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Spontaneous supersymmetry breaking with anomalous $U(1)$ symmetry in metastable vacua and moduli stabilization

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We show that in (anomalous) $U(1)$ gauge theories with the Fayet-Iliopoulos term and with generic interactions, there are metastable vacua in which supersymmetry (SUSY) is spontaneously broken even without $U(1)_R$ symmetry. In this vacua, various hierarchical structures such as Yukawa hierarchy can be explained by the smallness of the Fayet-Iliopoulos parameter. It is shown that, by adding just one positively charged field to phenomenologically viable models, spontaneous SUSY breaking is realized. Moreover, we propose a new scenario for the stabilization of the moduli in the SUSY breaking models. It is a new feature that the moduli can be stabilized without the superpotential being dependent on them.

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I. INTRODUCTION

The minimal supersymmetric standard model (MSSM) is one of the most promising candidates for a model beyond the standard model (SM) [1–3]. It has several attractive features. For example, the weak scale can be stabilized by supersymmetry (SUSY), three gauge couplings meet at a scale that strongly implies the supersymmetric grand unified theory (GUT) [4–6], and the lightest supersymmetric particle (LSP) can be dark matter. However, there are a lot of unsatisfactory features. One of them is that the number of parameters is more than 100. If we introduce these parameters generically, various flavor changing neutral current (FCNC) processes and CP violating observables, like electric dipole moments of the electron and the neutron, become too large to be consistent with the experimental bounds [7–10]. Moreover, it is not known why the supersymmetric Higgs mass parameter μ is of the same order as the SUSY breaking scale [11]. Most of these unsatisfied features are strongly related to SUSY breaking. Therefore, it is important to understand the origin of SUSY breaking in the MSSM in order to solve these problems. Moreover, the LHC is expected to reveal some features of SUSY breaking, so it is important to examine various SUSY breaking mechanisms before the LHC gives the results.

The (anomalous) $U(1)_A$ gauge theories with the Fayet-Iliopoulos (FI) term [12] are often used to explain the hierarchical structures of Yukawa couplings [13–19]. This is quite reasonable because the hierarchical structures can be explained under the assumption that all the interactions which are allowed by the symmetry are introduced with $O(1)$ coefficients. Moreover, it has been pointed out that the $U(1)_A$ symmetry can play an important role even in breaking the grand unified group [19–24]. This is also natural because the serious fine-tuning problem called the doublet-triplet splitting problem can be solved under the same assumption in which the generic interactions are introduced [19,25].

In the literature, it has been argued that (anomalous) $U(1)_A$ symmetry with the FI term can play an important role even in breaking SUSY spontaneously [26]. In order to break SUSY with generic interactions, the $U(1)_R$ symmetry must be imposed [27]. In other words, without $U(1)_R$ symmetry, SUSY vacua appear in general. However, the above phenomenological models often have no $U(1)_R$ symmetry. Therefore, it is important to examine spontaneous SUSY breaking without $U(1)_R$ symmetry. This may be possible if we consider the metastable vacua [28–31].[1] In this paper, we point out that even if generic interactions are introduced in (anomalous) $U(1)_A$ gauge theory with the FI term and without $U(1)_R$ symmetry, SUSY can be spontaneously broken in metastable vacua in which various hierarchical parameters are determined by the smallness of the FI parameter. If generic interactions are introduced with $O(1)$ coefficients, almost all the scales can be determined by the symmetry of the theory [i.e., the $U(1)_A$ charges]. We calculate the various scales, including several SUSY breaking scales in some examples. One of the most interesting features of the metastable spontaneous SUSY breaking proposed in this paper is that by adding just one positively charged field to the phenomenologically viable models mentioned in the previous paragraph, spontaneous SUSY breaking is realized. This makes us expect more complete models in which, in addition to the previous advantages of the models with anomalous $U(1)_A$ symmetry, SUSY breaking is also controlled by the anomalous $U(1)_A$ gauge symmetry.

One of the most important problems in the phenomenology of the superstring theory is the moduli stabilization

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[1] In Ref. [31], metastable SUSY breaking with the FI term is discussed, though in their model $U(1)_R$ symmetry is imposed.

problem [32], though there have been several attempts to solve this problem in various scenarios in which SUSY is dynamically broken by the strong dynamics of supersymmetric QCD (SQCD) [33–39]. Especially in models with anomalous $U(1)_A$ symmetry, the gauge anomaly is canceled by the shift of the moduli, and in general, the FI parameter is determined by the vacuum expectation value (VEV) of the moduli [40–43]. Therefore, in the context of the SUSY breaking models with anomalous $U(1)_A$ symmetry, it is interesting and challenging to consider the moduli stabilization simultaneously. We propose a possibility to stabilize the moduli in the SUSY breaking scenario. As a result, we can obtain a SUSY breaking scenario in which SUSY is spontaneously broken and the moduli can be stabilized without the superpotential being dependent on the moduli. In the literature, the moduli-dependent superpotential, which is induced by nonperturbative effects or by the flux compactification [44], plays an essential role in stabilizing the moduli. However, in the moduli stabilization mechanism proposed in this paper, the superpotential does not include the moduli.[2] This feature is quite important because the moduli-dependent superpotential generically spoils the SUSY zero mechanism [19,25,45] which plays an important role in building realistic models.

