

Asymptotically AdS₃ solutions to topologically massive gravity at special values of the coupling constants

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We study exact solutions to cosmological topologically massive gravity coupled to topologically massive electrodynamics at special values of the coupling constants. For the particular case of the so-called chiral point $l\mu_G = 1$, vacuum solutions (with vanishing gauge field) are exhibited. These correspond to a one-parameter deformation of general relativity solutions, and are continuously connected to the extremal Bañados-Teitelboim-Zanelli black hole with bare constants $J = -lM$. At the chiral point this extremal Bañados-Teitelboim-Zanelli black hole turns out to be massless, and thus it can be regarded as a kind of *ground state*. Although the solution is not asymptotically AdS₃ in the sense of Brown-Henneaux boundary conditions, it does obey the weakened asymptotic recently proposed by Grumiller and Johansson. Consequently, we discuss the holographic computation of the conserved charges in terms of the stress tensor in the boundary. For the case where the coupling constants satisfy the relation $l\mu_G = 1 + 2l\mu_E$, electrically charged analogues to these solutions exist. These solutions are asymptotically AdS₃ in the strongest sense, and correspond to a logarithmic branch of self-dual solutions previously discussed in the literature.

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I. INTRODUCTION

In the last year and a half there has been a revived interest in three-dimensional gravity. This was mainly motivated by E. Witten's proposal [1] that Einstein gravity in AdS₃ is holographically dual to a holomorphically factorizable CFT₂. This idea has attracted considerable attention, and led to intense debate [2–5]. Another model of three-dimensional gravity that has attracted much attention recently is topologically massive gravity (TMG), which corresponds to three-dimensional Einstein gravity coupled to a gravitational Chern-Simons term without torsion [6]; namely,

$$I_G = \frac{1}{2\kappa^2} \int_{\mathcal{M}} d^3x \sqrt{-g} \left(R + \frac{2}{l^2} \right) + \frac{1}{4\kappa^2 \mu_G} \times \int_{\mathcal{M}} d^3x \epsilon^{\lambda\mu\nu} \Gamma_{\lambda\sigma}^{\rho} \left(\partial_{\mu} \Gamma_{\rho\nu}^{\sigma} + \frac{2}{3} \Gamma_{\mu\rho}^{\gamma} \Gamma_{\nu\gamma}^{\sigma} \right) + \mathcal{B} \quad (1)$$

with $l^{-2} = -\Lambda$ and $\kappa^2 = 8\pi G$, and where \mathcal{B} stands for the boundary term, which we are not writing explicitly here [see (16) below]. The three-dimensional gravity theory defined by (1) contains a local massive graviton degree of freedom [6,7], and it also admits black hole solutions [8], which makes TMG a very interesting model to be explored.

One of the interesting properties of TMG is that its holographic description [9] in terms of a CFT₂ captures several interesting features of the AdS₃/CFT₂ realization. As in the case of Einstein gravity in AdS₃, the asymptotic isometry group of TMG in this background is generated by

two copies of the Virasoro algebra with nontrivial central extension. When the gravitational Chern-Simons coupling μ_G takes the special value $\mu_G = 1/l$, the central charge of left-moving excitations in the boundary theory vanishes. This led to the authors of [10] to conjecture that at $\mu_G = 1/l$ the boundary conformal field theory that is dual to TMG is chiral. This affirmation gave rise to an interesting discussion about the existence of bulk local degrees of freedom at the chiral point; see Refs. [11–24].

In addition to AdS₃, other backgrounds of TMG have recently shown to be of great interest. In particular, warped versions of AdS₃ have led to fabulous applications such as the description of extremal four-dimensional Kerr black holes [25–30]. The connection of these backgrounds to Gödel black holes [28] are also very interesting.

Here, we will be concerned with topologically massive gravity coupled to its electromagnetic analog, the topologically massive electrodynamics (TME). The gauge theory action is given by the Maxwell term coupled to the Abelian Chern-Simons term; namely,

$$I_E = -\frac{1}{4} \int_{\mathcal{M}} d^3x \sqrt{-g} F_{\mu\nu} F^{\mu\nu} + \frac{\mu_E}{4} \times \int_{\mathcal{M}} d^3x \epsilon^{\mu\nu\rho} A_{\mu} F_{\nu\rho}. \quad (2)$$

In this paper, we will consider the special case $l\mu_G = 1 - 2\epsilon$ with $\epsilon = -l\mu_E$. The reason why we are particularly interested in this relation between coupling constants is that such theories admit a class of solution with interest-

ing properties. For instance, particular features of exact solutions at $l\mu_G = 1$ ($\varepsilon = 0$) were noticed even before the *chiral gravity conjecture* [15] was formulated; see for instance [31]. At these points of the space of parameters several exact solutions reported in the literature are seen to coincide, and it is precisely when this degeneracy happens that new solutions with interesting properties usually come up. In particular, at the chiral point the solutions we will describe here correspond to an asymptotically AdS₃ solutions of TMG in a vacuum (with vanishing gauge field). We discuss the theory at the chiral point in Sec. II, where we present these vacuum solutions and discuss their properties in detail. In Sec. III, we generalize the solutions to the case of TMG charged under the TME theory with $l\mu_G = 1 + 2l\mu_E$. The charged solutions turn out to be asymptotically AdS₃, with a gauge field configuration that diverges at the horizon. We also discuss the relation between the solutions we present here with self-dual solutions previously reported in the literature. We summarize the results in Sec. IV.

II. VACUUM SOLUTIONS AT THE CHIRAL POINT

A. Topologically massive gravity and its solution

The equations of motion of TMG follow from varying (1) with respect to the metric $g_{\mu\nu}$. In the presence of matter [consider in particular (2)], these equations read

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \frac{1}{l^2}g_{\mu\nu} + \frac{1}{\mu_G}C_{\mu\nu} = \kappa^2 T_{\mu\nu}, \quad (3)$$

where $\Lambda = -l^2$, $T_{\mu\nu}$ is the stress tensor of the electromagnetic field, and $C_{\mu\nu}$ is the Cotton tensor, given by

$$C_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu}^{\alpha\beta}\nabla_{\alpha}R_{\beta\nu} + \frac{1}{2}\varepsilon_{\nu}^{\alpha\beta}\nabla_{\alpha}R_{\mu\beta}. \quad (4)$$

In three-dimensions the Weyl tensor identically vanishes, and the Cotton tensor is the one that comes to play its role: It is a traceless tensor that vanishes if and only if the metric is locally conformally flat. A traceless condition implies that all the solutions of the field equations satisfy

$$R = -\frac{6}{l^2} - 2\kappa^2 T_{\mu}^{\mu},$$

and one finds that all three-dimensional Einstein manifolds solve (3).

