Covariant light-front approach for B_c transition form factors

Wei Wang,[1] Yue-Long Shen,[2] and Cai-Dian Lü[1,3]

[1]*Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, People's Republic of China*
[2]*Institute of Information Science and Engineering, Ocean University of China, Qingdao 266100, People's Republic of China*
[3]*Kavli Institute for Theoretical Physics China, Chinese Academy of Sciences, Beijing 100190, People's Republic of China*
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In the covariant light-front quark model, we investigate the form factors of B_c decays into D, D^*, D_s, D_s^*, η_c, J/ψ, B, B^*, B_s, and B_s^* mesons. The form factors in the spacelike region are directly evaluated. To extrapolate the form factors to the physical region, we fit the form factors by adopting a suitable three-parameter form. At the maximally recoiling point, $b \to u$, d, and s transition form factors are smaller than $b \to c$ and $c \to d$, s form factors, while the $b \to u$, d, s, and c form factors at the zero-recoiling point are close to each other. In the fitting procedure, we find that parameters in $A_2^{B_cB^*}$ and $A_2^{B_cB_s^*}$ strongly depend on decay constants of B^* and B_s^* mesons. Fortunately, semileptonic and nonleptonic B_c decays are not sensitive to these two form factors. We also investigate branching fractions, polarizations of the semileptonic B_c decays. $B_c \to (\eta_c, J/\psi)l\nu$ and $B_c \to (B_s, B_s^*)l\nu$ decays have much larger branching fractions than $B_c \to (D, D^*, B, B^*)l\nu$. For the three kinds of $B_c \to Vl\nu$ decays, longitudinal contributions are comparable with the transverse contributions. These predictions will be tested on the ongoing and forthcoming hadron colliders.

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I. INTRODUCTION

B meson decays provide a golden place to extract magnitudes and phases of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, which can test the origins of CP violation in and beyond the standard model (SM). There has been remarkable progress in the study of semileptonic and nonleptonic B meson decays. Experimentally, the two B factories have accumulated more than 10^9 $B\bar{B}$ events. Some rare decays with branching fractions of the order 10^{-7} have been observed. On the theoretical side, great success has also been achieved: apart from contributions proportional to the form factors, the so-called nonfactorizable diagrams and some other radiative corrections are taken into account. All of them make B physics suitable for the precise test of the SM and the search of new phenomena (see Ref. [1] for a recent review).

Compared with B mesons, the B_c meson is heavier: the mass of a $B_c\bar{B}_c$ pair has exceeded the threshold of $\Upsilon(4S)$, thus B_c mesons cannot be produced on the B factories. But the B_c meson has a promising prospect on the hadron colliders. The CERN LHC experiment, which is scheduled to run in the very near future, will produce plenty of B_c events. With more data accumulated in the future, the study on B_c mesons will be of great importance. The B_c meson can decay not only via the $b \to q$ ($q = u$, d, s, c) transition like the lighter $B_{u,d,s}$ mesons, but also through the $c \to q$ ($q = u$, d, s) transitions. The CKM matrix element in the $c \to s$ transition $|V_{cs}| \sim 1$ is much larger than the CKM matrix element $|V_{cb}| \sim 0.04$ in the $b \to c$ transition. Although the phase space in $c \to d$, s decays is smaller than that in $b \to c$ transition, the former decays provide about 70% to the decay width of B_c. This results in a larger decay width and a much smaller lifetime for the B_c meson: $\tau_{B_c} < \frac{1}{3}\tau_B$. The two heavy b and $\bar{c}$ quarks can annihilate to provide a new kind of weak decays with sizable partial decay widths. The purely leptonic annihilation decay $B_c \to l\bar{\nu}$ can be used to extract the decay constant of B_c and the CKM matrix element V_{cb}.

Semileptonic B_c decays are much simpler than nonleptonic decays: the leptonic part can be straightforwardly evaluated using perturbation theory leaving only hadronic form factors. In two-body nonleptonic B_c decays, most channels are also dominated by the B_c transition form factors. Thus the B_c transition form factors have already received considerable theoretical interest [2–16]. In the present work, we will use the light-front quark model to analyze these form factors. The light-front QCD approach has some unique features, which are particularly suitable to describe a hadronic bound state [17]. Based on this approach, a light-front quark model with many advantages is developed [18–22]. This model provides a relativistic treatment of the hadron and also gives a full treatment of the hadron spin by using the so-called Melosh rotation. The light-front wave functions, which describe the hadrons in terms of their fundamental quark and gluon degrees of freedom, are independent of the hadron momentum and thus are explicitly Lorentz invariant. In the covariant light-front quark model [22], the spurious contribution, which is dependent on the orientation of the light-front, becomes irrelevant in the study of decay constants and form factors and that makes the light-front quark model more self-consistent. This covariant model has been successfully extended to investigate the decay constants and form factors of the s-wave and p-wave mesons [23–25], the heavy quarkonium [26].

Our paper is organized as follows. The formalism of the covariant light-front quark model is presented in the next section. Numerical results for the form factors and decay rates of semileptonic B_c decays are given in Sec. III. We also compare our predictions of form factors with those evaluated in the literature. Our conclusions are given in Sec. IV. In Appendix A, we give the relation between the form factors defined in various studies on B_c decays and the widely used Bauer-Stech-Wirbel (BSW) form factors [27]. In Appendix B, we collect some specific rules when performing the p^- integration.

II. COVARIANT LIGHT-FRONT QUARK MODEL

$B_c \to P$ and V (P, V denotes a pseudoscalar and a vector meson, respectively) form factors induced by vector and axial-vector currents are defined by

$$\langle P(P'')|V_\mu|B_c(P')\rangle = f_+(q^2)P_\mu + f_-(q^2)q_\mu, \tag{1}$$

$$\langle V(P'', \varepsilon''^*)|V_\mu|B_c(P')\rangle = \epsilon_{\mu\nu\alpha\beta}\varepsilon''^{*\nu}P^\alpha q^\beta g(q^2), \tag{2}$$

$$\langle V(P'', \varepsilon''^*)|A_\mu|B_c(P')\rangle = -i\{\varepsilon''^*_\mu f(q^2) + \varepsilon''^* \cdot P[P_\mu a_+(q^2) + q_\mu a_-(q^2)]\}, \tag{3}$$

where $P = P' + P''$, $q = P' - P''$, and the convention $\epsilon_{0123} = 1$ is adopted. The vector and axial-vector currents are defined as $\bar\psi\gamma_\mu\psi'$ and $\bar\psi\gamma_\mu\gamma_5\psi'$. In the $b \to q$ ($q = u, d, s$, and c) transition, ψ and ψ' denote the q quark field and the b quark field, respectively; while in the $c \to q'$ ($q' = u, d$, and s) transition, ψ and ψ' denote the q' quark field and the c quark field, respectively. In the literature, the BSW [27] form factors are more frequently used:

$$\langle P(P'')|V_\mu|B_c(P')\rangle = \left(P_\mu - \frac{m_{B_c}^2 - m_P^2}{q^2}q_\mu\right)F_1^{B_cP}(q^2) + \frac{m_{B_c}^2 - m_P^2}{q^2}q_\mu F_0^{B_cP}(q^2), \tag{4}$$

$$\langle V(P'', \varepsilon''^*)|V_\mu|B_c(P')\rangle = -\frac{1}{m_{B_c} + m_V}\epsilon_{\mu\nu\alpha\beta}\varepsilon''^{*\nu}P^\alpha q^\beta \times V^{B_cV}(q^2), \tag{5}$$

$$\begin{aligned}\langle V(P'', \varepsilon''^*)|A_\mu|B_c(P')\rangle = i\Big\{&(m_{B_c} + m_V)\varepsilon''^*_\mu A_1^{B_cV}(q^2) \\ &- \frac{\varepsilon''^* \cdot P}{m_{B_c} + m_V}P_\mu A_2^{B_cV}(q^2) \\ &- 2m_V\frac{\varepsilon''^* \cdot P}{q^2}q_\mu[A_3^{B_cV}(q^2) \\ &- A_0^{B_cV}(q^2)]\Big\}.\end{aligned} \tag{6}$$

These two kinds of form factors are related to each other via

$$\begin{aligned}F_1^{B_cP}(q^2) &= f_+(q^2),\\ F_0^{B_cP}(q^2) &= f_+(q^2) + \frac{q^2}{m_{B_c}^2 - m_P^2}f_-(q^2),\\ V^{B_cV}(q^2) &= -(m_{B_c} + m_V)g(q^2),\\ A_1^{B_cV}(q^2) &= -\frac{f(q^2)}{m_{B_c} + m_V},\\ A_2^{B_cV}(q^2) &= (m_{B_c} + m_V)a_+(q^2),\\ A_3^{B_cV}(q^2) - A_0^{B_cV}(q^2) &= \frac{q^2}{2m_V}a_-(q^2),\end{aligned} \tag{7}$$

with $A_3^{B_cV}(0) = A_0^{B_cV}(0)$, and

$$A_3^{B_cV}(q^2) = \frac{m_{B_c} + m_V}{2m_V}A_1^{B_cV}(q^2) - \frac{m_{B_c} - m_V}{2m_V}A_2^{B_cV}(q^2). \tag{8}$$

