

## Note on the QCD evolution of generalized form factors

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Generalized form factors of hadrons are objects appearing in moments of the generalized parton distributions. Their leading-order DGLAP-ERBL QCD evolution is exceedingly simple and the solution is given in terms of matrix triangular structures of linear equations where the coefficients are the evolution ratios. We point out that this solution has a practical importance in analyses where the generalized form factors are basic objects, *e.g.*, the lattice-gauge studies or models. It also displays general features of their evolution.

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Generalized parton distributions (GPDs) (for notations and discussion see the extensive reviews [1–9] and references therein) carry very rich information on the internal structure of hadrons. In particular, moments of the GPDs in the  $X$  variable, according to the polynomiality feature, can be written as polynomials in the  $\xi$  variable, assuming the form

$$\int_{-1}^1 dX X^{2j} F^{ns}(X, \xi, t) = 2 \sum_{i=0}^j A_{2j+1,2i}(t) \xi^{2i}, \quad (1)$$

$$\int_{-1}^1 dX X^{2j+1} F^s(X, \xi, t) = 2 \sum_{i=0}^{j+1} A_{2j+2,2i}(t) \xi^{2i},$$

with  $j = 0, 1, \dots$ . Here  $X$  is the average fraction of the target's momentum carried by the struck quark, and  $\xi$  is the fraction of the momentum along the light cone passed to the target. We use the so-called symmetric notation, where  $X \in [-1, 1]$  and  $\xi \in [0, 1]$ . Indices  $ns$  and  $s$  denote the nonsinglet and singlet distributions. The singlet GPD consists of the quark and gluon parts,  $F^s(X, \xi, t) = (F^{s,q}(X, \xi, t), XF^{s,G}(X, \xi, t))$ . The *generalized form factors*  $A_{nk}$  (GFFs) for the quark operators are the matrix elements between states  $i$  and  $f$  of the form (we take the even-parity operator and a spin 0 target)

$$\langle f(p') | \bar{\psi}(0) \gamma^{\{\mu} \overleftarrow{D}^{\mu_1} \overleftarrow{D}^{\mu_2} \dots \overleftarrow{D}^{\mu_{n-1}} \} \psi(0) | i(p) \rangle$$

$$= 2 P^{\{\mu} P^{\mu_1} \dots P^{\mu_{n-1}} \} A_{n0}(t) \quad (2)$$

$$+ 2 \sum_{\substack{k=2 \\ \text{even}}}^n q^{\{\mu} q^{\mu_1} \dots q^{\mu_{k-1}} P^{\mu_k} \dots P^{\mu_{n-1}} \} 2^{-k} A_{nk}(t),$$

with  $n = 1, 2, 3, \dots$ ,  $k = 0, 2, \dots, n$ ,  $P = (p + p')/2$ ,  $q = p' - p$ ,  $t = q^2$ , and  $\psi$  denoting the quark field. The symbol  $\overleftarrow{D}$  is the QCD covariant derivative,  $\overleftarrow{D} = \frac{1}{2}(\overleftarrow{D} - \overrightarrow{D})$ , and  $\{\dots\}$  denote the symmetrization of indices and the subtraction of traces for each pair of indices  $\mu_1 \dots \mu_{n-1}$ .

The factor of  $2^{-k}$  is conventional. A similar expression can be written for the singlet gluon GFFs [7].

For the simplest case of a spin-0 target with positive charge such as *e.g.*  $\pi^+$ ,  $A_{10}(t)$  is the charge form factor, while  $A_{20}(t)$  and  $-A_{22}(t)$  are the quark components of the gravitational form factors [10,11].