Let us begin by considering the theory at the chiral point $l\mu_G = \pm 1$. At this point, we can consider solutions with vanishing gauge field, and the coupling μ_E then takes an arbitrary value. More precisely, at the chiral point $l\mu_G = 1$ one finds a vacuum solution of TMG, whose metric reads

$$ds^2 = -N^2(r)dt^2 + \frac{dr^2}{N^2(r)} + r^2(N_{\phi}(r)dt - d\phi)^2 + N_k^2(r)(dt - ld\phi)^2, \quad (5)$$

where

$$N^2(r) = \frac{r^2}{l^2} - \kappa^2 M + \frac{\kappa^4 M^2 l^2}{4r^2}, \quad N_{\phi}(r) = \frac{\kappa^2 M l}{2r^2}, \quad (6)$$

and

$$N_k^2(r) = k \log((r^2 - \kappa^2 M l^2 / 2) / r_0^2), \quad (7)$$

where k and r_0 are two real arbitrary constants. We use the convention $\epsilon^{tr\phi} = +1$. It is not hard to verify that (5) solves (3) in a vacuum when $l\mu_G = 1$.

That is, metric (5) represents an exact solution of topologically massive gravity that emerges at the chiral point. The Cotton tensor associated to this solution is proportional to k , so that it is a genuine solution to TMG in the sense that it does not solve the Einstein equation, except for the particular case $k = 0$, where the metric becomes the extremal Bañados-Teitelboim-Zanelli (BTZ) black hole [32,33]. For all values of k the metric is clearly circularly symmetric and static, and thus compatible with $SO(2) \times \mathbb{R}$ symmetry.

In its Arnowitt-Deser-Misner form, the metric reads

$$ds^2 = -\mathcal{N}_{\perp}^2(r)dt^2 + \frac{dr^2}{N^2(r)} + \mathcal{R}^2(r)(d\phi - \mathcal{N}_{\phi}(r)dt)^2, \quad (8)$$

where we have defined

$$\mathcal{N}_{\perp}^2(r) = N^2(r) - r^2 N_{\phi}^2(r) - N_k^2(r) + \mathcal{R}^2(r) \mathcal{N}_{\phi}^2(r), \quad (9)$$

and

$$\begin{aligned} \mathcal{R}^2(r) &= r^2 + l^2 N_k^2(r), \\ \mathcal{N}_{\phi}(r) &= \mathcal{R}^{-2}(r)(r^2 N_{\phi}(r) + l N_k^2(r)). \end{aligned} \quad (10)$$

Metric (5) is actually nicely behaved. Despite the abstruse form of the off-diagonal component $g_{\phi t}$, the determinant of the metric is clearly $\det g = -r^2$, and the metric is Lorentzian for all values of the radial coordinate r . The metric seems to present a horizon at $r^2 = \kappa^2 M l^2 / 2$. Nevertheless, for $k \neq 0$ the metric in its form (5) is not defined for $r^2 \leq \kappa^2 M l^2 / 2$ (for $k = 0$ region $r^2 < \kappa^2 M l^2 / 2$ would correspond to the interior of the BTZ black hole). Let us analyze this aspect together with the geodesic structure in more detail: At $r^2 = \kappa^2 M l^2 / 2$, function N_k^2 diverges, while N^2 vanishes. Then, by analyzing the geodesic equation for massive particles, one observes that the divergence of N_k^2 contributes to the radial effective potential with a term like $\sim -(k/r^2) \log(r^2 - \kappa^2 M l^2 / 2)$. This means that for $k > 0$, massive particles are scattered back when they approach $r^2 = \kappa^2 M l^2 / 2$, and this means that at least for positive k , the ‘‘horizon’’ is not actually there. In fact, for $k > 0$ the circle $r^2 = \kappa^2 M l^2 / 2$ turns out to be located at an infinite geodesic distance from any point. For $k < 0$ the geodesic distance to a point at $r^2 = \kappa^2 M l^2 / 2$ turns out to be finite. However, by taking a look at the angular component of the geodesic equation one realizes

that the trajectories of massive particles wind indefinitely around the circle defined by $r^2 = \kappa^2 M l^2 / 2$ and thus these geodesic cannot be extended across this circle [34].

From (5) we also notice that g_{tt} vanishes at $r^2 = \kappa^2 M l^2 + k l^2 \log((r^2 - \kappa^2 M l^2 / 2) / r_0^2)$, and this always happens if $k \leq 0$. In particular, we know that for the spinning BTZ (i.e. $k = 0$) the radius $r = \kappa^2 M l^2$ defines its ergosphere [33]. For $k > 0$, however, metric function g_{tt} only vanishes if the parameters satisfy

$$\kappa^2 M \geq 2k(1 - \log(l^2 k / r_0^2)). \quad (11)$$

For instance, let us consider the case $M = 0$, for which the metric (5) takes the simple form

$$\begin{aligned} ds^2 &= \frac{l^2}{r^2} dr^2 + \frac{r^2}{l^2} (d\phi^2 - dt^2) + k \log\left(\frac{r^2}{l^2}\right) (dt - d\phi)^2 \\ &= \frac{l^2}{r^2} dr^2 + \frac{r^2}{l^2} dx^+ dx^- + k \log\left(\frac{r^2}{l^2}\right) (dx^-)^2, \end{aligned} \quad (12)$$

where we defined $x^\pm = \phi \pm t$, we absorbed a factor l in ϕ , and fixed r_0 . From this expression we observe that if $k < 0$ the component g_{tt} vanishes at $r^2 = -2|k|l^2 \log(r/r_0)$, and that $g_{\phi\phi}$ may also vanish depending on r_0 . On the other hand, if $k > 0$ then the component $g_{\phi\phi}$ vanishes at $r^2 = -2kl^2 \log(r/r_0)$, and g_{tt} may also vanish.