In the covariant light-front quark model, we will work in the $q^+ = 0$ frame and employ the light-front decomposition of the momentum $P' = (P'^-, P'^+, P'_\perp)$, where $P'^\pm = P'^0 \pm P'^3$, so that $P'^2 = P'^+P'^- - P'^2_\perp$. The incoming and outgoing mesons have the momenta $P' = p'_1 + p_2$ and $P'' = p''_1 + p_2$ and the masses M' and M'', respectively. For the B_c transition form factors, $M' = m_{B_c}$. The quark and antiquark inside the incoming (outgoing) meson have the masses $m_1'^{(\prime\prime)}$ and m_2 and the momenta $p_1'^{(\prime\prime)}$ and p_2, respectively. These momenta can be expressed in terms of the internal variables $(x_i, p'_\perp)$ as

$$p'^+_{1,2} = x_{1,2}P'^+, \qquad p'_{1,2\perp} = x_{1,2}P'_\perp \pm p'_\perp, \tag{9}$$

with $x_1 + x_2 = 1$. Using these internal variables, one can define some useful quantities for the incoming meson:

$$\begin{aligned}M_0'^2 &= (e'_1 + e_2)^2 = \frac{p'^2_\perp + m'^2_1}{x_1} + \frac{p'^2_\perp + m_2^2}{x_2},\\ \tilde M'_0 &= \sqrt{M_0'^2 - (m'_1 - m_2)^2},\\ e_i^{(\prime)} &= \sqrt{m_i^{(\prime)2} + p'^2_\perp + p'^2_z},\\ p'_z &= \frac{x_2M'_0}{2} - \frac{m_2^2 + p'^2_\perp}{2x_2M'_0},\end{aligned} \tag{10}$$

where $e_i^{(\prime)}$ can be interpreted as the energy of the quark or the antiquark and M'_0 can be viewed as the kinematic invariant mass of the meson system. The definition of the internal quantities for the outgoing meson is similar. To compute the hadronic amplitudes, we require the Feynman rules for the meson-quark-antiquark vertices ($i\Gamma'_M$):

$$i\Gamma'_P = H'_P\gamma_5, \tag{11}$$

$$i\Gamma'_V = iH'_V\left[\gamma_\mu - \frac{1}{W'_V}(p'_1 - p_2)_\mu\right]. \qquad (12)$$

For the outgoing meson, one should use $i(\gamma_0 \Gamma'^{\dagger}_M \gamma_0)$ for the relevant vertices.

In the conventional light-front quark model, the constituent quarks are required to be on mass shell and physical quantities can be extracted from the plus component of the current matrix elements. However, this framework suffers from the problem of noncovariance because of the missing zero-mode contributions. In order to solve this problem, Jaus proposed the covariant light-front approach which provides a systematical way to deal with the zero-mode contributions [22]. Physical quantities such as decay constants and form factors can be calculated in terms of Feynman momentum loop integrals which are manifestly covariant. For example, the lowest order contribution to a form factor is depicted in Fig. 1 and the $P \to P$ transition amplitude is given by

$$\mathcal{B}^{PP}_\mu = -i^3 \frac{N_c}{(2\pi)^4}\int d^4 p'_1 \frac{H'_P(H''_P)}{N'_1 N''_1 N_2} S^{PP}_\mu, \qquad (13)$$

where $N'^{(\prime\prime)}_1 = p'^{(\prime\prime)2}_1 - m'^{(\prime\prime)2}_1$, and $N_2 = p_2^2 - m_2^2$.

$$\begin{aligned} S^{PP}_\mu &= \mathrm{Tr}[\gamma_5(\not p''_1 + m''_1)\gamma_\mu(\not p'_1 + m'_1)\gamma_5(-\not p_2 + m_2)] \\ &= 2p'_{1\mu}[M'^2 + M''^2 - q^2 - 2N_2 - (m'_1 - m_2)^2 \\ &\quad - (m''_1 - m_2)^2 + (m'_1 - m''_1)^2] + q_\mu[q^2 - 2M'^2 \\ &\quad + N'_1 - N''_1 + 2N_2 + 2(m'_1 - m_2)^2 - (m'_1 - m''_1)^2] \\ &\quad + P_\mu[q^2 - N'_1 - N''_1 - (m'_1 - m''_1)^2]. \end{aligned} \qquad (14)$$

In practice, we use the light-front decomposition of the loop momentum and perform the integration over the minus component using the contour method. If the covariant vertex functions are not singular when performing the integration, the transition amplitude will pick up the singularities in the antiquark propagator. The integration then leads to

$$\begin{aligned} N'^{(\prime\prime)}_1 &\to \hat N'^{(\prime\prime)}_1 = x_1(M'^{(\prime\prime)2} - M'^{(\prime\prime)2}_0), \\ H'^{(\prime\prime)}_M &\to h'^{(\prime\prime)}_M, \qquad W''_M \to w''_M, \\ \int \frac{d^4 p'_1}{N'_1 N''_1 N_2} H'_P H''_P S &\to -i\pi \int \frac{dx_2 d^2 p'_\perp}{x_2 \hat N'_1 \hat N''_1} h'_P h''_P \hat S, \end{aligned} \qquad (15)$$

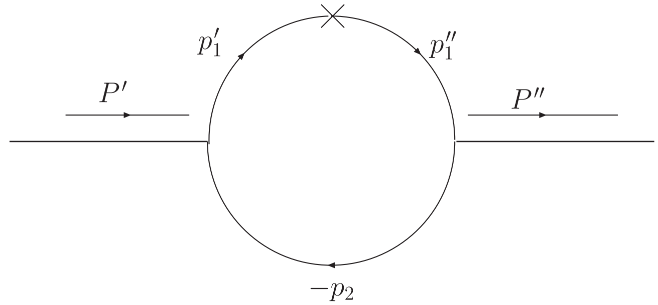

FIG. 1. Feynman diagram for $B_c \to P$ and V decay amplitudes. The X in the diagram denotes the vector or axial-vector transition vertex while the meson-quark-antiquark vertices are given in the text.

where

$$M''^2_0 = \frac{p''^2_\perp + m''^2_1}{x_1} + \frac{p''^2_\perp + m_2^2}{x_2}, \qquad (16)$$

with $p''_\perp = p'_\perp - x_2 q_\perp$. The explicit forms of h'_M and w'_M for the pseudoscalar and vector meson are given by

$$\begin{aligned} h'_P &= h'_V = (M'^2 - M'^2_0)\sqrt{\frac{x_1 x_2}{N_c}}\frac{1}{\sqrt{2}\tilde M'_0}\varphi', \\ w'_V &= M'_0 + m'_1 + m_2, \end{aligned} \qquad (17)$$

where φ' is the light-front wave function for pseudoscalar and vector mesons. After this integration, the conventional light-front model is recovered but manifestly the covariance is lost as it receives additional spurious contributions proportional to the lightlike four vector $\tilde\omega = (0, 2, \mathbf{0}_\perp)$. The undesired spurious contributions can be eliminated by the inclusion of the zero-mode contribution which amounts to performing the p^- integration in a proper way. The specific rules under this p^- integration are derived in Refs. [22,23] and the relevant ones in this work are collected in Appendix B.

Using Eqs. (14)–(17) and taking the advantage of the rules in Refs. [22,23], we obtain expressions for the $P \to P$ form factors:

$$f_+(q^2) = \frac{N_c}{16\pi^3}\int dx_2 d^2 p'_\perp \frac{h'_P h''_P}{x_2 \hat N'_1 \hat N''_1}[x_1(M'^2_0 + M''^2_0) + x_2 q^2 - x_2(m'_1 - m''_1)^2 - x_1(m'_1 - m_2)^2 - x_1(m''_1 - m_2)^2],$$

$$\begin{aligned} f_-(q^2) &= \frac{N_c}{16\pi^3}\int dx_2 d^2 p'_\perp \frac{2h'_P h''_P}{x_2 \hat N'_1 \hat N''_1}\Big\{-x_1 x_2 M'^2 - p'^2_\perp - m'_1 m_2 + (m''_1 - m_2)(x_2 m'_1 + x_1 m_2) \\ &\quad + 2\frac{q\cdot P}{q^2}\left(p'^2_\perp + 2\frac{(p'_\perp\cdot q_\perp)^2}{q^2}\right) + 2\frac{(p'_\perp\cdot q_\perp)^2}{q^2} - \frac{p'_\perp\cdot q_\perp}{q^2}[M''^2 - x_2(q^2 + q\cdot P) \\ &\quad - (x_2 - x_1)M'^2 + 2x_1 M'^2_0 - 2(m'_1 - m_2)(m'_1 + m''_1)]\Big\}. \end{aligned} \qquad (18)$$

