α-vacuum and inflationary bispectrum

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In this paper, we discuss the non-Gaussianity originated from the α-vacuum on the cosmic microwave background anisotropy. For the α-vacuum, there exists a correlation between points in the acausal two patches of de Sitter spactime. Such kind of correlation can lead to large local form non-Gaussianity in the α-vacuum. For the single field slow-roll inflationary scenario, the space-time is in a quasi-de Sitter phase during the inflation. We will show that the α-vacuum in this case will lead to non-Gaussianity with a distinguished feature, of a large local form and a very different shape.

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I. INTRODUCTION

In the standard hot big bang cosmology, there are several tough problems, including the flatness, isotropy and homogeneity, horizon, and topological defects problems. The hot big bang theory is unable to answer these problem in a natural way. Inflation, as an add-on, is remarkably successful in solving these problems. It gives a natural initial condition for our observed universe [1–3]. Furthermore the quantum fluctuations during inflation seed wrinkles in the cosmic microwave background (CMB) and today's large scale structure [4–8]. As a result, inflation predicts a nearly scale invariant Gaussian CMB spectrum, which has been confirmed very well in the experiments [9].

However, inflation as a successful scenario in the very early universe has its own difficulties. One of the problems with inflation is that there are too many inflationary models, which cannot be distinguished by the scalar power spectrum and power spectrum index from the CMB observation. It is essential to find more powerful signatures which could distinguish various models from each other. Moreover, inflation has some conceptual problems, the cosmological singularity problem and Trans-Planckian physics being two of them. In a sense, the inflation scenario is not really a fundamental theory. The trouble with it mainly comes from our ignorance of the physics of the very early universe, which should be governed by a quantum gravity theory.

The rapid development of precise experiment cosmology opens new windows to the very early universe. The scalar spectral index and its running, the gravitational wave, non-Gaussianity, and the isocurvature perturbation in the CMB [9] are among the important probes. These probes will not only constrain a large amount of inflation models and make the paradigm more clear, but also shed light on various other issues beyond the usual inflation scenario. These issues include the initial conditions of the inflation model, Trans-Planckian physics, and alternative models to inflation, etc.

Among the probes, non-Gaussianity is one of the most important ones. It contains much information: magnitude, shape, sign, and even running. In principle, it could distinguish various inflation models. The deviation from the Gaussian distribution of the CMB in the Wilkinson Microwave Anisotropy Probe (WMAP) observation is parametrized by $f_{\rm NL}$ [10],

$$\zeta = \zeta_g + \tfrac{3}{5} f_{\rm NL} \zeta_g^2, \tag{1}$$

where ζ is the curvature perturbation in the uniform density slices, and the subscript g denotes the Gaussian distribution. In the WMAP five-year data [9], two kinds of non-Gaussianity, local form and equilateral form, have been analyzed

$$\begin{aligned} -9 < f_{\rm NL}^{\rm local} < 111 \qquad (95\%{\rm CL}), \\ -151 < f_{\rm NL}^{\rm equil} < 253 \qquad (95\%{\rm CL}). \end{aligned} \tag{2}$$

The central value of the local form non-Gaussianity is 51. If the value of the local form non-Gaussianity is confirmed by future experiments, such as the Planck satellite, then it will be a great challenge to many inflation models, including the most well-studied single field inflation models.

In fact, the non-Gaussianity in the CMB spectrum may come from various sources during the evolution. The temperature fluctuation $\frac{\Delta T}{T}$ is the observable in the CMB observation. During inflation, the quantum fluctuation of the inflatons $\delta\phi$ are generated, and the modes of the fluctuation grow with the exponentially expanding universe. After the fluctuations leave the horizon, the decoherence effect makes the quantum fluctuations to be the classical ones. In the large scale, the physical freedom of scalar perturbation is curvature perturbation ζ. If we follow a mode,

$$\begin{matrix}\text{Initial condition} \\ \text{(Vacuum)}\end{matrix} \rightarrow \delta\phi \rightarrow \zeta \rightarrow \frac{\Delta T}{T}. \tag{3}$$

All the transformations are linear at first order, thus the temperature fluctuations are Gaussian. Meanwhile, any deviation from linearity in these transformations and the changes in the initial condition will influence the final observable.

(i) Let us first consider the last stage of the transformation $\zeta \to \frac{\Delta T}{T}$. The fluctuations in the gravitational potential on the last scattering surface result in temperature fluctuations in the CMB, which is known as the Sachs-Wolfe effect. The nonlinear Sachs-Wolfe effect generates $f_{\rm NL}$ of order one.

(ii) The curvature perturbation ζ is conserved in the single field inflation, while in the multiple field case, the entropy perturbation changes the evolution of ζ. It will suppress the perturbation conversion factor for ζ in this process. That is why in the curvaton mechanism [11] and new ekyrotic models [12] the large local form non-Gaussianity is possible. (The other important reason for ekyrotic models generating large non-Gaussianity is that the slow-roll condition breaks down.)

(iii) The primordial non-Gaussianity, which resides on the quantum fluctuation of the scalar field [13], can be from the microphysics deep in the horizon. Since the observation requires the potential of the inflaton to be slow roll, the interaction of the inflaton is weak, and non-Gaussianity is only the order of the slow-roll parameter $f_{\rm NL} \sim \mathcal{O}(\epsilon, \eta)$. The picture will change when the modified gravity and noncanonical action are considered, such as in ghost inflation [14], Dirac-Born-Infeld (DBI) inflation [15], and k inflation [16]. The nonlinearity in the action can produce a large equilateral form non-Gaussianity in the CMB. On the other hand, the backreaction argument [13] explains why microphysics in the horizon cannot have large local form non-Gaussianity.

(iv) The initial condition could be another important source of non-Gaussianity. One attempt is to consider the thermal vacuum in the inflationary cosmology [17], in which the equilateral and local form non-Gaussianity are both $\gtrsim \mathcal{O}(1)$ in some cases. In this paper, we will consider one parameter family of vacuum states, called the α-vacuum, in de Sitter space-time [18,19] and quasi-de Sitter space-time in inflation. In these vacua, there are correlations between points in the acausal two patches of de Sitter space-time. We will show that the α-vacuum can induce large local form non-Gaussianity.

As we know, the standard treatment in the scalar driven inflation scenario is based on semiclassical gravity, in which the background is described by classical Einstein gravity and the perturbations are quantized in the background. During the slow-roll period, since the evolution of the background is in a quasi-de Sitter phase, the perturbations could be treated as the quantum field in a de Sitter space-time. In the expanding background of inflation, the quantum modes are stretching across the horizon. The modes observed in the CMB could be in the trans-Planckian region at the very early time. If the inflation lasts about 70 e-foldings, the perturbation which we observe in our horizon, at that time, is deep inside the horizon. And the wavelength of this perturbation is smaller than the Planck scale. The semiclassical description is not applicable for the perturbation. It is necessary to consider the stringy effect or some other quantum gravity effects on inflation. The trans-Planckian physics in inflation was first raised in [20]. As in black hole physics, an efficient way to count the Trans-Planckian effect is to modify the dispersion relations. Various dispersion relations and their physical implications have been widely studied [21]. Another way to discuss the trans-Planckian physics is based on the space-time uncertainty from quantum gravity, such as string theory. This noncommutative effect will impact the power spectrum and gravitational waves of the CMB [22,23] and also modify the non-Gaussianity [24].

There is another attempt to address the trans-Planckian physics in inflation, first suggested by Danielsson [25]. The new important ingredient in the discussion is the introduction of one parameter family of the α-vacuum state in the inflationary background. In de Sitter space-time, the Bunch-Davies vacuum is the standard vacuum which is invariant under the de Sitter space isometry group. However, due to the absence of the globally defined time-like Killing vector, the vacuum in de Sitter space-time cannot be defined uniquely. Similarly, the choice of the vacuum in the inflationary background is subtle and may induce observable signature on CMB data. It was argued that the effective field theory and semiclassical gravity are applicable from the length of the new physical scale cutoff to the large scale of the universe. And it was also assumed that the modes were generated one by one at the Planck scale or new physics scale Λ such that the initial conditions are imposed at a mode-dependent time instead of in the infinite past. This is the motivation to the introduction of the α-vacuum in inflation. Its physical implications on inflation have been intensely studied. The order of the correction to the power spectrum has been discussed in [26–35]. For example, in [32] using the method of effective field theory the authors found that the correction was $\sim\mathcal{O}(\frac{H^2}{\Lambda^2})$, and in [31] the authors calculated the correction of power spectrum when the modes are initially created by the adiabatic vacuum state, and found that the correction was $\sim\mathcal{O}(\frac{H^3}{\Lambda^3})$. The careful analysis of these different corrections can be seen in [34]. For the non-Gaussianity from the trans-Planckian physics it was first roughly analyzed in [36], and in [37] its folded form was analyzed in the effective field theory. In this paper we follow the treatments in [25,30]. In [30] the authors evaluated the α-vacuum effect in the general single field inflation background and found an oscillating dependence on the wave number k for the power spectrum. The reason is that during inflation the Hubble scale is not constant, and the coefficient for the α-vacuum state sensitively relies on k. We will show that this k-dependence lead to a distinguished feature of non-Gaussianity.

This paper is organized as follows: in Sec. II, we first discuss the vacuum states in de Sitter space, the relation between the Euclidean vacuum and α-vacuum. Then we introduce different correlation functions and explain the property of the antipodal correlation in de Sitter space. In Secs. III and IV, we review the Lagrangian formalism to compute the power spectrum and three-point correlation of the curvature perturbation. Section V is the main result of this paper. We evaluate the local form and equilateral form non-Gaussianity for the Euclidean vacuum and α-vacuum in both de Sitter space-time and inflationary backgrounds. We also draw the shapes of non-Gaussianity in each case. Finally, we conclude in Sec. VI.

II. VACUUM STATE IN DE SITTER SPACE

The space-time of inflation is a quasi-de Sitter space-time, which can be conventionally described by the Friedmann-Robertson-Walker (FRW) coordinate,

$$ds^2 = dt^2 - e^{2Ht}dx^2, \tag{4}$$

where H is the Hubble scale. In this section, to illustrate the feature of the α-vacuum clearly, we mainly discuss the de Sitter space in which H is simply a constant. Note that the metric (4) actually covers only half of the de Sitter space-time.

The equation of motion for a scalar field in the background takes the form

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} - \nabla^2\delta\phi + \frac{\partial V}{\partial\delta\phi} = 0. \tag{5}$$

The scalar field could be an inflaton, for which the mass of the scalar field is much less than the Hubble scale $m \ll H$. The complete solution of (5) can be expressed in momentum space [38],

$$\delta\phi(\tau, \mathbf{x})_k = \tfrac{1}{2}\pi^{1/2}H(-\tau)^{3/2}e^{i\mathbf{k}\cdot\mathbf{x}}[c_1(k)\mathrm{H}_\nu^{(1)}(-k\tau) + c_2(k)\mathrm{H}_\nu^{(2)}(-k\tau)], \tag{6}$$

where $\tau = -\frac{1}{aH}$ is the conformal time in de Sitter space,

$$\nu = \sqrt{\frac{9}{4} - 12\frac{m^2}{H^2}}, \tag{7}$$

and $\mathrm{H}_\nu^{(1)}_{(2)}$ are Hankel functions of the first and second kind. In the limit of $\eta \to -\infty$ for a fixed k, which means that a mode of the scalar field is deep in the Hubble horizon,

$$\mathrm{H}^{(1)}_{(2)}(-k\eta) \to \left(-\frac{2}{\pi k\eta}\right)^{1/2} e^{(\mp)ik\eta}, \tag{8}$$

up to some constant phase factor.