The organization of this paper is as follows. In the second section, we will compare two simple spontaneous SUSY breaking models with anomalous $U(1)_A$ symmetry. $U(1)_R$ symmetry is imposed in one model and not in the other. The latter model has metastable SUSY breaking vacua in which the hierarchical couplings such as Yukawa couplings can be realized. In the third section, we extend the metastable SUSY breaking model without $U(1)_R$ symmetry to more general models. Applying these general results to phenomenologically viable models, it is easily understood that by adding one positively charged field to the models, spontaneous SUSY breaking is realized. And in the fourth section, we will consider the moduli stabilization. Finally, we will give a summary and discussions.

II. SPONTANEOUS SUSY BREAKING WITH ANOMALOUS $U(1)_A$ SYMMETRY

In this section, we consider SUSY breaking models with anomalous $U(1)_A$ gauge symmetry. And we show that there are metastable SUSY breaking vacua in a simple model with generic interactions without $U(1)_R$ symmetry. In the vacua, hierarchical couplings can be obtained.

Before we consider the SUSY breaking model without $U(1)_R$ symmetry, let us recall what happens with $U(1)_R$ symmetry [12]. For simplicity, we consider a model which contains two fields S and Θ, where S has positive integer $U(1)_A$ charge s and Θ has negative $U(1)_A$ charge $\theta = -1$. (In this paper, we use the lowercase letter as the charge for the field denoted by the uppercase letter.) We assign R charges for S and Θ, as shown in Table I. In this model, the generic superpotential becomes

$$W = S\Theta^s \tag{1}$$

where the coefficients are neglected. (In this paper, we usually neglect the coefficients in the interactions and take the cutoff $\Lambda = 1$.) The F-terms and the D-term in this model,

$$F_S^* = -\frac{\partial W}{\partial S} = -\Theta^s, \qquad F_\Theta^* = -\frac{\partial W}{\partial \Theta} = -sS\Theta^{s-1},$$
$$D_A = -g(\xi^2 - |\Theta|^2 + s|S|^2), \tag{2}$$

where ξ is a FI parameter, cannot vanish simultaneously because the F-flatness conditions result in the vanishing VEV of Θ under which the D-flatness condition cannot be satisfied. Therefore SUSY is spontaneously broken in this model. The VEVs of these fields, and the F and D terms are determined by the minimization of the potential

$$V = |F_S|^2 + |F_\Theta|^2 + \tfrac{1}{2}D_A^2 \tag{3}$$

as

$$\langle S\rangle = 0, \qquad \langle\Theta\rangle = \lambda, \tag{4}$$

$$\langle F_S\rangle \sim \lambda^s, \qquad \langle F_\Theta\rangle = 0, \qquad \langle D_A\rangle \sim \frac{s}{g}\lambda^{2s-2}, \tag{5}$$

when $\xi \ll 1$. Here, $\lambda \equiv \langle\Theta\rangle/\Lambda \sim \xi/\Lambda$, and without loss of generality, we can take the VEV of Θ to be real because of the $U(1)_A$ symmetry. The typical SUSY breaking scale $\lambda^s\Lambda$ must be around the weak scale, which is obtained, for example, when $s \sim 24$ for $\lambda \sim 0.22$ and $\Lambda = 2 \times 10^{18}$ GeV.

What happens if we do not impose $U(1)_R$ symmetry? The quantum numbers are given as in Table II. Then, the generic superpotential becomes

$$W(S\Theta^s) = \sum_{n=1} a_n (S\Theta^s)^n. \tag{6}$$

TABLE I. The quantum numbers of the superpotential W and the fields S and Θ.

	W	S	Θ
$U(1)_A$	0	$s > 0$	-1
$U(1)_R$	2	2	0

[2]Generically, the symmetry allows exponential-type interactions of the moduli and inverse power-type interactions of the introduced fields, which may be induced by the nonperturbative effects of the strong dynamics or of the string. In this paper, we consider the case in which such interactions are not induced or are sufficiently small if they exist, because these interactions spoil the SUSY zero mechanism which plays an important role in solving various phenomenological problems. Such an assumption may be reasonable in our setup because, in order to break SUSY, we do not require any strong coupling gauge theory which has a dynamical scale larger than the weak scale.