The GPDs undergo the QCD evolution with a change of the renormalization scale. Unlike the parton distribution functions (PDFs) or the distribution amplitudes (DAs), it is nontrivial to pass to the space of moments where the evolution is diagonal and then invert the transformation. This issue makes the case different and more complicated already at LO from the case of the PDFs, where the Mellin moments are used, or the DAs, where the Gegenbauer moments diagonalize the evolution. Theoretical tools have been developed to achieve the task, such as the Shuvaev transformations [12], the dual representation of the GPDs [13,14] or techniques based on the conformal moments complemented with the Mellin-Barnes transformations [15,16]. One may also solve the DGLAP-ERBL equations numerically [17,18]. Such approaches are needed, if the whole GPD is demanded. Frequently, however, one is only interested or has access to a limited number of GFFs. The purpose of this note is to point out that in fact the LO DGLAP-ERBL evolution of the GFFs is inherently much simpler and straightforward to implement in practice, without any need of complicated mathematical transformations. The result discussed in this paper is implicitly present in numerous works concerning the evolution of GPDs ([7] and references therein), but nevertheless we find it practical to present its explicit form, useful for those dealing with the GFFs only and not the full GPDs.

Our starting point is the work of Kivel and Mankiewicz [19,20], which elaborates the formalism of Balitsky and Braun [21] on the QCD string operators in the coordinate space. We use Eqs. (18) and (20) from Ref. [19] for the LO DGLAP-ERBL evolution of the GPDs from the scale  $\mu_0$ , where one assumes they are known, to the scale  $\mu$ :

$$\mathcal{F}^{ns}(\beta, \xi, t; \mu^2) = \frac{1}{\sqrt{\pi}} \left( \frac{2}{\beta\xi} \right)^{3/2} \sum_{n=0}^{\infty} (-1)^n \left( \frac{3}{2} + 2n \right) L_{2n+1} J_{(3/2)+2n}(\beta\xi) \int_0^1 d\omega F^{ns}(\omega, \xi, t; \mu_0^2) C_{2n}^{3/2} \left( \frac{\omega}{\xi} \right). \quad (3)$$

$$\mathcal{F}^s(\beta, \xi, t; \mu^2) = \frac{1}{\sqrt{\pi}} \left( \frac{2}{\beta\xi} \right)^{3/2} \sum_{n=0}^{\infty} (-1)^n \left( \frac{5}{2} + 2n \right) L_{2n+2} J_{(5/2)+2n}(\beta\xi) \int_0^1 d\omega F^s(\omega, \xi, t; \mu_0^2) C_{2n+1}^{3/2} \left( \frac{\omega}{\xi} \right),$$

where

$$\mathcal{F}^{ns}(\beta, \xi, t; \mu^2) = \frac{2}{\pi} \int_0^1 d\omega F^{ns}(\omega, \xi, t; \mu^2) \cos(\omega\beta),$$

$$\mathcal{F}^s(\beta, \xi, t; \mu^2) = \frac{2}{\pi} \int_0^1 d\omega F^s(\omega, \xi, t; \mu^2) \sin(\omega\beta). \quad (4)$$

For the nonsinglet case the quantity  $L_{2n+1}$  denotes the evolution ratio. For the singlet case  $L_{2n+2}$  forms a two-dimensional matrix in the quark-gluon space. Explicitly,

$$L_{2n+1} = \left( \frac{\alpha(\mu^2)}{\alpha(\mu_0^2)} \right)^{\gamma_n^{qq}/(2\beta_0)}, \quad L_{2n+2} = \left( \frac{\alpha(\mu^2)}{\alpha(\mu_0^2)} \right)^{\Gamma_n/(2\beta_0)},$$

$$\Gamma_n = \begin{pmatrix} \gamma_n^{qq} & \gamma_n^{qG} \\ \gamma_n^{Gq} & \gamma_n^{GG} \end{pmatrix}, \quad (5)$$

where the  $\gamma$ 's denote the appropriate anomalous dimensions. Our further procedure is based on the observation