When a mode is deep in the horizon, the spatial scale is much smaller than the Hubble scale so that the curvature effect is negligible and the scalar field is well described by quantum field theory in Minkowski space. As $\eta \to -\infty$, we could choose the vacuum as in the flat space and positive frequency modes from the Hankel function of the second kind $\mathrm{H}_\nu^{(2)}$, i.e. $c_2(k) = 0$. This is called thermal vacuum or Euclidean vacuum in de Sitter space-time.

The scalar field is quantized by the canonical method,

$$\delta\phi = \sum_n (\delta\phi_n a_n + \delta\phi_n^* a_n^\dagger), \tag{9}$$

where n denotes all the quantum numbers of the modes, and $\{a_n, a_n^\dagger\}$ are the annihilation and creation operators satisfying the commutative relation

$$[a_n, a_m^\dagger] = (2\pi)^3\delta_{mn}. \tag{10}$$

The Euclidean vacuum state is defined to be

$$a_n|\Omega\rangle = 0. \tag{11}$$

Let us consider a mode k as $\eta \to -\infty$. The physical wavelength of the modes is smaller than the Planck scale. It is natural to set a physical cutoff for momentum p_c. When $p > p_c$, one has to consider the trans-Planckian effect. The influence of new degrees of freedom and a new physical law could be effectively encoded in the change of dispersion relation [21], space-time noncommutativity [22,23], or some other ways. When $p < p_c$, the solution of the Klein-Gorden equation is reliable, and the solution of the scalar field is the linear combination of $\delta\phi(\eta, \mathbf{x})_k^\pm$ in (6). A new set of modes for the trans-Plankian effect is expressed as the combination of the Euclidean modes by a Bogoliubov transformation [18,39] (Mottola-Allen transform),

$$\tilde{\delta}\phi_n \equiv N_\alpha(\delta\phi_n + e^\alpha\delta\phi_n^*), \qquad N_\alpha = \frac{1}{\sqrt{1 - e^{\alpha+\alpha^*}}}, \tag{12}$$

where α is a complex number with $\mathrm{Re}\alpha < 0$ to denote the rotation of field space, and N_α is derived from the Wronskian condition or the rule of Bogoliubov transformation. Since

$$\delta\phi = \sum_n(\delta\phi_n a_n + \delta\phi_n^* a_n^\dagger) = \sum_n(\tilde{\delta}\phi_n\tilde{a}_n + \tilde{\delta}\phi_n^*\tilde{a}_n^\dagger),$$

the equation of $\tilde{a}_n$ can be expressed as

$$\tilde{a}_n = N_\alpha(a_n - e^{\alpha^*}a_n^\dagger). \tag{13}$$

Thus the new vacuum called the α-vacuum is defined as follows:

$$\tilde{a}_n|\alpha\rangle = 0, \tag{14}$$

where the α-vacuum is still de Sitter invariant, just like the Euclidean one. The Bogoliubov transform can be implemented by a unitary transform [19],

$$\tilde{a}_n = \mathcal{S}a_n\mathcal{S}^\dagger, \tag{15}$$

where

$$\mathcal{S} = \exp\left\{\sum_n c(a_n^\dagger)^2 - \bar{c}(a_n)^2\right\},$$
$$c(\alpha) = \frac{1}{4}\left(\ln\tanh\frac{-\mathrm{Re}\alpha}{2}\right)e^{-i\,\mathrm{Im}\alpha}. \tag{16}$$

The relation between the two vacua is

$$|\alpha\rangle = \mathcal{S}|\Omega\rangle. \tag{17}$$

Let us turn to the Green functions which are useful in studying the power spectrum and non-Gaussianity. There are several kinds of Green functions, but all of them can be expressed by the Wightman function.

In the Euclidean vacuum, the Wightman function can be expressed as

$$G_E(x, x') = \langle\Omega|\delta\phi(x)\delta\phi(x')|\Omega\rangle = \sum_n \delta\phi_n(x)\delta\phi_n^*(x'). \tag{18}$$

For a distance much smaller than the Hubble scale, G_E takes the form in Minkowski space-time,

$$G_E(x, x') \sim \frac{1}{(t - t' - i\epsilon)^2 - |\vec{x} - \vec{x}'|^2}. \tag{19}$$

Near the light cone, the Green function is divergent.

The metric (4) only covers one-half of the whole de Sitter space-time. To illustrate the character of the Wightman function for the α-vacuum, we must extend the analysis to the whole de Sitter space-time. The de Sitter space-time can be constructed as a hyperboloid in the five-dimensional flat space-time,

$$-(X^0)^2 + (X^1)^2 + (X^2)^2 + (X^3)^2 + (X^4)^2 = \frac{1}{H^2}, \tag{20}$$

where X^i is the coordinate in the flat five-dimensional space-time. Thus de Sitter space-time is a maximal symmetric space-time with constant curvature, and its symmetry group is $O(1, 4)$. Define the antipodal point of X as $X_A = -X$. In the Euclidean vacuum, the modes of scalar field can be chosen to obey the rule [18,39],

$$\delta\phi_n(x_A) = \delta\phi_n^*(x), \tag{21}$$

where x_A denotes the antipodal point to x in de Sitter space as in Fig. 1.

In the α-vacuum, the Wightman function takes the form

$$G_\alpha(x, x') = \langle\alpha|\delta\phi(x)\delta\phi(x')|\alpha\rangle = \sum_n \tilde{\delta}\,\phi_n(x)\tilde{\delta}\phi_n^*(x'). \tag{22}$$

With Eqs. (18) and (21), G_α is of the form

$$G_\alpha(x, x') = N_\alpha^2[G_E(x, x') + e^{\alpha+\alpha^*}G_E(x', x) + e^{\alpha^*}G_E(x, x'_A) + e^{\alpha}G_E(x_A, x')]. \tag{23}$$

The Wightman function has some special properties. First, the Green function contains singularity at antipodal points $\{x_A, x\}$. The singularity cannot be observed because of the separation by the horizon. Second, the correlation is not acausal, because when one calculates the retarded (advanced) Green function below, the correlation from acausal patches does not exist. Finally, the Wightman function contains the correlation between points in the two patches of de Sitter space-time. It brings correction to the power spectrum. And most importantly, it influences the shape of non-Gaussianity, which makes it much different from the Euclidean vacuum.

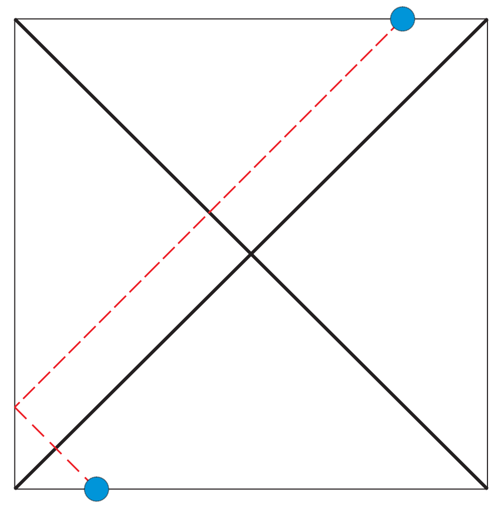

FIG. 1 (color online). The Penrose diagram of de Sitter space, where the two blue points are antipodal points in the space. The left upper part of the diagram is the spatial flat patch for an observer on the left-hand boundary.

In order to calculate the power spectrum and non-Gaussianity from the α-vacuum, it is better to use the Green functions in momentum space. The power spectrum of the scalar field can be read from the two-point correlator in momentum space,

$$\begin{aligned}\langle\alpha|\delta\phi(\mathbf{k}, \eta)\delta\phi(\mathbf{k}', \eta')|\alpha\rangle &= (2\pi)^3\delta(\mathbf{k} - \mathbf{k}')\delta\tilde{\phi}_k(\eta) \times \tilde{\delta}\phi_k^*(\eta') \\ &= (2\pi)^3\delta(\mathbf{k} - \mathbf{k}')N_\alpha^2[\delta\phi_k(\eta) + e^{\alpha}\delta\phi_k^*(\eta)][\delta\phi_k^*(\eta') + e^{\alpha^*}\delta\phi_k(\eta')],\end{aligned} \tag{24}$$

where $\delta\phi_k$ is the mode of scalar field in momentum field. In the Euclidean vacuum, for the leading order approximation

$$\delta\phi_k(\eta) = (-H\eta)\left(1 - \frac{i}{k\eta}\right)\frac{e^{-ik\eta}}{\sqrt{2k}}. \tag{25}$$

When the modes cross the horizon, the quantum fluctuation is transformed to a classical one, and the curvature perturbation is conserved for large scale. Thus we take the time η at the horizon crossing time, which is a good approximation to calculate the power spectrum in the single field case. For $k\eta \ll 1$,

$$\langle\alpha|\delta\phi(\mathbf{k},\eta)\delta\phi(\mathbf{k}',\eta')|\alpha\rangle \sim (2\pi)^3\delta(\mathbf{k}-\mathbf{k}')\frac{H^2}{2k^3}N_\alpha^2 \times(1+e^{\alpha+\alpha^*}-e^{\alpha}-e^{\alpha^*}) = (2\pi)^3\delta(\mathbf{k}-\mathbf{k}')\frac{2\pi^2}{k^3}\mathrm{P}(k). \tag{26}$$

Thus the power spectrum of the scalar field can be written as

$$\mathrm{P}(k) = \frac{H^2}{(2\pi)^2}\frac{1+e^{\alpha+\alpha^*}-2\,\mathrm{Re}\,e^{\alpha}}{1-e^{\alpha+\alpha^*}}. \tag{27}$$

The leading order correction of the power spectrum comes from the Re e^{α}. From the Wightman function in coordinate space, it is easy to see that the contribution is from $G_E(x, x'_A)$. Meanwhile, if we use a physical cutoff to set an initial condition of the modes, the power spectrum should depend on the cutoff scale Λ, which could be the string scale, Planck scale, or others.

