Radiative corrections to the three-body region of the Dalitz plot of baryon semileptonic decays with angular correlation between polarized emitted baryons and charged leptons: The initial-baryon rest frame case

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We complement the results for the radiative corrections to the $\hat{\mathbf{s}}_2 \cdot \hat{\boldsymbol{l}}$ angular correlation of baryon semileptonic decays of Neri *et al.* [Phys. Rev. D **78**, 054018 (2008)] with the final results in the rest frame of the decaying baryon. In addition, we present an analytical result which was not possible to obtain in Neri *et al.*'s work.

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In a recent paper [1], we obtained the radiative corrections (RC) to the Dalitz plot of the semileptonic decay of a spin-1/2 baryon A

$$A^{-}_{0}(p_1) \rightarrow B^{0}_{+}(p_2) + \ell^-(l) + \bar{\nu}_\ell(p_\nu), \tag{1}$$

when the angular correlation $\hat{\mathbf{s}}_2 \cdot \hat{\boldsymbol{l}}$ between the spin $\hat{\mathbf{s}}_2$ of the emitted baryon B and the direction $\hat{\boldsymbol{l}}$ of the emitted charged lepton ℓ is observed. As is customary, the results were presented in the rest frame of B where $p_2 = (M_2, 0, 0, 0)$. However, due to experimental conditions, it may be more convenient to produce such RC in the rest frame of A where $p_1 = (M_1, 0, 0, 0)$. It is not possible to translate directly the final result of Ref. [1] into the final result of the latter RC. The calculation must be retaken starting at earlier stages. In this paper we shall complement the analysis of Ref. [1] and present the final result for the RC to the $\hat{\mathbf{s}}_2 \cdot \hat{\boldsymbol{l}}$ angular correlation in the rest frame of A. To emphasize the above change of frame, although a rather simple Lorentz transformation is involved at the transition amplitude level [basically, specializing 4-vectors to either the rest frame of B or the rest frame of A, e.g., $p_1 = (E_1, \mathbf{p}_1)$, $p_2 = (M_2, \mathbf{0})$ in the first frame or $p_1 = (M_1, \mathbf{0})$, $p_2 = (E_2, \mathbf{p}_2)$ in the second frame], the reader should appreciate that a very long and tedious calculation is required to obtain the expression (ready to be used in experimental analyses) of the decay rate with RC of $A \rightarrow B\ell\nu$ with $\hat{\mathbf{s}}_2 \cdot \hat{\boldsymbol{l}}$ as an observable in either frame.

We shall follow the same procedure and use the same conventions and notation of Ref. [1]. Accordingly, the four-momenta and masses of the particles involved in process (1) will be denoted by $p_1 = (E_1, \mathbf{p}_1)$, $p_2 = (E_2, \mathbf{p}_2)$, $l = (E, \boldsymbol{l})$, and $p_\nu = (E^0_\nu, \mathbf{p}_\nu)$ and by M_1, M_2, m, and m_ν, respectively. We shall omit a detailed discussion, which can be found in Ref. [1]. Let us just recall that our results for the virtual part will be model-independent, gauge invariant, and finite in the ultraviolet and will contain the infrared divergence. In order to avoid repetition of long expressions, it will be convenient to trace a close parallelism with the analysis of the RC for the angular correlation $\hat{\mathbf{s}}_1 \cdot \hat{\boldsymbol{l}}$ between the spin $\hat{\mathbf{s}}_1$ of A and the direction $\hat{\boldsymbol{l}}$ of ℓ which were obtained in Ref. [2].

Without further ado, the differential decay rate with virtual RC in the rest frame of A including the $\hat{\mathbf{s}}_2 \cdot \hat{\boldsymbol{l}}$ correlation and covering the two charge assignments of A^{-}_{0} is

$$d\Gamma_V = d\Gamma'_V - d\Gamma^{(s)}_V. \tag{2}$$

The unpolarized part was already calculated in Ref. [2]. It has the form

$$d\Gamma'_V = d\Omega\left[A'_0 + \frac{\alpha}{\pi}(A'_1\phi + \phi' A''_1)\right]. \tag{3}$$

