and $\mathbf{L}^{(2)}$ and explicit expressions may be found in [7]. There are oscillations in P_L and the mode-coupling term (second line), but the integral in the first line has a wide kernel so the oscillations are suppressed. As it happens, F is peaked around $\mathbf{k}_1 \approx \mathbf{k}_2 \approx \mathbf{k}/2$, which helps to explain why this term in the third line leads to a peak shift. If P_L contains an oscillatory piece, e.g., $\sin(kr)$, then the third term contains a piece schematically of the form $\sin^2(kr/2) \sim 1 + \cos(kr)$, which oscillates out of phase with P_L. It is the sum of the two out of phase components that leads to a shift in the peak of $\xi(r)$ or the phasing of the harmonics in $P_{\rm obs}(k)$.

It is now straightforward, though tedious, to repeat these steps for the reconstructed field. The formalism of Ref. [7] must be generalized to allow two displacements ($\mathbf{s}$ and $\mathbf{\Psi}$). Again there are three contributions, P_{ss}, P_{dd}, and P_{sd}, and three smoothings, Σ_{ss}, Σ_{dd}, and Σ_{sd} of the same form as before. The term proportional to P_L becomes

$$P_{\rm recon}(k) = \{e^{-k^2\Sigma_{ss}^2/2}\mathcal{S}^2(k) + 2e^{-k^2\Sigma_{sd}^2/2}\mathcal{S}(k)[1-\mathcal{S}(k)] + e^{-k^2\Sigma_{dd}^2/2}[1-\mathcal{S}(k)]^2\}P_L(k), \quad (37)$$

directly comparable to the result of the toy model.

The leading contribution to the mode-coupling term is the same as in standard perturbation theory, and is strictly positive, coming from $\langle\delta^{(2)}\delta^{(2)}\rangle$. Recalling the relation between $\delta^{(2)}_{\rm recon}$ and $\delta^{(2)}$ we need to replace F in the mode-coupling term with $\hat{F}$ subject to

$$2\hat{F}(\mathbf{k}_1,\mathbf{k}_2) \equiv \mathbf{k}\cdot\mathbf{L}^{(2)}(\mathbf{k}_1,\mathbf{k}_2) + \mathbf{k}\cdot\mathbf{L}^{(1)}(\mathbf{k}_1)\mathbf{k}\cdot\mathbf{L}^{(1)}(\mathbf{k}_2) \times [1-\mathcal{S}(\mathbf{k}_1)-\mathcal{S}(\mathbf{k}_2)] \quad (38)$$

where $\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}$. The piece of the mode-coupling integral which shifts the peak comes from $\mathbf{k}_1 \approx \mathbf{k}_2 \approx \mathbf{k}/2$ where $\mathbf{k}\cdot\mathbf{L}_2 = 0$ and

$$\hat{F}(\mathbf{k}/2,\mathbf{k}/2) = 2\{1 - 2\mathcal{S}(\mathbf{k}/2)\}. \quad (39)$$

Since the term in $\{\cdots\}$ is bounded between -1 and 1, $\hat{F}^2 < F^2$ for all k, suppressing the reconstructed mode-coupling term relative to the corresponding term in the nonlinear power spectrum. This explains why the miscalibrations in the acoustic scale were reduced after reconstruction in [8].

IV. COMMENTS

It is now generally understood ([4,6,7] and this work) that the dominant effect of the nonlinear evolution of matter perturbations on the baryon oscillations is to damp the higher harmonics, $P_{\rm obs}(k) = \exp(-k^2\Sigma^2/2)P_{\rm lin}(k) + \cdots$, or equivalently, smooth the feature in the correlation function. Eisenstein *et al.* [9] proposed a reconstruction method, demonstrated on simulations, that undoes this nonlinear smoothing and appears to restore the linear power spectrum. Motivated by recent progress in Lagrangian perturbation theory [7,12], we revisit this algorithm in order to better understand why it works as well as its shortcomings. Our principal conclusions are as follows:

(i) The field generated by the reconstruction process is *not* the linear density field at second order. Note that this is a general statement, independent of assumptions about the smoothing of the initial density field.

(ii) Reconstruction does reduce the damping of the oscillations, by about a factor of 2 when the input density field is smoothed on the nonlinear scale.

(iii) Reconstruction also reduces the mode-coupling terms which introduce an out of phase component of the oscillations or shift the peak.

(iv) The reconstructed power spectrum is the sum of three power spectra (the autopower spectra of the displaced and shifted fields, and their cross spectrum), each of which have different damping terms [Eq. (32)]. An appropriate model for the reconstructed power spectrum should take this into account, instead of modeling it as a single damping scale.

(v) When the smoothing scale is close to the nonlinear scale, the correlation between the shifted and displaced fields plays a crucial role.

Our results suggest a number of natural extensions. The effects of bias and redshift space distortions have been incorporated into the Lagrangian formalism [7,12], and could therefore be folded in to the LPT formulation of reconstruction. We have observed that the reconstructed density field is not the linear density field; an interesting possibility is to explore whether higher order reconstruction schemes actually yield dividends. Even within the context of the existing reconstruction schemes, it is possible that a different weighting of the three power spectra may yield improved accuracy in measuring the distance scale. We leave these avenues open for future investigation.

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