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Majoron emission in muon and tau decays revisited

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In models where the breaking of lepton number is spontaneous a massless Goldstone boson, the Majoron (J), appears. We calculate the theoretically allowed range for the branching ratios of Majoron-emitting charged lepton decays, such as Br($\mu \to eJ$) and Br($\mu \to eJ\gamma$), in a supersymmetric model with spontaneous breaking of R-parity. Br($\mu \to eJ$) is maximal in the same region of parameter space for which the lightest neutralino decays mainly invisibly. A measurement of Br($\mu \to eJ$) thus potentially provides information on R-parity violation complementary to accelerator searches. We also briefly discuss existing bounds and prospects for future improvements on the Majoron coupling to charged leptons.

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I. INTRODUCTION

Spontaneous breaking of lepton number leads to a massless Goldstone boson, the Majoron (J) [1–3]. There are two well-known experimental probes for the Majoron: The first is the invisible width of the Z^0 boson, very precisely measured at LEP [4]. The second is neutrinoless double beta decay [5]. The NEMO-3 Collaboration, for example, has published limits on half-lives for Majoron-emitting neutrinoless double beta decay for a number of isotopes [6]. In addition, there are different astrophysical constraints on the Majoron from the cooling of red giant stars and supernovae [7,8].

Another interesting possibility to search for Majorons, namely, charged lepton decays with Majoron emission, has attracted considerably less attention. Indeed, the limits on $l_i \to l_j J$ quoted by the Particle Data Group [4] are all based on experimental data which is now more than 20 years old. Probably this apparent lack of interest from the experimental side is due to the fact that both, the triplet [2] and the doublet Majoron [3], are ruled out by LEP data, while the (classical) singlet Majoron model [1] predicts Majoron-neutrino and Majoron-charged-lepton couplings which are unmeasurably small.

Supersymmetry (SUSY) is a theoretically well-motivated extension of the standard model. With the first data taking of the LHC only months away, searches for SUSY will gain momentum soon. One of the many virtues of SUSY is the fact that the minimal supersymmetric extension of the standard model (MSSM) provides an interesting candidate for the cold dark matter (CDM), usually assumed to be the lightest neutralino (χ_1^0) *if R-parity is conserved*. A stable, electrically neutral lightest supersymmetric particle (LSP) will escape the detector and lead to the famous missing momentum signal, upon which standard SUSY searches are based. If R-parity is broken, the LSP decays and the CDM candidate is lost. For explicit R-parity violation ($\not{R}_p$), the missing energy signal is degraded, but a larger number of jets and charged lepton final states should make discovery of $\not{R}_p$ a (comparatively) easy task. In *spontaneous* $\not{R}_p$ (s-$\not{R}_p$), however, the lightest neutralino can decay invisibly through $\chi_1^0 \to J\nu$. As pointed out in [9,10], if the scale of $\not{R}_p$ is low, this decay mode can be easily dominant and s-$\not{R}_p$ can be confused with a standard MSSM with R_p conserved.

Here, we revisit $l_i \to l_j J$ within spontaneous R-parity violation. Our calculation is based on the model of [11]. In this model the Majoron is mainly singlet, thus escaping the LEP bounds. This is different from the original spontaneous model [3], which used the left-sneutrinos to break R-parity. Nevertheless, in our model the Majoron can play an important role phenomenologically. In [12] $l_i \to l_j J$ was calculated for a tau neutrino mass of $m_{\nu_\tau} \simeq$ MeV. Here we show that (a) despite the fact that current neutrino mass bounds are of the order of eV or less, theoretically $\mu \to eJ$ can be (nearly) arbitrarily large in s-$\not{R}_p$, and (b) $\mu \to eJ$ is large in the same part of SUSY parameter space where the invisible neutralino decay is large, making the discovery of R-parity violation at the LHC difficult. Br($\mu \to eJ$) thus gives complementary information to accelerator experiments.

At the same time, the MEG experiment [13] has started taking data. MEG is optimized to search for Br($\mu \to e\gamma$) with a sensitivity of Br($\mu \to e\gamma$) $\sim$ (few)10^{-14}. While the impressive statistics of the experiment should allow, in principle, to improve the existing bound on Br($\mu \to eJ$) [4] by a considerable margin, the experimental triggers and cuts make it necessary to resort to a search for the radiative

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Majoron emission mode, $\text{Br}(\mu \to eJ\gamma)$, if one wants to limit (or measure) the Majoron-charged-lepton coupling. We therefore also calculate $\text{Br}(\mu \to eJ\gamma)$.

In the next section, we will briefly discuss s-$\not{R}_p$ and give an approximative, analytical estimation of $\text{Br}(l_i \to l_j J)$ and $\text{Br}(l_i \to l_j J\gamma)$. We then present our numerical results in Sec. (III), showing the correlation of $\text{Br}(l_i \to l_j J)$ with the invisible branching ratio of the lightest neutralino decay. In Sec. (IV) we then turn to a brief discussion of existing experimental bounds and comment on how a limit on $\text{Br}(\mu \to eJ\gamma)$ might be used to put a bound on the Majoron-charged-lepton coupling. We then close with a brief conclusion in Sec. (V).