Namely, any polynomial of $x \equiv S\Theta^s$ is allowed for the superpotential $W(x)$. It is known that such a model has SUSY vacua. Actually, among the F-terms and the D-term

$$\begin{aligned} F_S^* &= -\frac{\partial W}{\partial S} = -\frac{\partial W}{\partial x}\Theta^s, \\ F_\Theta^* &= -\frac{\partial W}{\partial \Theta} = -\frac{\partial W}{\partial x} s\Theta^{s-1}S, \\ D_A &= -g(\xi^2 - |\Theta|^2 + s|S|^2), \end{aligned} \tag{7}$$

the F-flatness conditions can be satisfied by taking the VEV of $S\Theta^s$ to be $\partial W/\partial x = 0$, and D-flatness can be satisfied by choosing the VEV of Θ. Generically, the VEVs of S and Θ become $O(1)$. However, it has not been emphasized that this model has metastable SUSY breaking vacuum at $\langle\Theta\rangle \sim \xi$ and $\langle S\rangle \sim 0$ if $\xi \ll 1$. Note that such VEVs play an important role in solving various phenomenological problems [13–25]. The reason for the metastability is, roughly speaking, that in the region $\langle S\rangle$, $\langle\Theta\rangle \ll 1$, the superpotential becomes approximately $W = S\Theta^s$, which is nothing but the superpotential in the spontaneous SUSY breaking model with $U(1)_R$ symmetry.

In order to estimate the VEVs $\langle S\rangle = S_r e^{i\phi_s}$, $\langle\Theta\rangle = \Theta$, $\langle F_S\rangle$, $\langle F_\Theta\rangle$, and $\langle D_A\rangle$ and see the metastability of this vacuum, we must examine the potential

$$V = \left|\frac{\partial W}{\partial x}\right|^2 (\Theta^{2s} + s^2 S_r^2 \Theta^{2(s-1)}) + \frac{g^2}{2}(\xi^2 - \Theta^2 + sS_r^2)^2, \tag{8}$$

where $\frac{\partial W}{\partial x} = \sum_{n=1} a_n n (S_r e^{i\phi_s}\Theta^s)^{n-1}$. Let us discuss the vacuum which is slightly away from $\langle S\rangle = 0$ and $\langle\Theta\rangle = \xi \ll 1$. Then, it is sufficient to examine the superpotential up to the second order as $W = a_1(S\Theta^s) + a_2(S\Theta^s)^2$ because of the smallness of the VEVs of S and Θ^s. The stationary conditions

$$\frac{\partial V}{\partial \Theta} = 0, \qquad \frac{\partial V}{\partial S_r} = 0, \qquad \frac{\partial V}{\partial \phi_s} = 0 \tag{9}$$

lead to

$$\langle D_A\rangle \sim \frac{s|a_1|^2}{g}\Theta^{2s-2} \sim \frac{s}{g}\lambda^{2s-2}, \tag{10}$$

$$S_r \sim -\frac{\Theta^{s+2}}{2s^2|a_1|^2}(a_2 a_1^* e^{i\phi_s} + \text{H.c.}) \sim \frac{1}{s^2}\lambda^{s+2}, \tag{11}$$

$$a_1^* a_2 e^{i\phi_s} - a_1 a_2^* e^{-i\phi_s} = 0. \tag{12}$$

If we define $\delta\Theta = \Theta - \xi$, Eq. (10) implies $\delta\Theta \sim -\frac{s|a_1|^2}{2g}\xi^{2s-3}$. This is consistent with our assumption that $\delta\Theta/\xi \ll 1$. Then, the VEVs of the auxiliary fields are determined as $-F_S^* \sim a_1\lambda^s$ and $-F_\Theta^* \sim \frac{\lambda^{2s+1}A}{sa_1^*}$, where $A \equiv -a_1 a_2^* e^{-i\phi_s} = -a_1^* a_2 e^{i\phi_s}$. Moreover, the stability condition requires $a_1^* a_2 e^{i\phi_s} = -|a_1 a_2|$.

TABLE II. The $U(1)_A$ charges of the fields S and Θ.

	S	Θ
$U(1)_A$	$s > 0$	-1

One of the biggest differences between the above two SUSY breaking models is the value of the VEV of the positively charged field S. (It is obvious that the difference in the VEV of F_Θ is caused by the difference in the VEV of S.) Therefore, we will briefly examine the reason for the difference below. In the model with $U(1)_R$ symmetry the VEV of S is vanishing, while in the model without $U(1)_R$ symmetry, S has a nonvanishing VEV, though the VEV is smaller than the typical SUSY breaking scale $F_S/\Lambda \sim \lambda^s$. Without $U(1)_R$ symmetry, the superpotential includes higher dimensional operators like $S^2\Theta^{2s}$. This term leads to a tadpole term $\langle F_S\rangle\lambda^{2s}S$ after obtaining the nonvanishing VEV of F_S, which results in the nonvanishing VEV of S. Namely, the VEV $\langle S\rangle$ is decided by the tadpole and the mass term $\lambda^{s-1}S\Theta$ as