Now, let us move on and discuss the asymptotic behavior of (5). In the large r limit, metric (5) takes the asymptotic form

$$g_{tt} = -\frac{r^2}{l^2} + \mathcal{O}(\log(r)) + \mathcal{O}(1), \quad g_{rr} = \frac{l^2}{r^2} + \mathcal{O}(r^{-4}), \quad (13)$$

$$g_{\phi\phi} = r^2 + \mathcal{O}(\log(r)) + \mathcal{O}(1), \quad (14)$$

$$g_{\phi t} = \mathcal{O}(\log(r)) + \mathcal{O}(1).$$

We observe from this large r expansion that this solution is not asymptotically AdS₃ according to the definition given by Brown and Henneaux in [35]. Nevertheless, (5) does still obey the weakened AdS₃ asymptotic recently proposed by Grumiller and Johansson in [12,21]. To see this, let us set $l = 1$ for notational convenience, and define the new coordinates $x^\pm = \phi \pm t$ and $y = r^{-1}$. In terms of these coordinates, the large r expansion of (5) reads

$$\begin{aligned} g_{--} &= \mathcal{O}(\log(y)) + \mathcal{O}(1), & g_{-+} &= y^{-2} + \mathcal{O}(1), \\ g_{yy} &= y^{-2} + \mathcal{O}(1), \end{aligned} \quad (15)$$

together with $g_{++} = 0$ and $g_{y\pm} = 0$. It is worth noticing that asymptotic behavior (15) is strictly included in the boundary conditions proposed in [12,21], which, in addition, would also permit a next-to-leading behavior like $g_{+y} = \mathcal{O}(y)$ and $g_{y-} = \mathcal{O}(y \log(y))$. These weakened boundary conditions were recently discussed within the context of *chiral gravity*, and these were shown to be consistent with conformal asymptotic symmetry. In turn,

this would permit to define a consistent stress tensor in the boundary. Our solution can be thought of as a realization of the boundary conditions of [12,21].

B. Conserved charges and boundary terms

Because the off-diagonal term in (5) grows logarithmically $\sim 2k \log(r)$ at large distances [36], it turns out that metric (5) is not asymptotically AdS₃ in the sense of [35]. However, we can still proceed to compute conserved charges of this solution by holographic methods. After all, the solution is still asymptotically AdS₃ in the sense of the boundary conditions recently proposed in [12,21]. Then, we can resort to the method of defining an effective stress tensor induced on the boundary $\partial\mathcal{M}$, as in the case of asymptotically locally AdS₃ solutions [39] (see also the seminal paper [40]).

Consider the action with the boundary term,

$$\begin{aligned} I_G &= \frac{1}{2\kappa^2} \int_{\mathcal{M}} d^3x \sqrt{-g} \left(R + \frac{2}{l^2} \right) + \frac{1}{\kappa^2} \int_{\partial\mathcal{M}} d^2y \sqrt{-\gamma} K \\ &\quad + \frac{1}{4\kappa^2 \mu_G} \int_{\mathcal{M}} d^3x \epsilon^{\lambda\mu\nu} \Gamma_{\lambda\sigma}^\rho \left(\partial_\mu \Gamma_{\nu\rho}^\sigma + \frac{2}{3} \Gamma_{\mu\tau}^\sigma \Gamma_{\nu\rho}^\tau \right), \end{aligned} \quad (16)$$

where $K = \text{Tr}K = K_i^i$ is the trace of the extrinsic curvature K_{ij} . Here, we see the Gibbons-Hawking term \mathcal{B} appears. This action can be expressed in terms of Gaussian coordinates $ds^2 = d\eta^2 + \gamma_{ij} dx^i dx^j$, with $K_{ij} = \frac{1}{2} \partial_\eta \gamma_{ij}$. This reads [9,12]

$$\begin{aligned} I_G &= \frac{1}{2\kappa^2} \int_{\mathcal{M}} d^2y d\eta \sqrt{-\gamma} \left(R^{(2)} + K^2 - \text{Tr}(K^2) + \frac{2}{l^2} \right) \\ &\quad + \frac{1}{4\kappa^2 \mu_G} \int_{\mathcal{M}} d^2y d\eta \epsilon^{ij} \left(-2K_i^l \partial_\eta K_{jl} + \Gamma_{in}^l \partial_\eta \Gamma_{jl}^n \right. \\ &\quad \left. + 2K_k^n \Gamma_{in}^l \Gamma_{jl}^k + K_n^l \partial_j \Gamma_{il}^n + \Gamma_{jn}^l \partial_i K_l^n \right), \end{aligned} \quad (17)$$

where $\text{Tr}K^2 = K_i^j K_j^i$. In this expression, the Gibbons-Hawking term does not appear because it cancels against a total derivative coming from the bulk contribution. Expression (17) turns out to be an action for the metric γ_{ij} , which corresponds to the induced metric in the boundary. The stress tensor T^{ij} associated to the boundary manifold [40] is then obtained by varying (17) with respect to γ_{ij} and evaluating it on-shell; namely, $\delta I_G = \frac{1}{2} \times \int_{\partial\mathcal{M}} d^2x \sqrt{-\gamma} T^{ij} \delta \gamma_{ij}$. The conserved charges computed with this stress tensor [see (20) below] diverge and then it is necessary to regularize the action by adding an appropriate counterterm [9]. Such counterterm turns out to be a cosmological constant term in the boundary; namely,

$$\Delta I_G = -\frac{1}{l\kappa^2} \int d^2y \sqrt{-\gamma}, \quad (18)$$

which only depends on geometric quantities of the boundary, not affecting the equations of motion in the bulk. A similar counterterm can be seen to appear when analyzing

other backgrounds of TMG. For instance, if one considers the warped AdS₃ black hole of [8,27], the counterterm in the boundary is also given by (18) but replacing the overall factor $-1/l\kappa^2$ by a μ_G -dependent factor that coincides with that in (18) when $\mu_G = 3/l$ [41].