II. SPONTANEOUS *R*-PARITY BREAKING

The model we consider [11] extends the particle spectrum of the MSSM by three additional singlet superfields, $\hat{\nu}^c$, $\hat{S}$ and $\hat{\Phi}$, with lepton number assignments of $L = -1, 1, 0$, respectively. The superpotential can be written as

$$\mathcal{W} = h_U^{ij}\hat{Q}_i\hat{U}_j\hat{H}_u + h_D^{ij}\hat{Q}_i\hat{D}_j\hat{H}_d + h_E^{ij}\hat{L}_i\hat{E}_j\hat{H}_d + h_\nu^i\hat{L}_i\hat{\nu}^c\hat{H}_u - h_0\hat{H}_d\hat{H}_u\hat{\Phi} + h\hat{\Phi}\hat{\nu}^c\hat{S} + \frac{\lambda}{3!}\hat{\Phi}^3. \tag{1}$$

Strictly speaking only $\hat{\nu}^c$ is necessary to spontaneously break R_p. The inclusion of $\hat{S}$ and $\hat{\Phi}$ allows to construct a superpotential which purely consists of trilinear terms, thus potentially solving also the μ problem of the MSSM. Note that the symmetries of the model, as defined in [11], allow also to add bilinear terms with dimension of mass to Eq. (1). However, we omit such terms here for the sake of keeping the number of free parameters of the model at the minimum. One could justify the absence of such bilinears—in the same way as is usually done in the NMSSM—by introducing a discrete Z_3 symmetry. We note, however, that our numerical results on the charged lepton decays are not affected by the presence or absence of these terms. At low energy various fields acquire vacuum expectation values (vevs). Besides the usual MSSM Higgs boson vevs v_d and v_u, these are $\langle\Phi\rangle = v_\phi/\sqrt{2}$, $\langle\tilde{\nu}^c\rangle = v_R/\sqrt{2}$, $\langle\tilde{S}\rangle = v_S/\sqrt{2}$ and $\langle\tilde{\nu}_i\rangle = v_{L_i}/\sqrt{2}$. Note, that $v_R \neq 0$ generates effective bilinear terms $\epsilon_i = h_\nu^i v_R/\sqrt{2}$ and that v_R, v_S and v_{L_i} violate lepton number as well as R-parity. The observed smallness of neutrino masses guarantees that the $\not{R}_p$ operators generated by these vevs are small. The smallness of ϵ_i implies that either h_ν^i or v_R is small, but not necessarily both.

Details of the model, such as mass matrices and couplings, can be found in [14,15]; for the phenomenology of the LSP decay in this model see [9,10]. For brevity in the following we will concentrate on only a few relevant aspects of the phenomenology of this model: the neutrino mass matrix, the lightest neutralino decay to Majorons and charged lepton decays.

The main motivation to study R-parity breaking supersymmetry certainly is that $\not{R}_p$ generates neutrino masses and thus contains a possible explanation for the observed neutrino oscillation data. For the spontaneous model defined in Eq. (1), the effective neutrino mass matrix at tree-level can be cast into a very simple form:

$$-(\mathbf{m}_{\nu\nu}^{\mathbf{eff}})_{ij} = a\Lambda_i\Lambda_j + b(\epsilon_i\Lambda_j + \epsilon_j\Lambda_i) + c\epsilon_i\epsilon_j. \tag{2}$$

Here, $\Lambda_i = \epsilon_i v_d + v_{L_i}\mu$, with $\mu = h_0 v_\phi/\sqrt{2}$. The coefficients a, b and c are defined as

$$\begin{aligned} a &= \frac{m_\gamma h^2 v_\phi}{4\sqrt{2}\,\text{Det}(M_H)}\left(-hv_R v_S + \frac{1}{2}\lambda v_\phi^2 + h_0 v_d v_u\right), \\ b &= \frac{m_\gamma h^2 \mu}{4\,\text{Det}(M_H)} v_u(v_u^2 - v_d^2), \\ c &= \frac{h^2\mu}{\text{Det}(M_H)} v_u^2(2M_1M_2\mu - m_\gamma v_d v_u). \end{aligned} \tag{3}$$

$\text{Det}(M_H)$ is the determinant of the (7, 7) matrix of the heavy neutral states (the four MSSM states, $\tilde{B}$, $\tilde{W}$ and $\tilde{H}_{u,d}$, plus the three fermionic components of the new singlet superfields of Eq. (1))

$$\text{Det}(M_H) = \frac{1}{16}h_0h^2v_\phi^2\Big[4(2M_1M_2\mu - m_\gamma v_d v_u) \times \left(-hv_Rv_S + \frac{1}{2}\lambda v_\phi^2 + h_0 v_d v_u\right) - h_0 m_\gamma(v_u^2 - v_d^2)^2\Big] \tag{4}$$

and $v^2 = v_u^2 + v_d^2$. The "photino" mass parameter is defined as $m_\gamma = g^2M_1 + g'^2M_2$. Since μ is fixed to $\mu = h_0v_\phi/\sqrt{2}$ the determinant of the matrix of the heavy states is $\text{Det}(M_H) \propto v_\phi^3$ in the limit of large v_ϕ. One can easily fit the observed neutrino masses and angles using Eq. (2), see [10] and the short discussion in the next section.