Similarly, the $P \to V$ transition amplitudes are given by

$$\mathcal{B}_\mu^{PV} = -i^3 \frac{N_c}{(2\pi)^4} \int d^4 p_1' \frac{H_P'(iH_V'')}{N_1' N_1'' N_2} S_{\mu\nu}^{PV} \varepsilon''^{*\nu}, \tag{19}$$

where

$$\begin{aligned}
S_{\mu\nu}^{PV} &= (S_V^{PV} - S_A^{PV})_{\mu\nu} = \mathrm{Tr}\Big[\Big(\gamma_\nu - \frac{1}{W_V''}(p_1'' - p_2)_\nu\Big)(\not p_1'' + m_1'')(\gamma_\mu - \gamma_\mu\gamma_5)(\not p_1' + m_1')\gamma_5(-\not p_2 + m_2)\Big] \\
&= -2i\epsilon_{\mu\nu\alpha\beta}\{p_1'^\alpha P^\beta(m_1'' - m_1') + p_1'^\alpha q^\beta(m_1'' + m_1' - 2m_2) + q^\alpha P^\beta m_1'\} + \frac{1}{W_V''}(4p_{1\nu}' - 3q_\nu - P_\nu) i\epsilon_{\mu\alpha\beta\rho} p_1'^\alpha q^\beta P^\rho \\
&\quad + 2g_{\mu\nu}\{m_2(q^2 - N_1' - N_1'' - m_1'^2 - m_1''^2) - m_1'(M''^2 - N_1'' - N_2 - m_1''^2 - m_2^2) - m_1''(M'^2 - N_1' - N_2 - m_1'^2 - m_2^2) \\
&\quad - 2m_1' m_1'' m_2\} + 8p_{1\mu}' p_{1\nu}'(m_2 - m_1') - 2(P_\mu q_\nu + q_\mu P_\nu + 2q_\mu q_\nu)m_1' + 2p_{1\mu}' P_\nu(m_1' - m_1'') \\
&\quad + 2p_{1\mu}' q_\nu(3m_1' - m_1'' - 2m_2) + 2P_\mu p_{1\nu}'(m_1' + m_1'') + 2q_\mu p_{1\nu}'(3m_1' + m_1'' - 2m_2) + \frac{1}{2W_V''}(4p_{1\nu}' - 3q_\nu - P_\nu) \\
&\quad \times\{2p_{1\mu}'[M'^2 + M''^2 - q^2 - 2N_2 + 2(m_1' - m_2)(m_1'' + m_2)] + q_\mu[q^2 - 2M'^2 + N_1' - N_1'' + 2N_2 - (m_1 + m_1'')^2 \\
&\quad + 2(m_1' - m_2)^2] + P_\mu[q^2 - N_1' - N_1'' - (m_1' + m_1'')^2]\}.
\end{aligned} \tag{20}$$

The above equations give the expression for $P \to V$ form factors:

$$\begin{aligned}
g(q^2) &= -\frac{N_c}{16\pi^3} \int dx_2 d^2 p_\perp' \frac{2h_P' h_V''}{x_2 \hat{N}_1' \hat{N}_1''} \Big\{x_2 m_1' + x_1 m_2 + (m_1' - m_1'')\frac{p_\perp' \cdot q_\perp}{q^2} + \frac{2}{w_V''}\Big[p_\perp'^2 + \frac{(p_\perp' \cdot q_\perp)^2}{q^2}\Big]\Big\}, \\
f(q^2) &= \frac{N_c}{16\pi^3} \int dx_2 d^2 p_\perp' \frac{h_P' h_V''}{x_2 \hat{N}_1' \hat{N}_1''} \Big\{2x_1(m_2 - m_1')(M_0'^2 + M_0''^2) - 4x_1 m_1'' M_0'^2 + 2x_2 m_1' q\cdot P + 2m_2 q^2 - 2x_1 m_2(M'^2 + M''^2) \\
&\quad + 2(m_1' - m_2)(m_1' + m_1'')^2 + 8(m_1' - m_2)\Big[p_\perp'^2 + \frac{(p_\perp' \cdot q_\perp)^2}{q^2}\Big] + 2(m_1' + m_1'')(q^2 + q\cdot P)\frac{p_\perp' \cdot q_\perp}{q^2} \\
&\quad - 4\frac{q^2 p_\perp'^2 + (p_\perp' \cdot q_\perp)^2}{q^2 w_V''}\Big[2x_1(M'^2 + M_0'^2) - q^2 - q\cdot P - 2(q^2 + q\cdot P)\frac{p_\perp' \cdot q_\perp}{q^2} - 2(m_1' - m_1'')(m_1' - m_2)\Big]\Big\},
\end{aligned} \tag{21}$$

$$\begin{aligned}
a_+(q^2) &= \frac{N_c}{16\pi^3} \int dx_2 d^2 p_\perp' \frac{2h_P' h_V''}{x_2 \hat{N}_1' \hat{N}_1''} \Big\{(x_1 - x_2)(x_2 m_1' + x_1 m_2) - [2x_1 m_2 + m_1'' + (x_2 - x_1)m_1']\frac{p_\perp' \cdot q_\perp}{q^2} \\
&\quad - 2\frac{x_2 q^2 + p_\perp' \cdot q_\perp}{x_2 q^2 w_V''}[p_\perp' \cdot p_\perp'' + (x_1 m_2 + x_2 m_1')(x_1 m_2 - x_2 m_1'')]\Big\}, \\
a_-(q^2) &= \frac{N_c}{16\pi^3} \int dx_2 d^2 p_\perp' \frac{h_P' h_V''}{x_2 \hat{N}_1' \hat{N}_1''} \Big\{2(2x_1 - 3)(x_2 m_1' + x_1 m_2) - 8(m_1' - m_2)\Big[\frac{p_\perp'^2}{q^2} + 2\frac{(p_\perp' \cdot q_\perp)^2}{q^4}\Big] - [(14 - 12x_1)m_1' \\
&\quad - 2m_1'' - (8 - 12x_1)m_2]\frac{p_\perp' \cdot q_\perp}{q^2} + \frac{4}{w_V''}\Big([M'^2 + M''^2 - q^2 + 2(m_1' - m_2)(m_1'' + m_2)](A_3^{(2)} + A_4^{(2)} - A_2^{(1)}) \\
&\quad + Z_2(3A_2^{(1)} - 2A_4^{(2)} - 1) + \frac{1}{2}[x_1(q^2 + q\cdot P) - 2M'^2 - 2p_\perp' \cdot q_\perp - 2m_1'(m_1'' + m_2) - 2m_2(m_1' - m_2)] \\
&\quad \times (A_1^{(1)} + A_2^{(1)} - 1)q\cdot P\Big[\frac{p_\perp'^2}{q^2} + \frac{(p_\perp' \cdot q_\perp)^2}{q^4}\Big](4A_2^{(1)} - 3)\Big)\Big\}.
\end{aligned}$$

The functions $A_1^{(1)}$, $A_2^{(1)}$, $A_3^{(2)}$, $A_4^{(2)}$, and Z_2 are given in Appendix B. Expressions for the BSW form factors can be directly obtained through the simple relation given in Eq. (7).

III. NUMERICAL RESULTS

The $\bar{q}q$ meson state is described by the light-front wave function which can be obtained by solving the relativistic

Schrödinger equation. But in fact except for some limited cases, the exact solution is not obtainable. In practice, we usually prefer to employ a phenomenological wave function to describe the hadronic structure. In this work, we will use the simple Gaussian-type wave function which has been extensively examined in the literature:

$$\varphi' = \varphi'(x_2, p'_\perp) = 4\left(\frac{\pi}{\beta'^2}\right)^{3/4}\sqrt{\frac{dp'_z}{dx_2}}\exp\left(-\frac{p'^2_z + p'^2_\perp}{2\beta'^2}\right),$$
$$\frac{dp'_z}{dx_2} = \frac{e'_1 e_2}{x_1 x_2 M'_0}. \tag{22}$$

The parameter β', which describes the momentum distribution, is expected to be of order $\Lambda_{\rm QCD}$. It is usually fixed by the meson's decay constant whose analytic expression in the covariant light-front model is given in [23]. The decay constant of $f_{J/\psi}$ can be determined by the leptonic decay width

$$\Gamma_{ee} \equiv \Gamma(J/\psi \to e^+e^-) = \frac{4\pi\alpha^2_{em}Q^2_c f^2_{J/\psi}}{3m_{J/\psi}}, \tag{23}$$

where $Q_c = +2/3$ denotes the electric charge of the charm quark. Using the measured results for the electronic width of J/ψ [28]:

$$\Gamma_{ee} = (5.55 \pm 0.14 \pm 0.02)\ \text{keV}, \tag{24}$$

we obtain $f_{J/\psi} = (416 \pm 5)$ MeV. Under the factorization assumption, the decay constant of η_c has been extracted by the CLEO Collaboration from $B \to \eta_c K$ decays [29]:

$$f_{\eta_c} = (335 \pm 75)\ \text{MeV}, \tag{25}$$

where the central value is about 20% smaller than that of J/ψ. In this work, we will assume the same decay constant for η_c as that of J/ψ. We also introduce an uncertainty of 20% to this value. Decay constants for charged pseudoscalars are usually derived through the purely leptonic decays:

$$\Gamma(P \to l\bar\nu) = \frac{G^2_F|V_{\rm CKM}|^2}{8\pi} f^2_P m^2_l m_P\left(1 - \frac{m^2_l}{m^2_P}\right)^2. \tag{26}$$