Including the counterterm (18), and in the case of asymptotically AdS₃ spaces, the boundary stress tensor takes the form

$$2\kappa^2 T^{ij} = 2\left(K^{ij} - \gamma^{ij} \text{Tr}K - \frac{1}{l} \gamma^{ij}\right) + \frac{1}{\mu_G} \epsilon^{k(i} (\gamma^{j)l} \partial_\eta K_{kl} + 2\partial_\eta K_k^j). \quad (19)$$

This expression can be used to compute conserved charges associated to isometries on the boundary $\partial\mathcal{M}$. One is mainly concerned with the conserved charges that are associated to Killing vectors ∂_t and ∂_ϕ , which correspond to the mass and the angular momentum, respectively. To define the charges it is convenient to make use of the Arnowitt-Deser-Misner formalism adapted to the boundary $\partial\mathcal{M}$. Then, the charges are defined by [40]

$$Q[\xi] = \int ds \xi^i u^j T_{ij}, \quad (20)$$

where ds is the volume element of the constant- t surfaces at the boundary, u is a unit vector orthogonal to the constant- t surfaces, and ξ is the Killing vector that generates the isometry in $\partial\mathcal{M}$.

To see how it works, let us consider the BTZ solution, whose metric is

$$ds^2 = -N^2(r)dt^2 + \frac{dr^2}{N^2(r)} + r^2(d\phi + N_\phi(r)dt)^2, \quad (21)$$

with

$$N^2(r) = \frac{r^2}{l^2} - \kappa^2 M + \frac{\kappa^4 J^2}{4r^2}, \quad N_\phi(r) = \frac{\kappa^2 J}{2r^2}. \quad (22)$$

It is straightforward to compute the mass and the angular momentum of ([32]) following the recipe described above. The mass and the angular momentum of BTZ black hole in TMG are then given by

$$M_{\text{BTZ}} = M + \frac{J}{l^2 \mu_G}, \quad J_{\text{BTZ}} = J + \frac{M}{\mu_G}, \quad (23)$$

respectively. It is well known [8,9] that this result differs from the charges of the same solution for GR, which are recovered if $1/\mu_G = 0$. In particular, these values for the mass and angular momentum in TMG imply that at the chiral point $\mu_G = 1/l$ all the BTZ black holes in TMG fulfill the relation $J_{\text{BTZ}} = lM_{\text{BTZ}} = lM + J$. More specifically, if $J = -lM$ at the chiral point both the mass and the angular momentum vanish.

Then, we can use the same idea to compute the mass and angular momentum of (5). It yields

$$M_{(k)} = \frac{6\pi k}{\kappa^2}, \quad J_{(k)} = -\frac{6\pi lk}{\kappa^2}. \quad (24)$$

This is consistent with the fact that (5) is a perturbation of the extremal BTZ black hole with $J = -lM$ at the chiral point $\mu_G = 1/l$. Recall that BTZ black hole with bare parameters obeying $J = -lM$ in chiral gravity have zero mass and zero angular momentum, and then we interpret it as the *ground state* for (5). Notice that, as long as Newton constant is positive, the BTZ black hole in TMG has positive mass, and our solution (5) has also positive mass for $k > 0$. Conversely, if we adopt the *wrong sign* for the Newton constant [what amounts to change $\kappa^2 \rightarrow -\kappa^2$ in (1) but keeping $\kappa^2 M$ unchanged] then the BTZ black hole turns out to have negative mass, while (5) has positive mass for $k < 0$.

Before concluding this section, let us mention that at the point $l\mu_G = -1$ one also finds a vacuum solution of TMG with the form

$$ds^2 = -N^2(r)dt^2 + \frac{dr^2}{N^2(r)} + r^2(N_\phi(r)dt + d\phi)^2 + N_k^2(r)(r^2 - \kappa^2 M l^2/2)(dt + ld\phi)^2. \quad (25)$$

Unlike solution (5), this metric tends to that of the extremal BTZ black hole when r approaches the horizon $r^2 = \kappa^2 M l^2/2$. The off-diagonal term in (25), however, grows in a more drastic way, behaving like $\sim 2kr^2 \log r$ at large distances.

Also, a charged solution at the chiral point exists, and it has a form like (5) and (25) with its charge associated to k . Now, we move on to discuss charged solutions.

III. CHARGED SOLUTIONS WITH A CHERN-SIMONS TERM

A. The solutions

In this section, we will show that solution (5) admits a natural generalization when TMG is coupled to TME (2), if the coupling constants satisfy

$$l\mu_G = 1 + 2l\mu_E. \quad (26)$$

For further convenience, we define the parameter $\varepsilon = -l\mu_E = \frac{1}{2}(1 - l\mu_G)$, which is an arbitrary real number. In particular, the theory at the chiral point corresponds to $\varepsilon = 0$ and $\varepsilon = 1$. For the case $l\mu_G = 1 + 2l\mu_E = 1 - 2\varepsilon > 1$, the metric of the charged solution takes the form

$$ds^2 = -N^2(r)dt^2 + \frac{dr^2}{N^2(r)} + r^2(d\phi - N_\phi(r)dt)^2 - N_Q^2(r)(dt - ld\phi)^2, \quad (27)$$

with

$$N^2(r) = \frac{r^2}{l^2} - \kappa^2 M + \frac{\kappa^4 M^2 l^2}{4r^2}, \quad N_\phi(r) = \frac{\kappa^2 M l}{2r^2},$$

and with

$$N_Q^2(r) = \frac{1}{2}\kappa^2 Q^2 (r^2 - \kappa^2 M l^2 / 2)^{-l\mu_E} \times \log((r^2 - \kappa^2 M l^2 / 2) / r_0^2),$$

and the electromagnetic field takes the form

$$A(r) = A_0 (r^2 - \kappa^2 M l^2 / 2)^{-l\mu_E / 2} (dt + l d\phi),$$

$$A_0^2 = Q^2 \frac{2(l\mu_E + 1)}{l\mu_E(2l\mu_E + 1)}. \quad (28)$$

Again, metric (27) corresponds to a deformation of the extremal BTZ black hole, which corresponds to the uncharged case $Q = 0$. If $l\mu_E > 0$, function N_Q^2 in (27) diverges at the horizon $r^2 = \kappa^2 M l^2 / 2$, but the curvature invariants remain constant. In fact, for all the solutions we find the Ricci scalar

$$R = -\frac{6}{l^2}, \quad (29)$$

and the Kretschmann scalar,

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = R_{\mu\nu}R^{\mu\nu} = \frac{12}{l^4}; \quad (30)$$

and one also finds $C_{\mu\nu}C^{\mu\nu} = 0$.