$$A_{2k+1,2l} = k\Gamma(2k) \sum_{m=0}^k (4m+3) L_{2m+1} \sum_{j=k-l}^k \frac{2^{2(j-k)} (-1)^{m-j} \Gamma(j+m+\frac{3}{2}) A_{2j+1,2(j-k+l)}^0}{\Gamma(2j+1)\Gamma(m-j+1)\Gamma(k-m+1)\Gamma(k+m+\frac{5}{2})}, \quad (8)$$

for  $k = 0, 1, 2, \dots$  and  $l = 0, 1, \dots, k$ , or, explicitly,

$$A_{10} = L_1 A_{10}^0, \quad A_{32} = \frac{1}{5} (L_1 - L_3) A_{10}^0 + L_3 A_{32}^0,$$

$$A_{54} = \frac{1}{105} (9L_1 - 14L_3 + 5L_5) A_{10}^0 + \frac{2}{3} (L_3 - L_5) A_{32}^0 + L_5 A_{54}^0,$$

...

$$A_{30} = L_3 A_{30}^0, \quad A_{52} = \frac{2}{3} (L_3 - L_5) A_{30}^0 + L_5 A_{52}^0,$$

...

$$A_{50} = L_5 A_{50}^0, \quad (9)$$

where we have grouped the equations in the growing difference of the indices  $n$  and  $i$  in  $A_{ni}$ . The ellipses denote equations with  $n \geq 5$ . Since  $L_1 = 1$ , the vector form factor, of course, does not evolve. All other form factors in Eq. (9) change. While the *standard* form factors  $A_{n0}$  retain their shape, *i.e.*  $A_{n0}(t)/A_{n0}(t=0)$  is not altered by the evolution, other genuine *generalized* form factors involve linear combinations and both their value at  $t=0$  and their shape do change. Analogously, for the singlet case

that  $\mathcal{F}(\beta, \xi, t; \mu^2)$  is the generating function of the the GFF's. We then expand Eq. (3) and (4) in  $\beta$  around 0 using the series expansions

$$J_m(z) = \sum_{l=0}^{\infty} \frac{(-1)^l}{l!(m+l)!} \left( \frac{z}{2} \right)^{2l+m}, \quad (6)$$

$$C_m^\lambda(z) = \sum_{l=0}^{\lfloor m/2 \rfloor} \frac{(-1)^l (2z)^{m-2l} (\lambda)_{m-l}}{l!(m-2l)!}, \quad (7)$$

where  $(\cdot)_n$  denotes the Pochhammer symbol, and finally use the polynomiality property (1). As a result, polynomials in  $\beta$  and  $\xi$  are obtained on both sides of Eq. (3). For subsequent values of the powers of  $\beta$  we compare the coefficients of powers of  $\xi$ . As a result, the equations for the form factors from Eq. (1) and (2) follow immediately. We use the short-hand notation  $A_{nk} = A_{nk}(X, \xi, t; \mu)$  and  $A_{nk}^0 = A_{nk}(X, \xi, t; \mu_0)$ . For the nonsinglet case

$$A_{2k+2,2l} = \Gamma(2k+2) \sum_{m=0}^k (4m+5) L_{2m+2} \sum_{j=k-l}^k \frac{2^{2j-2k-1} (-1)^{m-j} \Gamma(j+m+\frac{5}{2}) A_{2(j+1),2(j-k+l)}^0}{\Gamma(2j+2)\Gamma(m-j+1)\Gamma(k-m+1)\Gamma(k+m+\frac{7}{2})}, \quad (10)$$

for  $k = 0, 1, 2, \dots$  and  $l = 0, 1, \dots, k+1$ , or, explicitly,

$$A_{22} = L_2 A_{22}^0, \quad A_{44} = \frac{3}{7} (L_2 - L_4) A_{22}^0 + L_4 A_{44}^0,$$

$$A_{66} = \frac{5}{231} (11L_2 - 18L_4 + 7L_6) A_{22}^0 + \frac{10}{11} (L_4 - L_6) A_{44}^0 + L_6 A_{66}^0,$$

...

