Neutralino annihilation into massive quarks with supersymmetric QCD corrections

Björn Herrmann

Institut für Theoretische Physik und Astrophysik, Universität Würzburg, Am Hubland, D-97074 Würzburg, Germany

Michael Klasen and Karol Kovařík*

*Laboratoire de Physique Subatomique et de Cosmologie, Université Joseph Fourier/CNRS-IN2P3/INPG,
53 Avenue des Martyrs, F-38026 Grenoble, France*

We compute the full $\mathcal{O}(\alpha_s)$ supersymmetric (SUSY)-QCD corrections for neutralino annihilation into massive quarks through gauge or Higgs bosons and squarks in the minimal supersymmetric standard model, including the known resummation of logarithmically enhanced terms. The numerical impact of the corrections on the extraction of SUSY mass parameters from cosmological data is analyzed for gravity-mediated SUSY-breaking scenarios and shown to be sizable, so that these corrections must be included in common analysis tools.

I. INTRODUCTION

The astrophysical and cosmological evidence gathered in recent years supports the hypothesis of dark matter in our Universe. In particular, models of structure formation favor cold dark matter (CDM) consisting of weakly interacting massive particles (WIMPs) with nonrelativistic velocities. The five-year data of the Wilkinson Microwave Anisotropy Probe (WMAP), combined with the results of supernovae experiments and baryonic acoustic oscillation data, constrain the relic density Ω of CDM in the Universe at 95% (2σ) confidence level to [1]

$$0.1097 < \Omega_{\mathrm{CDM}} h^2 < 0.1165. \qquad (1)$$

Here, h denotes the present Hubble expansion rate H_0 in units of $100 \text{ km s}^{-1} \text{Mpc}^{-1}$.

In contrast to the standard model (SM) itself, its various extensions can provide viable candidates for WIMPs. In the minimal supersymmetric standard model (MSSM) with R-parity conservation, the lightest supersymmetric particle (LSP) is such a candidate, if it is a color singlet and electrically neutral. In many scenarios, in particular, those where supersymmetry (SUSY) breaking is mediated by gravity, the LSP is the lightest neutralino and thus a suitable dark matter candidate in a large part of the parameter space. To calculate the number density n of the relic particle, one has to solve the Boltzmann equation

$$\frac{dn}{dt} = -3Hn - \langle \sigma_{\mathrm{ann}} v \rangle (n^2 - n_{\mathrm{eq}}^2), \qquad (2)$$

where n_{eq} is the density of the relic particle in thermal equilibrium and v is the relative velocity of the annihilating pair.

The thermally averaged annihilation cross section $\langle \sigma_{\mathrm{ann}} v \rangle$ includes all (co-)annihilation processes of the dark matter particle into SM particles. It is dominated by two-particle final states, most notably by fermion-antifermion pairs and by combinations of gauge ($W^\pm$, Z^0) and Higgs bosons (h^0, H^0, A^0, $H^\pm$) [2]. The nature of the final state contributing the most depends strongly on the region of parameter space. Fermion final states have the clear advantage that they are always kinematically allowed. Their leading contribution is proportional to the mass of the fermion, so that we focus our attention here on the annihilation into the massive quarks of the third generation.

The relic density of the lightest neutralino depends on the SUSY-breaking parameters of the MSSM, which determine the nature of the neutralino as well as the couplings and masses that appear in the annihilation cross section. The number of free parameters is often reduced to a few universal parameters imposed at the high scale, e.g., the five parameters m_0, $m_{1/2}$, A_0, $\tan\beta$, and $\mathrm{sgn}(\mu)$ in minimal supergravity (mSUGRA). Using the experimental limits in Eq. (1), one can then constrain the parameter space in a complementary way to collider and low-energy experiments.

This analysis is made possible by public computer codes, which perform the calculation of the dark matter relic density within models beyond the SM. The most popular and developed codes are DARKSUSY [3] and MICROMEGAS [4]. The implemented processes are mostly calculated at leading order, and higher order corrections are only included for some very sensitive quantities. However, owing to the large magnitude of the strong coupling constant, all QCD and SUSY-QCD corrections significantly affect the annihilation cross section.