The experimental results for the decay constants of charmed mesons are averaged as [30]

$$f_{D_s} = (273 \pm 10)\ \text{MeV}, \qquad f_D = (205.8 \pm 8.9)\ \text{MeV}. \tag{27}$$

As clearly shown in the above equation, the uncertainties for these decay constants are less than 5%. It provides a solid foundation for the precise study on B_c transition form factors. In the heavy quark limit, the decay constant f_{D^*} of a vector heavy meson D^* is related to that of a pseudoscalar meson through

$$f_{D^*} = f_D \times \sqrt{\frac{m_D}{m_{D^*}}}, \tag{28}$$

where m_D and m_{D^*} denote the masses of the pseudoscalar and vector mesons, respectively. That implies $f_{D^*} < f_D$ since $m_{D^*} > m_D$. In the following, we will use the same values for the decay constant of the vectors and pseudoscalars. To compensate the differences, we will also introduce an uncertainty of 10% to the decay constants. Decay constants for the bottom mesons are employed by

$$f_{B_c} = (400 \pm 40)\ \text{MeV}, \qquad f_B = (190 \pm 20)\ \text{MeV},$$
$$f_{B_s} = (230 \pm 20)\ \text{MeV}. \tag{29}$$

These values are slightly smaller than results provided by lattice QCD [31]:

$$f_{B_s} = (253 \pm 8 \pm 7)\ \text{MeV},$$
$$f_{B_c} = (489 \pm 4 \pm 3)\ \text{MeV}. \tag{30}$$

Decay constants of the vector B mesons are used as $f_{B^*} = (210 \pm 20)$ MeV and $f_{B^*_s} = (260 \pm 20)$ MeV which are about 10% larger than those of the pseudoscalar B mesons. Shape parameters β's determined from these decay constants, together with the constituent quark masses used in the calculation, are shown in Table I. The consistent quark masses are close to the ones used in Refs. [23,24]. To estimate the uncertainties caused by these quark masses, we will introduce the uncertainties of 0.03 and 0.1 GeV to the light quark masses and the heavy quark masses, respectively. The masses (in units of GeV) of hadrons are used as [28]

$$m_{B_c} = 6.276, \qquad m_D = 1.8645, \qquad m_{D^*} = 2.0067,$$
$$m_{D_s} = 1.9682, \qquad m_{D^*_s} = 2.112, \qquad m_{\eta_c} = 2.9804,$$
$$m_{J/\psi} = 3.0969, \qquad m_B = 5.279, \qquad m_{B^*} = 5.325,$$
$$m_{B_s} = 5.3675, \qquad m_{B^*_s} = m_{B^*} + m_{B_s} - m_B. \tag{31}$$

If a light meson is emitted in exclusive nonleptonic decays, only the form factor at the maximally recoiling point ($q^2 \simeq 0$) is required but the q^2-dependent behavior in the full $q^2 > 0$ region is required in semileptonic B_c decays. Because of the condition $q^+ = 0$ imposed during the course of calculation, form factors can be directly studied only at spacelike momentum transfer $q^2 = -q^2_\perp \le 0$, which are not relevant for the semileptonic processes. It

TABLE I. Input parameters m_q and β' (in units of GeV) in the Gaussian-type light-front wave function (22). Uncertainties of β' are from the decay constants as discussed in the text.

$m_{u,d}$	m_s	m_c	m_b
0.25	0.37	1.4	4.8
β'_D	β'_{D^*}	β'_{D_s}	$\beta'_{D^*_s}$
$0.466^{+0.022}_{-0.021}$	$0.366^{+0.010}_{-0.010}$	$0.600^{+0.026}_{-0.025}$	$0.438^{+0.010}_{-0.010}$
β'_B	β'_{B^*}	β'_{B_s}	$\beta'_{B^*_s}$
$0.555^{+0.048}_{-0.048}$	$0.528^{+0.033}_{-0.034}$	$0.626^{+0.045}_{-0.045}$	$0.599^{+0.033}_{-0.032}$
β'_{η_c}	$\beta'_{J/\psi}$	β'_{B_c}	
$0.814^{+0.092}_{-0.086}$	$0.632^{+0.005}_{-0.005}$	$0.890^{+0.075}_{-0.074}$	

has been proposed in [23] to parametrize form factors as explicit functions of q^2 in the spacelike region and one can analytically extend them to the timelike region. To shed light on the momentum dependence, we will choose the parametrization for the b quark decays:

$$F(q^2) = F(0) \exp(c_1 \hat{s} + c_2 \hat{s}^2), \tag{32}$$

where $\hat{s} = q^2/m_{B_c}^2$ and F denotes any one of the form factors F_1, F_0 and V, A_0, A_1, and A_2. But for $c \to u$, d, and s transitions, we find that the fitted values for the two parameters c_1 and c_2 are not stable and thus we adopt the optional three-parameter form:

$$F(q^2) = \frac{F(0)}{1 - \frac{q^2}{m_{\rm fit}^2} + \delta(\frac{q^2}{m_{\rm fit}^2})^2}. \tag{33}$$

In the procedure to fit the form factors $A_2^{B_cB^*}$ and $A_2^{B_cB_s^*}$, we find that the shape parameters ($m_{\rm fit}$, δ) strongly depend on the decay constants f_{B^*} and $f_{B_s^*}$. In this case, our predictions on these two form factors are unreliable; thus we refrain from predicting these two form factors. Fortunately, the ambiguity of $A_2^{B_cB^*}$ and $A_2^{B_cB_s^*}$ will not affect the physical quantities in various physical decay channels. As we can see from Eq. (8), the masses of B^* and B_s^* mesons are very close to that of B_c; thus the second term on the right-hand side is negligible. The form factor A_0, which is relevant for the nonleptonic $B_c \to B^*(B_s^*)P$ decays, receives small contributions from A_2. Contributions from A_2 to the $B_c \to B^*(B_s^*)l\nu$ decays and $B_c \to B^*(B_s^*)V$ decays are also small which will be shown in the following.

Our predictions of the remanent form factors are collected in Tables II and III. The first kind of uncertainties shown in these tables is from those in decay constants of the B_c meson and the final mesons, while the second kind of uncertainties is from those in the constituent quark masses. Several remarks are given in order. First, from these two tables, we can see that the $B_c \to D$, D^*, D_s, and D_s^* form factors at the maximally recoiling point ($q^2 = 0$) are smaller than the other ones. It can be understood as follows. In $B_c \to D$, D^*, D_s, and D_s^* transitions, the initial charm quark is almost at rest and its momentum is of order m_c. In the final state, the meson moves very fast and the charm quark tends to have a very large momentum of order m_b. In this transition, the overlap between the wave functions is limited which will produce small values for the form factors. In $B_c \to \eta_c$, J/ψ transitions, the spectator charm antiquark in η_c, J/ψ play the same role with the charm quark generated from the weak vertex. The light-front wave function of the charmonium is expected to have a maximum at

$$E = \frac{m_{B_c}^2 + m_{\eta_c}^2}{4m_{B_c}} \sim \frac{m_{B_c}}{4} \approx m_c.$$

The overlap between the initial and final states' light-front wave functions in $B_c \to \eta_c$ and J/ψ becomes larger, which certainly induces larger form factors. It is also similar for the $B_c \to B$ and B_s form factors. Secondly, the $B_c \to D_s$ and η_c form factors at the zero-recoiling point are close to each other. The initial charm quark is almost at rest and its momentum is of order m_c. In these two kinds of transitions, the charm spectator in the final states tends to posses a momentum of order m_c. The over-

TABLE II. $B_c \to D$, D^*, D_s, D_s^*, η_c, and J/ψ form factors in the light-front quark model. The uncertainties are from the B_c decay constants and the decay constant of the final state mesons.