Next, let us analyze the retarded Green function to prove that the antipodal point does not break the causality. The retarded Green function is defined as

$$G_R(x,x') = i\theta(t-t')\langle\mathrm{VAC}|[\delta\phi(x),\delta\phi(x')]|\mathrm{VAC}\rangle, \tag{28}$$

where

$$\theta(t) = \begin{cases}1 & t>0\\ 0 & t<0\end{cases}. \tag{29}$$

In the Euclidean vacuum, the retarded Green function in momentum space takes the form

$$G_{RE}(\eta,\tau) = i(2\pi)^3\delta(\mathbf{k}-\mathbf{k}')\begin{cases}0 & \eta<\tau\\ \delta\phi_k^*(\tau)\delta\phi_k(\eta)-\delta\phi_k(\tau)\delta\phi_k^*(\eta) & \eta>\tau\end{cases}. \tag{30}$$

And in the α-vacuum, the retarded Green function takes the form

$$G_{R\alpha}(\eta,\tau) = i(2\pi)^3\delta(\mathbf{k}-\mathbf{k}')\begin{cases}0 & \eta<\tau\\ \tilde{\delta\phi}_k^*(\tau)\tilde{\delta\phi}_k(\eta)-\tilde{\delta\phi}_k(\tau)\tilde{\delta\phi}_k^*(\eta) & \eta>\tau\end{cases}. \tag{31}$$

According to Eq. (12), the Green function can be expressed by the one in the Euclidean vacuum,

$$G_{R\alpha}(\eta,\tau) = N_\alpha^2[G_{RE}(\eta,\tau)+e^{\alpha+\alpha^*}G_{RE}^*(\eta,\tau)]. \tag{32}$$

From the above equation, the retarded Green function in the α-vacuum does not contain the correlation from the two patches of de Sitter space-time, so the causality is kept in the vacuum.

III. ARNOWITT-DESER-MISNER (ADM) FORMALISM AND CURVATURE PERTURBATION IN INFLATIONARY BACKGROUND

During inflation, the Hubble radius is changing slowly and the space-time is not exactly a de Sitter space-time. As usual, we have slow-roll parameters,

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}^2}{2H^2} \simeq \frac{1}{2}\left(\frac{V'}{V}\right)^2, \quad \eta = -\frac{\ddot{\phi}}{H\dot{\phi}}+\frac{1}{2}\frac{\dot{\phi}^2}{H^2} \simeq \left(\frac{V''}{V}\right) = \frac{V''}{3H^2}. \tag{33}$$

The condition ϵ, $\eta \ll 1$ indicates that the velocity and the acceleration of the inflaton is quite small. Despite the small deviation from pure de Sitter space-time, one can still define the α-vacuum. However, the physical implications of the α-vacuum to the CMB spectrum are very different. For example, in de Sitter space the power spectrum has a constant correction with a magnitude of $\mathcal{O}(H/\Lambda)$ [25], where Λ is the scale of the new physics, while in inflationary background the correction is dependent of the wave number k, so that the power spectrum oscillates with k [30].

For simplicity, we just analyze the single field inflationary models with canonical action. If there exists only one scalar field in quasi-de Sitter space-time, then there is just one perturbation freedom $\delta\phi$ from the scalar field. However, there are also four scalar perturbation freedoms from the metric $\delta g_{\mu\nu}$. $\delta\phi$ and $\delta g_{\mu\nu}$ do not decouple for scalar perturbation. Gauge invariance removes two of the scalar degrees of freedom by time and spatial reparametrizaions $x_i \rightarrow x_i + \partial_i\epsilon(t,x)$ and $t \rightarrow t+\epsilon(t,x)$ [40]. The constraints in the action remove two other freedoms. Thus, there is only one scalar degree of freedom left. Therefore, we should choose a convenient gauge and discuss the only physical freedom in the single field inflationary model.

In general, the space-time can be decomposed using the ADM formalism [41], and the metric takes the form

$$ds^2 = -\mathrm{N}^2dt^2 + h_{ij}(dx^i+\mathrm{N}^idt)(dx^j+\mathrm{N}^jdt), \tag{34}$$

where h_{ij} is the metric of three dimensional spatial slices. The lapse N and the shift vector N_i contain the freedom of the scalar perturbation, such as time reparametrization and spatial reparametrization. With the 3 + 1 decomposition, the extrinsic curvature of the spatial slice is

$$K_{ij} = \mathrm{N}\Gamma^0_{ij} = \frac{1}{2\mathrm{N}}(\dot{h}_{ij}-\nabla_i\mathrm{N}_j-\nabla_j\mathrm{N}_i), \tag{35}$$

where Γ^0_{ij} is the Christoffel symbol in four dimension space-time. And the intrinsic curvature of the spatial slices takes the form:

$$R^{(3)} = R - K_{ij}K^{ij} - K^2, \tag{36}$$

where

$$K = K^i_i. \tag{37}$$

To simplify the action, we introduce another parameter $E_{ij} \equiv \mathrm{N}K_{ij}$, so the standard Einstein-Hilbert action can be written as

$$S = \frac{1}{2}\int d^4x\sqrt{h}[\mathrm{N}R^{(3)} - 2\mathrm{N}V + \mathrm{N}^{-1}(E_{ij}E^{ij} - E^2) + \mathrm{N}^{-1}(\dot{\phi} - \mathrm{N}^i\partial_i\phi)^2 - \mathrm{N}h^{ij}\partial_i\phi\partial_j\phi], \tag{38}$$

where $h = \det h_{ij}$. In the action, there is no time derivative of N or N_i, so they are Lagrangian multipliers which can be solved directly as the constraint equations.

It is convenient to choose the comoving gauge, in which the inflaton perturbation vanishes in the spatial slices,

$$\delta\phi = 0, \quad h_{ij} = a^2 e^{2\zeta}\delta_{ij}. \tag{39}$$

The spatial metric h_{ij} is the nonperturbative form [13,42], and the tensor perturbation is omitted for considering only scalar perturbation. In this gauge, ζ is the physical degree of freedom, which is constant outside the horizon in single field inflation.

In the comoving gauge, the constraint equation is

$$\nabla_i[\mathrm{N}^{-1}(\mathrm{E}^i_j - \delta^i_j\mathrm{E})] = 0,$$
$$R^{(3)} - 2V - \mathrm{N}^{-2}(\mathrm{E}_{ij}\mathrm{E}^{ij} - \mathrm{E}^2) - \mathrm{N}^{-2}\dot{\phi}^2 = 0. \tag{40}$$

From these constraints and the equations of the background, the action can be expanded to second order, third order, and even higher order of ζ. In order to get the action order by order, we need to expand the N and N_i first. For the shift vector N_i, it can always be decomposed as

$$\mathrm{N}_i = \partial_i\psi + \tilde{\mathrm{N}}_i, \tag{41}$$

where $\partial_i\tilde{\mathrm{N}}_i = 0$, and ψ denotes the scalar perturbation of metric g_{0i}. These Lagrangian multipliers can be decomposed in powers of ζ,

$$\mathrm{N} = 1 + \alpha_1 + \alpha_2 + \cdots, \qquad \psi = \psi_1 + \psi_2 + \cdots,$$
$$\tilde{\mathrm{N}}_i = \tilde{\mathrm{N}}^{(1)}_i + \tilde{\mathrm{N}}^{(2)}_i + \cdots, \tag{42}$$

where the subscript denotes the order, for example $\alpha_n \sim \mathcal{O}(\zeta^n)$. From the constraint equation on N_i (40), we could get

$$\alpha_1 = \frac{\dot{\zeta}}{H}, \qquad \partial^2\tilde{\mathrm{N}}^{(1)}_i = 0. \tag{43}$$

Using the appropriate boundary condition, $\tilde{\mathrm{N}}_i$ can be set to 0. From the viewpoint of inflationary perturbation, $\tilde{\mathrm{N}}_i$ represents the vector perturbation of the metric, which vanishes in the boundary. From the constraint equation on N (40), using the equation of $R^{(3)}$

$$R^{(3)} = -2a^{-2}e^{-2\zeta}[(\partial\zeta)^2 + 2\partial^2\zeta], \tag{44}$$

and Friedmann equation of the background, the first order of ψ is given by

$$\psi_1 = -\frac{\zeta}{H} + a^2\frac{\dot{\phi}^2}{2H^2}\partial^{-2}\dot{\zeta}. \tag{45}$$

To get the action to the quadratic order of ζ, it is enough to expand N and N_i to the first order of ζ, because the second order term in N and N_i will multiply the zero order of the constraint equation which is zero. With the same reason, to get the cubic action for ζ, we do not need to expand the N and N_i to cubic order. The second order expansion of N and N_i in the cubic action vanish or reduce to total derivatives. So the action to the quadratic and cubic order of ζ can be obtained by substituting N, N_i to the first order into the action and then expanding the action to the second and third order of ζ.

After integrating by parts, the quadratic action of ζ takes the form as

$$S_2 = \frac{1}{2}\int d^4x\frac{\dot{\phi}^2}{H^2}[a^3\dot{\zeta}^2 - a(\partial\zeta)^2]. \tag{46}$$

To get the field equation and solve the curvature perturbation for different modes, a rescale field is defined as

$$v \equiv z\zeta, \qquad z \equiv a\sqrt{2\epsilon}. \tag{47}$$

The equation of motion is

$$v''_k + \left(k^2 - \frac{z''}{z}\right)v_k = 0, \tag{48}$$

where the prime $'$ denotes the derivative with the conformal time τ. Considering the slow-roll condition the expression of the conformal time is somehow different from the value in de Sitter space,

$$d\tau = \frac{d(aH)}{(aH)^2(1-\epsilon)}. \tag{49}$$

The conformal time can be written as

$$\tau \simeq -\frac{1}{aH(1-\epsilon)} \simeq -\frac{1+\epsilon}{aH}. \tag{50}$$

For the power-law inflation, the conformal time takes an exact form, $\tau = -1/(aH)(1-\epsilon)$. In the field equation,

$$\frac{z''}{z} \simeq 2a^2H^2\left(1 + \frac{5}{2}\epsilon - \frac{3}{2}\eta\right), \tag{51}$$

so the solution of (48) is the form of Bessel functions,

$$v_k = \tfrac{1}{2}(-\pi\tau)^{1/2}[c_1(k)\mathrm{H}^{(1)}_\nu(-k\tau) + c_2(k)\mathrm{H}^{(2)}_\nu(-k\tau)], \tag{52}$$

where

$$\nu = \frac{4}{9} + \epsilon - 3\eta. \tag{53}$$

The solution is similar to (6). In the case of the single field inflation, there is only one degree of freedom for the scalar perturbation, so using scalar field perturbation $\delta\phi$ as the physical freedom or using the curvature perturbation, the two kinds of descriptions are the same. On the large scale, the curvature perturbation ζ is conserved in the single field case, while the scalar field decays to other fields at the end of inflation. Thus it is more clear to use ζ as the physical freedom to describe the perturbation. The results of the power spectrum from the two descriptions are the same up to a factor 2ϵ.