$$\langle S\rangle \sim \frac{\text{coefficient of tadpole}}{\text{mass}^2} \tag{13}$$

as seen in Fig. 1. It is obvious that the larger tadpole term leads to lower potential energy. Therefore, the phase of the VEV of S is determined so that the tadpole term $W|_{\theta^2} + \text{H.c.} \ni SS\Theta^{2s}|_{\theta^2} + \text{H.c.} \sim SF_S\lambda^{2s} + \text{H.c.} = 2\lambda^{3s}S\cos\phi_s$ becomes maximal, i.e., $\cos\phi_s = \pm 1$. (Here we take the coefficients in the superpotential to be real for simplicity.) The signature of $\cos\phi_s$ is determined so that the absolute value of $\partial W/\partial x = a_1 + a_2 S_r e^{i\phi_s}\Theta^s$ is minimal; i.e., the sign becomes negative if a_1 has the same sign as a_2. (Here we use the notation that S and Θ are positive.) Since the mass term is given by $|\frac{\partial W}{\partial \Theta}|^2 \ni \lambda^{2s-2}|S|^2$, the VEV $\langle S\rangle$ becomes $\langle S\rangle \sim \lambda^{s+2}$ from Eq. (13). If $\lambda^s\Lambda$ is the weak scale and the cutoff Λ is much larger than the weak scale, the VEV $\langle S\rangle$ would be much smaller than the cutoff Λ and the VEV $\langle\Theta\rangle$. As a result, the values of these VEVs are approximately satisfied with the expected VEV relations that are important in solving phenomenological problems.

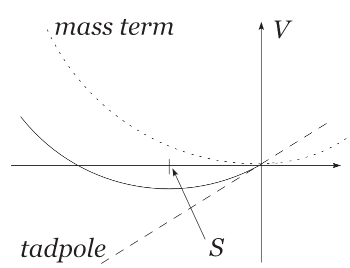

FIG. 1. This figure shows the potential of S. The VEV of S is determined by the tadpole and the mass term of S. The solid line shows the sum of the contributions from the tadpole and mass terms.

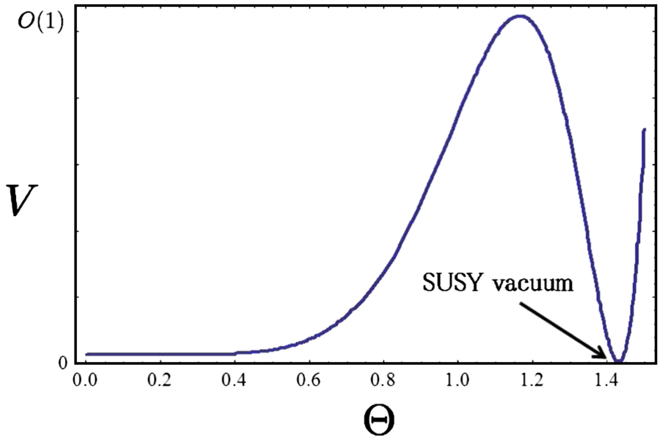

FIG. 2 (color online). This figure shows the potential of Θ. Here, we take $s = 4$ and $\xi = 0.2$.

We show the schematic form of the potential in this model in Figs. 1 and 2. The potential of Θ rapidly increases above $\Theta = 1$ because of $|F_S|^2$. Figure 3 shows the magnification of the potential around the origin.

In the last part of this section, we estimate the lifetime of the metastable vacuum by following the arguments in Ref. [46], with the values $s = 24$ for $\lambda = 0.22$. The lifetime of the metastable vacuum is approximately given by

$$\tau \propto e^P, \tag{14}$$

where P is dimensionless and can be given by

$$P = \frac{(\sqrt{V_h}\Delta)^4}{\epsilon'^3}. \tag{15}$$

Δ shows the distance from the supersymmetric vacuum to the metastable vacuum. V_h shows the height of the barrier wall between the metastable vacuum and SUSY vacua, and ϵ' shows the potential height of the metastable vacuum. The values for this model become $\Delta \sim O(\Lambda)$, $V_h \sim O(\Lambda^4)$, $\epsilon' \sim O(\lambda^{2s}\Lambda^4) \sim O(M_{\rm SUSY}^2\Lambda^2)$, where $M_{\rm SUSY}$ is the SUSY breaking scale. Therefore, we can estimate P as

