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Kinematic constraints to the transition redshift from supernovae type Ia union data

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The kinematic approach to cosmological tests provides direct evidence to the present accelerating stage of the Universe that does not depend on the validity of general relativity, as well as on the matter-energy content of the Universe. In this context, we consider here a linear two-parameter expansion for the decelerating parameter, $q(z) = q_0 + q_1 z$, where q_0 and q_1 are arbitrary constants to be constrained by the union supernovae data. By assuming a flat Universe we find that the best fit to the pair of free parameters is $(q_0, q_1) = (-0.73, 1.5)$ whereas the transition redshift is $z_t = 0.49^{+0.14}_{-0.07}(1\sigma)\ ^{+0.54}_{-0.12}(2\sigma)$. This kinematic result is in agreement with some independent analyses and more easily accommodates many dynamical flat models (like ΛCDM).

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I. INTRODUCTION

It is now widely believed that the Universe at redshifts smaller than unity underwent a "dynamic phase transition" from decelerating to accelerating expansion which has been corroborated by several independent analyses. In the context of the general relativity theory such a phenomenon can be interpreted as a dynamic influence of some sort of dark energy whose main effect is to change the sign of the universal decelerating parameter $q(z)$.

The most direct observation supporting the present accelerating stage of the Universe comes from the luminosity distance versus redshift relation measurements using supernovas (SNe) type Ia [1,2]. Initially, they were interpreted in light of ΛCDM scenarios using either background or inhomogeneous luminosity distances [3]. However, independent theoretical and observational/statistical analyses point to more general models whose basic ingredient is a negative-pressure dark energy component [4].

Nowadays, the most accepted cosmic picture is an expanding flat (or nearly flat) spatial geometry whose dynamics is driven by an exotic component called dark energy, 3/4 of composition, and 1/4 for matter component (baryons plus dark). Among a number of possibilities to describe this dark energy component, the simplest and most theoretically appealing way is by means of a positive cosmological constant Λ. Other possible candidates are a vacuum decaying energy density, or a time varying Λ-term [5], a time varying relic scalar field slowly rolling down its potential [6], the so-called "X-matter," an extra component simply characterized by an equation of state $p_x = \omega\rho_x$ [7], the Chaplygin gas whose equation of state is given by $p = -A/\rho$ where A is a positive constant [8]. For scalar field and XCDM scenarios, the ω parameter may be a function of the redshift [9], or still, as it has been recently discussed, it may violate the dominant energy condition and assume values < -1 when the extra component is named phantom cosmology [10]. It should be stressed, however, that all these models are based on the validity of general relativity or some of its scalar-tensorial generalizations.

On the other hand, Turner and Riess [11] have discussed an alternative route—sometimes called the kinematic approach—in order to obtain information about the beginning of the present accelerating stage of the Universe with no assumption concerning the validity of general relativity or even of any particular metric gravitational theory (in this connection see also Weinberg [12]). Although considering that such a method does not shed light on the physical or geometrical properties of the new energetic component causing the acceleration, it allows one to assess the direct empirical evidence for the transition deceleration/acceleration in the past, as provided by SNe type Ia measurements. Many authors have constrained values for the transition redshift (z_t), explored implications on the cosmic acceleration, or yet, used it as a trustworthy discriminator for cosmology. This value is obtained without supposing any energy components (baryons, dark matter, dark energy), or any other cause for acceleration [13–17].

More recently, Mortsell and Clarkson [18] applied a kinematic approach to determine if the Copernican assumption is violated. Moreover, by using a Taylor expansion of the scale factor, it was found that the acceleration today is detected to an accuracy $>12\sigma$. It was also claimed, with basis on the ratio of the scale of the baryon acoustic oscillations as imprinted in the cosmic microwave background and in the large-scale distribution of galaxies, that a flat or negatively curved universe decelerates at high redshifts.

In this paper, by adopting the kinematic approach for which the full gravitational theory also does not play a prominent role, we investigate the cosmological implications on the transition redshift z_t and deceleration parameters from the Supernovae Cosmology Project (SCP) union sample [2].

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II. LUMINOSITY DISTANCE AND SAMPLE

To begin with, let us assume that the spatially flat Friedman-Robertson-Walker (FRW) metric geometry, as motivated by inflation and the WMAP results [19]. Following standard lines [11], the luminosity distance is kinematically defined by the following integral expression (in our units $c = 1$).

$$D_L(z) = (1+z)\int_0^z \frac{du}{H(u)} = \frac{(1+z)}{H_0}\int_0^z \exp\left[-\int_0^u [1+q(u)]d\ln(1+u)\right]du, \tag{1}$$

where the $H(z) = \dot{a}/a$ is the Hubble parameter, and, $q(z)$, the deceleration parameter.