F	$F(0)$	$F(q^2_{\rm max})$	c_1	c_2
$F_1^{B_cD}$	$0.16^{+0.02+0.02}_{-0.02-0.01}$	$1.10^{+0.07+0.11}_{-0.07-0.10}$	$3.46^{+0.24+0.19}_{-0.22-0.19}$	$0.90^{+0.05+0.06}_{-0.05-0.06}$
$F_0^{B_cD}$	$0.16^{+0.02+0.02}_{-0.02-0.01}$	$0.59^{+0.02+0.05}_{-0.02-0.05}$	$2.41^{+0.22+0.17}_{-0.20-0.17}$	$0.47^{+0.04+0.04}_{-0.04-0.04}$
$V^{B_cD^*}$	$0.13^{+0.01+0.02}_{-0.02-0.02}$	$1.16^{+0.08+0.16}_{-0.07-0.14}$	$4.21^{+0.30+0.25}_{-0.27-0.25}$	$1.09^{+0.07+0.07}_{-0.06-0.07}$
$A_0^{B_cD^*}$	$0.09^{+0.01+0.01}_{-0.01-0.01}$	$0.79^{+0.06+0.09}_{-0.05-0.08}$	$4.18^{+0.30+0.27}_{-0.27-0.27}$	$0.96^{+0.06+0.08}_{-0.07-0.06}$
$A_1^{B_cD^*}$	$0.08^{+0.01+0.01}_{-0.01-0.01}$	$0.42^{+0.02+0.05}_{-0.01-0.04}$	$3.18^{+0.28+0.24}_{-0.25-0.23}$	$0.65^{+0.06+0.06}_{-0.04-0.05}$
$A_2^{B_cD^*}$	$0.07^{+0.01+0.01}_{-0.01-0.01}$	$0.51^{+0.01+0.07}_{-0.01-0.06}$	$3.78^{+0.26+0.23}_{-0.24-0.23}$	$0.80^{+0.04+0.06}_{-0.04-0.05}$
$F_1^{B_cD_s}$	$0.28^{+0.02+0.02}_{-0.02-0.02}$	$1.24^{+0.04+0.09}_{-0.05-0.09}$	$2.78^{+0.17+0.14}_{-0.16-0.14}$	$0.72^{+0.03+0.04}_{-0.03-0.04}$
$F_0^{B_cD_s}$	$0.28^{+0.02+0.02}_{-0.02-0.02}$	$0.68^{+0.01+0.04}_{-0.01-0.04}$	$1.72^{+0.15+0.12}_{-0.14-0.12}$	$0.27^{+0.05+0.02}_{-0.05-0.02}$
$V^{B_cD_s^*}$	$0.23^{+0.02+0.03}_{-0.02-0.02}$	$1.36^{+0.07+0.16}_{-0.07-0.14}$	$3.63^{+0.23+0.21}_{-0.21-0.21}$	$0.95^{+0.04+0.06}_{-0.04-0.06}$
$A_0^{B_cD_s^*}$	$0.17^{+0.01+0.01}_{-0.01-0.01}$	$0.94^{+0.06+0.08}_{-0.05-0.08}$	$3.58^{+0.23+0.22}_{-0.21-0.23}$	$0.83^{+0.06+0.06}_{-0.04-0.06}$
$A_1^{B_cD_s^*}$	$0.14^{+0.01+0.02}_{-0.01-0.01}$	$0.51^{+0.01+0.04}_{-0.01-0.04}$	$2.62^{+0.21+0.19}_{-0.19-0.19}$	$0.53^{+0.03+0.04}_{-0.03-0.04}$
$A_2^{B_cD_s^*}$	$0.12^{+0.01+0.02}_{-0.01-0.02}$	$0.57^{+0.01+0.06}_{-0.02-0.06}$	$3.18^{+0.19+0.18}_{-0.18-0.18}$	$0.66^{+0.03+0.04}_{-0.04-0.04}$
$F_1^{B_c\eta_c}$	$0.61^{+0.03+0.01}_{-0.04-0.01}$	$1.09^{+0.00+0.05}_{-0.02-0.05}$	$1.99^{+0.22+0.08}_{-0.20-0.08}$	$0.44^{+0.05+0.02}_{-0.05-0.02}$
$F_0^{B_c\eta_c}$	$0.61^{+0.03+0.01}_{-0.04-0.01}$	$0.86^{+0.02+0.04}_{-0.03-0.04}$	$1.18^{+0.26+0.09}_{-0.24-0.09}$	$0.17^{+0.09+0.02}_{-0.09-0.02}$
$V^{B_cJ/\psi}$	$0.74^{+0.01+0.03}_{-0.01-0.03}$	$1.45^{+0.03+0.09}_{-0.04-0.08}$	$2.46^{+0.13+0.10}_{-0.13-0.10}$	$0.56^{+0.02+0.03}_{-0.03-0.03}$
$A_0^{B_cJ/\psi}$	$0.53^{+0.01+0.02}_{-0.01-0.02}$	$1.02^{+0.02+0.07}_{-0.02-0.07}$	$2.39^{+0.13+0.11}_{-0.13-0.11}$	$0.50^{+0.02+0.02}_{-0.03-0.02}$
$A_1^{B_cJ/\psi}$	$0.50^{+0.01+0.02}_{-0.02-0.02}$	$0.80^{+0.00+0.05}_{-0.01-0.05}$	$1.73^{+0.12+0.12}_{-0.12-0.12}$	$0.33^{+0.01+0.02}_{-0.02-0.02}$
$A_2^{B_cJ/\psi}$	$0.44^{+0.02+0.02}_{-0.03-0.02}$	$0.81^{+0.02+0.05}_{-0.03-0.04}$	$2.22^{+0.11+0.11}_{-0.10-0.11}$	$0.45^{+0.01+0.02}_{-0.01-0.02}$

TABLE III. Results for the $B_c \to B$, B^*, B_s, and B_s^* form factors in the light-front quark model. The uncertainties are from the B_c decay constants and the decay constant of the final state mesons.

F	$F(0)$	$F(q^2_{\rm max})$	$m_{\rm fit}$	δ
$F_1^{B_cB}$	$0.63^{+0.04+0.03}_{-0.05-0.03}$	$0.96^{+0.05+0.08}_{-0.07-0.07}$	$1.19^{+0.09+0.01}_{-0.09-0.01}$	$0.33^{+0.04+0.01}_{-0.04-0.01}$
$F_0^{B_cB}$	$0.63^{+0.04+0.03}_{-0.05-0.03}$	$0.81^{+0.02+0.06}_{-0.03-0.05}$	$1.52^{+0.22+0.02}_{-0.19-0.02}$	$0.52^{+0.16+0.02}_{-0.10-0.02}$
$V^{B_cB^*}$	$3.29^{+0.17+0.32}_{-0.21-0.30}$	$4.89^{+0.19+0.61}_{-0.27-0.53}$	$2.65^{+0.13+0.05}_{-0.14-0.06}$	$1.75^{+0.27+0.10}_{-0.22-0.11}$
$A_0^{B_cB^*}$	$0.47^{+0.01+0.04}_{-0.01-0.04}$	$0.68^{+0.01+0.07}_{-0.02-0.07}$	$0.99^{+0.04+0.04}_{-0.04-0.04}$	$0.31^{+0.03+0.02}_{-0.03-0.02}$
$A_1^{B_cB^*}$	$0.43^{+0.01+0.04}_{-0.01-0.04}$	$0.57^{+0.00+0.06}_{-0.01-0.06}$	$1.16^{+0.07+0.03}_{-0.07-0.03}$	$0.27^{+0.03+0.01}_{-0.03-0.02}$
$F_1^{B_cB_s}$	$0.73^{+0.03+0.03}_{-0.04-0.03}$	$1.01^{+0.02+0.07}_{-0.04-0.06}$	$1.35^{+0.07+0.01}_{-0.08-0.01}$	$0.35^{+0.04+0.00}_{-0.04-0.01}$
$F_0^{B_cB_s}$	$0.73^{+0.03+0.03}_{-0.04-0.03}$	$0.87^{+0.00+0.05}_{-0.02-0.05}$	$1.77^{+0.24+0.04}_{-0.20-0.04}$	$0.60^{+0.23+0.04}_{-0.14-0.04}$
$V^{B_cB_s^*}$	$3.62^{+0.12+0.31}_{-0.15-0.29}$	$4.93^{+0.14+0.53}_{-0.19-0.47}$	$2.94^{+0.11+0.04}_{-0.11-0.05}$	$1.78^{+0.25+0.07}_{-0.21-0.08}$
$A_0^{B_cB_s^*}$	$0.56^{+0.00+0.04}_{-0.01-0.04}$	$0.75^{+0.00+0.07}_{-0.01-0.07}$	$1.13^{+0.03+0.04}_{-0.04-0.04}$	$0.33^{+0.03+0.02}_{-0.03-0.02}$
$A_1^{B_cB_s^*}$	$0.52^{+0.00+0.04}_{-0.01-0.04}$	$0.64^{+0.00+0.06}_{-0.01-0.06}$	$1.33^{+0.07+0.03}_{-0.07-0.03}$	$0.28^{+0.03+0.01}_{-0.03-0.01}$

laps of the wave functions in $B_c \to D_s$ and η_c transitions are expected to be of similar size. Thirdly, the SU(3) symmetry breaking effects in $B_c \to D$ and D_s and $B_c \to D^*$ and D_s^* form factors are quite large, as the decay constant of D_s is about one-third larger than that of the D meson. But in $B_c \to B$, B_s and $B_c \to B^*$, B_s^* transitions, the SU(3) breaking effect is small, because the decay constants $f_{B^{(*)},B_s^{(*)}}$ are of similar size. Fourthly, since the uncertainties from decay constants of D, D_s, and J/ψ are very small, the relevant uncertainties to the form factors are also very small.