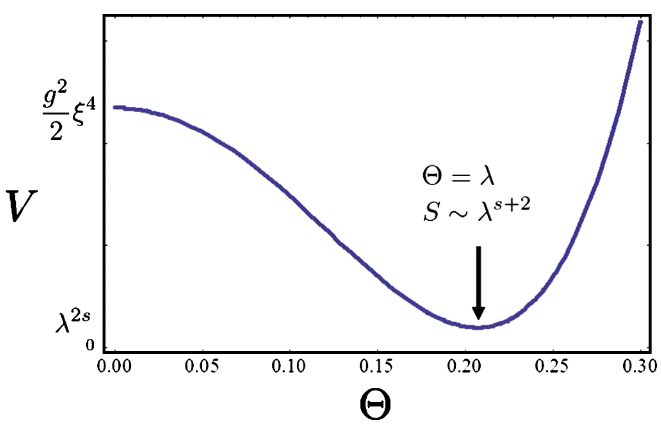

FIG. 3 (color online). This figure shows the magnification of the potential in Fig. 2 around the origin.

$$P \sim \frac{\Lambda^{12}}{M_{\rm SUSY}^6\Lambda^6} \sim \frac{\Lambda^6}{M_{\rm SUSY}^6}. \tag{16}$$

Then we obtain $P \sim 10^{96}$. Therefore, the lifetime of the metastable vacuum is much larger than the age of our Universe.

III. GENERAL CASES

In this section, we will extend the SUSY breaking model discussed in the previous section to more general ones. We introduce n_+ positively charged fields $S_i (i = 1, \cdots, n_+)$ and n_- negatively charged fields $Z_j (j = 1, \cdots, n_- - 1)$ and Θ as shown in Table III.

One of the $n_+ + n_-$ complex F-flatness conditions becomes trivial because of $U(1)_A$ gauge symmetry, but we have the real D-flatness condition for $U(1)_A$. Then the VEVs of $n_+ + n_-$ complex fields are generically fixed by these conditions, except for one real field which corresponds to the Nambu-Goldstone mode of $U(1)_A$ symmetry if all the conditions are independent. Therefore, if we introduce the generic superpotential $W(S, Z, \Theta)$, namely, if the above conditions become independent, then, in general, there are SUSY vacua at which all the VEVs of the fields are of order 1 if all the coefficients are of order 1. However, as discussed in Refs. [19,25], when $n_+ \le n_- - 1$, other SUSY vacua appear at which all positively charged fields S_i have vanishing VEVs and the negatively charged fields Z_j and Θ have nonvanishing VEVs which are not larger than $O(\xi) \ll 1$. When all positively charged fields have vanishing VEVs, the F-flatness conditions of negatively charged fields are trivially satisfied because $\partial W/\partial Z_j$ have positive charges. Therefore, the n_+ F-flatness conditions and the D-flatness condition of $U(1)_A$,

$$\frac{\partial W}{\partial S_i} = 0, \qquad D_A = g\left(\xi^2 - |\Theta|^2 + \sum_j z_j |Z_j|^2\right) = 0, \tag{17}$$

constrain the n_- VEVs of negatively charged fields Z_j and Θ. If $n_+ \le n_- - 1$, these conditions can be satisfied in general, and therefore, there are SUSY vacua. Because of the D-flatness conditions, the nonvanishing VEVs cannot be larger than $\xi \ll 1$. (In this paper, we call such vacua small vacua.) In particular, when $n_+ = n_- - 1$, all the VEVs are determined by their charges as

$$\langle S_i \rangle = 0, \qquad \langle Z_j \rangle \sim \lambda^{-z_j}. \tag{18}$$

Since the generic superpotential can be rewritten as $W(\tilde{S}_i, \tilde{Z}_j)$, where $\tilde{S}_i = S_i \Theta^{s_i}$ and $\tilde{Z}_j = Z_j \Theta^{z_j}$, the

TABLE III. The $U(1)_A$ charges for the fields S_i, Z_j, and Θ. Here $i = 1 \sim n_+$, $j = 1 \sim (n_- - 1)$.

	S_i	Z_j	Θ
$U(1)_A$	$s_i > 0$	$z_j < -1$	-1

F-flatness conditions of S_i,

$$\frac{\partial W}{\partial S_i} = \Theta^{s_i} \frac{\partial W}{\partial \tilde{S}_i}, \tag{19}$$

give solutions as $\langle \tilde{Z}_j \rangle = O(1)$ because $W(\tilde{S}_i, \tilde{Z}_j)$ have $O(1)$ coefficients. The equations $\langle \tilde{Z}_j \rangle = O(1)$ mean that $\langle Z_j \rangle \sim \lambda^{-z_j}$.[3] Usually, this is the case in most of the phenomenologically viable models.