The full expressions for A'_0, A'_1, A''_1, ϕ, and ϕ' can be found in Eqs. (B1), (B2), (B3), (11), and (12), respectively, of this reference. Here the phase space factor $d\Omega = (1/2)(G_V^2/2)dE_2 dE d\Omega_\ell d\phi_2 2M_1/(2\pi)^5$ is one-half the phase space factor of Eq. (25) of this same reference. The polarization appears in

$$d\Gamma^{(s)}_V = d\Omega\left[A^{(s)}_0 + \frac{\alpha}{\pi}(B'\phi + \phi' B'')\right]\hat{\mathbf{s}}_2 \cdot \hat{\boldsymbol{l}}. \tag{4}$$

$\hat{\mathbf{s}}_2$ is introduced with the spin projector $\Sigma(s_2) = (1 - \gamma_5 \not{s}_2)/2$ applied to the u_B spinor. The 4-vector s_2 obeys $s_2 \cdot s_2 = -1$ and $s_2 \cdot p_2 = 0$. The trace calculation will lead to products $s_2 \cdot a$, with $a = l, p_\nu, p_1$. These products are specialized to the rest frame of A using the relation [3]

$$s_2 \cdot a = \mathbf{s}_2^A \cdot \left[\frac{\mathbf{p}_2}{E_2} a_0 - \mathbf{a}\right]$$
$$= \hat{\mathbf{s}}_2^B \cdot \left[\frac{\mathbf{p}_2}{M_2}\left(a_0 - \frac{\mathbf{p}_2 \cdot \mathbf{a}}{E_2 + M_2}\right) - \mathbf{a}\right], \tag{5}$$

which corresponds to the Lorentz transformation from the rest frame of B to the rest frame of A. In the first equality of (5) it is understood that $\mathbf{p}_2$, $\mathbf{s}_2^A$, and the components of the 4-vector $a = (a_0, \mathbf{a})$ are specialized in the rest frame of A. In the second equality of (5) only $\hat{\mathbf{s}}_2^B$ still remains in the rest frame of B. The upper indices A and B on the spin of the emitted baryon emphasize this fact. In addition, the contributions of the correlations $\hat{\mathbf{s}}_2^B \cdot \hat{\mathbf{p}}_2$ and $\hat{\mathbf{s}}_2^B \cdot \hat{\mathbf{p}}_\nu$ to the $\hat{\mathbf{s}}_2^B \cdot \hat{\boldsymbol{l}}$ correlation over the Dalitz plot are taken into account with the substitution rule [2] $\hat{\mathbf{s}}_2^B \cdot \hat{\mathbf{p}} \to (\hat{\mathbf{s}}_2^B \cdot \hat{\boldsymbol{l}})(\hat{\mathbf{p}} \cdot \hat{\boldsymbol{l}})$, with $\hat{\mathbf{p}} = \hat{\mathbf{p}}_2, \hat{\mathbf{p}}_\nu$, which is valid under integration of the variables, other than E and E_2, contained in $d\Omega$. These steps will be extended to the bremsstrahlung part, where $\hat{\mathbf{p}}$ can also stand for the direction of the photon 3-momentum.

The results in Eq. (4) are new. Their explicit expressions are

$$A_0^{(s)} = Q_1'\left[p_2 y_0 \frac{M_1}{M_2}\right] + Q_2'\left[\frac{p_2 y_0}{M_2}\left(E - \frac{p_2 l y_0}{E_2 + M_2}\right) - l\right], \tag{6}$$

$$B' = -D_4 E_\nu^0 l + D_3 E(p_2 y_0 + l), \tag{7}$$

$$B'' = -D_3 E \hat{\boldsymbol{l}} \cdot \mathbf{p}_\nu, \tag{8}$$

where $E_\nu^0 = M_1 - E - E_2$ and $y_0 = (E_\nu^{0^2} - p_2^2 - l^2)/2p_2 l$, with