In this paper, we extend our previous work [5] by computing the full QCD and SUSY-QCD corrections to neutralino annihilation into top and bottom quark-antiquark pairs. As an example, we present the impact of the radiative corrections in mSUGRA scenarios where the

*kovarik@lpsc.in2p3.fr

annihilation into top quark pairs is dominant. Moreover, we include a study of bottom quark final states at small $\tan\beta$, where we find agreement with previous results of a similar calculation [6]. Since light quark final states do not lead to sizable contributions in the analyzed mSUGRA scenarios, their discussion is postponed to a later publication.

II. CALCULATION DETAILS

The annihilation of neutralinos into quarks proceeds at tree level through an exchange of the Z-boson and Higgs bosons in the s-channel or of scalar quarks (squarks) in the t- and u-channels (see Fig. 1). To reach a sufficient annihilation rate, the cross section has to be enhanced either by an s-channel resonance or by a small squark mass.

For these processes, we calculate the full one-loop QCD and SUSY-QCD corrections involving the exchange of a gluon or a gluino. These affect the vertices including Z- and Higgs bosons and a quark-antiquark pair as well as those with neutralinos, quarks, and squarks. In addition, one has to include the box diagrams arising from the exchange of a gluon or a gluino between the final state quarks in the t- and u-channels (see Fig. 2). All relevant divergent integrals are evaluated in the modified dimensional reduction scheme ($\overline{\text{DR}}$). We use on-shell renormalization of the wave functions and couplings in order to eliminate the ultraviolet divergences. The squark mixing matrix is also renormalized on shell as proposed in Ref. [7]. In the Yukawa couplings, however, we use the quark masses defined in the $\overline{\text{DR}}$ renormalization scheme. Full details of the calculation will be presented in a forthcoming paper.

For the bottom quark, we start from the input value $m_b^{\overline{\text{MS}}}(m_b)$, evolve to the scale Q using three-loop SM renormalization group equations, change from the $\overline{\text{MS}}$ to the $\overline{\text{DR}}$ scheme using the corresponding relation at two loops [8], and finally include the MSSM threshold corrections comprising the sbottom-gluino and stop-chargino one-loop contributions. The latter are considerably enhanced for large $\tan\beta$ or large A_b, so that they have to be resummed to all orders of perturbation theory [9]. Denoting the resummable part by Δ_b and the finite one-loop remainder by Δm_b, the bottom quark mass is then given by

FIG. 2. SUSY-QCD loop diagrams contributing to the annihilation of neutralinos into quarks. We leave out the diagrams for the u-channel contributions and the crossed box diagrams.

$$m_b^{\text{MSSM}}(Q) = \frac{m_b^{\overline{\text{DR}}}(Q)}{1 + \Delta_b} - \Delta m_b. \tag{3}$$

The $\overline{\text{DR}}$-mass of the top quark is obtained from the on-shell value $m_t = 172.4$ GeV [10] by subtracting the finite part of the on-shell mass counterterm.

The infrared divergence connected to the exchange of a massless gluon is also regularized dimensionally. To cancel the infrared divergent poles, we include the bremsstrahlung process with an additional gluon in the final state. To allow for integration of the divergent bremsstrahlung matrix element and for the cancellation of the infrared divergences, we use the dipole subtraction formalism for massive partons [11].

We have performed several checks of the calculation. In particular, we have compared our analytical calculation of the virtual part with the one produced by the packages FEYNARTS and FORMCALC [12] and the QCD corrections to the s-channel Higgs-boson exchange with the known results of Ref. [13].