What happens if $n_+ > n_- - 1$? Since the number of constraints is larger than the number of variables, there is no solution, and therefore, small vacua cannot be in the supersymmetric vacua, as discussed in Refs. [19,25]. However, as discussed in the previous section, small vacua can be metastable. Let us figure out what happens if $n_+ = n_-$. One of the F- or D-flatness conditions cannot be satisfied. If the F-flatness condition of the largest charged field S_{n_+} is not satisfied and the other F- and D-flatness conditions are almost satisfied, the vacuum energy becomes the lowest in the small vacua because Eq. (19) gives $|F_{S_i}| \sim \lambda^{s_i}$. Therefore, the vacuum energy becomes $V \sim |F_{S_{n+}}|^2 \sim \lambda^{2s_{n_+}}$, which can be very small if the maximal charge $s_{n_+} \gg 1$. This feature may give an explanation of the large hierarchy between the SUSY breaking scale and the Planck scale. It is reasonable to expect that the potential energy becomes larger than $\lambda^{2s_{n_+}}$ between the small vacua and the SUSY vacua at which all the VEVs are of order 1. When the VEVs become larger than ξ, the D-flatness condition requires the nonvanishing VEVs for positively charged fields. Then all the F-terms, including those of negatively charged fields, can contribute to the vacuum energy, which generically becomes larger than $\lambda^{2s_{n_+}}$. Note that if we add one positively charged field to the phenomenologically viable model in which $n_+ = n_- - 1$, we can obtain the model in which SUSY is spontaneously broken by the metastable vacua.

IV. MODULI STABILIZATION IN A MODEL WITH ANOMALOUS $U(1)_A$ SYMMETRY

In the previous sections, we have assumed that the FI parameter ξ is a constant. However, in the context of supergravity or the superstring, the FI parameter is dynamically determined; i.e., ξ depends on the VEV of the moduli (or dilaton) D [40–43]. Actually, since $U(1)_A$ gauge symmetry is given by

$$V_A \to V_A + \frac{i}{2}(\Lambda - \Lambda^\dagger), \tag{20}$$

$$D \to D + \frac{i}{2}\delta_{\rm GS}\Lambda, \tag{21}$$

where Λ is a gauge parameter chiral superfield. A dimensionless parameter $\delta_{\rm GS}$, which is proportional to trQ_A, is positive when tr$Q_A > 0$. The Kähler potential $K_D(D + D^\dagger - \delta_{\rm GS} V_A)$ is invariant under $U(1)_A$ gauge symmetry, and the FI term can be given as

$$\int d^4\theta K_D(D + D^\dagger - \delta_{\rm GS} V_A) = -\left(\frac{\delta_{\rm GS} K_D'}{2}\right) D_A + \cdots \equiv \xi^2 D_A + \cdots, \tag{22}$$

where we take the sign of the trace of the anomalous $U(1)_A$ charge trQ_A so that $\xi^2 > 0$. [Since tr $Q_A > 0$ in most of the phenomenologically viable models, the positivity of the FI parameter requires $K_D' < 0$, which is consistent with the stringy tree-level Kähler potential of the moduli, $K_D = -\ln(D + D^\dagger - \delta_{\rm GS} V_A)$.]

The stabilization of the moduli is one of the important issues in string theory and/or in models with anomalous $U(1)_A$ gauge symmetry. In this section, we will examine a new possibility for the moduli stabilization by using the moduli-dependent potential through the FI parameter ξ, which is obtained in the previous sections as

$$V \sim |F_S|^2 \sim \xi^{2s}. \tag{23}$$

First, we examine the stabilization by the deformation of the Kähler potential of the moduli K_D from $K_D = -\ln(D + D^\dagger - \delta_{\rm GS} V_A)$, which can be obtained by stringy calculation at tree level. However, unfortunately we found it impossible. The point is simple. It is shown that $\xi^{2s}(D)$ is a monotonically decreasing function for D. Actually,

$$\frac{\partial \xi^{2s}}{\partial D} = (\xi^{2s})' = s K_D''(K_D')^{s-1}\left(-\frac{\delta_{\rm GS}}{2}\right)^s < 0, \tag{24}$$

where K_D'' is positive because it becomes the coefficient of the moduli kinetic term. This result shows that it is difficult to stabilize the moduli by the deformation of K_D.

Next, we will consider the deformation of the Kähler potential of S from the canonical form. Since the scalar potential of the moduli can be obtained as

$$V \sim \left(\frac{\partial^2 K_S}{\partial S \partial S^\dagger}\right)^{-1} \left|\frac{\partial W}{\partial S}\right|^2 \sim \left(\frac{\partial^2 K_S}{\partial S \partial S^\dagger}\right)^{-1} \xi^{2s}(D), \tag{25}$$

the moduli can be stabilized, as shown in Fig. 4 if $\frac{\partial^2 K_S}{\partial S \partial S^\dagger}$ becomes much smaller than 1 at $\langle D \rangle = D_0$.