$$A_{20} = L_2 A_{20}^0, \quad A_{42} = \frac{3}{7} (L_2 - L_4) A_{20}^0 + L_4 A_{42}^0, \quad (11)$$

$$A_{64} = \frac{5}{231} (11L_2 - 18L_4 + 7L_6) A_{20}^0 + \frac{10}{11} (L_4 - L_6) A_{42}^0 + L_6 A_{64}^0,$$

...

$$A_{40} = L_4 A_{40}^0, \quad A_{62} = \frac{10}{11} (L_4 - L_6) A_{40}^0 + L_6 A_{62}^0,$$

...

$$A_{60} = L_6 A_{60}^0,$$

Note the identical structure of the first two groups in the above equation. Again, the shape of the form factors  $A_{n0}$  does not change. The sets of Eqs. (9) and (11), although implicitly present in schemes involving the conformal moments, have not, to our knowledge, been written explicitly and their practical importance has not been recognized. Since all quantities on the right-hand side are known, from any practical point of view the problem of the leading-order DGLAP-ERBL evolution of the GFFs is solved.

Expressions for higher values of  $n$  may be obtained from the general expressions, however, it is the lowest form factors which are most relevant, as they can be obtained in the Euclidean lattice studies for the pion [22–24] and the nucleon [25–28]. Equations (9) and (11) are useful for the evolution of higher GFFs which eventually will be measured on the lattice as the accuracy is increased, as well as for various model calculations, where the results need to be evolved in order to compare to the data [29].

Sequences of equations in (9) and (11) separated by ellipses form (infinite) triangular matrix structures. Mixing occurs between the  $ns$  or  $s$  form factors where the difference  $n-i$  in  $A_{ni}$  is fixed, for instance  $A_{10}, A_{32}, A_{54}$ , etc. This corresponds, according to Eq. (2), to the mixing of the  $t$ -channel states of the same angular momentum. Indeed,  $n-i$  is the number of Lorentz indices of the  $t$ -channel momentum  $q$ . These triangular matrix equations may be diagonalized, yielding the combinations

$$A_{10}, \quad A_{32} - \frac{1}{5}A_{10}, \quad A_{54} - \frac{2}{3}A_{32} + \frac{1}{21}A_{10}, \quad (12)$$

etc., which evolve autonomously with  $L_n$ . The coefficients in the above combinations are proportional to the coefficients of the Gegenbauer polynomial  $C_n^{3/2}(z)$ . In fact, we are simply recovering the well known fact [7] that the conformal moments of the GPDs,

$$\frac{\Gamma(3/2)\Gamma(n+1)}{2^{n+1}\Gamma(n+3/2)} \int_{-1}^1 dX \xi^n C_n^{3/2}(X/\xi) F^{ns,s}(X, \xi, t), \quad (13)$$

evolve autonomously at LO. The point is, however, that the diagonalization of Eq. (12) is not necessary for the evolution of the GFFs, as for any practical purpose one can simply apply Eq. (9) and (11).

Asymptotically, as  $\mu^2 \rightarrow \infty$  we have (for the positive-parity operator)  $L_1 = 1$  and  $L_n \rightarrow 0$  for  $n > 1$ . Hence, in the nonsinglet channel

$$\begin{aligned} A_{10} &\rightarrow A_{10}^0, & A_{32} &\rightarrow \frac{1}{5}A_{10}^0, & A_{54} &\rightarrow \frac{9}{105}A_{10}^0 & \dots \\ A_{30} &\rightarrow 0, & A_{52} &\rightarrow 0, & \dots & A_{50} &\rightarrow 0, & \dots \end{aligned} \quad (14)$$