III. NUMERICAL ANALYSIS

To evaluate the impact of the QCD and SUSY-QCD corrections on the neutralino relic density, we have chosen three typical parameter points in mSUGRA shown in Table I. They were selected so that the top and bottom quark-antiquark final states dominate the total annihilation cross section and the relic density lies within the WMAP range of Eq. (1). Moreover, the SUSY particle mass spectrum resulting from the high-scale parameters respects the experimental mass exclusion limits, and the branching ratio of the decay $b \rightarrow s\gamma$ lies within 2σ of the experimental bound BR$(b \rightarrow s\gamma) = (3.52 \pm 0.25) \times 10^{-4}$, obtained
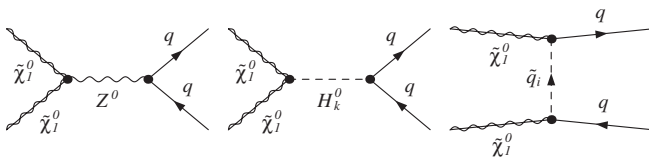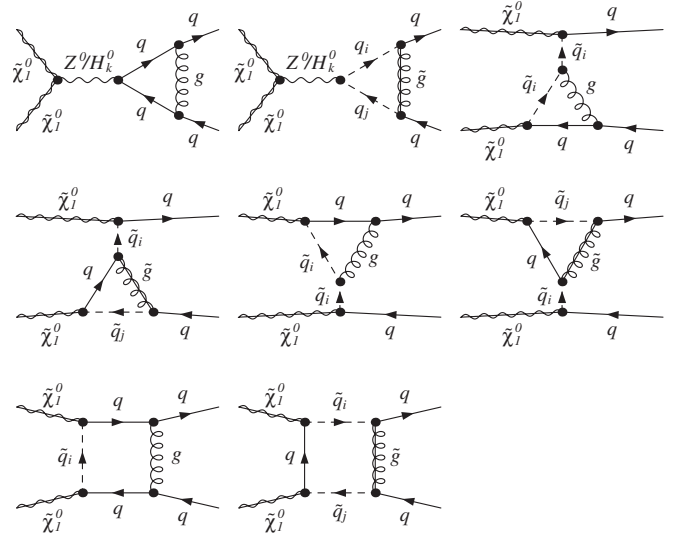

FIG. 1. Tree-level diagrams for the annihilation of neutralinos into quarks. The crossed u-channel diagram is not shown. We denote the neutral Higgs bosons (h^0, H^0, A^0) by H_k^0.

TABLE I. High-scale mSUGRA parameters (in GeV) together with the corresponding neutralino relic density and the contributions of the massive quark-antiquark final states to the annihilation cross section for our selected scenarios.

	m_0	$m_{1/2}$	A_0	$\tan\beta$	$\text{sgn}(\mu)$	$\Omega_{\text{CDM}}h^2$	$b\bar{b}$	$t\bar{t}$
1	1800	131	-1500	10	$+$	0.116	86%	$\cdots$
2	5800	536	-1500	10	$+$	0.111	$\cdots$	72%
3	3200	520	0	50	$+$	0.110	8%	71%

from combined measurements by *BABAR*, Belle, and CLEO [14].

The high-scale parameters are evolved down to the electroweak scale using SPHENO 2.2.3 [15]. The neutralino relic density is calculated with MICROMEGAS 2.1, where we have included the full radiative QCD and SUSY-QCD corrections to the annihilation cross section. Our first parameter point at low $\tan\beta$ has been chosen near the bulk region with a low fermion mass parameter $m_{1/2}$, but a rather large scalar mass m_0 in order to avoid the constraint coming from $\text{BR}(b \to s\gamma)$. The remaining top-quark dominated points lie both in the focus point region, where m_0 is very large. At large $\tan\beta$, there is also a contribution from bottom quarks due to their important coupling to the *CP*-odd Higgs boson.

In the top panels of Figs. 3–5, we show the numerical results for the relevant annihilation cross sections as a function of the relative momentum p_{cm}, that is related to the center-of-mass energy $\sqrt{s}$ and the neutralino mass $m_{\tilde\chi}$ through $s = 4(p_{\text{cm}}^2 + m_{\tilde\chi}^2)$. Shown are the leading order results with $\overline{\text{DR}}$ Yukawa couplings (dash-dotted lines), the approximation already included in MICROMEGAS (dashed lines), and the result including our full one-loop QCD and SUSY-QCD corrections (solid lines). We also show, in arbitrary units, the Boltzmann distribution function involved in the calculation of the thermal average at the freeze-out temperature (shaded areas). It indicates which center-of-mass momenta contribute most to the relic density. For example, in Fig. 3 one sees that an important contribution comes also from energies below the light Higgs-boson resonance, which is clearly visible in the top panel.