From a phenomenological point of view the most important difference between spontaneous and explicit R-parity violating models is the appearance of the Majoron. The pseudoscalar sector of the model we consider has eight different eigenstates. Two of them are Goldstone bosons. The standard one is eaten by the Z^0 boson; the remaining state is identified with the Majoron. In the limit $v_{L_i} \ll v_R, v_S$ the Majoron profile is given by the simple expression

$$R_{Jm}^{P^0} \simeq \left(0, 0, \frac{v_{L_k}}{V}, 0, \frac{v_S}{V}, -\frac{v_R}{V}\right). \tag{5}$$

Here, $V = \sqrt{v_R^2 + v_S^2}$ and terms of order $\frac{v_L^2}{Vv}$, where $v_L^2 = \sum_i v_{L_i}^2$, have been neglected.

Majorons are weakly coupled, thus potentially lead to a decay mode for the lightest neutralino which is invisible. Neutralino-Majoron couplings can be calculated from the general coupling $\chi_i^0 - \chi_j^0 - P_k^0$

$$\mathcal{L} = \frac{1}{2}\bar{\chi}_i^0(O_{Lij}^{nnp}P_L + O_{Rij}^{nnp}P_R)\chi_j^0 P_k^0. \tag{6}$$

Mixing between the neutralinos and the neutrinos then leads to a coupling $\chi_1^0 - \nu_k - J$. In the limit $v_R, v_S \gg \epsilon_i, v_{L_i}$ one can derive a very simple approximation formula for $O_{\tilde{\chi}_1^0\nu_k J}$. It is given by [10]

$$|O_{\tilde{\chi}_1^0\nu_k J}| \simeq -\frac{\tilde{\epsilon}_k}{V}N_{14} + \frac{\tilde{v}_{L_k}}{2V}(g'N_{11} - gN_{12}) + \cdots, \tag{7}$$

where the dots stand for higher-order terms neglected here and N is the matrix which diagonalizes the (MSSM) neutralino mass matrix. $\tilde{\epsilon} = U_\nu^T \cdot \vec{\epsilon}$ and $\tilde{v}_L = U_\nu^T \cdot \vec{v}_L$. Here $(U_\nu)^T$ is the matrix which diagonalizes either the part of the (3, 3) effective neutrino mass matrix, proportional to a or c, depending on which gives the larger eigenvalue. Equation (7) shows that for constant $\tilde{\epsilon}$ and $\tilde{v}_L$, $O_{\tilde{\chi}_1^0\nu_k J} \to 0$ as v_R goes to infinity. This is as expected, since for $v_R \to \infty$ the spontaneous model approaches the explicit bilinear model. We note that, in addition to the Majoron, there is also a rather light singlet scalar, called the "scalar partner" of the Majoron in [15], S_J. The lightest neutralino has a coupling $O_{\tilde{\chi}_1^0\nu_k S_J}$, which is of the same order as $O_{\tilde{\chi}_1^0\nu_k J}$. Since S_J decays to nearly 100% to two Majorons, this decay mode contributes sizably to the invisible width of the lightest neutralino; for more details see [10].

The decays $l_i \to l_j J$ can be calculated from the general coupling $\chi_i^+ - \chi_j^- - P_k^0$. In the limit of small R-parity violating parameters the relevant interaction Lagrangian for the $l_i - J - l_j$ coupling is given by

$$\mathcal{L} = \bar{l}_i(O_{LijJ}^{ccp}P_L + O_{RijJ}^{ccp}P_R)l_j J \tag{8}$$

with

$$O_{RijJ}^{ccp} = -\frac{i(h_E)^{jj}}{\sqrt{2}V}\left[\frac{v_d v_L^2}{v^2}\delta_{ij} + \frac{1}{\mu^2}(C_1\Lambda_i\Lambda_j + C_2\epsilon_i\epsilon_j + C_3\Lambda_i\epsilon_j + C_4\epsilon_i\Lambda_j)\right]$$
$$O_{LijJ}^{ccp} = (O_{RjiJ}^{ccp})^*. \tag{9}$$

The C coefficients are different combinations of MSSM parameters

$$C_1 = \frac{g^2}{2\,\mathrm{Det}_+^2}(-g^2 v_d v_u^2 - v_d\mu^2 + v_u M_2\mu)$$
$$C_2 = -2v_d \qquad C_3 = -\frac{g^2 v_d v_u}{\mathrm{Det}_+} \qquad C_4 = 1 - \frac{g^2 v_d v_u}{2\,\mathrm{Det}_+} \tag{10}$$

where Det_+ is the determinant of the MSSM chargino mass matrix $\mathrm{Det}_+ = M_2\mu - \frac{1}{2}g^2 v_d v_u$. Equation (9) shows that one expects large partial widths to Majorons, if v_R is low.