Although generalizable for nonzero curvature, Eq. (1) is not a crude approximation as one may think at first sight. In the framework of a flat FRW-type universe, it is an exact expression for the luminosity distance which depends on the epoch-dependent deceleration parameter, $q(z)$, as well as on the present Hubble constant, H_0. The simplest way to work with the coupled definitions (1) and (2) as a kinematic model for the SN type Ia data is by adopting parametric representations for $q(z)$. As one may check, in the case of a linear two-parameter expansion for $q(z) = q_0 + zq_1$ [20], the integral (1) can be represented in terms of a special function as (see [17])

$$D_L(z) = \frac{(1+z)}{H_0} e^{q_1} q_1^{q_0-q_1}[\gamma(q_1 - q_0, (z+1)q_1) - \gamma(q_1 - q_0, q_1)], \tag{2}$$

where $q_0 = q(z = 0)$ is the present value of the deceleration parameter, q_1 is the derivative in the redshift evaluated at $z = 0$, and γ is the incomplete gamma function with the condition $q_1 - q_0 > 0$ must be satisfied (for more details, see Cunha and Lima [17]).

Now, by using the above expressions we may get information about q_0, q_1 and, therefore, about the global behavior of $q(z)$. Note also that a positive transition redshift, z_t, is obtained only for positive signs of q_1 (the variation rate of q_0) since q_0 is negative and the dynamic transition (from decelerating to accelerating) happens at $q(z_t) = 0$, or equivalently, $z_t = -q_0/q_1$.

In the statistical analysis below we consider the most complete data set we have right now, the SCP union sample [2]. The union SNe compilation is a new data set of low-redshift nearby-Hubble-flow SNe and new analysis procedures to work with several heterogeneous compilations' SNe Ia. It includes 13 independent sets, and, after selection cuts, the robust compilation obtained is composed by 307 SNe Ia events distributed over the redshift interval $0.015 \leq z \leq 1.55$.

III. STATISTICAL ANALYSIS AND RESULTS

The analysis below is based on the luminosity distance as given by [1] with the "linear expansion" for $q(z)$. The primary aim is to limit the parameters q_0 and q_1 by using the union compilation data as above discussed. All the results are derived by marginalizing the likelihood function over the nuisance parameter, H_0, thereby obtaining the contours and the associated probabilities.

In Fig. 1 we show the theoretical predictions of the kinematic approach to the residual Hubble diagram with respect to an eternally coasting Universe model ($q(z) \equiv 0$). The different models are characterized by the selected values of q_0 and q_1, as depicted in the diagram. Let us consider the maximum likelihood that can be determined from a χ^2 statistics for a given set of parameters (H_0, q_0, q_1). In what follows we investigate the bounds arising on the empirical $q(z)$ parameters and the probability of the redshift transition for each SNe type Ia sample. By marginalizing the likelihood function over the nuisance parameter, H_0, the contours and the probabilities of the transition redshift for each sample are readily computed.

In Fig. 2(a), we see that the SCP union sample strongly favors a universe with recent acceleration ($q_0 < 0$) and previous deceleration ($dq/dz > 0$). With two free parameters the confidence region is $0.7 \leq q_1 \leq 2.2$ and $-0.93 \leq q_0 \leq -0.52$ with (68%) confidence level. It should be remarked that the presence of a forbidden region forming a trapezium. The horizontal line on the top is defined by $q_1 = 0$ which leads to an infinite (positive or negative) transition redshift while the segment at 45° is the infinite future ($z_t = -1$). The values of z_t associated with the horizontal segment in the bottom are always smaller than -1 (no transition region), in fact $-1.5 \leq z_t \leq -1$. Finally, one may conclude that the vertical segment is associated