The center panels of Figs. 3–5 show the corresponding predictions for the neutralino relic density as a function of the gaugino mass parameter $m_{1/2}$ around our parameter points 1, 2, and 3, respectively. The horizontal bands indicate the experimental limits of Eq. (1). Because of the enhanced cross section, the relic density is reduced by about the same amount, and the region of parameter space agreeing with the constraints is modified.

In the bottom panels we finally show the cosmologically favored regions in the m_0-$m_{1/2}$ plane that result from a relic density calculation using the cross section at the tree level, the approximation included in MICROMEGAS, and the full one-loop SUSY-QCD corrected cross section. Note that the
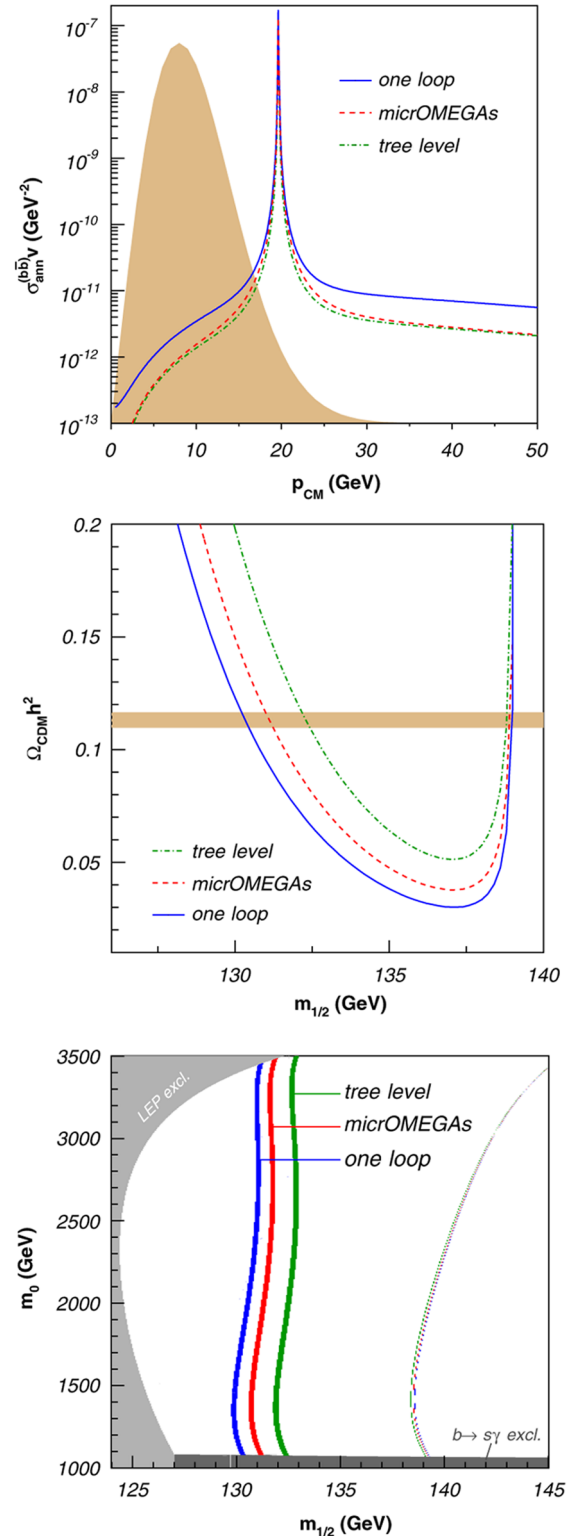