In order to realize such a situation, let us take a more generic Kähler potential of the S field as

$$K_S = S^\dagger S f(D + D^\dagger - \delta_{\rm GS} V_A), \tag{26}$$

where $f(x)$ is a function of x. If the function $f(x)$ is given by

[3] Of course, even if $n_+ \leq n_- - 1$, it is possible that there are no such vacua; i.e., under the assumption that all the positively charged fields have vanishing VEVs, all the F- and D-flatness conditions cannot be satisfied. For example, if one positive charge s_1 is smaller than all the magnitudes of the negative charges z_j, then the F-flatness condition of S_1 and the D-flatness condition cannot be satisfied simultaneously. Here, we do not consider such extreme cases.

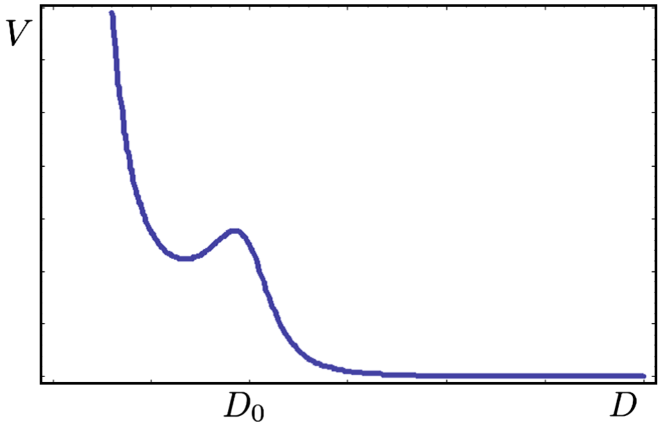

FIG. 4 (color online). The potential of the moduli.

$$f(x) = c(x - x_0)^2 + \epsilon, \tag{27}$$

where $c \sim O(1)$, $0 < \epsilon \ll 1$, then the function $\frac{\partial^2 K_S}{\partial S \partial S^\dagger}$ becomes much smaller than 1 at $x = x_0$. The moduli potential (25) can be rewritten by using the Kähler potential (26) as

$$\begin{aligned} V &\sim \left(\frac{\partial^2 K_S}{\partial S \partial S^\dagger}\right)^{-1} \left|\frac{\partial W}{\partial S}\right|^2 \\ &\sim \frac{\xi^{2s}(x)}{(c(x - x_0)^2 + \epsilon)} \\ &\sim \frac{1}{(c(x - x_0)^2 + \epsilon)} \left(\frac{\delta_{\rm GS}}{2x}\right)^s, \end{aligned} \tag{28}$$

where the last equality is obtained from Eq. (22) and $K_D(x) \sim -\ln x$. It is easily shown that, if the condition

$$\epsilon < \frac{c}{s(s+2)} x_0^2 \tag{29}$$

is satisfied, the moduli potential has a local minimum at $x = x_-$ and a local maximum at $x = x_+$, as shown in Fig. 4, where

$$\begin{aligned} x_\pm &\equiv \left(\frac{s+1}{s+2}\right) x_0 \{1 \pm \sqrt{1-\gamma}\}, \\ \gamma &\equiv \frac{s(s+2)}{(s+1)^2}\left(1 + \frac{\epsilon}{c x_0^2}\right). \end{aligned} \tag{30}$$

Let us estimate the scales of $F_S \sim -(\frac{\partial^2 K_S}{\partial S \partial S^\dagger})^{-1} \frac{\partial W}{\partial S}$ and D_A. At the metastable vacuum $x = x_-$, $\frac{\partial^2 K_S(x)}{\partial S \partial S^\dagger}$ can be estimated as

$$\begin{aligned} \frac{\partial^2 K_S(x)}{\partial S \partial S^\dagger}\bigg|_{(x=x_-)} &= \frac{c x_0^2}{(s+2)^2}\left(2 - \frac{s(s+2)\epsilon}{c x_0^2}\right. \\ &\quad \left. + 2\sqrt{1 - \frac{s(s+2)\epsilon}{c x_0^2}}\right) \end{aligned} \tag{31}$$

$$\sim \begin{cases} \frac{4}{(s+2)^2} c x_0^2 & (\epsilon \ll \frac{c x_0^2}{s(s+2)}) \\ \frac{1}{(s+2)^2} c x_0^2 & (\epsilon \sim \frac{c x_0^2}{s(s+2)}). \end{cases} \tag{32}$$

Namely, $\frac{\partial^2 K_S}{\partial S \partial S^\dagger} \sim \frac{x_0^2}{s^2} \ll 1$ for $s \gg 1$ and $c \sim 1$. Therefore, $F_S \sim s^2 \lambda^s / x_0^2$. From the scalar potential

$$V \sim \frac{\partial^2 K_S}{\partial S \partial S^\dagger} |F_S|^2 + \frac{1}{2} D_A^2, \tag{33}$$

we obtain

$$D_A \sim \frac{s^3}{x_0^2} \lambda^{2s-2}. \tag{34}$$

In the previous sections, we obtained $D_A \sim s\lambda^{2s-2}$, which is much larger than $|F_S|^2 \sim \lambda^{2s}$. In the scenario in which the FI term is dynamically determined, the ratio $D_A/|F_S|^2$ becomes smaller.