In the literature, there already exist lots of studies on B_c transition form factors [2–16] and their results are collected in Tables IV and V. Since J/ψ can be easily reconstructed by a lepton pair on the hadron collider, the $B_c \to J/\psi$ form factors have been widely studied in many theoretical frameworks. In a very recent paper [15], the authors have derived two kinds of wave functions for the charmonium state under harmonic oscillator potential and Coulomb potential. They also used these wave functions to investigate the $B_c \to \eta_c$, J/ψ form factors under the perturbative QCD approach. Compared with their results, our predictions are typically smaller. The main reason is that they have used a much larger decay constant f_{B_c}. Regardless of this effect, our results are consistent with theirs. Results collected in Table IV (including ours) have large differences which can be discriminated by the future LHC experiments. The $B_c \to D_s$, D_s^* is described as the flavor changing neutral current $b \to s$ transition at the quark level which is purely loop effects in the SM. As a consequence, this transition has a very small Wilson coefficient and the $B_c \to D_s$ and D_s^* form factors are less studied in the literature. Similar to the $b \to u$, s, and c transitions, predictions of the $c \to u$ and s transition form factors have large differences between different methods. As indicated from these two tables, results evaluated in Refs. [8,9,12,14] are different with the other ones and ours to a large extent. In Ref. [9], all of the results except for the B_c to charmonium transitions are larger than the other results: the authors have taken into account the α_s/v corrections and the form factors are enhanced by 3 times due to the Coulomb renormalization of the quark-meson vertex for the heavy quarkonium B_c. Moreover, small decay constants for the B meson are adopted which also give large form factors: $f_B = 140$–170 MeV, $f_{B^*}/f_B = 1.11$, and $f_{B_s}/f_B = 1.16$. In Ref. [14], the authors have chosen the chiral correlation functions to derive the form factors in the light-cone sum rules. Although only the twist-2 distribution amplitudes contribute and contributions from the twist-3 distribution amplitudes vanish, uncertainties of the continuum and the higher resonance interpolated by both the axial-vector current and the vector current are expected to be larger. In Ref. [12], the authors also adopted the three-point QCD sum rules but different correlation functions are chosen. The form factors $A_2^{B_cB^*}$ and $A_2^{B_cB_s^*}$ in Ref. [8] have different signs with the other results. The large differences in different models can be used to distinguish them in the future.

At the quark level, the $B_c \to P(V)l\bar{\nu}$ decays are described as $b \to c(u)W^- \to c(u)l\bar{\nu}$ or $c \to d(s)W^+ \to d(s)l^+\nu$. Integrating out the highly off-shell intermediate degrees of freedom at tree level, the effective electroweak Hamiltonian for $b \to ul\bar{\nu}_l$ transition, as an example, is

$$\mathcal{H}_{\rm eff}(b \to ul\bar{\nu}_l) = \frac{G_F}{\sqrt{2}} V_{ub}\bar{u}\gamma_\mu(1-\gamma_5)b\bar{l}\gamma^\mu(1-\gamma_5)\nu_l. \tag{34}$$

Since radiative corrections due to strong interactions only happen between the b quark and the u quark, they characterize the interactions at the low energy, and the Wilson coefficient which contains the physics above the m_b scale is not altered. With the masses of leptons taken into account, the differential decay widths of $B_c \to Pl\bar{\nu}$ and $B_c \to Vl\bar{\nu}$ ($l = e$, μ, and τ) are given by

TABLE IV. $B_c \to D$, D^* and $B_c \to D_s$, D_s^* form factors at $q^2 = 0$ evaluated in the literature.

	$F_1^{B_cD} = F_0^{B_cD}$	$A_0^{B_cD^*}$	$A_1^{B_cD^*}$	$A_2^{B_cD^*}$	$V^{B_cD^*}$
DW [2] [a]	0.154	0.156	0.145	0.134	0.224
CNP [3]	0.13	0.05	0.11	0.17	0.25
NW [7]	0.1446	0.094	0.100	0.105	0.175
IKS [8]	0.69	0.47	0.56	0.64	0.98
Kiselev [9][b]	0.32 [0.29]	0.35 [0.37]	0.43 [0.43]	0.51 [0.50]	1.66 [1.74]
EFG [10]	0.14	0.14	0.17	0.19	0.18
HZ [14]	0.35	0.05	0.32	0.57	0.57
DSV [16]	0.075	0.081	0.095	0.11	0.16
	$F_1^{B_cD_s} = F_0^{B_cD_s}$	$A_0^{B_cD_s^*}$	$A_1^{B_cD_s^*}$	$A_2^{B_cD_s^*}$	$V^{B_cD_s^*}$
Kiselev [9][b]	0.45 [0.43]	0.47 [0.52]	0.56 [0.56]	0.65 [0.60]	2.02 [2.27]
DSV [16]	0.15	0.16	0.18	0.20	0.29

[a]We quote the results with $\omega = 0.6$ GeV.
[b]The nonbracketed (bracketed) results are evaluated in sum rules (potential model).

TABLE V. $B_c \to \eta_c$, J/ψ, B, B^*, B_s, and B_s^* form factors at $q^2 = 0$ evaluated in the literature.

	$F_1^{B_c\eta_c} = F_0^{B_c\eta_c}$	$A_0^{B_cJ/\psi}$	$A_1^{B_cJ/\psi}$	$A_2^{B_cJ/\psi}$	$V^{B_cJ/\psi}$
DW [2][a]	0.420	0.408	0.416	0.431	0.591
CNP [3]	0.20	0.26	0.27	0.28	0.38
KT [4]	0.23	0.21	0.21	0.23	0.33
KLO [6][b]	0.66	0.60	0.63	0.69	1.03
NW [7]	0.5359	0.532	0.524	0.509	0.736
IKS [8]	0.76	0.69	0.68	0.66	0.96
Kiselev [9][c]	0.66 [0.7]	0.60 [0.66]	0.63 [0.66]	0.69 [0.66]	1.03 [0.94]
EFG [10]	0.47	0.40	0.50	0.73	0.49
IKS2 [11]	0.61	0.57	0.56	0.54	0.83
HNV [13]	0.49	0.45	0.49	0.56	0.61
HZ [14]	0.87	0.27	0.75	1.69	1.69
SDY [15]	0.87	0.27	0.75	1.69	1.69
DSV [16]	0.58	0.58	0.63	0.74	0.91
	$F_1^{B_cB} = F_0^{B_cB}$	$A_0^{B_cB^*}$	$A_1^{B_cB^*}$	$A_2^{B_cB^*}$	$V^{B_cB^*}$
DW [2][a]	0.662	0.682	0.729	1.240	5.690
CNP [3]	0.3	0.35	0.34	0.23	1.97
NW [7]	0.4504	0.269	0.291	0.538	1.94
IKS [8][d]	0.58	0.35	0.27	−0.60	3.27
Kiselev [9][c]	1.27 [1.38]	0.55 [0.51]	0.84 [0.81]	4.06 [4.18]	15.7 [15.9]
EFG [10]	0.39	0.20	0.42	2.89	3.94
AS [12]	…	0.28	0.17	−1.10	0.09
HNV [13][e]	0.39	0.34	0.38	0.80	1.69
HZ [14]	0.90	0.27	0.90	7.9	7.9
DSV [16]	0.41	0.42	0.63	2.74	4.77
	$F_1^{B_cB_s} = F_0^{B_cB_s}$	$A_0^{B_cB_s^*}$	$A_1^{B_cB_s^*}$	$A_2^{B_cB_s^*}$	$V^{B_cB_s^*}$
DW [2][a]	0.715	0.734	0.821	1.909	5.657
CNP [3]	0.30	0.39	0.38	0.35	2.11
CKM [5][f]	0.403	0.433	0.487	1.155	3.367
NW [7]	0.5917	0.445	0.471	0.787	2.81
IKS [8][d]	0.61	0.39	0.33	−0.40	3.25
Kiselev [9][c]	1.3 [1.1]	0.56 [0.47]	0.69 [0.70]	2.34 [3.51]	12.9 [12.9]
EFG [10]	0.50	0.35	0.49	2.19	3.44
HNV [13][e]	0.58	0.52	0.55	0.98	2.29
HZ [14]	1.02	0.36	1.01	9.04	9.04
DSV [16]	0.55	0.57	0.79	3.24	5.19

[a]We quote the results with $\omega = 0.6$ GeV.
[b]We quote the values where the Coulomb corrections are taken into account.
[c]The nonbracketed (bracketed) results are evaluated in sum rules (potential model).
[d]We add a minus sign to the form factors F_1, A_0, A_1, and A_2
[e]We add a minus sign for their predictions on the form factors.
[f]We quote the results which correspond to $m_b = 4.9$ GeV and $\omega = 0.4$ GeV.

$$\frac{d\Gamma(B_c \to P l \bar{\nu})}{dq^2} = \left(\frac{q^2 - m_l^2}{q^2}\right)^2 \frac{\sqrt{\lambda(m_{B_c}^2, m_P^2, q^2)} G_F^2 |V_{\rm CKM}|^2}{384 m_{B_c}^3 \pi^3} \frac{1}{q^2} \{(m_l^2 + 2q^2)\lambda(m_{B_c}^2, m_P^2, q^2) F_1^2(q^2) + 3m_l^2 (m_{B_c}^2 - m_P^2)^2 F_0^2(q^2)\}, \tag{35}$$

$$\frac{d\Gamma_L(B_c \to V l \bar{\nu})}{dq^2} = \left(\frac{q^2 - m_l^2}{q^2}\right)^2 \frac{\sqrt{\lambda(m_{B_c}^2, m_V^2, q^2)} G_F^2 |V_{\rm CKM}|^2}{384 m_{B_c}^3 \pi^3} \frac{1}{q^2} \left\{3m_l^2 \lambda(m_{B_c}^2, m_V^2, q^2) A_0^2(q^2) + (m_l^2 + 2q^2) \times \left|\frac{1}{2m_V}\left[(m_{B_c}^2 - m_V^2 - q^2)(m_{B_c} + m_V) A_1(q^2) - \frac{\lambda(m_{B_c}^2, m_V^2, q^2)}{m_{B_c} + m_V} A_2(q^2)\right]\right|^2\right\}, \tag{36}$$

TABLE VI. Branching ratios (in units of %) and polarizations $\frac{\Gamma_L}{\Gamma_T}$ of $B_c \to Ml\nu$ decays. The first kind of uncertainties is from the B_c decay constants and the decay constant of the final state mesons, while the second one is from the quark masses. The last kind of uncertainties is from the decay width of B_c and the CKM matrix element V_{ub}. The mass difference between an electron and a muon does not provide sizable effects in $B_c \to D^{(*)} l\bar{\nu}$ and $B_c \to \eta_c(J/\psi) l\bar{\nu}$ decays, but it does in $B_c \to B^{(*)} l\nu$ and $B_c \to B_s^{(*)} l\nu$ decays.