It is worth noticing that (29) holds even for charged solutions. This implies that the gauge field configuration is such that $T_\mu^\mu = 0$. We will return to this point below. It is also interesting that the Kretschmann scalar turns out to be independent of the parameters of the solution Q and M . This is a curious fact since solutions of Einstein gravity coupled to matter yielding a traceless stress tensor generically depend on the integration constants of the solution [42]. The fact that both the Ricci and Kretschmann scalars take the same value for all the members of the family of metrics (27) could lead us to suspect that all these geometries correspond to discrete quotients of the same (vacuum) space. However, this cannot be the case for *all* the solutions since the case $Q = 0$ [resp $k = 0$ in (5)] is locally AdS₃, while $Q \neq 0$ has a nonvanishing Cotton tensor.

The asymptotic behavior of (27) is determined by the following expansion:

$$g_{tt} = -\frac{r^2}{l^2} + \mathcal{O}(1) + \mathcal{O}(r^{-2l\mu_E} \log(r)),$$

$$g_{rr} = \frac{l^2}{r^2} + \mathcal{O}(r^{-4}),$$

$$g_{\phi\phi} = r^2 + \mathcal{O}(1) + \mathcal{O}(r^{-2l\mu_E} \log(r)),$$

$$g_{\phi t} = \mathcal{O}(1) + \mathcal{O}(r^{-2l\mu_E} \log(r)),$$

and $g_{r\phi} = g_{rt} = 0$. That is, solutions (5) are asymptotically AdS₃ for $l\mu_E = -\varepsilon > 0$. However, for $l\mu_E > 0$, the gauge field (28) diverges at the horizon $r^2 = \kappa^2 M l^2 / 2$. It turns out that solutions for which the gauge field vanishes at the horizon (e.g. for $l\mu_E < 0$) diverges dramatically at

the boundary, and *vice versa*, and thus no hair is allowed in this sense.

B. The logarithmic branch of self-dual solutions

As mentioned, the fact that the Ricci scalar of (27) takes the value $R = -6l^{-2}$ tells us that these charged solutions satisfy the traceless condition $T_\mu^\mu = 0$, which in three-dimensions implies $F_{\mu\nu}F^{\mu\nu} = 0$. This is reminiscent of the self-dual solutions discussed by Ait Moussa and Clément in [43]. Then, a natural question is whether our solutions are somehow related to those of [43]. We will see that, even though solutions (27) were not considered in [43], these can be obtained starting from the ones considered in that paper by taking the limit $l\mu_G \rightarrow 1 + 2l\mu_E$ appropriately, and then extending the manifold. To see this, let us define the coordinate

$$\frac{2}{\mu_E} \rho = r^2 - \frac{\kappa^2 M l^2}{2}.$$

In terms of the new radial coordinate ρ the metric (27) takes the form

$$ds^2 = -\left(\frac{2}{l^2 \mu_E} \rho + \frac{1}{2} Q^2 \rho^{-l\mu_E} \log(\rho) - \frac{1}{2} \mathcal{M}\right) dt^2$$

$$- l(Q^2 \rho^{-l\mu_E} \log(\rho) - \mathcal{M}) dt d\phi + \frac{l^2 d\rho^2}{4\rho^2}$$

$$+ l^2 \left(\frac{2}{l^2 \mu_E} \rho - \frac{1}{2} Q^2 \rho^{-l\mu_E} \log(\rho) + \frac{1}{2} \mathcal{M}\right) d\phi^2, \quad (31)$$

where we defined

$$Q^2 = (\mu_E/2)^{l\mu_E} \kappa^2 Q^2, \quad \mathcal{M} = \kappa^2 M.$$

In [43], similar solutions were considered for the case $l\mu_G - 1 \neq 2l\mu_E$, and these have the slightly different form

$$ds^2 = -\left(\frac{2}{l^2 \mu_E} \rho + \frac{1}{2} Q^2 \rho^{-l\mu_E} + \frac{1}{2} \mathcal{J} \rho^{(1-l\mu_G)/2} - \frac{1}{2} \mathcal{M}\right) dt^2$$

$$- l(Q^2 \rho^{-l\mu_E} + \mathcal{J} \rho^{(1-l\mu_G)/2} - \mathcal{M}) dt d\phi$$

$$+ \frac{l^2 d\rho^2}{4\rho^2} + l^2 \left(\frac{2}{l^2 \mu_E} \rho - \frac{1}{2} Q^2 \rho^{-l\mu_E} - \frac{1}{2} \mathcal{J} \rho^{(1-l\mu_G)/2} + \frac{1}{2} \mathcal{M}\right) d\phi^2. \quad (32)$$

Therefore, solutions (27) arise in the limit $l\mu_G \rightarrow 1 + 2l\mu_E$ of (32). At the point (26), two independent solutions to the field equations degenerate and thus the logarithmic form $\sim Q^2 \rho^{-l\mu_E} \log \rho$ stands as a new linear independent solution. The other solution $\sim \mathcal{J} \rho^{-l\mu_E} = \mathcal{J} \rho^{(1-l\mu_G)/2}$ contributes by setting the scale ρ_0 [related to r_0 in (27)], where the logarithm vanishes.

The case (32) we consider here is somehow special. It is continuously connected to the vacuum solutions (5) and

(25). Likely, solution (5) can be associated to a particular limit of solutions studied in [44].

Notice that for $Q = 0$ the region $-l^2\mu_E\kappa^2M/4 \leq \rho < 0$ corresponds to the region inside the horizon of the extremal BTZ black hole. Recall that for $k < 0$ the point $\rho = 0$ is at a finite geodesic distance from any point located at $\rho > 0$, and the geodesics end there. On the other hand, for $k > 0$ the point $\rho = 0$ is at infinite geodesic distance, as it happens for the self-dual solutions considered in [43].

C. Reduced field equations

The relation with the self-dual solutions (32) suggests that we could use the techniques used [43] to rederive our solutions (31). The idea in [43] was to reduce the field equations of TMG to a relativistic dynamical system, which is easily solved by choosing the appropriate ansatz.