When $\tau \rightarrow -\infty$, the Hubble radius is infinite relative to the modes k, the term $\frac{z''}{z}$ can be omitted, and the gravitational effect is negligible, so $v_k \propto e^{-ik\tau}$ in the Euclidean vacuum. On the other hand, when $k \rightarrow +\infty$, the contribution to the v_k comes from both the negative and positive frequency in the α-vacuum,

$$v_k = c_1 \frac{e^{-ik\tau}}{\sqrt{2k}} + c_2 \frac{e^{+ik\tau}}{\sqrt{2k}}. \tag{54}$$

We may have the question why the equation of motion is still effective as $k \rightarrow +\infty$. At the scale Λ, the new physics and some new freedom will emerge. The new physics scale is set to the Planck scale, string scale, or any other ones. In this paper, we assume that the new physics scale $\Lambda > H$, and Λ is constant. Thus at least on large scales, the field equation (48) is reliable. There are two functions c_1 and c_2, which depend on the new physics. What is the most important, we can determine the $c_1(k)$, $c_2(k)$ from the boundary condition at $k/a = \Lambda$. In other words, we can effectively choose the appropriate boundary condition to take into account new physics, even without knowing its nature. Whatever the new physics, it is in the short distance. On the large scale, the solution just contains the new variable c_1 and c_2. Information of the new physics will give a different value of c_1 and c_2 [43]. This change of initial condition will eventually show up in the CMB anisotropy.

Considering the modes with wavelength larger than the new physics scale but smaller than the Hubble radius,

$$v'_k = -i\sqrt{\frac{k}{2}} c_1 e^{-ik\tau} + i\sqrt{\frac{k}{2}} c_2 e^{+ik\tau}. \tag{55}$$

With the limit value of v_k (54) and v'_k, and the boundary condition at $k/a = p_c = \Lambda$, c_1 and c_2 can be determined,

$$\begin{aligned} c_1 &= \frac{\sqrt{2k}}{2} e^{ik\tau_c}\left[v_k(\tau_c) + \frac{i}{k} v'_k(\tau_c)\right],\\ c_2 &= \frac{\sqrt{2k}}{2} e^{-ik\tau_c}\left[v_k(\tau_c) - \frac{i}{k} v'_k(\tau_c)\right], \end{aligned} \tag{56}$$

where τ_c is the time when the k mode is at the boundary, thus it depends on k, which can be solved by $k/a = p_c = \Lambda$. As in [25,30] we set the boundary condition as the wave function at the scale Λ only containing an emergent wave, and at momentum $p = p_c$, the quantum fluctuation of scalar field takes the form as

$$\frac{d\delta\phi}{dt} = -ip_c \delta\phi. \tag{57}$$

The calculations of $\delta\phi$ and ζ in single field inflation are similar, so at the boundary

$$\frac{1}{a}\frac{d(v_k/a)}{d\tau} = -i\frac{k}{a^2} v_k. \tag{58}$$

With the relation (58) and the expression for c_1 and c_2 (56), we could obtain

$$\begin{aligned} c_1 &= \frac{1}{2}\left[2 + i\left(\frac{Ha}{k}\right)_c\right]\sqrt{2k} e^{ik\tau_c} v_k(\tau_c),\\ c_2 &= -\frac{i}{2}\left(\frac{Ha}{k}\right)_c \sqrt{2k} e^{-ik\tau_c} v_k(\tau_c), \end{aligned} \tag{59}$$

where $(Ha/k)_c = H_c/p_c = H_c/\Lambda$. Note that in de Sitter space the Hubble scale H is constant, while in slow-roll inflation the Hubble radius is changing $H = H_0 a^{-\epsilon}$. Thus in slow-roll inflation c_1 and c_2 are dependent of k.

In order to compare with the situations of de Sitter space, we require $c_2/c_2 = e^\alpha$ in the α-vacuum. The solution of the field equation (48) takes the form as (52) up to an unimportant overall phase factor. Thus the parameter e^α is

$$e^\alpha = \frac{c_2}{c_1} = -e^{-2ik\tau_c}\frac{i}{2\frac{\Lambda}{H} + i}, \tag{60}$$

from which we know that in de Sitter space e^α is independent of k, while in slow-roll inflation e^α is dependent of k. The magnitude of e^α is

$$|e^\alpha| = \sqrt{\frac{1}{4\frac{\Lambda^2}{H^2} + 1}} \sim \frac{H}{2\Lambda}. \tag{61}$$

The scalar power spectrum is

$$\begin{aligned} \mathrm{P}_\zeta &= \frac{k^3}{2\pi^2}\left|\frac{v_k}{z}\right|^2\\ &\simeq \frac{1}{8\pi^2}\frac{H^2}{\epsilon}\frac{1 + e^{\alpha+\alpha^*} - 2\,\mathrm{Re} e^\alpha}{1 - e^{\alpha+\alpha^*}}(-k\tau)^{3-2\nu}, \end{aligned} \tag{62}$$

where $\nu = \frac{3}{2} + 3\epsilon - \eta$. If $\Lambda \gg H$,

$$\begin{aligned} \mathrm{Re}\, e^\alpha &= -\frac{1}{4\frac{\Lambda^2}{H^2} + 1}\sin\left[2\frac{\Lambda}{H(1-\epsilon)}\right]\\ &\quad + \frac{2\frac{\Lambda}{H}}{4\frac{\Lambda^2}{H^2} + 1}\cos\left[2\frac{\Lambda}{H(1-\epsilon)}\right]\\ &\simeq \frac{H}{2\Lambda}\cos\left[2\frac{\Lambda}{H(1-\epsilon)}\right], \end{aligned} \tag{63}$$

where H is the value when the momentum of mode k is p_c. It is clear that $\frac{\Lambda}{H}$ depends on k. The exact relation can be derived from

$$\Lambda = p_c = \frac{k}{a(\tau_c)}, \qquad H = H_0 a^{-\epsilon}, \tag{64}$$

then

$$\frac{\Lambda}{H} \propto k^{\epsilon}. \tag{65}$$

Therefore, in slow-roll inflation, the correction to the power spectrum will oscillate with the variable wave number k. The correction is $\mathcal{O}(\frac{H}{\Lambda})$, and the correction is likely to be observed in future experiments.

IV. THREE-POINT CORRELATOR IN EUCLIDEAN VACUUM

As we discussed in the last section, to get the cubic action of ζ we only need to expand the Lagrangian multipliers N and N_i to the first order of ζ. The cubic action could be obtained by substituting the N and N_i to the ADM action and expanding the action to the third order of ζ. Then by integrating by parts many times and using some technique such as field redefinition, the action can be further simplified. From the cubic action, the three-point correlator is simply obtained by using the path integral formalism at the tree level.

Integrating by parts and dropping the total derivatives, the cubic action can be written as [13]

$$\begin{aligned} S_3 = \frac{1}{2}\int d^4x a^3\bigg[&\frac{2}{a^2}\frac{\dot{H}}{H^2}\zeta(\partial\zeta)^2 - \dot{\phi}^2\frac{\dot{\zeta}^3}{H^3} \\ &- \frac{4}{a^4}\partial^2\psi_1\partial_i\zeta\partial_i\psi_1 - \frac{3}{a^4}\zeta\partial^2\psi_1\partial^2\psi_1 \\ &+ \frac{1}{a^4}\frac{\dot{\zeta}}{H}\partial^2\psi_1\partial^2\psi_1 + \frac{3}{a^4}\zeta\partial_i\partial_j\psi_1\partial_i\partial_j\psi_1 \\ &- \frac{1}{a^4}\frac{\dot{\zeta}}{H}\partial_i\partial_j\psi_1\partial_i\partial_j\psi_1\bigg]. \end{aligned} \tag{66}$$

At first glance, the leading order term in the cubic action is $\mathcal{O}(\epsilon^0)$, but after careful integration by parts, all the terms $\mathcal{O}(\epsilon^0)$ and $\mathcal{O}(\epsilon^1)$ will cancel out, so that the leading order of the cubic action is $\mathcal{O}(\epsilon^2)$. If substituting the equation of ψ_1 (45), after integration by parts, the action has terms like $\ddot{\zeta}$. It is convenient to use the field equation from the quadratic action,

$$\frac{\delta L}{\delta\zeta}\bigg|_1 = a\bigg(\frac{d\partial^2\chi}{dt} + H\partial^2\chi - \epsilon\partial^2\zeta\bigg), \tag{67}$$

where

$$\partial^2\chi \equiv a^2\epsilon\dot{\zeta}, \tag{68}$$

χ is the second term in ψ_1 (45). The final result of the cubic action is

$$S_3 = \int d^4x[4a^5\epsilon^2 H\dot{\zeta}^2\partial^{-2}\dot{\zeta} + 2f(\zeta)\delta L/\delta\zeta|_1], \tag{69}$$

where

$$f(\zeta) = \frac{-2\eta + 3\epsilon}{4}\zeta^2 + \frac{1}{2}\epsilon\partial^{-2}(\zeta\partial^2\zeta) + \cdots. \tag{70}$$

Here we omit the terms in $f(\zeta)$ which contains the derivative of ζ because ζ is conserved on the large scale, and any derivative of ζ has no contribution. In the cubic action, the contribution from the $f(\zeta)$ terms is obtained from the redefinition of $\zeta\zeta_n + f(\zeta_n)$. With the redefinition

$$S_2[\zeta]S_2[\zeta_n] - \int d^4x 2f(\zeta_n)\delta L/\delta\zeta|_1, \tag{71}$$

the second terms in the cubic action are canceled. When we calculate the three-point correlator, both the contributions coming from the cubic ζ and the contributions from the redefinition should be taken into account.

The three-point correlator could be computed using the path integral formalism in the interaction picture,

$$\begin{aligned} \langle\zeta(t,\mathbf{k}_1)\zeta(t,\mathbf{k}_2)\zeta(t,\mathbf{k}_3)\rangle_{\text{tree}} = i\int_{t_0}^{t} dt'\langle[&\zeta(t,\mathbf{k}_1)\zeta(t,\mathbf{k}_2) \\ &\times\zeta(t,\mathbf{k}_3), L_3(t')]\rangle, \end{aligned} \tag{72}$$

where t_0 is some time that the mode is deep inside the horizon. The integration can be divided into three parts. The first part is from the period during which the modes are deep inside the horizon. In this range, the modes oscillate rapidly, so the contribution is simply zero. In Euclidean vacuum the value of t_0 is set to $-\infty$, while in the α-vacuum the value of t_0 is at the boundary for new physics. We assume that $\Lambda \gg H$, so that t_0 in the α-vacuum is also deep inside the horizon. In both situations, the contribution from this part is zero. The second part is the region well outside the horizon. Because the value of ζ is constant, the contribution only contains the redefinition of ζ. The third region is near the horizon, where we use the solution of the field equation (52) and compute the three-point correlator from the path integral.