	$B_c \to De\bar{\nu}_e$	$B_c \to D\mu\bar{\nu}_\mu$	$B_c \to D\tau\bar{\nu}_\tau$	$B_c \to \eta_c e\bar{\nu}_e$	$B_c \to \eta_c \mu\bar{\nu}_\mu$
$\mathcal{BR}$	$0.0030^{+0.0005+0.0005+0.0007}_{-0.0004-0.0004-0.0007}$	$0.0030^{+0.0005+0.0005+0.0007}_{-0.0004-0.0004-0.0007}$	$0.0021^{+0.0003+0.0003+0.0005}_{-0.0003-0.0003-0.0005}$	$0.67^{+0.04+0.04+0.10}_{-0.07-0.04-0.10}$	$0.67^{+0.04+0.04+0.10}_{-0.07-0.04-0.10}$
	$B_c \to \eta_c\tau\bar{\nu}_\tau$	$B_c \to Be\bar{\nu}_e$	$B_c \to B\mu\bar{\nu}_\mu$	$B_c \to B_s e\bar{\nu}_e$	$B_c \to B_s\mu\bar{\nu}_\mu$
$\mathcal{BR}$	$0.190^{+0.005+0.014+0.029}_{-0.012-0.013-0.029}$	$0.109^{+0.014+0.013+0.017}_{-0.016-0.012-0.017}$	$0.104^{+0.013+0.013+0.016}_{-0.015-0.012-0.016}$	$1.49^{+0.10+0.15+0.23}_{-0.13-0.14-0.23}$	$1.41^{+0.09+0.14+0.21}_{-0.12-0.14-0.21}$
	$B_c \to D^*e\bar{\nu}_e$	$B_c \to D^*\mu\bar{\nu}_\mu$	$B_c \to D^*\tau\bar{\nu}_\tau$	$B_c \to J/\psi e\bar{\nu}_e$	$B_c \to J/\psi\mu\bar{\nu}_\mu$
$\mathcal{BR}$	$0.0045^{+0.0005+0.0010+0.0011}_{-0.0004-0.0008-0.0010}$	$0.0045^{+0.0005+0.0010+0.0011}_{-0.0004-0.0008-0.0010}$	$0.0027^{+0.0003+0.0006+0.0007}_{-0.0002-0.0005-0.0006}$	$1.49^{+0.01+0.15+0.23}_{-0.03-0.14-0.23}$	$1.49^{+0.01+0.15+0.23}_{-0.03-0.14-0.23}$
$\frac{\Gamma_L}{\Gamma_T}$	$0.68^{+0.02+0.02+0.00}_{-0.02-0.02-0.00}$	$0.68^{+0.02+0.02+0.00}_{-0.02-0.02-0.00}$	$0.70^{+0.01+0.02+0.00}_{-0.01-0.02-0.00}$	$1.04^{+0.00+0.02+0.00}_{-0.00-0.02-0.00}$	$1.04^{+0.00+0.02+0.00}_{-0.00-0.02-0.00}$
	$B_c \to J/\psi\tau\bar{\nu}_\tau$	$B_c \to B^*e\bar{\nu}_e$	$B_c \to B^*\mu\bar{\nu}_\mu$	$B_c \to B_s^*e\bar{\nu}_e$	$B_c \to B_s^*\mu\bar{\nu}_\mu$
$\mathcal{BR}$	$0.370^{+0.002+0.042+0.056}_{-0.005-0.038-0.056}$	$0.141^{+0.002+0.029+0.021}_{-0.004-0.026-0.021}$	$0.134^{+0.002+0.028+0.020}_{-0.004-0.025-0.020}$	$1.96^{+0.00+0.34+0.30}_{-0.03-0.32-0.30}$	$1.83^{+0.00+0.32+0.28}_{-0.03-0.30-0.28}$
$\frac{\Gamma_L}{\Gamma_T}$	$0.81^{+0.01+0.01+0.00}_{-0.01-0.01-0.00}$	$1.07^{+0.01+0.02+0.00}_{-0.01-0.03-0.00}$	$1.06^{+0.01+0.02+0.00}_{-0.01-0.02-0.00}$	$1.14^{+0.01+0.02+0.00}_{-0.01-0.02-0.00}$	$1.11^{+0.01+0.01+0.00}_{-0.01-0.02-0.00}$

$$\frac{d\Gamma^{\pm}(B_c \to Vl\bar{\nu})}{dq^2} = \left(\frac{q^2 - m_l^2}{q^2}\right)^2 \frac{\sqrt{\lambda(m_{B_c}^2, m_V^2, q^2)}G_F^2 |V_{\mathrm{CKM}}|^2}{384 m_{B_c}^3 \pi^3} \times \left\{(m_l^2 + 2q^2)\lambda(m_{B_c}^2, m_V^2, q^2) \times \left|\frac{V(q^2)}{m_{B_c} + m_V} \mp \frac{(m_{B_c} + m_V)A_1(q^2)}{\sqrt{\lambda(m_{B_c}^2, m_V^2, q^2)}}\right|^2\right\}, \tag{37}$$

where the superscript $+$ $(-)$ denotes the right-handed (left-handed) states of vector mesons. $\lambda(m_{B_c}^2, m_i^2, q^2) = (m_{B_c}^2 + m_i^2 - q^2)^2 - 4m_{B_c}^2 m_i^2$ with $i = P$ and V. The combined transverse and total differential decay widths are given by

$$\frac{d\Gamma_T}{dq^2} = \frac{d\Gamma^+}{dq^2} + \frac{d\Gamma^-}{dq^2}, \qquad \frac{d\Gamma}{dq^2} = \frac{d\Gamma_L}{dq^2} + \frac{d\Gamma_T}{dq^2}. \tag{38}$$

As we have mentioned in the above, the form factors $A_2^{B_cB^*}$ and $A_2^{B_cB_s^*}$ only give small contributions to semileptonic B_c decays. In these two channels, the two small variables $m_{B_c}^2 - m_V^2$ and q^2 satisfy the inequality: $q^2 \leq q^2_{\mathrm{max}} = (m_{B_c} - m_V)^2 \ll (m_{B_c} - m_V)(m_{B_c} + m_V) = m_{B_c}^2 - m_V^2$. One can expand the decay width in terms of small variables. The variable $\lambda(m_{B_c}^2, m_V^2, q^2)$ can be expanded as $\lambda(m_{B_c}^2, m_V^2, q^2) = (m_B^2 - m_V^2)^2 - 4(m_B^2 + m_V^2)q^2 + q^4 \sim (m_{B_c}^2 - m_V^2)^2$. From Eq. (36), we can see that the contribution from A_2 to the longitudinal differential decay width contains a factor of $\lambda(m_{B_c}^2, m_V^2, q^2)$ while the A_1 term is of the order $\sqrt{\lambda(m_{B_c}^2, m_V^2, q^2)}$. Numerical results show that the ratios

$$\frac{\lambda(m_{B_c}^2, m_{B^*}^2, q^2)}{(m_{B_c} + m_{B^*})^2(m_{B_c}^2 - m_{B^*}^2 - q^2)}$$

and

$$\frac{\lambda(m_{B_c}^2, m_{B_s^*}^2, q^2)}{(m_{B_c} + m_{B_s^*})^2(m_{B_c}^2 - m_{B_s^*}^2 - q^2)}$$

are smaller than 0.083 and 0.075 in the full region for q^2, respectively. It implies that the form factors $A_2^{B_cB^*}$ and $A_2^{B_cB_s^*}$ can be safely neglected in the decay width.[1]