FIG. 3 (color online). The effects of the radiative corrections on the cross section as a function of center-of-mass momentum (top panel), the prediction of the neutralino relic density as a function of the gaugino mass parameter $m_{1/2}$ (center panel), and on the cosmologically favored regions in the m_0-$m_{1/2}$ plane (bottom panel) for our parameter point 1 with annihilation into bottom quarks through the light Higgs-boson resonance.
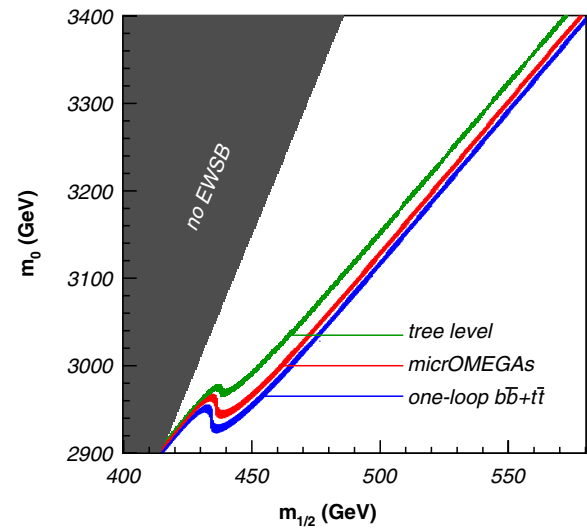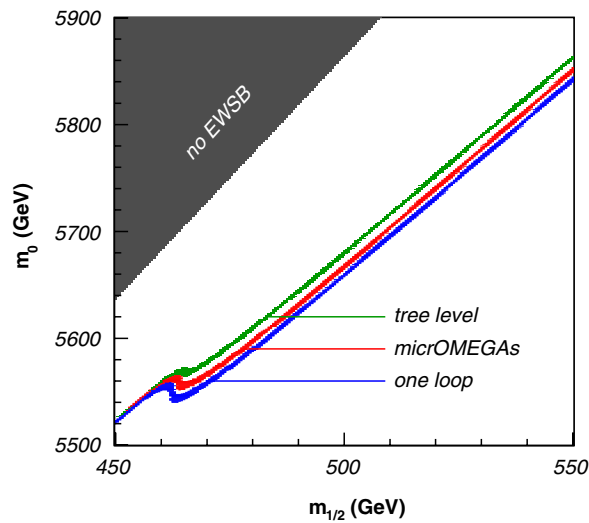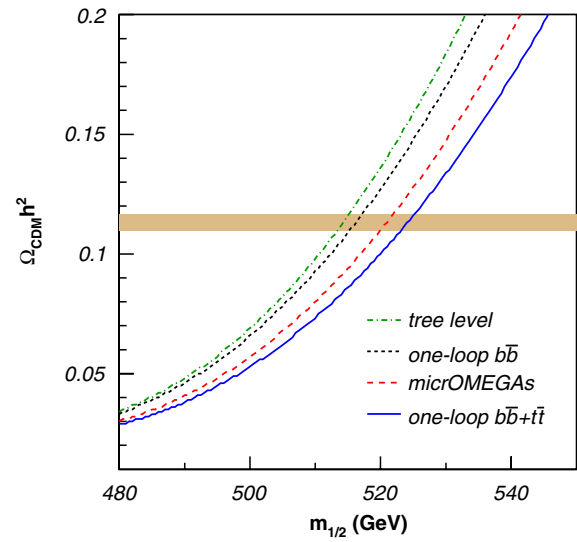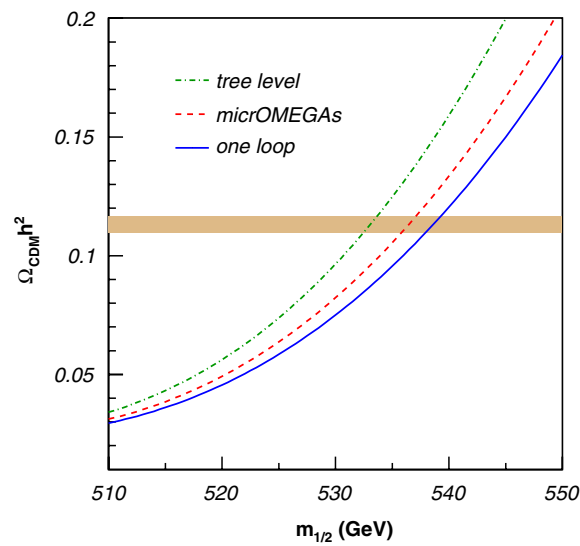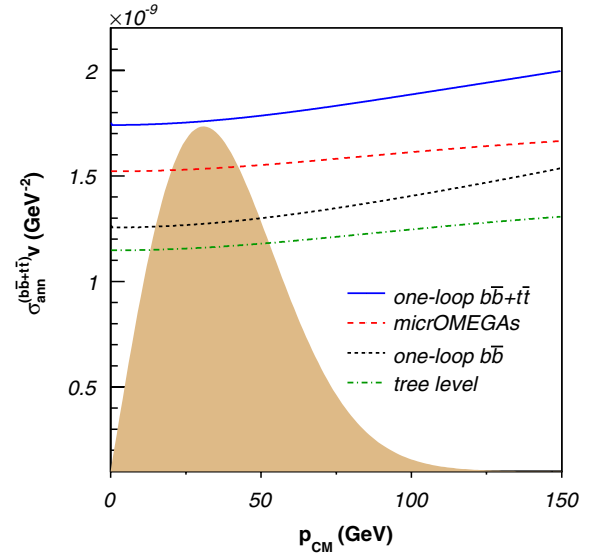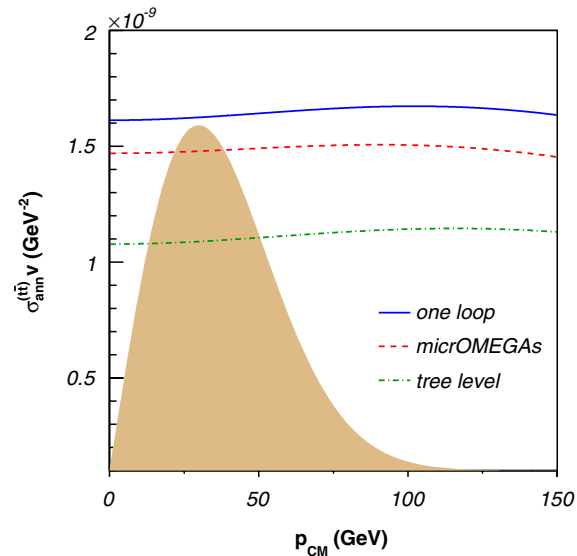
FIG. 4 (color online). Same as Fig. 3 for our scenario 2 with neutralino annihilation dominantly into top quarks.