For a charged lepton l_i, with polarization vector $\vec{P}_i$, the decay $l_i \to l_j J$ has a differential decay width given by

$$\frac{d\Gamma(l_i \to l_j J)}{d\cos\theta} = \frac{m_i^2 - m_j^2}{64\pi m_i^3}[|O_{LijJ}^{ccp}|^2(m_i^2 + m_j^2 \pm (m_i^2 - m_j^2)P_i\cos\theta) + |O_{RijJ}^{ccp}|^2 \times (m_i^2 + m_j^2 \mp (m_i^2 - m_j^2)P_i\cos\theta) + 4m_i m_j\,\mathrm{Re}(O_{LijJ}^{ccp*}O_{RijJ}^{ccp})] \tag{11}$$

where θ is the angle between the polarization vector $\vec{P}_i$ and the momentum $\vec{p}_j$ of the charged lepton in the final state, and $P_i = |\vec{P}_i|$ is the polarization degree of the decaying charged lepton.

In the limit $m_j \simeq 0$ the expression (11) simplifies to

$$\frac{d\Gamma(l_i \to l_j J)}{d\cos\theta} = \frac{m_i}{64\pi}|O_{LijJ}^{ccp}|^2(1 \pm P_i\cos\theta) \tag{12}$$

since $|O_{RijJ}^{ccp}|^2 \propto (h_E^{jj})^2 \propto m_j^2$. The angular distribution of the Majoron-emitting lepton decay is thus very similar to the standard model muon decay [4], up to corrections of the order $(m_j/m_i)^2$, which are negligible in practice.

We next consider the decay $\mu \to eJ\gamma$.[1] It is induced by the Feynman diagrams shown in Fig. (1).

In the approximation $m_e \simeq 0$ the partial decay width for the process $\mu \to eJ\gamma$ can be written as

$$\Gamma(\mu \to eJ\gamma) = \frac{\alpha}{64\pi^2}|O_{L\mu eJ}^{ccp}|^2 m_\mu \mathcal{I}(x_{\min}, y_{\min}) \tag{13}$$

where $\mathcal{I}(x_{\min}, y_{\min})$ is a phase space integral given by

$$\mathcal{I}(x_{\min}, y_{\min}) = \int dxdyf(x,y) = \int dxdy\frac{(x-1)(2-xy-y)}{y^2(1-x-y)}, \tag{14}$$

the dimensionless parameters x, y are defined as usual

$$x = \frac{2E_e}{m_\mu}, \quad y = \frac{2E_\gamma}{m_\mu} \tag{15}$$

and $x_{\min}$ and $y_{\min}$ are the minimal electron and photon energies measured in a given experiment.

Note that the integral $\mathcal{I}(x_{\min}, y_{\min})$ diverges for $y_{\min} = 0$. This infrared divergence is well known from the standard model radiative decay $\mu \to e\bar{\nu}\nu\gamma$, and can be taken care of in the standard way by introducing a nonzero photon mass m_γ. Note that in the limit $m_e = 0$ there also appears a colinear divergence, just as in the SM radiative decay. Since in any practical experiment there is a minimum measurable photon energy, $y_{\min}$, as well as a minimum

[1] Formulas for the radiative Majoron decays of the τ can be found from straightforward replacements.

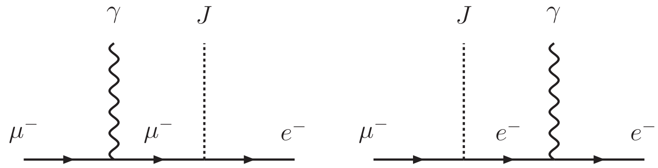

FIG. 1. Feynman diagrams for the decay $\mu \to eJ\gamma$. As in the standard model radiative decay $\mu \to e\bar{\nu}\nu\gamma$ these diagrams contain an infrared divergence for $m_\gamma = 0$, see text.

measurable photon-electron angle ($\theta_{e\gamma}$), neither divergence affects us in practice. We simply integrate from the minimum value of y up to $y_{\max}$ when estimating the experimental sensitivity of $\mathrm{Br}(\mu \to eJ\gamma)$ on the Majoron coupling.

In the calculation of the integral $\mathcal{I}(x_{\min}, y_{\min})$ one has to take into account not only the experimental cuts applied to the variables x and y, but also the experimental cut for the angle between the directions of electron and photon. This angle is fixed for kinematical reasons to

$$\cos\theta_{e\gamma} = 1 + \frac{2 - 2(x + y)}{xy}. \tag{16}$$

This relation restricts $x_{\max}$ to be $x_{\max} \le 1$ as a function of y (and vice versa) and to $x_{\max} < 1$ for $\cos\theta_{e\gamma} > -1$.

Using the formula for $\Gamma(\mu \to eJ)$, in the approximation $m_e \simeq 0$,

$$\Gamma(\mu \to eJ) = \frac{m_\mu}{32\pi} |O^{ccp}_{L\mu eJ}|^2, \tag{17}$$

one finds a very simple relation between the two branching ratios

$$\mathrm{Br}\,(\mu \to eJ\gamma) = \frac{\alpha}{2\pi}\,\mathcal{I}(x_{\min}, y_{\min})\mathrm{Br}(\mu \to eJ). \tag{18}$$

We will use Eq. (18) in Sec. (IV) when we discuss the relative merits of the two different measurements.