Integrating the differential decay widths over the variable q^2, one obtains partial decay widths and polarization fractions. The lifetime of the B_c meson and the relevant CKM matrix elements are used as [28]

$$\tau_{B_c} = (0.46 \pm 0.07)\ \mathrm{ps}, \qquad |V_{cb}| = 41.2 \times 10^{-3}, \qquad |V_{ub}| = (3.93 \pm 0.36) \times 10^{-3}, \qquad |V_{cd}| = 0.230, \qquad |V_{cs}| = 0.973, \tag{39}$$

where the small uncertainties in the other CKM matrix elements are neglected. Our predictions of branching ratios and polarization quantities $\frac{\Gamma_L}{\Gamma_T}$ in semileptonic B_c decays are given in Table VI. The three kinds of uncertainties are from the decay constants of the B_c meson and the meson in the final state, the constituent quark masses, and the lifetime of B_c together with the CKM matrix elements. The first kind of uncertainties in the $B_c \to (D, D_s, J/\psi)l\bar{\nu}$ decays is very small, as the uncertainties in decay constants of D and J/ψ are small. The different mass between the electron and the muon does not have sizable effects on $b \to u$ and c semileptonic decays, but the branching ratios of $c \to u$ and s transitions are altered by roughly 5%. Branching ratios of $B_c \to Pl\bar{\nu}$ decays are smaller than the corresponding $B_c \to Vl\bar{\nu}$ ones, partly because there are three kinds of polarizations for vector mesons. Among the four kinds of transitions at the quark level, there is an inequality in the chain:

$$\mathcal{BR}(B_c \to D^* l\nu) < \mathcal{BR}(B_c \to B^* l\nu) < \mathcal{BR}(B_c \to J/\psi l\nu) < \mathcal{BR}(B_c \to B_s^* l\nu), \tag{40}$$

where we have taken decays involving a vector meson as an example. To understand this inequation, three points are

[1] In $B_c \to (B^*, B_s^*)V$ decays, the analysis is similar: q^2 is replaced by the mass square of the vector meson m_V^2.

essential. The CKM matrix elements for these four kinds of decays are given as

$$|V_{ub}| \ll V_{cb} \ll |V_{cd}| \ll V_{cs}. \tag{41}$$

The form factors at the zero-recoiling point roughly respect

$$F(B_c \to D^*) < F(B_c \to J/\psi) \sim F(B_c \to B^*) \sim F(B_c \to B_s^*). \tag{42}$$

The phase spaces in $B_c \to D^*$ and $B_c \to J/\psi$ transitions are much larger than those in $B_c \to B^*$ and B_s^* transitions, which can compensate for the small CKM matrix element in the $B_c \to J/\psi l \bar{\nu}$ decay. These predictions will be tested at the ongoing and forthcoming hadron colliders.

IV. CONCLUSION

Because of the rich data, measurements on the CKM matrix elements are becoming more and more accurate. B_c meson decays provide another promising place to continue the errand in B meson decays. They also offer a new window to explore the structure of weak interactions. Although the B_c meson cannot be produced on the two B factories, it has a promising prospect on the ongoing and forthcoming hadron colliders. Because of these interesting features, we have studied the B_c transition form factors in the covariant light-front quark model, which are relevant for the semileptonic B_c decays.

Comparing our predictions with results for the form factors in the literature, we find large discrepancies which may be useful to distinguish various theoretical methods. Our results for the form factors A_2 in $B_c \to B^*$ and $B_c \to B_s^*$ transitions strongly depend on the decay constants of the B^* and B_s^* mesons, which give large theoretical uncertainties to the form factors. For $B_c \to BP$ decays, the relevant form factor A_0 is almost independent of A_2: $A_0 \simeq A_1$. For semileptonic B_c decays (also $B_c \to B^* V$ decays), contributions from A_2 are at least suppressed by a factor of 0.08 compared with those from A_1. Thus the large uncertainties from A_2 will not affect the physical observables.

$B_c \to D$, D^*, D_s, and D_s^* form factors at the maximally recoiling point are smaller than $B_c \to \eta_c$, J/ψ, B, B^*, B_s, and B_s^*, while the $B_c \to D$, D_s, and η_c form factors at zero-recoiling point are close to each other. The SU(3) symmetry breaking effects in $B_c \to D$, D_s and $B_c \to D^*$, D_s^* are quite large; but in $B_c \to B$, B_s and $B_c \to B^*$, B_s^* transitions, the SU(3) breaking effects are not large. Semileptonic $B_c \to (\eta_c, J/\psi) l\nu$ and $B_c \to (B_s, B_s^*) l\nu$ decays have much larger branching fractions than the other two kinds of semileptonic B_c decays. In the three kinds of $B_c \to V l \nu$ decays, contributions from the longitudinal polarized vector are comparable with those from the transversely polarized vector. These predictions will be tested at the ongoing and forthcoming hadron colliders.

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APPENDIX A: RELATIONS OF DIFFERENT DEFINITIONS OF FORM FACTORS

In the literature, various conventions for the $B_c \to V$ form factors have been adopted. In this Appendix, we will collect their conventions and compare them with the BSW form factors. In Refs. [3,4,6,9], the authors defined the $B_c \to V$ form factors as

$$\langle V(P'', \epsilon'')|V_\mu|\bar{B}_c(P')\rangle = -\epsilon_{\mu\nu\alpha\beta}\varepsilon''^{*\nu}P^\alpha q^\beta F_V(q^2), \tag{A1}$$

$$\langle V(P'', \epsilon'')|A_\mu|\bar{B}_c(P')\rangle = iF_0(q^2)\epsilon''^*_\mu + iF_+(q^2)(\epsilon''^* \cdot P)P_\mu + iF_-(q^2)(\epsilon''^* \cdot P)q_\mu. \tag{A2}$$

These form factors are related to the BSW form factors by

$$V^{PV} = (m_{B_c} + m_V)F_V, \qquad A_1^{PV} = \frac{F_0}{m_{B_c} + m_V}, \qquad A_2^{PV} = -(m_{B_c} + m_V)F_+, \tag{A3}$$

$$A_0 = \frac{m_{B_c} + m_V}{2m_V}A_1^{PV}(q^2) - \frac{m_{B_c} - m_V}{2m_V}A_2^{PV}(q^2) + \frac{q^2}{2m_V}F_-. \tag{A4}$$

The definition of form factors g, f, a_+, and a_- in Ref. [7] is similar to ours in Eqs. (1)–(3) except for a phase i. In Refs. [11,13], the following definition for the form factors is adopted:

$$\langle V(P'', \epsilon'')|V_\mu - A_\mu|\bar{B}_c(P')\rangle = \frac{i}{m_{B_c} + m_V}\epsilon''^*_\nu(-g^{\mu\nu}P \cdot qA_0 + P^\mu P^\nu A_+ + q^\mu P^\nu A_- + i\epsilon^{\mu\nu\rho\sigma}P_\rho q_\sigma V), \tag{A5}$$

where A_+ corresponds to the BSW form factor A_2^{PV} and their form factor A_0^{IKS2} is related to the BSW form factor A_1^{PV}:

$$A_1^{PV} = \frac{A_0^{IKS2}(m_{B_c} - m_V)}{m_{B_c} + m_V}. \tag{A6}$$

In Ref. [14], the $B_c \to V$ form factors are defined as

$$\langle V(P'', \epsilon'')|V_\mu - A_\mu|\bar{B}_c(P')\rangle = -i\epsilon''^*_\nu(m_{B_c} + m_V)A_1 + iP_\mu(\epsilon''^* \cdot q)\frac{A_+}{m_{B_c} + m_V} + iq_\mu(\epsilon''^* \cdot q)\frac{A_-}{m_{B_c} + m_V} + \epsilon_{\mu\nu\rho\sigma}\epsilon''^*_\nu q_\rho P_\sigma \frac{V}{m_{B_c} + m_V}. \tag{A7}$$

The form factors A_1^{PV} and V^{PV} are the same as the relevant BSW form factors; their form factor A_+ corresponds to the BSW form factor A_2^{PV}.

APPENDIX B: SOME SPECIFIC RULES UNDER THE p^- INTEGRATION

When performing the p^- integration, one needs to include the zero-mode contribution. This amounts to performing the integration in a proper way in this approach. To be more specific, for $\hat{p}'_1$ under integration we use the following rules [22,23]:

$$\hat{p}'_{1\mu} \doteq P_\mu A_1^{(1)} + q_\mu A_2^{(1)}, \qquad \hat{N}_2 \to Z_2,$$
$$\hat{p}'_{1\mu}\hat{p}'_{1\nu} \doteq g_{\mu\nu}A_1^{(2)} + P_\mu P_\nu A_2^{(2)} + (P_\mu q_\nu + q_\mu P_\nu)A_3^{(2)} + q_\mu q_\nu A_4^{(2)}, \tag{B1}$$

where the symbol $\doteq$ reminds us that the above equations are true only after integration. $A_j^{(i)}$ are functions of $x_{1,2}$, $p'^2_\perp$, $p'_\perp \cdot q_\perp$, and q^2, and their explicit expressions have been studied in Refs. [22,23]:

$$Z_2 = \hat{N}'_1 + m'^2_1 - m_2^2 + (1 - 2x_1)M'^2 + (q^2 + q \cdot P)\frac{p'_\perp \cdot q_\perp}{q^2},$$
$$A_1^{(1)} = \frac{x_1}{2}, \qquad A_2^{(1)} = A_1^{(1)} - \frac{p'_\perp \cdot q_\perp}{q^2},$$
$$A_1^{(2)} = -p'^2_\perp - \frac{(p'_\perp \cdot q_\perp)^2}{q^2}, \qquad A_3^{(2)} = A_1^{(1)}A_2^{(1)},$$
$$A_4^{(2)} = (A_2^{(1)})^2 - \frac{1}{q^2}A_1^{(2)}. \tag{B2}$$

We do not show the spurious contributions in Eq. (B2) since they are numerically vanishing.

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