$$Q_1' = (F_1^2 - G_1^2)\frac{EE_2 - \mathbf{p}_2 \cdot \boldsymbol{l}}{M_1} - (F_1^2 + G_1^2 - 2F_1G_1)\frac{M_2 E}{M_1} + (F_1G_2 + F_2G_1)\frac{EM_2^2 + (E - E_\nu^0)(EE_2 - \mathbf{p}_2 \cdot \boldsymbol{l})}{M_1^2}$$
$$- (F_1G_2 - F_2G_1)\frac{M_2(EE_2 - \mathbf{p}_2 \cdot \boldsymbol{l} + m^2)}{M_1^2} + (F_1G_3 + F_3G_1)\frac{m^2}{M_1^3}(EE_2 - \mathbf{p}_2 \cdot \boldsymbol{l} + M_2^2) - (F_1G_3 - F_3G_1)\frac{M_2 m^2}{M_1^2}$$
$$+ (F_1F_2 + G_1G_2)\frac{Ep_2^2 - E_2\mathbf{p}_2 \cdot \boldsymbol{l}}{M_1^2} - F_2G_2\frac{M_2}{M_1^2}(-l^2 - \mathbf{p}_2 \cdot \boldsymbol{l} + EE_\nu^0) - (F_2G_3 + F_3G_2)\frac{M_2 m^2 E_\nu^0}{M_1^3}$$
$$- F_3G_3\frac{M_2 m^2}{M_1^4}(EE_\nu^0 + l^2 + \mathbf{p}_2 \cdot \boldsymbol{l}), \tag{9}$$

$$Q_2' = (F_1^2 + G_1^2)\frac{M_2}{M_1}(M_1 - E_2) - (F_1^2 - G_1^2)\frac{E_2 M_1 - M_2^2}{M_1}$$
$$+ 2F_1G_1\frac{M_2}{M_1}(E_\nu^0 - E) + (F_1G_2 + F_2G_1)\frac{E_2}{M_1}(E_\nu^0 - E)$$
$$- (F_1G_2 - F_2G_1)\frac{M_2}{M_1}(E_\nu^0 - E)$$
$$- (F_1G_3 + F_3G_1)\frac{E_2 m^2}{M_1^2} + (F_1G_3 - F_3G_1)\frac{M_2 m^2}{M_1^2}$$
$$- (F_1F_2 + G_1G_2)\frac{p_2^2}{M_1}, \tag{10}$$

$D_3 = 2(f_1' g_1' - g_1'^2)$, and $D_4 = 2(f_1' g_1' + g_1'^2)$. In Eqs. (6)–(10) and hereafter, p_2 and l will denote the magnitudes of the corresponding 3-momenta. To avoid making the notation more cumbersome, we did not put primes on the form factors on the right-hand side of Eqs. (9) and (10). However, it must be kept in mind that it is the primed form factors f_1' and g_1', where all the model dependence has been absorbed, that appear in these equations. We have limited ourselves to put primes on Q_1' and Q_2', as a reminder of this fact.

To the virtual RC of Eq. (3), one must add the bremsstrahlung RC. It arises from the radiative decay

$$A \to B + \ell + \nu_\ell + \gamma, \tag{11}$$

where γ is a real photon with 4-momentum $k = (\omega, \mathbf{k})$ and which, in order to regulate the infrared divergence, is emitted with mass λ and with an additional longitudinal degree of freedom. The summation over its polarization is performed according to Ref. [4], and its model-independent contribution is controlled with the Low theorem [5]. The integrations over k are performed covariantly following Ref. [4].

Introducing the projector $\Sigma(s_2)$ in the radiative decay transition amplitude, following the usual steps of squaring it and summing over all polarizations including the summation over the γ polarization [4], and extracting the $\hat{\mathbf{s}}_2^B \cdot \hat{\boldsymbol{l}}$ correlation as explained in the virtual part, one obtains for the differential decay rate the result

$$d\Gamma_B = d\Gamma_B' - d\Gamma_B^{(s)}, \tag{12}$$

where $d\Gamma_B'$ is independent of $\hat{\mathbf{s}}_2^B$ and can be identified with one-half the unpolarized decay rate of Eq. (27) of Ref. [2]. The B spin-dependent part is given by

$$d\Gamma_B^{(s)} = \frac{\alpha}{\pi} d\Omega (I_0 B' + D_3 \rho_3 + D_4 \rho_4)\hat{\mathbf{s}}_2^B \cdot \hat{\boldsymbol{l}}. \tag{13}$$