III. NUMERICAL RESULTS

All numerical results shown in this section have been obtained using the program package SPheno [16], extended to include the new singlet superfields $\hat{\nu}^c$, $\hat{S}$ and $\hat{\Phi}$. We always choose the $\not{R}_p$ parameters in such a way that solar and atmospheric neutrino data [17] are fitted correctly. The numerical procedure to fit neutrino masses is the following. For any random choice of MSSM parameters, we can reproduce the "correct" MSSM value of μ for a random value of v_ϕ, by appropriate choice of h_0. For any random set of h, λ, v_S and v_R, we can then calculate those values of h^i_ν and v_{L_i}, using Eq. (2), such that the corresponding ϵ_i and Λ_i give correct neutrino masses and mixing angles. In the plots shown below we use Λ_i for the atmospheric scale and ϵ_i for the solar scale.

As shown previously [10] if the lightest neutralino is mainly a bino, the decay to Majoron plus neutrino is dominant if v_R is low. This is demonstrated again for a bino LSP in Fig. 2, to the left, for a sample point using mSugra parameters $m_0 = 280$ GeV, $m_{1/2} = 250$ GeV, $\tan\beta = 10$, $A_0 = -500$ GeV and $\mathrm{sgn}(\mu) = +$. We stress that this result is independent of the choice of mSugra parameters to a large degree [9]. A scan over v_ϕ has been performed in this plot, varying v_ϕ in the huge interval $[1, 10^2]$ TeV. Large values of v_ϕ lead to small values of the constant c in the neutrino mass matrix, see Eq. (2). Small c require, for constant neutrino masses, large values of ϵ_i, which in turn lead to a large invisible width of the neutralino. The largest values of v_ϕ (dark areas) therefore lead

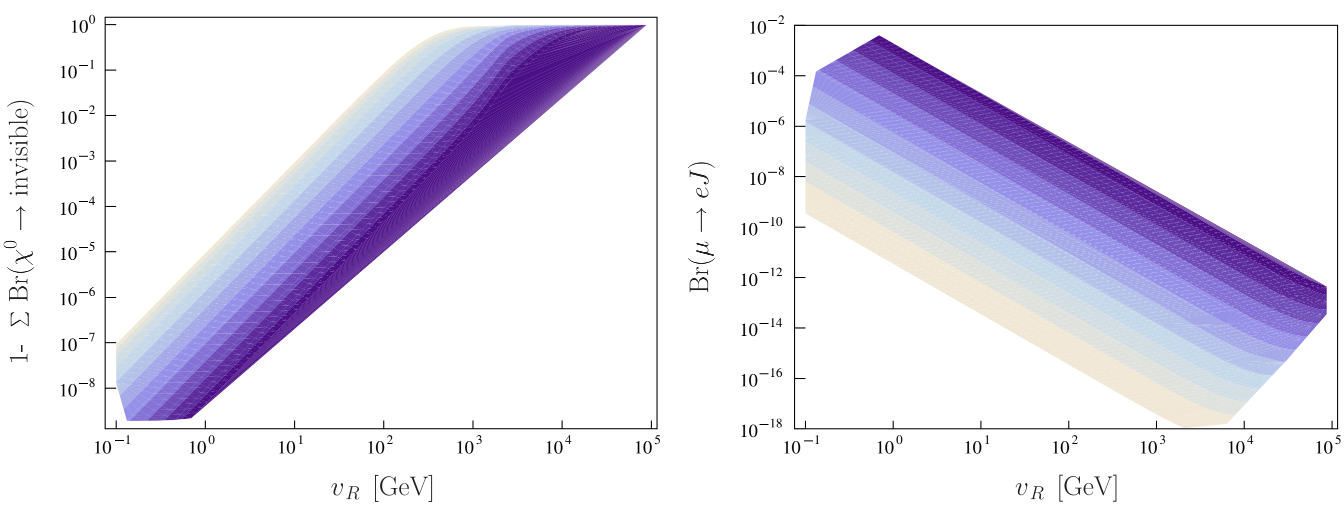

FIG. 2 (color online). Branching ratios for visible lightest neutralino decay (left) and branching ratio $\mathrm{Br}(\mu \to eJ)$ (right) versus v_R in GeV for a number of different choices of v_ϕ between $[1, 10^2]$ TeV indicated by the different colors. Darker colors indicate larger v_ϕ in a logarithmic scale. mSugra parameters are defined in the text. There is very little dependence on the actual mSugra parameters, however; see discussion and Fig. 3.