The leading order contribution to the three-point correlator in the Euclidean vacuum is as follows:

(i) *Contribution from* $\dot{\zeta}^2\partial^{-2}\dot{\zeta}$. We choose $t_0 = -\infty$ and $t = 0$, which will not influence the final results in the Euclidean and α-vacuum.

$$\begin{aligned} \langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = -i(2\pi)^3\delta\bigg(\sum_i\mathbf{k}_i\bigg)&\zeta_{k_1}(0)\zeta_{k_2}(0) \\ &\times\zeta_{k_3}(0)\int_{-\infty}^{0}d\tau g\frac{d}{d\tau}\zeta_{k_1}^*(\tau)\frac{d}{d\tau} \\ &\times\zeta_{k_2}^*(\tau)\partial^{-2}\frac{d}{d\tau}\zeta_{k_3}(\tau) + \text{perms} \\ &+ \text{c.c.}, \end{aligned} \tag{73}$$

where $K = k_1 + k_2 + k_3$, 'perms' denotes exchanging k_1, k_2, k_3, and c.c. represents the complex conjugate of the preceding terms. The prefactor $g = 4a^3\epsilon^2 H$. The three-point correlator from $\dot{\zeta}^2\partial^{-2}\dot{\zeta}$ is

$$(2\pi)^3\delta\Big(\sum_i \mathbf{k}_i\Big)\frac{H^4}{2^4\epsilon}\frac{1}{\prod_i k_i^3}\Big(\frac{k_1^2k_2^2}{K}\Big) + \text{perms} + \text{c.c.}$$
$$= (2\pi)^7\delta\Big(\sum_i \mathbf{k}_i\Big)(\mathrm{P}_\zeta)^2\frac{1}{\prod_i \mathrm{k}_i^3}\epsilon\Big(\frac{1}{\mathrm{K}}\sum_{i>j}\mathrm{k}_i^2\mathrm{k}_j^2\Big). \quad (74)$$

(ii) *The redefinition* $\zeta \mapsto \zeta_n + (-2\eta + 3\epsilon/4)\zeta_n^2$.
The three-point correlator from this contribution is

$$(2\pi)^7\delta\Big(\sum_i \mathbf{k}_i\Big)(\mathrm{P}_\zeta)^2\frac{1}{\prod_i \mathrm{k}_i^3}\frac{-2\eta + 3\epsilon}{8}\sum_i \mathrm{k}_i^3. \quad (75)$$

(iii) *The redefinition* $\zeta \mapsto \zeta_n + (\epsilon/2)\partial^{-2}(\zeta_n\partial^2\zeta_n)$.
The three-point correlator from this contribution is

$$(2\pi)^3\delta\Big(\sum_i \mathbf{k}_i\Big)\frac{H^4}{2^4\epsilon^2}\frac{1}{\prod_i k_i^3}\frac{1}{k_1k_2^2k_3^3} + \text{perms}$$
$$= (2\pi)^7\delta\Big(\sum_i \mathbf{k}_i\Big)(\mathrm{P}_\zeta)^2\frac{1}{\prod_i \mathrm{k}_i^3}\frac{\epsilon}{8}\sum_{i\neq j}\mathrm{k}_i\mathrm{k}_j^2. \quad (76)$$

Finally, taking all the contributions into account, we have the three-point correlator in the Euclidean vacuum [13],

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7\delta\Big(\sum_i \mathbf{k}_i\Big)(\mathrm{P}_\zeta)^2\frac{1}{\prod_i \mathrm{k}_i^3}\mathcal{A}, \quad (77)$$

where

$$\mathcal{A} = \epsilon\frac{1}{K}\sum_{i>j}k_i^2k_j^2 + \frac{-2\eta + 3\epsilon}{8}\sum_i k_i^3 + \frac{\epsilon}{8}\sum_{i\neq j}k_ik_j^2. \quad (78)$$

V. NON-GAUSSIANITY IN α-VACUUM

In this section, we analyze the shape of non-Gaussianity and especially its local form in the α-vacuum. We consider both the de Sitter space-time and quasi-de Sitter space-time and show that the local form non-Gaussianity in the quasi-de Sitter case has a distinctive feature.

The power spectrum and bispectrum are defined as

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle \equiv (2\pi)^3\delta(\mathbf{k}_1 + \mathbf{k}_2)\frac{2\pi^2}{k_1^3}\mathrm{P}_\zeta(k_1), \quad (79)$$

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_1)\rangle \equiv (2\pi)^3\delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)B_\zeta(k_1, k_2, k_3). \quad (80)$$

Non-Gaussianity measures the deviation of the CMB power spectrum from the Gaussian distribution,

$$\zeta = \zeta_g + \tfrac{3}{5}f_{\mathrm{NL}}(\zeta_g^2 - \langle\zeta_g^2\rangle), \quad (81)$$

in which f_{NL} characterizes the size of non-Gaussianity:

$$\frac{6}{5}f_{\mathrm{NL}} = \frac{\prod_i k_i^3}{\sum_i k_i^3}\frac{B_\zeta}{4\pi^4\mathrm{P}_\zeta^2}. \quad (82)$$

In the Euclidean vacuum, the three-point correlator is given by (77) and (78), so that the parameter of non-Gaussianity is given by

$$f_{\mathrm{NL}} = \frac{10}{3}\frac{1}{\sum_i k_i^3}\mathcal{A}. \quad (83)$$

The calculation of the three-point correlator in the above section can be extended to the α-vacuum. One can simply plug the value of ζ_k for the α-vacuum into Eq. (73) to evaluate the three-point correlator. To distinguish ζ's in different vacua, we use $\tilde{\zeta}_k$ to denote its value in the α-vacuum,

$$\langle\tilde{\zeta}(\mathbf{k}_1)\tilde{\zeta}(\mathbf{k}_2)\tilde{\zeta}(\mathbf{k}_3)\rangle = (2\pi)^7\delta\Big(\sum_i \mathbf{k}_i\Big)(\mathrm{P}_{\zeta'})^2\frac{1}{\prod_i \mathrm{k}_i^3}\mathcal{A}'. \quad (84)$$

Note that P_ζ is different in the two backgrounds, and it will give small correction to $\mathcal{A}'$. $\mathcal{A}'$ also contains two parts. In de Sitter space,

$$\mathcal{A}'^{(ds)} = N_\alpha^6(1 + 4\,\mathrm{Re}(e^\alpha))\mathcal{A}(k_1, k_2, k_3) + \tilde{\mathcal{A}}^{(ds)},$$
$$\tilde{\mathcal{A}}^{(ds)} = N_\alpha^3\,\mathrm{Re}(e^\alpha)[\mathcal{A}(-k_1, k_2, k_3) + \mathcal{A}(k_1, -k_2, k_3) + \mathcal{A}(k_1, k_2, -k_3) - 3\mathcal{A}(k_1, k_2, k_3)], \quad (85)$$

where $A(k_1, k_2, k_3)$ is defined in Eq. (78) and we neglect the higher order contribution from the slow-roll parameter and $\mathrm{Re}\,e^\alpha$. In de Sitter space, $\mathrm{Re}\,e^\alpha$ is independent of k, and $N_\alpha = 1/\sqrt{1 - e^{\alpha+\alpha^*}} \sim 1$. In the discussion below the value of N_α is set to 1, and $4\,\mathrm{Re}(e^\alpha)A(k_1, k_2, k_3)$ is the next order contribution to the first part of $\mathcal{A}'$, so we will also neglect it. In quasi-de Sitter space-time,

$$\mathcal{A}'^{(q)} = \mathcal{A}(k_1, k_2, k_3) + \tilde{\mathcal{A}}^{(q)},$$
$$\tilde{\mathcal{A}}^{(q)} = [\mathrm{Re}(e^\alpha_{\mathrm{k}_1})\mathcal{A}(-\mathrm{k}_1, \mathrm{k}_2, \mathrm{k}_3) + \mathrm{Re}(e^\alpha_{\mathrm{k}_2})\mathcal{A}(\mathrm{k}_1, -\mathrm{k}_2, \mathrm{k}_3) + \mathrm{Re}(e^\alpha_{\mathrm{k}_3})\mathcal{A}(\mathrm{k}_1, \mathrm{k}_2, -\mathrm{k}_3) - [\mathrm{Re}(e^\alpha_{\mathrm{k}_1}) + \mathrm{Re}(e^\alpha_{\mathrm{k}_2}) + \mathrm{Re}(e^\alpha_{\mathrm{k}_3})]\mathcal{A}(\mathrm{k}_1, \mathrm{k}_2, \mathrm{k}_3)], \quad (86)$$

where the index k_1, k_2, k_3 of e^α denotes its dependence of the wave number and the upper index (q) denotes quasi-de Sitter for short. The value of $\mathrm{Re}(e^\alpha)$ (63) sensitively depends on the variable k, since $\Lambda/H \gg 1$.

There are two forms of non-Gaussianity which are of particular importance in the data analysis of the WMAP.

One is the equilateral form and the other is the local form non-Gaussianity. The equilateral form requires $k_1 \sim k_2 \sim k_3$, and the three momentum vector composes an equilateral triangle. In this case, in the Euclidean vacuum and α-vacuum, $f_{\rm NL}$ are the same in the leading order approximation,

$$f_{\rm NL}^{\rm equil} \simeq \frac{10}{9}\left(\frac{23}{8}\epsilon - \frac{3}{4}\eta\right). \tag{87}$$

In both cases, the equilateral form non-Gaussianity $f_{\rm NL} \sim \mathcal{O}(\epsilon, \eta)$.

The local form non-Gaussianity requires that one of $k_i \ll$ the other two k. For instance, $-k_1 + k_2 + k_3 \sim k_3$. The three momentum vectors compose an isosceles triangle, $k_1 = k_2 \gg k_3$. The mode k_3 exits the horizon much earlier than the other two modes. In Euclidean vacuum, if we take the limit $k_1 = k_2 \gg k_3$, then

$$f_{\rm NL}^{\rm local} = \tfrac{5}{3}(3\epsilon - \eta) = \tfrac{5}{6}(1 - n_s), \tag{88}$$

where n_s is the spectral index,

$$n_s - 1 = \frac{d{\rm In}{\rm P}_\zeta}{d{\rm In}k}. \tag{89}$$

In Euclidean vacuum, ${\rm P}_\zeta \simeq H^2/8\pi^2\epsilon$, and $n_s - 1 = 2\eta - 6\epsilon$. This means that the local form non-Gaussianity in the Euclidean vacuum is the same order of ϵ and η.

The local form non-Gaussianity in the Euclidean vacuum can be estimated by the backreaction method [13]. The mode k_3 leaves the horizon earlier than the other two modes. The effect from the mode k_3 is rescaling the background space-time. The scale factor changes $a(t) \to a(t)e^{\zeta_3} \sim a(t)(1 + \zeta_3)$, so that the coordinates change accordingly $\delta x = \zeta_3 x$, where ζ_3 is the amplitude of the k_3 modes. The backreaction of the background will impact the modes deep in the horizon. The wave number decreases $\delta k = -\zeta_3 k$, and the modes will leave the horizon earlier,

$$\delta k = \delta a \cdot H = aH\delta tH. \tag{90}$$

According to the equation $\delta a = aH\delta t$, we get the relation $\delta t = -\zeta_3/H$. From the definition of $f_{\rm NL}$ (81), the local form non-Gaussianity is

$$f_{\rm NL}^{\rm local} = \frac{5}{3}\frac{\Delta\zeta}{\zeta_g^2} = \frac{5}{3}\frac{\Delta t\frac{d\zeta}{dt}}{\zeta_g^2} = \frac{5}{3}\frac{\frac{-\zeta_3}{H}\frac{d\zeta}{d{\rm Ink}}H}{\zeta_g^2} \simeq \frac{5}{6}(1 - n_s), \tag{91}$$

where the last equation uses the definition of the power index n_s. The backreaction method gives the same result as the one obtained in the direct way, but this method could be applied to other models. It indicates that the microphysics from the inflaton cannot have large local form non-Gaussianity in the Euclidean vacuum. Even if the action is in noncanonical form, such as DBI inflation [15] and K inflation [16], the local form non-Gaussianity is still $1 - n_s$ up to an order one constant.