Let us examine the concrete values of the parameters. To obtain $F_S \sim O(100\ \text{GeV})$, $s \sim 28$ is required for $\lambda = 0.2$, $\Lambda = 10^{18}$ GeV, $c = 1$, and $x_0 = 1$. Then, to satisfy the condition Eq. (29), the parameter ϵ must be smaller than 10^{-3}. The ratio $D_A/|F_S|^2$ becomes of order 1. This may be important in applying this mechanism of SUSY breaking to the realistic SUSY breaking models.

The lifetime of this metastable vacuum can easily be longer than the age of the Universe. Let us estimate the lifetime of the metastable vacuum by the successive substitution $V_h = V(x_+) - V(x_-)$, $\epsilon' = V(x_-)$, and $\Delta = x_+ - x_-$ in Eqs. (14) and (15). If we take the parameters $s = 28$, $c = 1$, $\epsilon = 10^{-3}$, and $x_0 = 1$, which satisfy the condition (29), then P can be estimated as

$$P > \frac{(\sqrt{V_h}\Delta)^4}{(\epsilon')^3} \sim 10^{31}. \tag{35}$$

Therefore, the lifetime of the metastable vacuum becomes much longer than the age of the Universe.

Note that we do not use extra SQCD dynamics to break SUSY and/or stabilize the moduli. In this scenario, the SUSY breaking scale, which is much smaller than the Planck scale, is obtained by the smallness of the FI parameter and the large anomalous $U(1)_A$ charge of the S field. Therefore, this new scenario for spontaneous SUSY breaking is economical. This is one of the most crucial differences between this scenario and the previously proposed scenarios [33–39].

V. DISCUSSION AND CONCLUSION

We proposed a new SUSY breaking scenario which can be applied to most of the phenomenologically viable models with anomalous $U(1)_A$ gauge symmetry. Even without $U(1)_R$ symmetry, which usually plays an essential role in breaking SUSY spontaneously, SUSY can be broken spontaneously because the SUSY breaking vacua are metastable. Moreover, we examined the moduli stabilization in this scenario. And we found that stabilization is possible by the deformation of the Kähler potential, though some tuning of parameters is required. It is important that the stabilization of the moduli can be realized without the superpotential being dependent on the moduli, because such a superpotential generically spoils the SUSY zero mechanism which plays a critical role in obtaining phenomenologically viable models.

One of the possible applications of this SUSY breaking scenario is that SUSY is spontaneously broken in the hidden sector by this scenario instead of the dynamical SUSY breaking scenario. Another interesting and important objective is to examine the possibility that this SUSY breaking mechanism is applied in the visible sector and that the realistic mass spectrum of superpartners of standard model particles is obtained at the same time; i.e., the hidden sector is not needed. Unfortunately, there are several obstacles. One of the most serious issues is that the gravity-mediated gaugino masses become $\lambda^{2s}\Lambda$, which is much smaller than the typical scalar fermion mass scale $F_S/\Lambda = \lambda^s\Lambda$. This is because the S field has a nonvanishing $U(1)_A$ charge. Gauge mediation is an interesting possibility to avoid this obstacle, though there is the μ problem. Another obstacle is that comparatively large D_A can induce FCNC that are too large, though stabilizing the moduli makes this issue milder. We think that this is an interesting and challenging future subject.

Anomalous $U(1)_A$ gauge symmetry plays an important role in solving various problems in the SUSY GUT scenario, such as the doublet-triplet splitting problem, the proton stability problem [19,25], an unrealistic GUT relation for the Yukawa couplings [18,19], the μ problem [47,48], etc., and in realizing the natural gauge coupling unification. It is quite impressive that these problems can be realized with a reasonable assumption that all terms which are allowed by the symmetry of the theory are introduced with $O(1)$ coefficients, and therefore, all the mass scales can be fixed by the symmetry of the theory. We thought that we needed an additional sector inducing SUSY breaking, which is called the "hidden sector," in any SUSY models with anomalous $U(1)_A$ symmetry. However, this may not be the case. By adding just one positively charged field to a phenomenologically viable model, spontaneous SUSY breaking is realized. Thus, we expect more complete models in which, in addition to the previous advantages of the models with anomalous $U(1)_A$ symmetry, SUSY breaking is also controlled by anomalous $U(1)_A$ gauge symmetry.

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