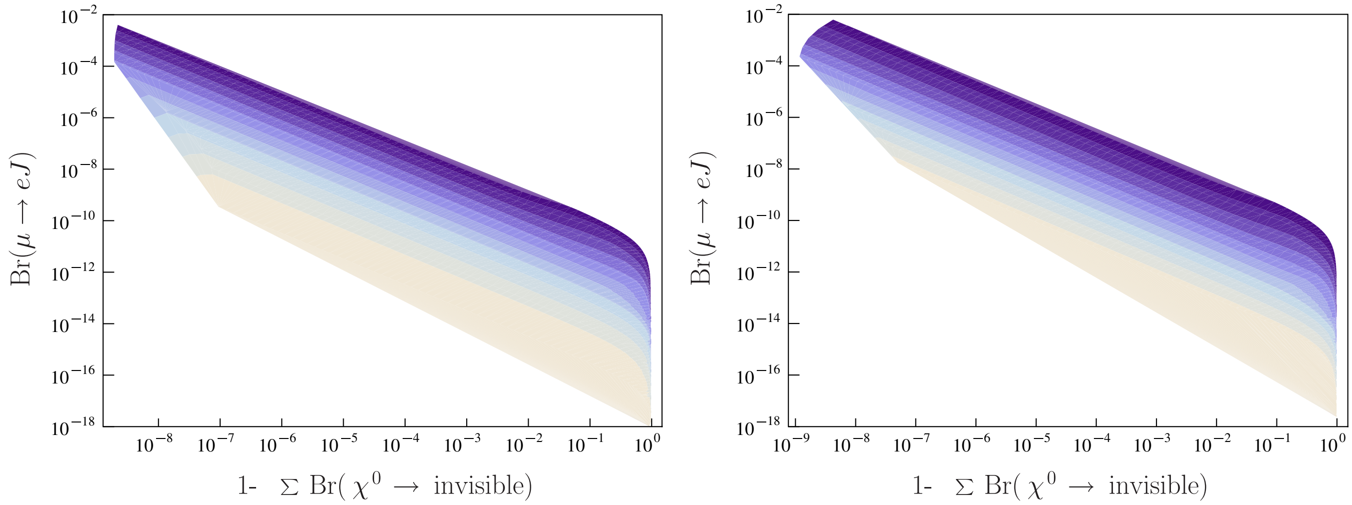

FIG. 3 (color online). Branching ratios for visible lightest neutralino decay versus branching ratio $\mathrm{Br}(\mu \to eJ)$ for two mSugra points, for various choices of v_ϕ, see Fig. 2. To the left, the same mSugra parameters as Fig. 2, to the right SPS1a'.

to the smallest visible neutralino decay branching ratios shown in Fig. 2.

Fig. 2, to the right, shows the branching ratio $\mathrm{Br}(\mu \to eJ)$ as a function of v_R for different values of v_ϕ. All parameters have been fixed to the same values as shown in the left figure. As the figure demonstrates, small values of v_R (and large values of v_ϕ) lead to large values of $\mathrm{Br}(\mu \to eJ)$. This agrees with the analytic expectation, compare to Eq. (9).

Our main result is shown in Fig. 3. In this figure we show $\mathrm{Br}(\mu \to eJ)$ versus the sum of all branching ratios of neutralino decays leading to at least one visible particle in the final state for two different choices of mSugra parameters. The similarity of the two plots shows that our result is only weakly dependent on the true values of mSugra parameters. We have checked this fact also by repeating the calculation for other mSugra points, although we do not show plots here. As expected $\mathrm{Br}(\mu \to eJ)$ anticorrelates with the visible bino decay branching fraction and thus probes a complementary part in the supersymmetric parameter space. An upper bound on $\mathrm{Br}(\mu \to eJ)$ will constrain the maximum branching ratio for invisible neutralino decay, thus probing the part of parameter space where spontaneous R-parity breaking is most easily confused with *conserved* R-parity at accelerators.

We have checked the points shown in the plots for various phenomenological constraints. LEP bounds are trivially fulfilled by $v_{L_i} < v_R$. Double beta decay bounds on $g_{\nu\nu J}$[2] are of the order of 10^{-4} [6] and, since the coupling $g_{\nu\nu J}$ is suppressed by two powers of R-parity violating parameters, are easily satisfied in our model. More interesting is the astrophysical limit on g_{eeJ}. Reference [7] quotes a bound of $g_{eeJ} \leq 3 \cdot 10^{-13}$. Although this bound is derived from the coupling of the Majoron to two electrons, thus constraining actually the products $v_{L_e}^2$, ϵ_e^2 and Λ_e^2, whereas $\mathrm{Br}(\mu \to eJ)$ is proportional to $\epsilon_e \epsilon_\mu$ and $\Lambda_e \Lambda_\mu$, it still leads to a (weak) constraint on $\mathrm{Br}(\mu \to eJ)$, since neutrino physics shows that two leptonic mixing angles are large. This requires that either $\epsilon_e \sim \epsilon_\mu$ *or* $\Lambda_e \sim \Lambda_\mu$. For the case studied in our plots, where ϵ_i generate the solar scale, $\tan^2\theta_\odot \simeq 1/2$ requires $\epsilon_e \sim \epsilon_\mu$. Numerically we then find that $g_{eeJ} \leq 3 \cdot 10^{-13}$ corresponds to an upper bound on $\mathrm{Br}(\mu \to eJ)$ of very roughly $\mathrm{Br}(\mu \to eJ) \lesssim (\mathrm{few}) \cdot 10^{-5}$. In case the neutrino data is fitted with $\vec{\epsilon}$ for the atmospheric scale, the corresponding bound is considerably weaker.