However, it is a totally different story in the α-vacuum. First, we consider the situation of de Sitter space-time, where ${\rm Re}\, e^\alpha$ is independent of k. The non-Gaussianity parameter $f_{\rm NL}$ contains the contributions from A and $\tilde{A}^{(ds)}$. The contributions from A are the same as in the Euclidean vacuum, which is omitted for conciseness. Thus $f_{\rm NL}$ from $\tilde{A}^{(ds)}$ is

$$\begin{aligned}\tilde{f}_{\rm NL} = {\rm Re}(e^\alpha)\frac{10}{3}\frac{1}{\sum_i {\rm k}_i^3}\Bigg[&-\frac{-2\eta + 3\epsilon}{8}\sum_i {\rm k}_i^3 - \frac{\epsilon}{8}\sum_{i\neq j}{\rm k}_i{\rm k}_j^2 \\ &+ \epsilon\sum_{i<j}k_i^2k_j^2\Bigg(\frac{1}{-k_1 + k_2 + k_3} + \frac{1}{k_1 - k_2 + k_3} \\ &+ \frac{1}{k_1 + k_2 - k_3} - \frac{3}{k_1 + k_2 + k_3}\Bigg)\Bigg].\end{aligned} \tag{92}$$

If we take the local form limit $k_3 \ll k_1 \sim k_2$, $-k_1 + k_2 + k_3 \sim k_3$, then

$$f_{\rm NL}^{\rm local} \simeq \frac{10}{3}{\rm Re}(e^\alpha)\epsilon\frac{{\rm k}_2}{{\rm k}_3}. \tag{93}$$

Although the prefactor ${\rm Re}(e^\alpha)$ is a small quantity $\sim\mathcal{O}(\epsilon\frac{H}{\Lambda})$, k_2/k_3 can be a huge number in the CMB window, say $k_{\rm max}/k_{\rm min} \sim 10^6$ for WMAP data, so that $f_{\rm NL}^{\rm local}$ could be of order one or even larger in the α-vacuum.

Second, we consider the local form non-Gaussianity in inflationary background, where ${\rm e}^\alpha$ strongly depends on k. The local form non-Gaussianity takes the form as

$$\begin{aligned}f_{\rm NL}^{\rm local} &\simeq \frac{5}{3}[{\rm Re}(e^\alpha_{{\rm k}_1}) + {\rm Re}(e^\alpha_{{\rm k}_2})]\epsilon\frac{{\rm k}_2}{{\rm k}_3} \simeq \frac{10}{3}{\rm Re}(e^\alpha_{{\rm k}_2})\epsilon\frac{{\rm k}_2}{{\rm k}_3} \\ &\simeq \frac{5}{3}\frac{\rm H}{\Lambda}\cos\left[2\frac{\Lambda}{{\rm H}(1-\epsilon)}\right]\epsilon\frac{{\rm k}_2}{{\rm k}_3}.\end{aligned} \tag{94}$$

Similar to de Sitter space-time, $f_{\rm NL}^{\rm local}$ is linear in $\epsilon\frac{k_2}{k_3}$ such that a large local form non-Gaussianity is possible. However, since ${\rm Re}(e^\alpha)$ is k dependent, $f_{\rm NL}^{\rm local}$ has a distinctive feature.

As a example, take Λ to be 10^{17} Gev which is the phenomenological string scale, $H(k_3) = 10^{15}$ Gev, the slow-roll parameter $\epsilon = 0.01$, $k_3 = 1$ (with unit $0.002\ {\rm Mpc}^{-1}$), and $1 \leqslant k_2 \lesssim 10^6$. In Fig. 2, we draw the local form non-Gaussianity: the light gray (red) line represents the one in de Sitter space-time, while the dark gray (blue) one represents the one in inflationary background. Obviously, the local form non-Gaussianity in two cases are different. Especially in the inflationary background, ${\rm Re}(e^\alpha_{{\rm k}_2})$ is oscillating with mode k, as shown in Fig. 3.

The possible largeness of the local form non-Gaussianity in the α-vacuum seems to violate the bound set by the backreaction argument. Why is the backreaction method not applicable in the α-vacuum? Generally speaking, the correlation from two patches of de Sitter space-time makes it so that the backreaction cannot give the whole effect of non-Gaussianity. In the α-vacuum the

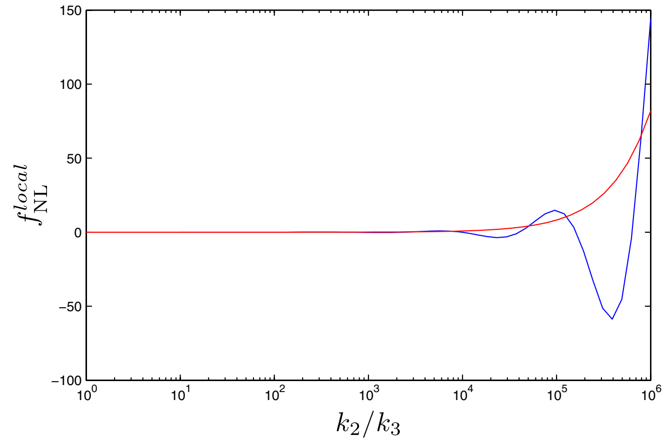

FIG. 2 (color online). The light gray (red) line represents $f_{\rm NL}^{\rm local}$ in de Sitter space and the dark gray (blue) line represents $f_{\rm NL}^{\rm local}$ quasi-de Sitter space.

FRW metric cannot describe the physics completely, and we must extend the space to the whole de Sitter space. It is clearer to see the correlation in space coordinates. For instance, one point x_1 is far outside the horizon, and the other two points x_2, x_3 are deep in the horizon. In the whole de Sitter space, the antipodal point x_{1A} is approaching the points x_2, x_3 deep in the horizon. When x_{1A} is near the light cone of x_2 or x_3, there is divergence in the tree-level three-point correlator.

If we carefully analyze $f_{\rm NL}$ in the α-vacuum (92), we will find that there is also divergence for the folded form non-Gaussianity [44]. Therefore, it is interesting to determine the shape of non-Gaussianity, which has the potential to distinguish different inflationary models if data analysis is accurate enough [45].

The definition of the shape is

$$\frac{A}{k_1 k_2 k_3}. \tag{95}$$

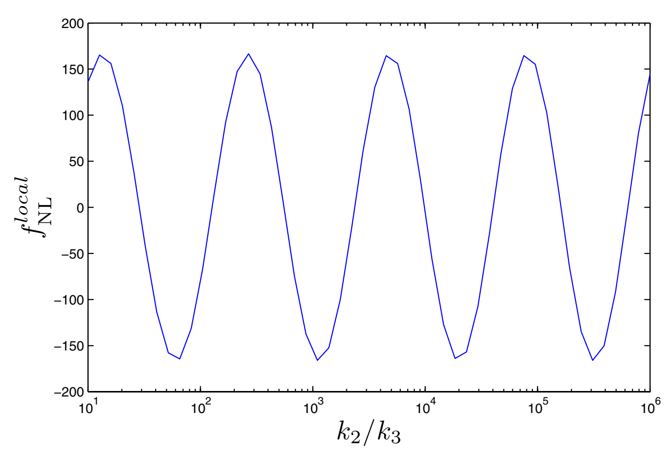

FIG. 3 (color online). $\frac{10}{3}\,\mathrm{Re}(e^{\alpha}_{k_2})\epsilon \times 10^6$, used to compare with Fig. 2.

We divide A into several parts. We use the subscripts ϵ and η to denote the parts proportional to ϵ or η. In any case, we always have the contribution without including the modifications from the α-vacuum:

$$\mathcal{A}_{\epsilon} = \epsilon\left(\frac{1}{K}\sum_{i>j} k_i^2 k_j^2 + \frac{3}{8}\sum_i k_i^3 + \frac{1}{8}\sum_{i\neq j} k_i k_j^2\right), \tag{96}$$

$$\mathcal{A}_{\eta} = \eta\left(\frac{-1}{4}\sum_i k_i^3\right), \tag{97}$$

which has been discussed in [13]. The modifications in de Sitter space-time are

$$\begin{aligned}\tilde{\mathcal{A}}_{\epsilon}^{(ds)} = \epsilon\,\mathrm{Re}(e^{\alpha})\bigg[&-\frac{3}{8}\sum_i k_i^3 - \frac{1}{8}\sum_{i\neq j} k_i k_j^2 \\ &+ \sum_{i<j} k_i^2 k_j^2 \bigg(\frac{1}{-k_1+k_2+k_3} + \frac{1}{k_1-k_2+k_3} \\ &+ \frac{1}{k_1+k_2-k_3} - \frac{3}{k_1+k_2+k_3}\bigg)\bigg],\end{aligned} \tag{98}$$

$$\tilde{\mathcal{A}}_{\eta}^{(ds)} = \eta\,\mathrm{Re}(e^{\alpha})\left(\frac{1}{4}\sum_i k_i^3\right). \tag{99}$$

The modifications in the quasi-de Sitter case are

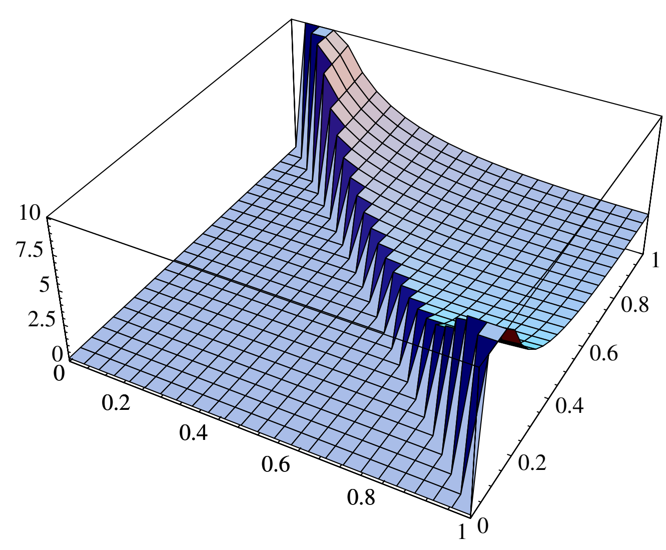

FIG. 4 (color online). $\mathcal{A}_{\epsilon}/k_1k_2k_3$. This is the shape for the Euclidean vacuum proportional to ϵ. With the shape of $A_{\eta}/k_1k_2k_3$, they give the leading contribution for the α-vacuum.