In the singlet channel we give the sum of the quark and gluon components, whose second moment for the assumed vector case is related to the momentum sum rule. Denoting  $S_{nk} = A_{nk}^q + A_{nk}^G$  we find asymptotically

$$\begin{aligned} S_{22} &\rightarrow S_{22}^0, & S_{44} &\rightarrow \frac{3}{7}S_{22}^0, & S_{66} &\rightarrow \frac{5}{21}S_{22}^0, & \dots \\ S_{20} &\rightarrow S_{20}^0, & S_{42} &\rightarrow \frac{3}{7}S_{20}^0, & S_{64} &\rightarrow \frac{5}{21}S_{20}^0, & \dots \\ S_{40} &\rightarrow 0, & S_{62} &\rightarrow 0, & \dots & S_{60} &\rightarrow 0, & \dots \end{aligned} \quad (15)$$

Equations (14) and (15) comply to the following LO asymptotic forms of the quark and gluon GPDs,

$$\begin{aligned} F^{ns} &\rightarrow \theta(\xi^2 - X^2) \frac{3}{2\xi} \left(1 - \frac{X^2}{\xi^2}\right) A_{10}(t), \\ F^{s,q} &\rightarrow \theta(\xi^2 - X^2) (S_{20}(t) + \xi^2 S_{22}(t)) \\ &\quad \times \frac{15}{4\xi^2} \frac{N_f}{4C_F + N_f} \frac{X}{\xi} \left(1 - \frac{X^2}{\xi^2}\right), \\ XF^{s,G} &\rightarrow \theta(\xi^2 - X^2) (S_{20}(t) + \xi^2 S_{22}(t)) \\ &\quad \times \frac{15}{16\xi} \frac{4C_F}{4C_F + N_f} \left(1 - \frac{X^2}{\xi^2}\right)^2. \end{aligned} \quad (16)$$

The lowest form factors determine these expressions. Note, however, that the gravitational form factors  $S_{20}(t)$  and  $S_{22}(t)$  need not be equal. Only for the special case  $S_{20}(t) = -S_{22}(t) = \theta(t)$  one recovers the typically written form with the common factor  $(1 - \xi^2)\theta(t)$ . Since  $S_{22}$  corresponds to the coupling of a scalar, and  $S_{20}$  to the traceless rank-2 tensor, there is no reason why  $S_{20}(t) = S_{22}(t)$  should hold. This issue is related to the lack of factorization of the  $t$ -dependence in GPDs.

In conclusion, we state that the generalization of the result to other channels is straightforward. For other probing operators one needs to simply use appropriate anomalous dimensions. For various targets (pion, nucleon) where different tensor couplings appear, one evolves the form factors separately for the independent structures.

Gravitational and higher-order GFFs may be obtained from chiral quark models of the GPDs for the pion [29–36] and for the nucleon [37,38]. A related quantity, the pion-photon transition distribution amplitude [39–42], has also been obtained in quark models Refs. [43–47] and its moments undergo the QCD evolution in a similar way. Dynamical calculations test the simplifying but *a priori* unjustified assumptions made in many phenomenological studies. In particular, the widely assumed factorization of the  $t$ -dependence is disproved. On the other hand, the reference scale,  $\mu_0$ , turns out to be very low in chiral quark models, around 320 MeV for the local models, and hence QCD evolution to experimentally accessible scales implies a long distance evolution. This is a case where the dilatation covariance may prove crucial, as it ensures the integrability of the renorm-group equations, hence the evolution path independence between two different scales [48].

Actually, much of the explicit simplicity of the GFF evolution of Eq. (9) and (11) is linked to the LO approxi-

mation and the conformal invariance of the evolution which represents faithfully the dilatation group, a feature not automatically guaranteed in the current factorization approaches at NLO, possibly inducing systematic errors. Renorm-group improvement of the GPDs might be implemented as previously done for the DGLAP evolution of the PDFs [48] as well as the ERBL evolution of the PDAs [49]. While these NLO complications prevent writing down a handy analytic solution for the GFF evolution, the problem can be reduced to a set of coupled differential equations. It

remains a tractable and much simpler alternative when a reduced set of GFFs is available. See also Refs. [15,16].

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