IV. EXPERIMENTAL CONSTRAINTS AND $\mathrm{Br}(\mu \to eJ\gamma)$

The Particle Data Group [4] cites [18] with an upper limit on the branching ratio of $\mathrm{Br}(\mu \to eX^0) \leq 2.6 \times 10^{-6}$, where X^0 is a scalar boson called the familon. This constraint does not apply to the Majoron we consider here, since it is derived from the decay of polarized muons in a direction opposite to the direction of polarization. The authors of [18] concentrated on this region, since it minimizes events from standard model β-decay. As shown in Eq. (12), the Majoron-emitting decay has a very similar angular distribution as the standard model decay, with the signal approaching zero in the data sample analyzed by [18]. Nevertheless, from the spin processed data shown in Fig. 7 of [18], which seems to be in good agreement with the SM prediction, it should in principle be possible to extract a limit on $\mathrm{Br}(\mu \to eJ)$. From this figure we estimate very roughly that this limit should be about 1 order of magnitude less stringent than the one for familon decay.

[2]We will use the symbol g when discussing experimental bounds, to differentiate from the model dependent couplings O_L^{ccp} and O_R^{ccp} defined in Sec. II.

For a better estimate a reanalysis of this data, including systematic errors, would be necessary.

Reference [19] searched for Majorons in the decay of $\pi \to e\nu J$, deriving a limit of $\mathrm{Br}(\pi \to e\nu J) \leq 4 \cdot 10^{-6}$. Since the experimental cuts used in this paper [19] are designed to reduce the standard model background from the decay chain $\pi \to \mu \to e$, the contribution from on-shell muons is reduced by about 5 orders of magnitude. The limit then essentially is a limit on the Majoron-neutrino-neutrino coupling, $g_{\nu\nu J}$, leaving only a very weak constraint on the coupling $g_{\mu e J}$. Also an analysis searching for $\mathrm{Br}(\mu \to eJ\gamma)$ has been published previously [20]. From a total data sample of $8.15 \cdot 10^{11}$ stopped muons over the live time of the experiment [20] derived a limit on $\mathrm{Br}(\mu \to eJ\gamma)$ of the order of $\mathrm{Br}(\mu \to eJ\gamma) \leq 1.3 \cdot 10^{-9}$. For the cuts used in this analysis, we calculate $\mathcal{I} \simeq 10^{-3}$. Thus, see Eq. (18), this limit translates into only a rather weak bound $\mathrm{Br}(\mu \to eJ) \leq 1.1 \cdot 10^{-3}$.

The MEG experiment [13] has a muon stopping rate of $(0.3\text{–}1) \cdot 10^8$ per second and expects a total of the order of 10^{15} muons over the expected live time of the experiment. An analysis of *electron only* events near the endpoint should therefore allow, in principle, to improve the existing limits on $\mathrm{Br}(\mu \to eJ)$ by an estimated (2–3) orders of magnitude, if systematic errors can be kept under control. However, the MEG experiment, as it is designed to search for $\mathrm{Br}(\mu \to e\gamma)$, uses a trigger that requires a photon in the event with a minimum energy of $E_\gamma^{\mathrm{min}} \geq 45$ MeV. MEG data can thus constrain the Majoron-charged-lepton coupling only via searching for $\Gamma(\mu \to eJ\gamma)$.

Figure 4 shows the value of the phase space integral $\mathcal{I}(x_{\mathrm{min}}, y_{\mathrm{min}})$ as a function of x_{min} for three different values of y_{min} and for two choices of $\cos\theta_{e\gamma}$. The MEG proposal describes the cuts used in the search for $\mu \to e\gamma$ as $x_{\mathrm{min}} \geq 0.995$, $y_{\mathrm{min}} \geq 0.99$ and $|\pi - \theta_{e\gamma}| \leq 8.4$ mrad. For these values we find a value of $\mathcal{I} \simeq 6 \cdot 10^{-10}$. A limit for $\mathrm{Br}(\mu \to e\gamma)$ of $\mathrm{Br}(\mu \to e\gamma) \leq 10^{-13}$ then translates into a limit of $\mathrm{Br}(\mu \to eJ) \leq 0.14$, obviously not competitive. To improve upon this bound, it is necessary to relax the cuts. For example, relaxing the cut on the opening angle to $\cos\theta_{e\gamma} = -0.99$, the value of the integral increases by more than 3 orders of magnitude for $x_{\mathrm{min}} = y_{\mathrm{min}} \geq 0.95$.

On the other hand, such a change in the analysis is prone to induce background events, which are avoided by the MEG cuts, specially designed for it. The MEG proposal discusses as the two most important sources of background: (a) prompt events from the standard model radiative decay $\mu \to e\nu\bar{\nu}\gamma$; and (b) accidental background from muon annihilation in flight. For the current experimental setup the accidental background is larger than the prompt background. Certainly, a better timing resolution of the experiment would be required to reduce this background. For the prompt background we estimate, using the formulas of [21], that for a total of 10^{13} muon events, one background event from the radiative decay will enter the analysis window for $x_{\mathrm{min}} = y_{\mathrm{min}} \simeq 0.96$ for the current cut on $\cos\theta_{e\gamma}$.