$I_0(E, E_2)$ is the infrared-divergent integral given in Eq. (26) of Ref. [2], and B' is identified with Eq. (7) of this work. The contributions which are different with respect to the corresponding ones of Ref. [2] are

$$\rho_3 = \frac{p_2 l}{4\pi}\int_{-1}^{1}dx\int_{-1}^{y_0}dy\int_0^{2\pi}\frac{d\phi_k}{D(1-\beta x)}\Big\{(p_2 y + l + \omega x)\times\Big[\frac{\beta^2(1-x^2)}{1-\beta x}+\frac{\omega}{E}\Big]+\frac{\beta^2(1-x^2)}{1-\beta x}\Big[xE-\frac{D}{\beta}\Big]\Big\} \quad (14)$$

and

$$\rho_4 = \frac{p_2 l}{4\pi}\int_{-1}^{1}dx\int_{-1}^{y_0}dy\int_0^{2\pi}\frac{d\phi_k}{D(1-\beta x)}\Big\{\Big[\frac{\beta^2(1-x^2)}{1-\beta x}\Big]l + E_\nu\Big[-\beta+\Big(\frac{1-\beta^2}{1-\beta x}-1-\frac{\omega}{E}\Big)x\Big]\Big\}, \quad (15)$$

where

$$D = E_\nu^0 + (\mathbf{p}_2 + \boldsymbol{l})\cdot\hat{\mathbf{k}} \quad (16)$$

and

$$\omega = \frac{p_2 l(y_0 - y)}{D}. \quad (17)$$

The neutrino energy in the presence of the real photon is $E_\nu = E_\nu^0 - \omega$. In Eqs. (14) and (15), $\mathbf{k}$ is referred to a coordinate axis system where $\hat{\boldsymbol{l}}$ points along the z direction and $\hat{\mathbf{p}}_2$ lies on the (x, z) plane. The integration over $\mathbf{k}$ is performed with the variables $y = \hat{\mathbf{p}}_2 \cdot \hat{\boldsymbol{l}}$ and $x = \hat{\mathbf{k}}\cdot\hat{\boldsymbol{l}}$ and the azimuthal angle ϕ_k. They are ready to be performed numerically.

In contrast with Ref. [1], where the integrals contained in the parts corresponding to Eqs. (14) and (15)—namely, Eq. (35) of this reference—were most of them new and required a substantial effort to be performed analytically, all of the integrals contained in Eqs. (14) and (15) have already been performed analytically in our previous work. It is here where the parallelism between the present analysis and the one for the $\hat{\mathbf{s}}_1\cdot\hat{\boldsymbol{l}}$ calculation [2] becomes very useful. One can compare Eqs. (14) and (15) with their counterparts Eqs. (34) and (35) of this last reference and observe that the integrals with the common factor D_4 correspond to the integrals in our ρ_3, while those with the common factor D_3 correspond to our ρ_4. Looking at the analytical results of the integrals in those Eqs. (34) and (35), given in Eqs. (46) and (47) of this same reference, we can establish the following connection:

$$\rho_3^{\hat{\mathbf{s}}_2^B\cdot\hat{\boldsymbol{l}}} = (\rho_2^l + \rho_4^l)^{\hat{\mathbf{s}}_1\cdot\hat{\boldsymbol{l}}} \quad (18)$$

and

$$\rho_4^{\hat{\mathbf{s}}_2^B\cdot\hat{\boldsymbol{l}}} = (\rho_1^l + \rho_3^l)^{\hat{\mathbf{s}}_1\cdot\hat{\boldsymbol{l}}}, \quad (19)$$

where $\rho_1^l, \ldots, \rho_4^l$ are given explicitly in Eqs. (48) and (49) of Ref. [2]. Here the upper labels $\hat{\mathbf{s}}_2^B\cdot\hat{\boldsymbol{l}}$ and $\hat{\mathbf{s}}_1\cdot\hat{\boldsymbol{l}}$ are introduced to stress this correspondence. Using these results, and after some rearrangement to get somewhat more compact expressions, the analytical forms of Eqs. (14) and (15) are

$$\rho_3 = \frac{p_2}{2}\Big[E^2(Y_2 - Y_3) - 2\theta_0 E + Z_1 + \frac{1}{2}m^2[2(1-\beta^2)\theta_2 - 5\theta_3] + \frac{1}{2}(3E^2 - 2l^2)\theta_4 - \frac{3}{2}El\theta_5 - (1-\beta^2)\frac{E}{2}\theta_6 + \frac{3}{2}E\theta_7 + \frac{\theta_9}{4} + l^2\theta_{10} - \frac{l}{2}\theta_{14} - \frac{1}{2}(4E + E_\nu^0)\eta_0 + \frac{\zeta_{21}}{2E}\Big], \quad (20)$$

$$\rho_4 = \frac{p_2}{2}\Big[l^2 Y_2 - \frac{1}{2}(2E - E_\nu^0)\eta_0 - \frac{1}{2}(E + 2E_\nu^0)\gamma_0 + \frac{Y_4 E}{2} + \frac{l^2}{2}\theta_3\Big]. \quad (21)$$