Consider with [43] the following parameterization of the metric

$$ds^2 = h_{ab}(\rho)dx^a dx^b + \frac{1}{\zeta^2 R^2(\rho)} d\rho^2, \quad (33)$$

$$A(\rho) = \psi_a(\rho)dx^a,$$

where $a, b = 0, 1$, with $x^0 = t$, $x^1 = \varphi$, and $h_{tt} + h_{\phi\phi} = 2T$, $h_{tt} - h_{\phi\phi} = 2X$, $h_{\phi t} = Y$. Here, $X^0 = T$, $X^1 = X$, and $X^2 = Y$ are functions of ρ that satisfy the Minkowski product $R^2 = \mathbf{X}^2 = -T^2 + X^2 + Y^2 = \eta_{ij}X^i X^j$ with $i, j = 0, 1, 2$. We also denoted $\psi_0 = A_t$ and $\psi_1 = A_\phi$ for convenience.

In terms of this variables, the action takes the form

$$\begin{aligned} I_G + I_E = & \frac{1}{2} \int d^2x \int d\rho \left(\frac{1}{2\kappa^2} \zeta \dot{X}_i \dot{X}^i + \frac{2}{\kappa^2 l^2} \zeta^{-1} \right. \\ & + \frac{1}{2\kappa^2 \mu_G} \zeta^2 \varepsilon_{ijk} X^i \dot{X}^j \dot{X}^k + \zeta \dot{\psi} \Sigma^0 \Sigma_i X^i \dot{\psi} \\ & \left. - \mu_E \psi \Sigma^0 \dot{\psi} \right), \end{aligned} \quad (34)$$

where $\mathbf{X} = (X^0, X^1, X^2)$, $X_i = \eta_{ij}X^j$ with $i, j, k = 0, 1, 2$. We also denoted $\dot{\mathbf{X}} = \partial_\rho \mathbf{X}$, $\ddot{\mathbf{X}} = \partial_\rho^2 \mathbf{X}$, etc. The components of the vector $\Sigma = (\Sigma^0, \Sigma^1, \Sigma^2)$ are given by $\Sigma^0 = \sigma^1$, $\Sigma^1 = i\sigma^2\sigma^1$, and $\Sigma^2 = \sigma^3$, where σ^i are the Pauli matrices acting on the two-component vectors $\psi = (\psi_0, \psi_1)$. Notice also that ζ stands in (34) as a Lagrange multiplier. The first two terms in the action above correspond to the Einstein-Hilbert term and the cosmological constant term, while the third one corresponds to the gravitational Chern-Simons term. On the other hand, the terms involving ψ come from the gauge field $A = \psi_a dx^a$ with $a = 0, 1$.

The products in (34) are defined as $\mathbf{X} \cdot \mathbf{Y} = X^i Y^j \eta_{ij}$, $(\mathbf{X} \wedge \mathbf{Y})^k = \eta^{kl} \varepsilon_{iji} X^i Y^j$, and then the action can be written as follows:

$$\begin{aligned} I_G + I_E = & \frac{1}{2} \int d^2x \int d\rho \left(\frac{1}{2\kappa^2 \mu_G} \zeta^2 \mathbf{X} \cdot (\dot{\mathbf{X}} \wedge \ddot{\mathbf{X}}) \right. \\ & + \frac{1}{2\kappa^2} \zeta \dot{\mathbf{X}}^2 + \zeta \dot{\psi} \Sigma^0 \Sigma \cdot \mathbf{X} \dot{\psi} - \mu_E \psi \Sigma^0 \dot{\psi} \\ & \left. + \frac{2}{\kappa^2 l^2} \zeta^{-1} \right). \end{aligned} \quad (35)$$

Varying this action with respect to ψ , we find

$$\frac{\partial}{\partial \rho} (\zeta (\Sigma \cdot \mathbf{X}) \dot{\psi} + \mu_E \psi) = 0. \quad (36)$$

This equation yields

$$\zeta \dot{\mathbf{S}}_E = \frac{2\mu_E}{R^2} \mathbf{X} \wedge \mathbf{S}_E, \quad \text{with} \quad \mathbf{S}_E = -\frac{\kappa^2}{2} \bar{\psi} \Sigma \psi. \quad (37)$$

Also, varying (35) with respect to \mathbf{X} we find

$$\begin{aligned} \ddot{\mathbf{X}} = & \frac{\zeta}{2\mu_G} (3(\dot{\mathbf{X}} \wedge \ddot{\mathbf{X}}) + 2(\mathbf{X} \wedge \ddot{\mathbf{X}})) \\ & - \frac{2\mu_E^2}{\zeta^2 R^2} \left(\mathbf{S}_E - \frac{2}{R^2} \mathbf{X} (\mathbf{S}_E \cdot \mathbf{X}) \right) \end{aligned} \quad (38)$$

and

$$\mathbf{S}_E \cdot \mathbf{X} = \frac{\zeta^2 R^2}{2\mu_E^2} \left(\mathbf{X} \cdot \ddot{\mathbf{X}} - \frac{3\zeta}{2\mu_G} \mathbf{X} \cdot (\dot{\mathbf{X}} \wedge \ddot{\mathbf{X}}) \right). \quad (39)$$

The Hamiltonian constraint comes from varying (35) with respect to ζ ,

$$\begin{aligned} \mathcal{H} = & \frac{1}{4\kappa^2} \left(\dot{\mathbf{X}}^2 + 2\mathbf{X} \cdot \ddot{\mathbf{X}} - \frac{\zeta}{\mu_G} \mathbf{X} \cdot (\dot{\mathbf{X}} \wedge \ddot{\mathbf{X}}) - \frac{4}{l^2 \zeta^2} \right) \\ = & 0. \end{aligned} \quad (40)$$

Now, let us look for solutions of the form

$$\mathbf{X}(\rho) = \mathbf{u}G(\rho) + \mathbf{v}F(\rho), \quad (41)$$

where F and G are functions of ρ , while \mathbf{u} and \mathbf{v} are two vectors such that $\mathbf{u} \cdot \mathbf{v} = \eta_{ij}u^i v^j = 0$, and $\mathbf{v}^2 = \eta_{ij}v^i v^j = 0$. This implies $\mathbf{u} \wedge \mathbf{v} = \lambda \mathbf{v}$, that is, $\eta^{kl} \varepsilon_{iji} u^i v^j = \lambda v^k$, where λ is an arbitrary constant. We can make the choice [45]

$$\begin{aligned} \mathbf{u} = & \frac{1}{2l} (1 - l^2, 1 + l^2, 0), \\ \mathbf{v} = & -\frac{1}{4} (1 + l^2, 1 - l^2, \mp 2l) \end{aligned} \quad (42)$$

and then $\mathbf{u}^2 = \eta_{ij}u^i u^j = 1$. Then, we have two possible choices for λ , namely, $\lambda = \pm 1$, which correspond to each possibility for the sign \pm in (42). This ambiguity in the sign will be ultimately related to the sign of $l\mu_G$.