A further relaxation of the cuts can lead, in principle, to much larger values for $\mathcal{I}(x_{\mathrm{min}}, y_{\mathrm{min}})$. However, the search for $\mathrm{Br}(\mu \to eJ\gamma)$ than necessarily is no longer background free. Since all the events from $\mu \to eJ\gamma$ lie along the line of $\cos\theta_{e\gamma}$ defined by Eq. (16), whereas events from the SM radiative mode fill all of the $\cos\theta_{e\gamma}$ space, such a strategy might be advantageous, given a large enough data sample.

Before closing this section, we mention that tau decays with Majoron emission are less interesting phenomenologically for two reasons. First, the existing experimental limits are much weaker for taus [22] $\mathrm{Br}(\tau \to \mu J) \leq 2.3\%$ and $\mathrm{Br}(\tau \to eJ) \leq 0.73\%$. And, second, although the coupling $\tau - \mu - J$ is larger than the coupling $\mu - e - J$ by a factor m_τ/m_μ, the total width of the tau is much larger than the width of the muon; thus the resulting

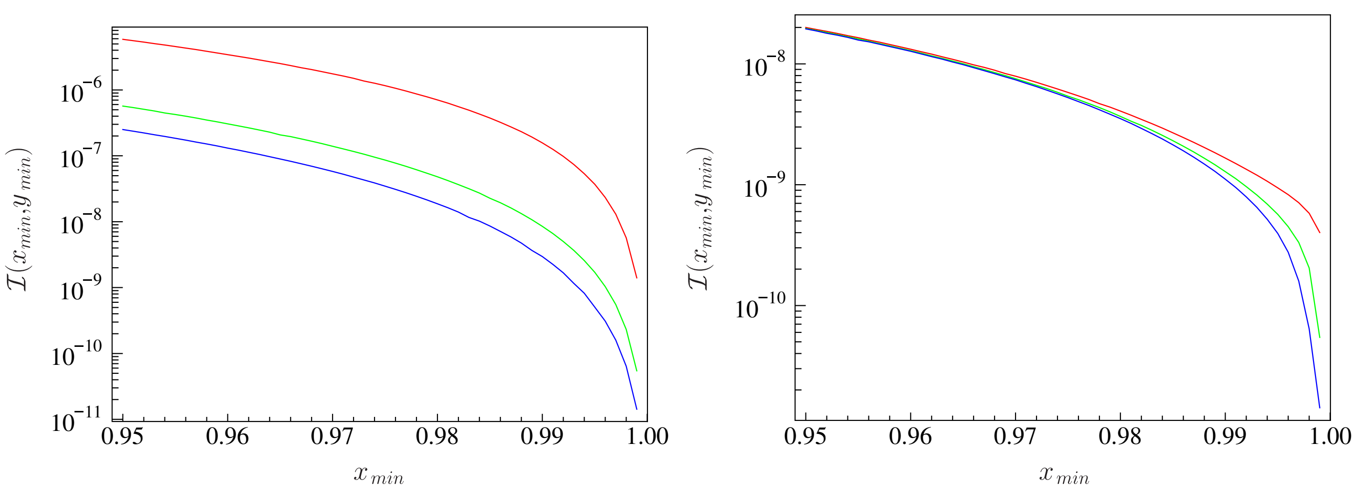

FIG. 4 (color online). The phase space integral for the decay $\mu \to eJ\gamma$ as a function of x_{min} for three different values of $y_{\mathrm{min}} = 0.95$, 0.99, 0.995 from top to bottom and for two different values of $\cos\theta_{e\gamma}$. To the left $\cos\theta_{e\gamma} = -0.99$, to the right $\cos\theta_{e\gamma} = -0.99997$.

theoretical predictions for tau branching ratios to Majorons are actually smaller than for the muon by a factor of approximately 10^4.

V. CONCLUSIONS

We have calculated branching ratios for exotic muon and tau decays involving Majorons in the final state. Branching ratios can be measurably large, if the scale of lepton number breaking is low. This result is independent of the absolute value of the neutrino mass. The lowest possible values of v_R (at large values of v_ϕ) are already explored by the existing limit on $\mathrm{Br}(\mu \to eJ)$.

We have briefly discussed the status of experimental limits. It will not be an easy task to improve the current numbers in future experiments. While MEG [13] certainly has a high number of muon events in the detector, a search for $\mathrm{Br}(\mu \to eJ\gamma)$ instead of $\mathrm{Br}(\mu \to eJ)$ suffers from a small value of the available phase space integral, given current MEG cuts. An improvement will only be possible, if a dedicated search by the experimentalists is carried out. Nevertheless, we believe this is a worthwhile undertaking, since measuring a finite value for $\mathrm{Br}(\mu \to eJ)$ will establish that R-parity is broken in a region of SUSY parameter space complementary to that probed by accelerator searches.

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