Enhancement of dark matter annihilation via Breit-Wigner resonance

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The Breit-Wigner enhancement of the thermally averaged annihilation cross section $\langle \sigma v \rangle$ is shown to provide a large boost factor when the dark matter annihilation process nears a narrow resonance. We explicitly demonstrate the evolution behavior of the Breit-Wigner enhanced $\langle \sigma v \rangle$ as the function of universe temperature for both the physical and unphysical pole cases. It is found that both of the cases can lead a large enough boost factor to explain the recent PAMELA, ATIC, and PPB-BETS anomalies. We also calculate the coupling of the annihilation process, which is useful for an appropriate model building to give the desired dark matter relic density.

I. INTRODUCTION

The existence of dark matter is by now well confirmed [1]. However, there is no candidate for the dark matter in the standard model. Understanding the nature of dark matter is one of the most challenging problems in particle physics and cosmology. The recent cosmological observations have established the concordance cosmological model where the present energy density consists of about 73% dark energy, 23% dark matter, and 4% atoms [2]. Currently, many dark matter search experiments are under way. These experiments can be classified as the direct dark matter searches and the indirect dark matter searches. The direct dark matter detection experiments may observe the elastic scattering of dark matter particles with nuclei. The indirect dark matter searches are designed to detect the dark matter annihilation productions, which include neutrinos, gamma rays, electrons, positrons, protons, and antiprotons. In addition, the CERN LHC searches are complementary to the direct and indirect dark matter detection experiments.

Recently, the indirect dark matter detection experiment PAMELA [3] reported an excess in the positron fraction from 10 to 100 GeV, but showed no excess for the antiproton data. The ATIC [4] and PPB-BETS [5] balloon experiments have also seen the excess in the $e^+ + e^-$ energy spectrum between 300 and 800 GeV. It is a natural idea that the dark matter annihilation can account for the PAMELA, ATIC, and PPB-BETS anomalies. However, the thermally averaged annihilation cross section $\langle \sigma v \rangle$ obtained from the observed relic density is far smaller than the required value from the PAMELA data. Therefore, one must resort to the large boost factor (about 100–1000) to explain the large positron flux. Current analysis on the clumpiness of dark matter structures indicates that the most probable boost factor should be less than 10–20 [6]. Considering the difficulty to yield a large boost factor, many authors investigate the decaying dark matter [7].

However, the PAMELA data require the lifetime of dark matter to be of the order of 10^{26} s. An alternative opinion is the nonperturbative Sommerfeld enhancement, which may provide a large boost factor as the weak force enhances the annihilation cross sections in the galactic halo [8].

The thermally averaged annihilation cross section $\langle \sigma v \rangle$ is a key quantity in the determination of the cosmic relic abundances of dark matter. On the other hand, $\langle \sigma v \rangle$ also determine the dark matter annihilation rate in the galactic halo. The only difference among the above two cases is the temperature T. For the relic density, $\langle \sigma v \rangle$ is usually evaluated at the freeze-out temperature $x \equiv m/T \approx 20$ (the averaged velocity $v \approx \sqrt{3/x}$), where m is the dark matter mass. The dark matter annihilation in the galactic halo occurs at $x \approx 3 \times 10^6$ ($v \approx 10^{-3}$). For nonrelativistic gases, $\langle \sigma v \rangle$ can usually be expanded in powers of x, $\langle \sigma v \rangle \propto x^{-k}$ [9]. For the s-wave annihilation ($k = 0$), $\langle \sigma v \rangle$ is a constant, which is independent of the temperature of the Universe. For the p-wave annihilation ($k = 1$), $\langle \sigma v \rangle$ will be decreased as the Universe evolution. Clearly, only if $k < 0$, $\langle \sigma v \rangle$ could be enhanced at the lower temperature. In such a case, one may obtain a large boost factor to explain the PAMELA, ATIC, and PPB-BETS anomalies. It is interesting to notice that when considering the annihilation cross section at a narrow resonance, we can derive a negative number for k, which then indicates a Breit-Wigner enhancement mechanism. Recently, such an enhancement has explicitly been analyzed in Ref. [10] (for the previous discussions, see Ref. [11]). In the past, many authors have studied the dark matter annihilation near a resonance [12,13].

In this paper, we try to further give a comprehensive analysis on such a Breit-Wigner enhancement. Instead of using the center of mass frame, we work in the cosmic comoving frame and adopt the usual single-integral formula to calculate $\langle \sigma v \rangle$. Except for checking the unphysical pole case, we will pay attention to the investigation for the physical pole case in which the cross section $\langle \sigma v \rangle$ is found

to have a maximum. In both cases, $\langle \sigma v \rangle$ will approach a constant as the Universe evolution. In terms of the observed dark matter abundance, we calculate the coupling of the annihilation process for the whole resonance parameter space. Hence, we derive the exact boost factor and find that both cases can lead a large enough boost factor to account for the PAMELA, ATIC, and PPB-BETS results.