In terms of the ansatz (41) and (42), the Hamiltonian constraint (40) reads

$$\dot{G}^2 + 2G\ddot{G} - \frac{4}{l^2 \mu_E^2} = 0. \quad (43)$$

On the other hand, the equations of motion give

$$\mathbf{S}_E = \frac{\lambda\mu_E}{4\mu_G} G^2 (-3\dot{F}\ddot{G} + 3\dot{G}\ddot{F} + 2G\ddot{F} - 2F\ddot{G})\mathbf{v} + (G\mathbf{u} + F\mathbf{v})G\ddot{G} - \frac{1}{2}(\ddot{G}\mathbf{u} + \ddot{F}\mathbf{v})G^2. \quad (44)$$

Varying with respect to ρ we obtain $\dot{\mathbf{S}}_E$. We find $\mathbf{X} \wedge \mathbf{S}_E$ with $R^2 = G^2$, and we can go back to (37) and find

$$\dot{\mathbf{S}}_E = \frac{2}{R^2} \mathbf{X} \wedge \mathbf{S}_E,$$

with $\zeta = \mu_E$.

Then, we can solve both for \mathbf{u} and for \mathbf{v} . Before doing this, let us further specify the ansatz

$$G(\rho) = a\rho \quad F(\rho) = \kappa^2 Q^2 \rho^\varepsilon \ln \rho - \kappa^2 M, \quad (45)$$

where Q and M are two arbitrary real constants, while a and ε are two real parameters to be determined. This ansatz automatically satisfies the equation for \mathbf{u} , and thus we only have to solve for \mathbf{v} . The Hamiltonian constraint (40) demands

$$a^2 = -\frac{4\Lambda}{\mu_E^2} = \frac{4}{l^2 \mu_E^2}, \quad (46)$$

which is only possible if $\Lambda = -l^{-2} < 0$. Then, we have $a = \pm 2/l\mu_E$.

On the other hand, from the equation for \mathbf{v} we find

$$\begin{aligned} & \frac{\mu_E}{\mu_G} a^2 \left(\varepsilon - \frac{1}{2} \right) \left(\frac{1}{2} \varepsilon \lambda a - 1 \right) \varepsilon (\varepsilon - 1) \\ &= a \left(\frac{\varepsilon}{2} a - \lambda \right) \varepsilon (\varepsilon - 1), \end{aligned} \quad (47)$$

and

$$\begin{aligned} & \frac{\mu_E}{\mu_G} a^3 \lambda \varepsilon \left(2\varepsilon^2 - \frac{9}{4}\varepsilon + \frac{1}{2} \right) + a^2 \varepsilon \left(-\frac{3}{2}\varepsilon + 1 \right) \\ &= \frac{\mu_E}{\mu_G} a^2 \left(3\varepsilon^2 - 3\varepsilon + \frac{1}{2} \right) - a(2\varepsilon - 1)\lambda. \end{aligned} \quad (48)$$

Let us first analyze the cases $0 \neq \varepsilon \neq 1$. From (47) we find that $\lambda = \pm 1$ and $a = \pm 2/\varepsilon$, which implies $l\mu_E = \pm \varepsilon$. Then, from both (47) and (48) we get $\mu_G/\mu_E = (2\varepsilon - 1)/\varepsilon$. That is,

$$l\mu_G = 2l\mu_E \mp 1. \quad (49)$$

For these cases $\varepsilon = 0$ and $\varepsilon = 1$, Eq. (47) is trivially satisfied and we get no restriction for a and μ_E . For $\varepsilon = 0$ Eq. (48) yields $l\mu_G = 1$, which corresponds to the chiral point. In fact, $\varepsilon = 0$ corresponds to the solution (5) since for this configuration we also find $\psi_0 = \psi_1 = 0$, so that

$$A_\mu = 0. \quad (50)$$

Similarly, for $\varepsilon = 1$ Eq. (48) implies $l\mu_G = -1$ with $\psi_t = \psi_\varphi = 0$, and this is solution (25).

For the generic case, the metric takes the form

$$\begin{aligned} ds^2 = & \left(\pm \frac{2}{l^2 \mu_E} \rho - \frac{1}{2} \kappa^2 Q^2 \rho^{\pm l\mu_E} \log(\rho) + \frac{1}{2} \kappa^2 M \right) dt^2 \\ & + l(\kappa^2 Q^2 \rho^\eta \log(\rho) - \kappa^2 M) dt d\varphi - l^2 \left(\pm \frac{2}{l^2 \mu_E} \rho \right. \\ & \left. + \frac{1}{2} \kappa^2 Q^2 \rho^{\pm l\mu_E} \log(\rho) - \frac{1}{2} \kappa^2 M \right) d\varphi^2 + \frac{l^2}{4\rho^2} d\rho^2 \end{aligned} \quad (51)$$

and the gauge field configuration is

$$\begin{aligned} A(\rho) &= \sqrt{\frac{2Q^2(1-\varepsilon)\rho^\varepsilon}{\varepsilon(2\varepsilon-1)}} (dt + l d\phi) \\ &= Q \sqrt{\frac{2(1 \mp l\mu_E)}{l\mu_E(2l\mu_E \mp 1)}} \rho^{\pm l\mu_E/2} (dt + l d\phi). \end{aligned} \quad (52)$$