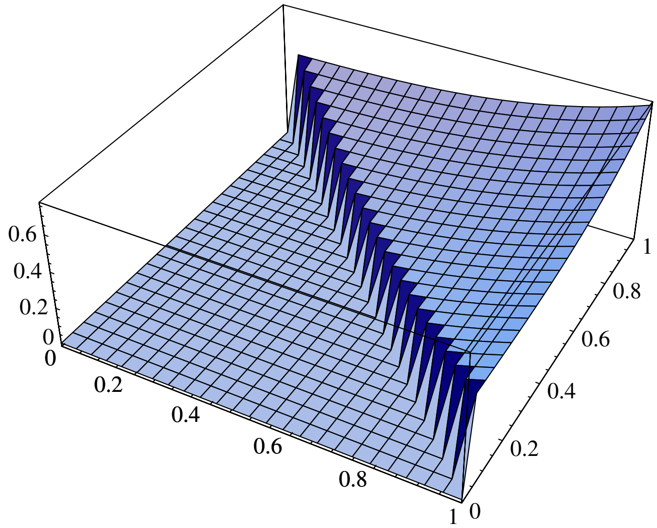

FIG. 5 (color online). $|\mathcal{A}_\eta|/k_1k_2k_3$. This is the shape for the Euclidean vacuum proportional to η.

$$\tilde{\mathcal{A}}_\epsilon^{(q)} = \epsilon\bigg[\frac{1}{3}[\mathrm{Re}(e^\alpha_{\mathrm{k}_1}) + \mathrm{Re}(e^\alpha_{\mathrm{k}_2}) + \mathrm{Re}(e^\alpha_{\mathrm{k}_3})] \times \bigg(-\frac{3}{8}\mathrm{k}_i^3\sum_i - \frac{1}{8}\sum_{i\neq j}\mathrm{k}_i\mathrm{k}_j^2\bigg) + \sum_{i<j}k_i^2k_j^2 \times \bigg(\frac{\mathrm{Re}(e^\alpha_{\mathrm{k}_1})}{-k_1+k_2+k_3} + \frac{\mathrm{Re}(e^\alpha_{\mathrm{k}_2})}{k_1-k_2+k_3} + \frac{\mathrm{Re}(e^\alpha_{\mathrm{k}_3})}{k_1+k_2-k_3} - \frac{\mathrm{Re}(e^\alpha_{\mathrm{k}_1}) + \mathrm{Re}(e^\alpha_{\mathrm{k}_2}) + \mathrm{Re}(e^\alpha_{\mathrm{k}_3})}{k_1+k_2+k_3}\bigg)\bigg], \tag{100}$$

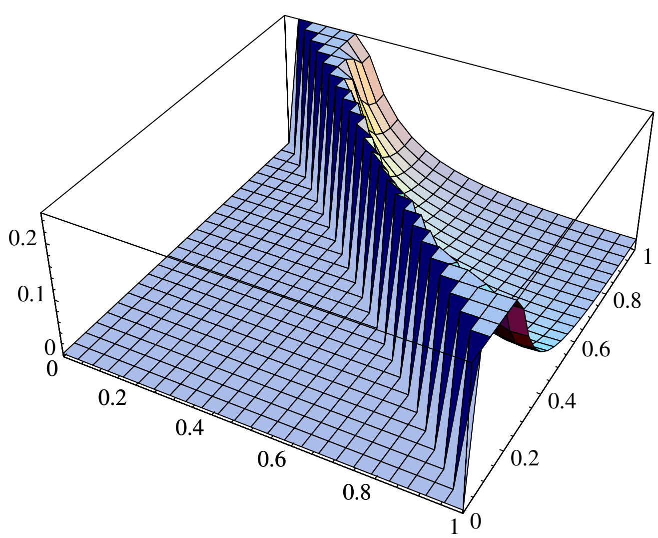

FIG. 6 (color online). $\tilde{\mathcal{A}}_\epsilon^{(ds)}/k_1k_2k_3$. This is the local α-vacuum shape for de Sitter space. The local form non-Gaussianity near the points (1, 0) and (0, 1) is large.

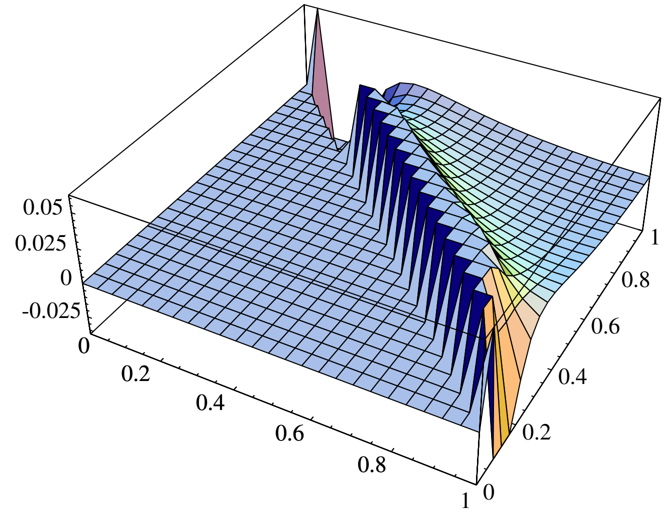

FIG. 7 (color online). $\tilde{\mathcal{A}}_\epsilon^{(q)}/k_1k_2k_3$. This is the local α-vacuum shape proportional to ϵ for inflationary space. The local form non-Gaussianity near the points (1, 0) and (0, 1) is large, and it reflects some oscillating character.

$$\tilde{\mathcal{A}}_\eta^{(q)} = \eta[\mathrm{Re}(e^\alpha_{\mathrm{k}_1}) + \mathrm{Re}(e^\alpha_{\mathrm{k}_2}) + \mathrm{Re}(e^\alpha_{\mathrm{k}_3})]\bigg(\frac{1}{12}\sum_i \mathrm{k}_i^3\bigg), \tag{101}$$

which are very different from the de Sitter space-time case.

It is more illustrative to draw the shapes of non-Gaussianity in various cases. Since $\tilde{\mathcal{A}}_\eta^{(ds)}$ and $\tilde{\mathcal{A}}_\eta^{(q)}$ are small numbers relative to $\mathcal{A}_\eta$, we will not draw their shapes. In Figs. 4 and 5 we draw the shapes of non-Gaussianity in the Euclidean vacuum. In Fig. 6, we draw the shape of $\tilde{\mathcal{A}}_\epsilon^{(ds)}$, and in Figs. 7 and 8 we draw the shape of $\tilde{\mathcal{A}}_\epsilon^{(q)}$. In all the figures, we use the following convention: $k_3 = 1$, the x-axis is k_1/k_3, the y-axis is k_2/k_3, and the

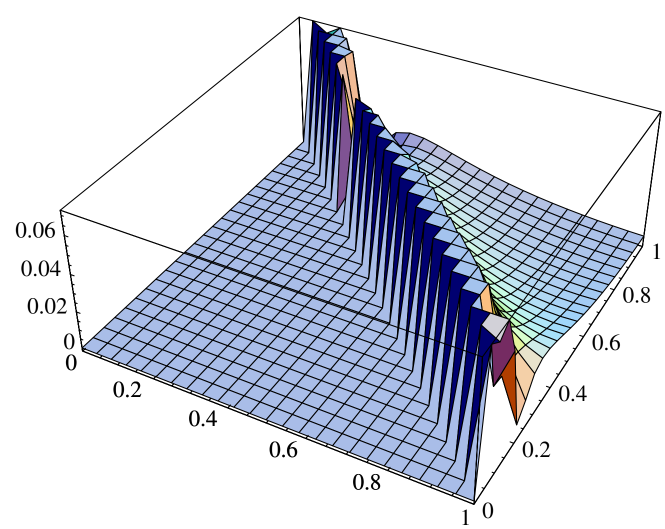

FIG. 8 (color online). $|\tilde{\mathcal{A}}_\eta^{(q)}|/k_1k_2k_3$. This is the local α-vacuum shape proportional to η for inflationary space. The local form is large and oscillating.

value of the z-axis is $\mathcal{A}/k_1k_2k_3$ up to a slow-roll parameter. The $x - y$ plane diagonal from (0, 1) to (1, 0) denotes the folded form, i.e. $k_1 + k_2 = k_3$. From the shapes of non-Gaussianity in the Euclidean vacuum, the folded form is finite, except the points (0, 1) and (1, 0). In contrast, the shapes of non-Gaussianity in the α-vacuum, the folded form is divergent. What is more, the $f_{\rm NL}$ for the folded form in the α-vacuum is divergent as shown in Eq. (92).

The local form non-Gaussianity in the shape is near the point (0, 1) and (1, 0). Figure 7 reflects the oscillating character in inflationary background.

VI. DISCUSSION

In this paper, we studied the physical implication of the α-vacuum on the CMB non-Gaussianity. We found that the α-vacuum may lead to large local form non-Gaussianity, and its signature is distinct. In de Sitter space-time, the local form is large which is proportional to k_2/k_3, and in the inflationary background, the local form could still be large but there is oscillating character for $f_{\rm NL}$. Another distinctive feature of the α-vacuum is that it leads to a divergent folded form non-Gaussianity. To illustrate the picture more clearly, we drew the shapes for different vacua and different backgrounds. We found that all of these figures have a dramatically different feature. For the de Sitter space-time the local form is large, and for the inflationary space-time the local form is not only large, but also oscillating. These are different from the standard slow-roll inflation, where the local form is proportional to the spectral index of scalar perturbation.

The observational feature from our study is different from [36] which only considered the α-vacuum correction from the de Sitter background. And even though our study has some overlaps with the one in [37], our method and main results are different. In [37], they used the effective field theory (EFT) method during inflation. Since the energy scale of inflation is very high, from the point of view of the EFT, some higher order derivative terms will be not negligible, and the correction to the non-Gaussianity originates from the stronger interaction at the beginning of the inflation. Comparing with trans-Planckian physics from the α-vacua, the EFT method could be another branch in inflationary cosmology. Our treatment focused on the influence of the fluctuation vacuum, while the EFT emphasized the action. From the view of experiments, due to different motivation, the final prediction of the observation is not the same. Reference [37] analyzed the folded form non-Gaussianities, which has not been analyzed in the WMAP5 yet. In our study, we concluded that the trans-Planckian effect would enhance the local form non-Gaussianities, which is the essential part of the WMAP data analysis.

There are various interesting issues to address on the implication of the α-vacuum on inflation. First of all, the loop effect in the α-vacuum is still an open question. Steven Weinberg has given a wonderful discussion about the loop effect in inflationary correlators [46–48]. In the α-vacuum the problem is focused on how to renormalize the scalar perturbation in the loop diagram. In some papers, it has been argued that the α-vacuum is not well defined in de Sitter space due to the divergence [49,50]. However, in [51] using the Schwinger-Keldysh formalism, a consistent renormalization method has been constructed to deal with the divergence. The other discussion on this issue can be found in [52]. It would be interesting to understand the loop effect and its physical implications in the α-vacuum.