The explicit forms of the functions θ_i, γ_0, η_0, ζ_{ij}, Y_i, and Z_1 need not be reproduced here. They are all found in Ref. [6].

One may say that Eqs. (13)–(15) are the Lorentz transformation of Eq. (37), the sum of Eqs. (42) and (44), and the sum of Eqs. (45) and (47) of Ref. [1], respectively. The reader may clearly see that there is no longer a simple Lorentz transformation that one can apply to the latter equations that readily produces the former equations. In addition, the analytical results (20) and (21) cannot be produced by a Lorentz transformation, because there is no analytical result in Ref. [1]. Even more, this analytical result uses many of the intermediate functions (θ_i, Y_i, etc.) of the $\hat{\mathbf{s}}_1\cdot\hat{\boldsymbol{l}}$ correlation of Ref. [6], to the extent that one can make the identification of Eqs. (18) and (19). Such an identification cannot be the result of a Lorentz transformation, because no transformation exists that changes $\hat{\mathbf{s}}_1$ into $\hat{\mathbf{s}}_2$.

Collecting the virtual and bremsstrahlung RC, our final result is

$$d\Gamma(A^- \to B^0 e^- \bar{\nu}) = d\Omega\Big\{\Big[A_0' + \frac{\alpha}{\pi}\Theta_I\Big] - \Big[A_0^{(s)} + \frac{\alpha}{\pi}\Theta_{II}\Big]\hat{\mathbf{s}}_2^B\cdot\hat{\boldsymbol{l}}\Big\}, \quad (22)$$

where the explicit forms of Θ_I and A_0' coincide with Φ_1 of Eq. (54) and A_0' of Eq. (B1) of Ref. [2], respectively. Our new results are $A_0^{(s)}$ of Eq. (6) and

$$\Theta_{II} = B'[\phi + I_0(E, E_2)] + B''\phi' + D_3\rho_3 + D_4\rho_4, \quad (23)$$

where all of the entries are defined above. The final result is infrared convergent.

Our main result Eq. (22) is not available in full elsewhere in the literature, although some parts of it are. We have kept all of them, new and old, in (22) for completeness sake. It is complementary to the final result Eq. (51) of Ref. [1]. With both of these expressions [Eqs. (51) and (22)], the experimental analysis of the $\hat{\mathbf{s}}_2\cdot\hat{\boldsymbol{l}}$ correlation with RC can be performed either in the rest frame of B or in the rest frame of A as it may be more convenient.

We should recall that the practical application in the Monte Carlo analysis may be to use the RC in the form

$$\Theta_i = a_i f_1^2 + b_i f_1 g_1 + c_i g_1^2 \tag{24}$$

and to calculate the numerical values of the coefficients a_i, b_i, and c_i throughout the Dalitz plot in the form of arrays. Such arrays would be fed into the Monte Carlo simulation as a matrix multiplication. This procedure should save a substantial computer effort. In Eq. (24), the index is $i = I, II$.

The present results complement the ones of Ref. [1]. The RC to the Dalitz plot when the $\hat{\mathbf{s}}_2^B \cdot \hat{\boldsymbol{l}}$ angular correlation is observed have been obtained both in the rest frame of the emitted baryon B and in the rest frame of the decaying baryon A. They cover the three-body region of the Dalitz plot, they can be used for all charge assignments of A and B [7], and l may be $e^{\pm}$, $\mu^{\pm}$, or $\tau^{\pm}$. They are presented in a form which is not compromised to fixing the values of the form factors at prescribed values. They provide a good approximation to RC of medium- (several tens of thousands) and low- (several thousands) statistics experiments in light- and heavy-quark baryon semileptonic decays, respectively. As a final remark, let us stress that, even if we use the same notation of our previous work, the expressions here apply only to the present case and there should arise no confusion.

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