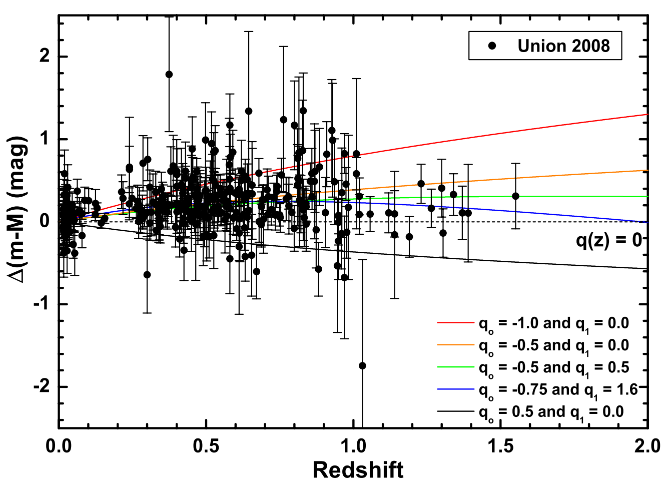

FIG. 1 (color online). Residual magnitude versus redshift is shown for 307 SNe type Ia from SCP union compilation. Data and kinematic models of the expansion history are shown relative to an eternally coasting model, $q(z) \equiv 0$.

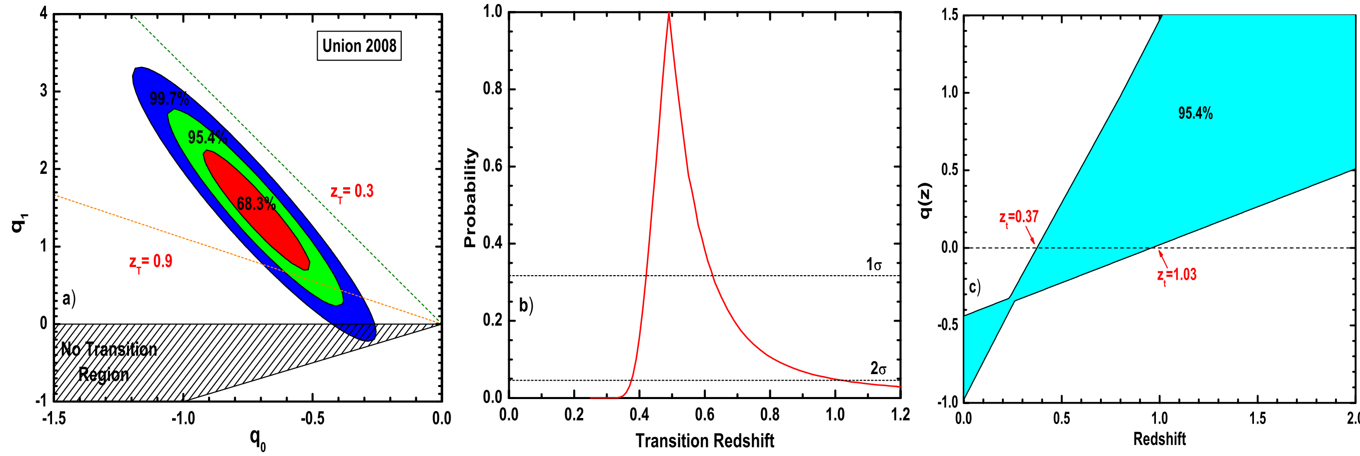

FIG. 2 (color online). a) The likelihood contours in the $q_0 - q_1$ plane for 307 SNe type Ia data. The contours correspond to 68%, 95%, and 99% confidence levels. The best fit to the pair $(q_0, q_1) = (-0.73, 1.5)$. b) Probability function for the past transition redshift for a two-parameter model of the expansion history, $q(z) = q_0 + q_1 z$. Our analysis furnishes the best fit $z_t = 0.49^{+0.14}_{-0.07}(1\sigma)\ ^{+0.54}_{-0.12}(2\sigma)$. c) Evolution of the decelerating parameter as a function of the redshift. In the panel the shadowed region means 2σ level for the SCP union sample. The dotted horizontal lines represent the coasting model ($q(z) \equiv 0$).

with $z_t \leq -1.5$, thereby demonstrating that the hachured trapezium is actually a physically forbidden region.