II. BREIT-WIGNER ENHANCEMENT

The PAMELA experiment observing no excess for the antiproton data indicates that dark matter will dominantly annihilate into the leptonic final states. In fact, the dark matter may first annihilate into some particles, such as the Higgs triplets in the left-right symmetric model [14], and then these particles decay into the charged leptons. For the purpose of this paper, we simply consider that two dark matter particles directly annihilate into a pair of charged leptons via S channel Higgs boson exchanging. Since the Breit-Wigner enhancement requires a narrow resonance, we follow Ref. [10] to introduce an auxiliary parameter δ ($|\delta| \ll 1$) to express the intermediate particle mass M

$$M^2 = 4m^2(1 - \delta). \qquad (1)$$

For the $\delta < 0$ case, one may obtain a physical pole. In this case, the exchanging particle may decay into both initial states and final states. For a given decay width Γ, one may write the following annihilation cross section:

$$4E_1E_2\sigma v = \frac{32\pi}{\sqrt{1 - \frac{4m^2}{M^2}}} \frac{s}{M^2} \frac{M^2\Gamma^2}{(s - M^2)^2 + M^2\Gamma^2} B_iB_f, \quad (2)$$

where B_i and B_f are the branching fractions of the resonance into the initial and final channels, respectively. Because of $B_i + B_f = 1$, one can directly obtain $B_iB_f \leq 0.25$. Here we have neglected the masses of final leptons. The parameter s is defined by $s \equiv (p_1 + p_2)^2$, where p_1 and p_2 are the four-momenta of initial dark matter particles. For the $\delta > 0$ case, we have an unphysical pole. In this case, the intermediate particle cannot decay into the initial dark matter particles. Therefore, we introduce a vertex α for the trilinear coupling among two dark matter particles and the exchanging particle to express the annihilation cross section

$$4E_1E_2\sigma v = 2\alpha^2 \frac{s}{M^2} \frac{M\Gamma}{(s - M^2)^2 + M^2\Gamma^2}. \qquad (3)$$

For the thermally averaged annihilation cross section $\langle \sigma v \rangle$, we adopt the usual single-integral formula

$$\langle \sigma v \rangle = \frac{1}{n_{EQ}^2} \frac{m}{64\pi^4 x} \int_{4m^2}^{\infty} \hat{\sigma}(s)\sqrt{s}K_1\left(\frac{x\sqrt{s}}{m}\right)ds, \quad (4)$$

with

$$n_{EQ} = \frac{g_i}{2\pi^2} \frac{m^3}{x} K_2(x); \qquad (5)$$

$$\hat{\sigma}(s) = 4E_1E_2\sigma v g_i^2 \sqrt{1 - \frac{4m^2}{s}}, \qquad (6)$$

where $K_1(x)$ and $K_2(x)$ are the modified Bessel functions. $g_i = 1$ is the internal degrees of freedom of dark matter particle. Since v of $\langle \sigma v \rangle$ is the Møller velocity, we work in the cosmic comoving frame. If one takes the center of mass frame [10], $\langle \sigma v \rangle$ should be multiplied by a factor $(1 + K_1^2(x)/K_2^2(x))/2$ [13].

For an analytical illustration, we redefine

$$s \equiv 4m^2(1 + z). \qquad (7)$$

Then the integration region of Eq. (4) becomes $0 \leq z < \infty$. The annihilation cross sections $4E_1E_2\sigma v$ in Eqs. (2) and (3) can be rewritten as

$$4E_1E_2\sigma v \propto \frac{1 + z}{(z + \delta)^2 + \gamma^2}, \qquad (8)$$

where γ is given by

$$\gamma \equiv \Gamma/M. \qquad (9)$$

For $x \geq 20$ and $|\delta|, \gamma \leq 0.1$, $\langle \sigma v \rangle$ in Eq. (4) can approximately be given by

$$\langle \sigma v \rangle \propto x^{3/2} \int_0^{z_{\text{eff}}} \frac{e^{-xz}\sqrt{z}}{(z + \delta)^2 + \gamma^2} dz. \qquad (10)$$