Expressions (51) and (52) correspond to solutions (27). Thus, we have rederived solutions (27) by using the method of [43]. This method also permits to compute conserved charges of the solutions in a rather systematic way. This amounts to calculate the so called superangular momentum \mathbf{J} , which is a current that gathers the conserved charges of this type of background with two commuting Killing vectors. The expression for such current is

$$\mathbf{J} = \mathbf{L} + \mathbf{S}_G + \mathbf{S}_E,$$

where

$$\begin{aligned} \mathbf{L} &= \frac{1}{2\kappa^2} \mathbf{X} \wedge \dot{\mathbf{X}}, \\ \mathbf{S}_G &= \frac{1}{4\kappa^2 \mu_G} (2\mathbf{X} \wedge (\mathbf{X} \wedge \ddot{\mathbf{X}}) - \dot{\mathbf{X}} \wedge (\mathbf{X} \wedge \dot{\mathbf{X}})). \end{aligned}$$

Evaluated in (51), these take the form

$$\begin{aligned} \mathbf{L} &= \frac{\lambda a}{2} (Q^2 (\varepsilon - 1) \rho^\varepsilon \log \rho + Q^2 \rho^\varepsilon + M) \mathbf{v}, \\ \mathbf{S}_G &= \frac{a^2}{4\mu_G} ((2\varepsilon - 1)(\varepsilon - 1) Q^2 \rho^\varepsilon \log \rho \\ &+ (4\varepsilon - 3) Q^2 \rho^\varepsilon - M) \mathbf{v}, \end{aligned}$$

and, from (44), we can also find the expression for \mathbf{S}_E . For the vacuum solution (5), which corresponds to the case $\varepsilon = 0$, we find

$$\mathbf{J} = -\frac{2k}{l\kappa^2} \mathbf{v}, \quad (53)$$

where we have set a to take a convenient value. Notice that (53) turns out to be proportional to k/κ^2 , like in (24). In [46] the computation of conserved charges from the expression for \mathbf{J} is discussed in detail, and the mass and angular momentum can be computed as quantities associated to Killing vectors ∂_t and ∂_ϕ , respectively. Remarkably, the mass and angular momentum computed with this method agree with our result (24), which was calculated by considering the stress tensor in the boundary

[47]. We will not give the details of this computation here; instead, we draw the reader's attention to the very interesting papers [8,43,44,48].

IV. SUMMARY

We have studied solutions to cosmological topologically massive gravity at special values of the coupling constants. First, we considered the theory at the chiral point, for which vacuum solution (5) was exhibited. This solution corresponds to a one-parameter deformation of GR solutions and is continuously connected to the extremal BTZ black hole. To be more precise, solution (5) has two parameters, k and M , and when $k = 0$ the solution turns out to be the extremal BTZ black hole with bare parameters satisfying $J = -lM$. It is well known that for all values of J and M , the BTZ black holes in TMG at the chiral point [15] satisfy the extremality relation $J_{\text{BTZ}} = lM_{\text{BTZ}} = lM + J$. In turn, the (massless) extremal BTZ with $lM + J = 0$ can be thought of as a kind of *ground state* of solutions (5), which are labeled by a real number k .

Solution (5) fails to be asymptotically AdS₃ in the sense of Brown-Henneaux boundary conditions [35], and this is because of a logarithmic damping at large distances. Nevertheless, it is still asymptotically AdS₃ in the sense of the boundary conditions recently proposed by Grumiller and Johansson in [12,21]. Then, the holographic computation of conserved charges in terms of the boundary stress tensor yielded (24), and the mass and angular momentum turn out to be proportional to k/κ^2 . Therefore, the sign of the mass of (5) can be chosen to be opposite to that of the BTZ black hole in this theory. That is, if one adopts the negative sign for the Newton constant (as it is usual in TMG) then the solutions with positive mass ($k < 0$) are those that allow geodesic to reach the radius $r^2 = \kappa^2 M l^2 / 2$ at a finite proper time, while for the case $k > 0$ that circle is at infinite geodesic distance.

We also considered solutions (27), which are charged analogues to (5) that exist when the coupling constants satisfy the relation $l\mu_G = 1 + 2l\mu_E$. Unlike vacuum solutions we found at the chiral point, their charged analogues may have a stronger damping at large distances and then represent asymptotically AdS₃ solutions in the sense of [35]. However, for asymptotically AdS₃ solutions both the gauge field and the effective potential of the geodesic equation for massive particles diverge at the horizon.

Like vacuum solutions, charged solutions (28) have constant scalar curvature $R = -6l^{-2}$. This implies that the corresponding gauge field configurations fulfill the condition $T_\mu^\mu = F_{\mu\nu}F^{\mu\nu} = 0$. This is reminiscent of the self-dual solutions studied in Ref. [43], and we have actually shown that solutions (27) can be thought of as a limiting procedure starting from the self-dual solutions of [43].

It is also remarkable that both solutions (5) and (27) have constant Kretschmann scalar, given by $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} =$

$12l^{-4}$. This means that all the quadratic invariants turn out to be independent of the two parameters of the solutions. Nevertheless, it is worth emphasizing that both M and Q still represent actual parameters labeling the solutions, as they enter in the computation of the charges in a nontrivial way, and, besides, parameter Q is the one that permits to interpolate between (27) and the extremal BTZ black hole.

Before concluding, let us comment on the relation between the solutions we discussed here and a class of pp -wave solutions recently discussed in the literature. Just recently, we were taught [49] that solution (5) can be obtained from one of the pp -wave solutions considered in [11,50] by an appropriate coordinate transformation, in addition to the compactification of the direction we denoted by ϕ . For instance, the metrics considered in [50] have the form

$$ds^2 = dR^2 + e^{2R} dx^+ dx^- + Rf(x^-)(dx^-)^2, \quad (54)$$

where $f(x^-)$ is an arbitrary function of x^- (for instance, see Eq. (3.21) of Ref. [50], with $l = 1$, $\mu = 1$, $\rho = R$, $u = x^+/2$, $v = x^-$, and $x^- = t - \phi$, with ϕ being compact; see also [11,31], where pp -wave solutions of TMG are discussed). It is easy to see that (54) can be written as our solution (5) by means of the appropriate coordinate transformation. For instance, consider the case $M = 0$, as in (12), which takes the form (54) by choosing $f = 2k$ and defining the radial coordinate $R = \log(r)$. A similar relation holds between (25) and the solution presented in Eq. (3.22) of [50] for the case $l\mu_G = -1$. This allows to interpret our charged solutions (27) as a generalization of some of the solutions considered in [11,50].

The solution we have presented here generalizes the extremal BTZ black hole solution at the chiral point, and it represents an exact realization of the boundary conditions proposed in [12,21].

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