FIG. 5 (color online). Same as Fig. 4 for our scenario 3 with neutralino annihilation into top and bottom quarks. The corrections to the $b\bar{b}$ final state only are also shown separately.

effect of the corrections is larger than the experimental errors, so that three distinct bands are observed. The grey shaded regions correspond to the points that are excluded due to mass limits, no electroweak symmetry breaking (EWSB), or the constraint from BR($b \to s\gamma$). Concerning the ''bulk'' region at low $\tan\beta$, the favored band is shifted to smaller values of $m_{1/2}$ and thus toward the region excluded by LEP mass limits. The cosmologically favored focus point region, however, is found at higher values of $m_{1/2}$ and lower values of m_0 after including the SUSY-QCD corrections.

Our additional corrections to the relic density for the case shown in Fig. 3 amount to about 10% with respect to the approximation implemented in MICROMEGAS. This is due to the low value of $\tan\beta$, which suppresses the part of the usually dominant resummed correction which is included in MICROMEGAS, making the rest of the SUSY-QCD correction relevant. In the case of top-quark final states, shown in Figs. 4 and 5, the additional corrections can amount up to 15%. Note that SUSY-QCD corrections for top-quark final states are not considered in MICROMEGAS. It is interesting that in the focus point scenario with large $\tan\beta$, the correction to the bottom quark final state has a sizable contribution to the total correction. As a result, the relative contribution of the annihilation into bottom quarks increases compared to the 8% at tree level. This is well visible in the center and bottom panels of Fig. 5 and shows the importance of the $\tan\beta$ enhanced corrections to the bottom Yukawa coupling.

IV. CONCLUSIONS

In summary, we have calculated the full one-loop QCD and SUSY-QCD corrections to neutralino annihilation into third generation quarks. A numerical evaluation has shown that the corrections have sizable effects on the annihilation cross section and in consequence on the extraction of SUSY mass parameters from cosmological data assuming that the lightest neutralino is the cold dark matter candidate. The induced difference is of the same order of magnitude as the experimental error from cosmological precision measurements, so that the full one-loop corrections should be taken into account when analyzing the SUSY parameter space with respect to cosmological data.

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