It is worthwhile to stress that the integration result of Eq. (10) is insensitive to x when z is negligible in $(z + \delta)^2 + \gamma^2$. In the $\delta > 0$ case, the effective integration upper bound is $z_{\text{eff}} \sim 4/x$, thus $\langle \sigma v \rangle$ can be enhanced as the universe evolution. For the $\delta < 0$ case, one may derive $z_{\text{eff}} \sim \max[4/x, 2|\delta|]$ when $|\delta| > \gamma$; we then find that $\langle \sigma v \rangle$ has a maximum at $x \sim 2/|\delta|$. If $|\delta| \ll \gamma$, one cannot obtain an obvious peak. When $x \gg 4/\max[|\delta|, \gamma]$, $\langle \sigma v \rangle$ will approach to a constant for both cases. Our numerical results [using Eq. (4)] in Figs. 1 and 2 explicitly demon-
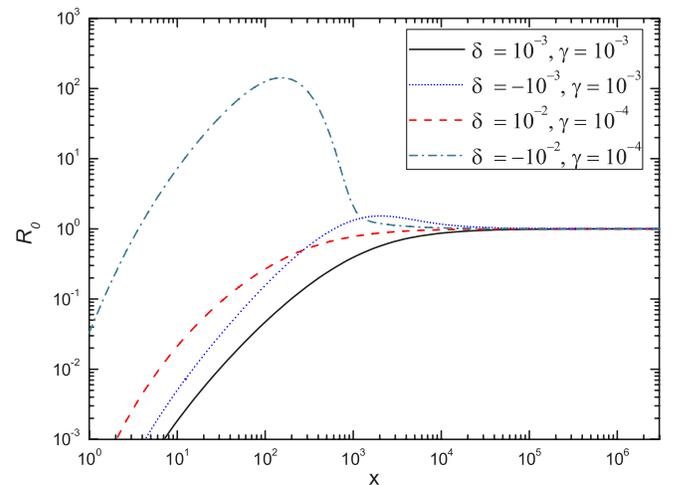
FIG. 1 (color online). Numerical illustration of the Breit-Wigner enhanced $\langle \sigma v \rangle$ as a function of x. The parameter R_0 is defined as $R_0 \equiv \langle \sigma v \rangle / \langle \sigma v \rangle_0$, with $\langle \sigma v \rangle_0$ denoting the thermally averaged annihilation cross section at $T = 0$.
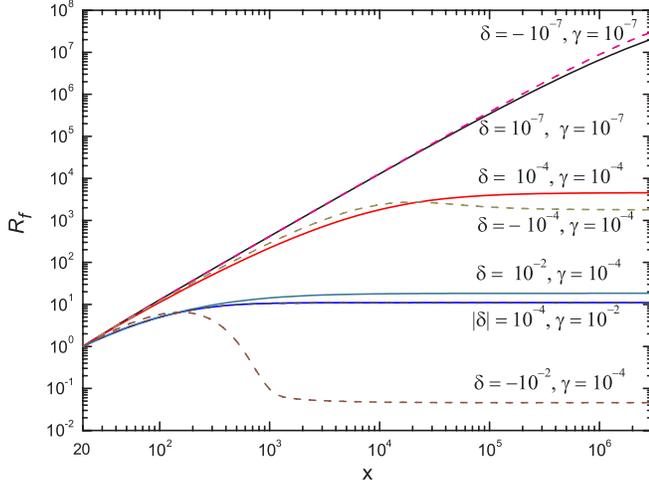
FIG. 2 (color online). Numerical illustration of the Breit-Wigner enhanced $\langle \sigma v \rangle$ as a function of x. The parameter R_f is defined as $R_f \equiv \langle \sigma v \rangle / \langle \sigma v \rangle_{x=20}$, with $\langle \sigma v \rangle_{x=20}$ denoting the thermally averaged annihilation cross section at the usual "freeze-out" time $x = 20$.

strate the above analysis. In Fig. 1, the parameter R_0 is defined as $R_0 \equiv \langle \sigma v \rangle / \langle \sigma v \rangle_0$, with $\langle \sigma v \rangle_0$ denoting the thermally averaged annihilation cross section at $T = 0$. One may easily see from Fig. 1 that the $\delta < 0$ case gives the larger R_0 at the higher temperature. In Fig. 2, we plot the ratio $R_f \equiv \langle \sigma v \rangle / \langle \sigma v \rangle_{x=20}$. It is seen that for small $|\delta|$ and γ with $|\delta| \sim \gamma < 10^{-3}$, both of the cases provide a significant enhancement for the thermally averaged annihilation cross section. For much smaller $|\delta| = \gamma = 10^{-7}$, the $\delta < 0$ case can give the larger enhancement. It should be mentioned that our results are independent of α, $B_i B_f$, and m.