In Fig. 2(b) (the center panel) one may see the probability of the associated transition redshift z_t, defined as $q(z_t) = 0$. It has been derived by summing the probability density in the q_0 versus the dq/dz plane along lines of constant transition redshift, $z_t = -q_0/(dq/dz)$. The resulting analysis yields $z_t = 0.49^{+0.135}_{-0.07}(1\sigma)\ ^{+0.54}_{-0.12}(2\sigma)$ for one free parameter which is in reasonable agreement with the value $z_t = 0.46 \pm 0.13$ [20]. In our analysis, the asymmetry in the probability of z_t is produced by a partially parabolic curve obtained when χ^2 is minimized. For this panel the central value $z_t = 0.49$ does not agree with the cosmic concordance ΛCDM model in 68.3% confidence level. However, it agrees with 95.4% (2σ) for this approach. Note that $z_t = 0.3$ (z_t for flat ΛCDM with $\Omega_m \simeq \Omega_\Lambda \simeq 0.5$) is outside of the allowed region with 95%, while, $z_t = 0.9$ (flat ΛCDM with $\Omega_m \simeq 0.2$ and $\Omega_\Lambda \simeq 0.8$) are well inside for the union sample.

At this point it is interesting to investigate how our analysis constrains the redshift evolution of the decelerating parameter itself. The basic results are displayed in Fig. 2(c); we see the evolution of the decelerating parameter as a function of the redshift for the parametrization $q(z) = q_0 + q_1 z$. The shadowed region denotes the 2σ region. The data favor the recent acceleration ($q_0 < 0$) and past deceleration ($q_1 > 0$) with high confidence level.

It is also interesting to compare the results derived here with another independent analysis. The first constraints using this parametrization were obtained by Riess and collaborators from 157 SNe the transition redshift was constrained to be at $z_t = 0.46 \pm 0.13$ [20]. More recently, using 182 SNe Riess *et al.* obtained $z_t = 0.43 \pm 0.07$ for the probability density in the q_0 vs q_1 plane along lines of constant transition redshift [21]. In an early paper, we studied this parametrization to three samples. For Astier data set 2006 we obtained $z_t = 0.61^{+3.68}_{-0.21}$, Gold sample 2007 the constraints were $z_t = 0.43^{+0.09}_{-0.05}$, and Davies data set 2007 $z_t = 0.60^{+0.28}_{-0.11}$. Besides that, Bayesian analysis was implemented to study this kinematic scenario by Elgarøy and Multamäki [22], and, a kinematical study from type Ia supernovae and X-ray cluster gas mass fraction measurements, by combining them they obtain significantly tighter results than using the SNe sample alone [23].

In Table I we summarize the recent results to the transition redshift z_t in the kinematic approach derived from different samples of SNe Ia data by the method presented here (see also Cunha and Lima [17] for earlier determinations using different samples). As shown there, for a fixed phenomenological law, $q(z) = q_0 + q_1 z$, the limits were derived separately for each sample of SNe type Ia data. Note that all the limits are strongly dependent on the specific data set considered. With this concern, we would also like to call attention to a recent article by Lima, Vitenti, and Reboucas [24]. By using a very different approach based on the violation of the energy conditions (null, weak, strong and dominant energy conditions [25]) combined with the union supernova data they reconstructed $q(z)$ and determined that the transition redshift falls on the interval $0.4 \lesssim z_t \lesssim 0.64$ at 1σ confidence

TABLE I. Limits to the transition redshift z_t.

Sample (data)	z_t (best-fit)	Confidence (1σ)	χ^2_{min}
Gold 2004 (157)	0.46	$0.33 \leq z_t \leq 0.59$	176
Astier 2006 (115)	0.61	$0.40 \leq z_t \leq 4.29$	113
Gold 2007 (182)	0.43	$0.38 \leq z_t \leq 0.52$	156
Davis 2007 (192)	0.60	$0.49 \leq z_t \leq 0.88$	195
This paper (307)	0.49	$0.42 \leq z_t \leq 0.63$	310

level, a result in reasonable agreement with our direct method discussed here (compare with the last line Table I).

IV. CONCLUSION

In this paper we have discussed the transition redshift obtained from a kinematical approach within a flat FRW standard line element. Our study strongly favors a universe with recent acceleration ($q_0 < 0$) and previous deceleration ($dq/dz > 0$) for an analysis which is independent of the matter-energy content of the Universe based on the phenomenological law, $q(z) = q_0 + q_1 z$, and the SCP data compilation [2]. In our analysis we use the analytical expression to the distance luminosity [17], as well as the excluded regions with $z_t < -1$ [Fig. 2(a)].

The likelihood function for the transition redshift was also discussed. In this case, the confidence regions in the bidimensional space parameter (q_0, q_1) do not cross the physically forbidden region. Our analysis provides independent evidence for a dynamical model in which a kinematic transition phase deceleration/acceleration happened at redshifts smaller than unity. Hopefully, the constraints on z_t will be considerably improved in the near future with the increase of supernova data at intermediate and high redshifts.

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