Second, the non-Gaussianity in the α-vacuum is sensitively dependent of the initial condition. In the case of single field inflation with higher derivative terms, the sound speed c_s is not one. For example, in the DBI inflation [15] and K inflation [16], the Lagrangian is not canonical, and the sound speed $c_s \ll 1$ in some situations. The sound horizon c_sH^{-1} may be smaller than the length scale of new physics. In this case, the initial condition at the new physics scale is chosen in the place larger than the sound Hubble scale. From the calculation of the non-Gaussianity, we know that in the path integral formalism the near horizon crossing region impacts the results of non-Gaussianity. The divergence of the non-Gaussianity in the α-vacuum may not exist as pointed out in the paper [36]. This situation also appears in some special trans-Planckian physics, such as noncommutative inflation [23], in the IR region the effective string scale is smaller than the Hubble scale.

Third, the different initial condition of the α-vacuum will dramatically change the correction of the power spectrum [31,32], thus it predicts different non-Gaussianity. Meanwhile, the trans-Planckian dispersion relation and noncommutative geometry will also give a different prediction for non-Gaussianity.

Fourth, it is also interesting to consider the physical implication of the α-vacuum in other inflationary models. One class of them is the multiple field inflation [53]. It has some very different signatures from the single field inflation: it has a non-negligible gravitational wave and large local form non-Gaussianity [54–58]. Another class of inflation models is inspired by string theory. In particular, DBI inflation is a very remarkable scenario. It may give large equilateral non-Gaussianity but no local form non-Gaussianity. It is worthwhile to discuss the α-vacuum effect in these inflationary models.

Finally, it could be expected that the trispectrum of the CMB from the α-vacuum is different from the one in the Euclidean vacuum. A more careful investigation would be valuable.

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[1] A. H. Guth, Phys. Rev. D **23**, 347 (1981).

[2] A. D. Linde, Phys. Lett. B **108**, 389 (1982).

[3] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).

[4] V. Mukhanov and G. Chibisov, JETP **33**, 532 (1981).

[5] A. H. Guth and S. Y. Pi, Phys. Rev. Lett. **49**, 1110 (1982).

[6] S. W. Hawking, Phys. Lett. B **115**, 295 (1982).

[7] A. A. Starobinsky, Phys. Lett. B **117**, 175 (1982).

[8] J. M. Bardeen, P. J. Steinhardt, and M. S. Turner, Phys. Rev. D **28**, 679 (1983).

[9] E. Komatsu *et al.*, arXiv:0803.0547 [Astrophys. J. Suppl. Ser. (to be published)].

[10] E. Komatsu and D. N. Spergel, Phys. Rev. D **63**, 063002 (2001); arXiv:astro-ph/0012197; E. Komatsu, arXiv:astro-ph/0206039.

[11] A. D. Linde and V. F. Mukhanov, Phys. Rev. D **56**, R535 (1997); D. H. Lyth and D. Wands, Phys. Lett. B **524**, 5 (2002); M. Li, C. Lin, T. Wang, and Y. Wang, arXiv:0805.1299; Q. G. Huang, Phys. Lett. B **669**, 260 (2008).

[12] K. Koyama, S. Mizuno, F. Vernizzi, and D. Wands, J. Cosmol. Astropart. Phys. 11 (2007) 024; E. I. Buchbinder, J. Khoury, and B. A. Ovrut, Phys. Rev. Lett. **100**, 171302 (2008); J. L. Lehners and P. J. Steinhardt, Phys. Rev. D **78**, 023506 (2008); Phys. Rev. D **77**, 063533 (2008).

[13] J. Maldacena, J. High Energy Phys. 05 (2003) 013.

[14] N. Arkani-Hamed, P. Creminelli, S. Mukhoyama, and M. Zaldarriaga, J. Cosmol. Astropart. Phys. 04 (2004) 001.

[15] E. Silverstein and D. Tong, Phys. Rev. D **70**, 103505 (2004); M. Alishahiha, E. Silverstein, and D. Tong, Phys. Rev. D **70**, 123505 (2004); X. Chen, Phys. Rev. D **71**, 063506 (2005); J. High Energy Phys. 08 (2005) 045; Phys. Rev. D **72**, 123518 (2005); M. Li, T. Wang, and Y. Wang, J. Cosmol. Astropart. Phys. 03 (2008) 028.

[16] J. Garriga and V. F. Mukhanov, Phys. Lett. B **458**, 219 (1999); C. Armendariz-Picon, T. Damour, and V. Mukhanov, Phys. Lett. B **458**, 209 (1999).

[17] B. Chen, Y. Wang, and W. Xue, J. Cosmol. Astropart. Phys. 05 (2008) 014; B. Chen, Y. Wang, W. Xue, and R. H. Brandenberger, arXiv:0712.2477.

[18] B. Allen, Phys. Rev. D **32**, 3136 (1985).

[19] R. Bousso, A. Maloney, and A. Strominger, Phys. Rev. D **65**, 104039 (2002).

[20] R.H. Brandenberger, Proceeding of the International School on Cosmology, Kish Island, Iran, 2000 (unpublished).

[21] J. Martin and R.H. Brandenberger, Phys. Rev. D **63**, 123501 (2001); R.H. Brandenberger and J. Martin, Mod. Phys. Lett. A **16**, 999 (2001); J.C. Niemeyer, Phys. Rev. D **63**, 123502 (2001); arXiv:astro-ph/0201511; J. Martin and R.H. Brandenberger, *Proceedings of the Ninth Marcel Grossmann Meeting on General Relativity*, edited by R. T. Jantzen, V. Gurzadyan, and R. Ruffini (World Scientific, Singapore, 2002); J.C. Niemeyer and R. Parentani, Phys. Rev. D **64**, 101301 (2001); M. Lemoine, M. Lubo, J. Martin, and J.P. Uzan, Phys. Rev. D **65**, 023510 (2001); R.H. Brandenberger, S.E. Joras, and J. Martin, Phys. Rev. D **66**, 083514 (2002).

[22] A. Kempf, Phys. Rev. D **63**, 083514 (2001); C. S. Chu, B. R. Greene, and G. Shiu, Mod. Phys. Lett. A **16**, 2231 (2001); F. Lizzi, G. Mangano, G. Miele, and M. Peloso, J. High Energy Phys. 06 (**2002**) 049; A. Kempf and J.C. Niemeyer, Phys. Rev. D **64**, 103501 (2001); S.F. Hassan and M. S. Sloth, Nucl. Phys. **B674**, 434 (2003); W. Xue, B. Chen, and Y. Wang, J. Cosmol. Astropart. Phys. 09 (2007) 011.

[23] R. Brandenberger and P. Ho, Phys. Rev. D **66**, 023517 (2002).

[24] K. Fang, B. Chen, and W. Xue, Phys. Rev. D **77**, 063523 (2008).

[25] U. H. Danielsson, Phys. Rev. D **66**, 023511 (2002); J. High Energy Phys. 07 (2002) 040.

[26] K. Goldstein and D. A. Lowe, Phys. Rev. D **67**, 063502 (2003);

[27] G. L. Alberghi, R. Casadio, and A. Tronconi, Phys. Lett. B **579**, 1 (2004).

[28] L. Bergstrom and U. H. Danielsson, J. High Energy Phys. 12 (2002) 038.

[29] N. Kaloper, M. Kleban, A. Lawrence, S. Shenker, and L. Susskind, J. High Energy Phys. 11 (2002) 037.

[30] R. Easther, B. R. Greene, W. H. Kinney, and G. Shiu, Phys. Rev. D **66**, 023518 (2002).

[31] J. C. Niemeyer, R. Parentani, and D. Campo, Phys. Rev. D **66**, 083510 (2002).

[32] N. Kaloper, M. Kleban, A. E. Lawrence, and S. Shenker, Phys. Rev. D **66**, 123510 (2002).

[33] R.H. Brandenberger and J. Martin, Int. J. Mod. Phys. A **17**, 3663 (2002).

[34] J. Martin and R.H. Brandenberger, Phys. Rev. D **68**, 063513 (2003).

[35] U. H. Danielsson, Phys. Rev. D **71**, 023516 (2005).

[36] X. Chen, M. X. Huang, S. Kachru, and G. Shiu, J. Cosmol. Astropart. Phys. 01 (2007) 002.

[37] R. Holman and A. J. Tolley, J. Cosmol. Astropart. Phys. 05 (2008) 001.

[38] N. Birrel and P. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England, 1982).

[39] E. Mottola, Phys. Rev. D **31**, 754 (1985).

[40] V. F. Mukhanov, H. A. Feldman, and R. H. Brandenberger, Phys. Rep. **215**, 203 (1992).

[41] R. Arnowitt, S. Deser, and C. W. Misner, arXiv:gr-qc/0405109.

[42] V. Acquaviva, N. Bartolo, S. Matarrese, and A. Riotto, Nucl. Phys. **B667**, 119 (2003); David Seery and James E. Lidsey, J. Cosmol. Astropart. Phys. 06 (2005) 003.

[43] A. A. Starobinsky, Pis'ma Zh. Eksp. Teor. Fiz. **73**, 415 (2001) [JETP Lett. **73**, 371 (2001)].

[44] The folded non-Gaussianity in the α-vacuum is also discussed in [36], in which they deal with $e^{\alpha} =$ constant.

[45] We would like to thank Xingang Chen for his careful explanation and discussion about the shape of non-Gaussianity.

[46] S. Weinberg, Phys. Rev. D **72**, 043514 (2005).

[47] S. Weinberg, Phys. Rev. D **74**, 023508 (2006).

[48] H. R. S. Cogollo, Y. Rodriguez, and C. A. Valenzuela-Toledo, J. Cosmol. Astropart. Phys. 08 (2008) 029.

[49] T. Banks and L. Mannelli, Phys. Rev. D **67**, 065009 (2003).

[50] M. B. Einhorn and F. Larsen, Phys. Rev. D **67**, 024001 (2003).

[51] K. Goldstein and D. A. Lowe, Nucl. Phys. **B669**, 325 (2003).

[52] H. Collins and R. Holman, Phys. Rev. D **71**, 085009 (2005); **74**, 045009 (2006); D. L. Nacir and F. D. Mazzitelli, Phys. Rev. D **76**, 024013 (2007).
[53] S. Dimopoulos, S. Kachru, J. McGreevy, and J. Wacker, J. Cosmol. Astropart. Phys. 08 (2008) 003; R. Easther and L. McAllister, J. Cosmol. Astropart. Phys. 05 (2006) 018.
[54] A. Linde and V. Mukhanov, Phys. Rev. D **56**, R535 (1997).
[55] F. Bernardeau and J. Uzan, Phys. Rev. D **66**, 103506 (2002).
[56] K. Enqvist, A. Jokinen, A. Mazumdar, T. Multamaki, and A. Vaihkonen, Phys. Rev. Lett. **94**, 161301 (2005).
[57] C. Gordon, D. Wands, B. A. Bassett, and R. Maartens, Phys. Rev. D **63**, 023506 (2000).
[58] D. Seery and J. E. Lidsey, J. Cosmol. Astropart. Phys. 09 (2005) 011; S. W. Li and W. Xue, arXiv:0804.0574; X. Gao, J. Cosmol. Astropart. Phys. 06 (2008) 029.