III. BOOST FACTOR AND COUPLINGS

The Breit-Wigner enhancement mechanism can change the indirect dark matter detection. On the other hand, the Breit-Wigner enhancement will affect the calculation of the dark matter relic density because the annihilation process does not freeze out even after the usual "freeze-out" time $x_f = 20$ [10]. More importantly, we can derive the size of α ($\delta > 0$) and $B_i B_f$ ($\delta < 0$) for given δ and γ from the observed dark matter abundance $\Omega_D h^2 = 0.1131 \pm 0.0034$ [2]. The values of α and $B_i B_f$ can help us to build appropriate models. With the help of Eqs. (2)–(4), one can calculate the thermally averaged annihilation cross section $\langle \sigma v \rangle$ in the galactic halo, which allows us to obtain the exact boost factor. It is worthwhile to stress that the Breit-Wigner enhancement does not affect the direct dark matter searches.

The evolution of dark matter abundance is given by the following Boltzmann equation [9]:

$$\frac{dY}{dx} = -\frac{x \mathbf{s}(x)}{H} \langle \sigma v \rangle (Y^2 - Y_{EQ}^2), \qquad (11)$$

where $Y \equiv n/\mathbf{s}(x)$ denotes the dark matter number density. The entropy density $\mathbf{s}(x)$ and the Hubble parameter H evaluated at $x = 1$ are given by

$$\mathbf{s}(x) = \frac{2\pi^2 g_*}{45} \frac{m^3}{x^3}; \qquad (12)$$

$$H = \sqrt{\frac{4\pi^3 g_*}{45}} \frac{m^2}{M_{\text{PL}}}, \qquad (13)$$

where $M_{\text{PL}} \simeq 1.22 \times 10^{19}$ GeV is the Planck energy. g_* is the total number of effectively relativistic degrees of freedom. Here we choose $g_* = 106.75$ for illustration. Using the result Y_0 of the integration of Eq. (11), we may obtain the dark matter relic density $\Omega_D h^2$

$$\Omega_D h^2 = 2.74 \times 10^8 \frac{m}{\text{GeV}} Y_0. \qquad (14)$$

Using the Boltzmann equation in Eq. (11), we numerically calculate α and $B_i B_f$ for the unphysical pole case and the physical pole case, respectively. Our numerical results (dashed lines) are shown in Figs. 3 and 4. Here we have taken $m = 1$ TeV. For the $\delta > 0$ case, we obtain 15 GeV $\lesssim \alpha \lesssim 4.9$ TeV, which can be easily satisfied. For the $\delta < 0$ case, the parameter $B_i B_f$ is far less than the upper bound 0.25 except for the lower left region. For most of the parameter range, $B_i B_f < 0.001$ indicates that the successful models must have a hierarchy between the initial branching factor B_i and the final branching factor B_f. If the dark matter mass m is enlarged by N times, α and
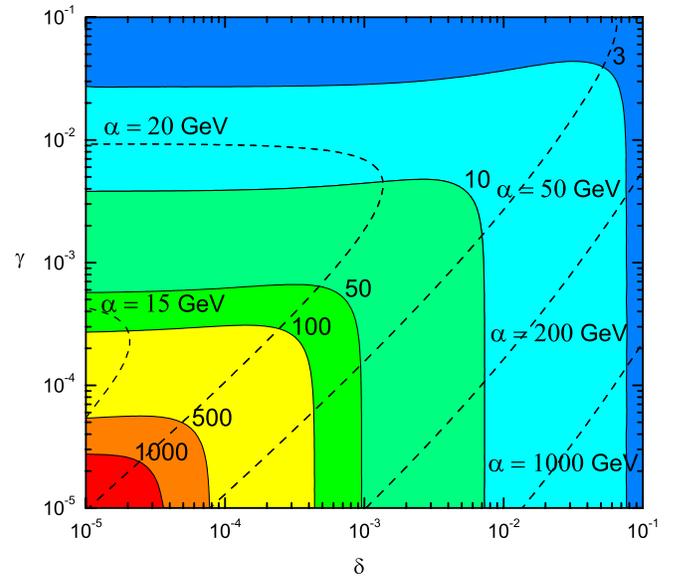
FIG. 3 (color online). Numerical illustration of the boost factor BF (solid lines) and the coupling α (dashed lines) on the δ and γ planes for the $\delta > 0$ case.
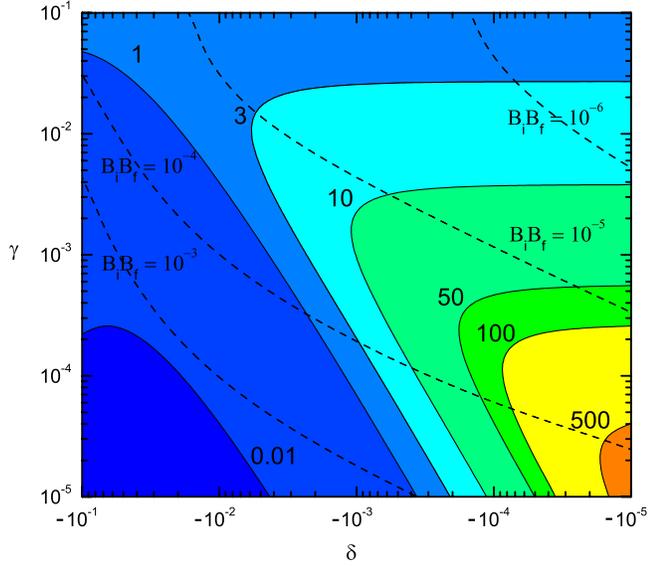
FIG. 4 (color online). Numerical illustration of the boost factor BF (solid lines) and the parameter $B_i B_f$ (dashed lines) on the δ and γ planes for the $\delta < 0$ case.

$B_i B_f$ in Figs. 3 and 4 should be approximately enlarged by N^2 times.

After obtaining α and $B_i B_f$, we calculate the thermally averaged annihilation cross section $\langle \sigma v \rangle$ in the galactic halo ($x \approx 3 \times 10^6$), which is shown in Figs. 3 and 4. We would like to emphasize that our results are insensitive to the dark matter mass m. Here we normalize $\langle \sigma v \rangle$ by the usual nonresonance annihilation cross section 10^{-9} GeV^{-2} to define the boost factor BF

$$ BF \equiv \frac{\langle \sigma v \rangle}{10^{-9} \text{ GeV}^{-2}}. \qquad (15) $$

It is clear that smaller $|\delta|$ and γ will provide larger boost factors. For the $\delta > 0$ case, the large boost factor ($BF \gtrsim 100$) requires $\delta, \gamma < \mathcal{O}(10^{-3})$. Our results have some differences from Fig. 4 in Ref. [10] for the region $BF < \mathcal{O}(10)$ even if we choose $g_* = 200$. For the $\delta < 0$ case, one may obtain $\delta, \gamma \lesssim \mathcal{O}(10^{-4})$ for $BF \gtrsim 100$. In the lower left region of Fig. 4, we find $BF \ll 1$, which implies that the indirect dark matter detection experiments will not find any signal of the dark matter annihilation. It is clear that both cases can provide a large enough boost factor to explain the PAMELA, ATIC, and PPB-BETS anomalies. This is one of our primary results.

IV. DISCUSSION AND CONCLUSION

We have evaluated the boost factor BF and the values of α, $B_i B_f$ in terms of the observed dark matter abundance $\Omega_D h^2$. If the parameter α^2 or $B_i B_f$ is enlarged by N times, the Breit-Wigner enhanced annihilation cross section will be enlarged by the same times. While the dark matter relic number density Y will be approximately suppressed by N times, one thus needs to introduce new dark matter candidates. Although we can obtain the larger boost factor, this scenario will give the smaller dark matter annihilation rate, which is proportional to $\langle \sigma v \rangle Y^2$. In fact, many models have several dominant annihilation processes, which may include the nonresonance and resonance cases. If the nonresonance annihilation processes determine the dark matter relic density, one cannot obtain α, $B_i B_f$, and BF from $\Omega_D h^2$. In this case, one may determine those parameters from other constraints; the required values for α, $B_i B_f$, and BF must be smaller than the predicted values in Figs. 3 and 4. In general, it still has some parameter space in this case to account for the PAMELA, ATIC, and PPB-BETS results.

In conclusion, we have made a comprehensive analysis based on the Breit-Wigner enhancement near the resonance point. In terms of the observed value of $\Omega_D h^2$, we have evaluated the exact thermally averaged annihilation cross section $\langle \sigma v \rangle$ in the galactic halo and the boost factor BF, and calculated the couplings α and $B_i B_f$ of the annihilation process for both the $\delta > 0$ and the $\delta < 0$ cases, respectively. The numerical results lead us to a general conclusion that both the $\delta > 0$ and the $\delta < 0$ cases can provide a large enough boost factor $BF \gtrsim 100$ to explain the PAMELA, ATIC, and PPB-BETS anomalies. It would be interesting to find a model with the appropriate coupling α or $B_i B_f$ for the annihilation processes to give the desired dark matter relic density.

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