Role of the tetraquark in the chiral phase transition

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We investigate the implications of a light tetraquark field on chiral symmetry restoration at nonzero temperature within a simple chirally symmetric model. In order for the chiral phase transition to be crossover, as shown by lattice QCD studies, a strong mixing between scalar quarkonium and tetraquark fields is required. This leads to a light (~ 0.4 GeV), predominantly tetraquark state, and a heavy (~ 1.2 GeV), predominantly quarkonium state in the vacuum, in accordance with recently advocated interpretations of spectroscopy data. The mixing even increases with temperature and leads to an interchange of the roles of the originally heavy, predominantly quarkonium state and the originally light, predominantly tetraquark state. Then, as expected, the scalar quarkonium is a light state when becoming degenerate in mass with the pion as chiral symmetry is restored at nonzero temperature.

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I. INTRODUCTION

In the last 30 years, theoretical and experimental work on the light scalar mesons with mass below $\sim$1.8 GeV initiated an intense debate about their nature. The issue is that too many scalar resonances have been identified than can be accommodated in a naive quark-antiquark picture. For instance, in the scalar-isoscalar channel there are five states: $f_0(600)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ [1]. In order to explain their nature, quarkonia, tetraquark, and mesonic molecular assignments, as well as a scalar glueball state with mass around 1.5 GeV suggested by lattice studies of quantum chromodynamics (QCD) [2], have been investigated in a variety of combinations and mixing patterns [3–6]. Yet, a clear answer to the question of which resonance should be dominantly identified as a scalar quarkonium (i.e., quark-antiquark) state is not at hand.

Nowadays evidence for a full nonet of scalars with mass below 1 GeV is mounting; these are the already mentioned isoscalars $f_0(600)$ and $f_0(980)$, as well as the isotriplet $a_0(980)$, and the two isodoublets of $K_0^*(800)$. As proposed long ago by Jaffe [7], a tetraquark assignment for these states can explain some puzzling properties, such as the mass ordering which is reversed compared to the expectation from a quark-antiquark picture, and the strong coupling of $a_0(980)$ and $f_0(980)$ to kaons [8]. Within this context the lightest scalar resonance $f_0(600)$ is interpreted as a tetraquark state $\frac{1}{2}[u, d][\bar{u}, \bar{d}]$, where the commutator indicates an antisymmetric flavor (as well as color) configuration of the diquark. Further indications of a non-quarkonium nature of the scalar states below 1 GeV, and thus of $f_0(600)$, are obtained from a large-N_c study in the framework of unitarized chiral perturbation theory [9] and in the lattice studies of Ref. [10].

If the light scalars are (predominantly) tetraquark states, the question is how to identify the lightest quarkonium state $\bar{n}n = 1/\sqrt{2}(\bar{u}u + \bar{d}d)$: the broad resonance $f_0(1370)$ is the first candidate. This assignment is also supported by the fact that the scalar quarkonia are p-wave states, and thus expected to lie above 1 GeV together with other p-wave quarkonia such as axial-vector and tensor mesons. The two isoscalars of the quarkonia nonet and the scalar glueball can mix, forming the states $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$; such scenarios have been discussed in Refs. [11]. While it is not clear if $f_0(1500)$ or $f_0(1710)$ carries the largest glueball amount, all the above cited works agree in the assignment of a dominant $\bar{n}n$ component to the resonance $f_0(1370)$.

For N_f massless quark flavors, the QCD Lagrangian [including effects from the $U(1)_A$ anomaly [12]] has a chiral $SU(N_f)_r \times SU(N_f)_l \times U(1)_V$ symmetry, $V = r + l$. If this symmetry is linearly realized, the mass eigenstates of QCD come in degenerate pairs, so-called chiral partners, which have the same quantum numbers except for parity and G-parity; e.g., the chiral partners of the scalar-isoscalar meson are the pseudoscalar isotriplet mesons, i.e., the pions. The chiral symmetry is spontaneously broken to $SU(N_f)_V$ in the vacuum [13], thus lifting the degeneracy of the chiral partners and rendering the pions Goldstone bosons. Since the pion is commonly regarded to be a quark-antiquark state, and if the resonance $f_0(1370)$ is predominantly a quarkonium state, the latter should be considered as the chiral partner of the pion, and not $f_0(600)$, as is usually assumed. Consequently, if the chiral symmetry of QCD is restored above some critical temperature T_c, as predicted by lattice QCD calculations [14], the resonance $f_0(1370)$, and not $f_0(600)$, should become degenerate in mass with the pion. Clarifying this issue is important for the interpretation of data from heavy-ion-collision experiments, whose major goal is to identify signatures for chiral symmetry restoration at nonzero temperature T.

The aim of this paper is to take a first step towards investigating this scenario of chiral symmetry restoration. We employ the toy model discussed in Ref. [6], which is the $N_f = 2$ limit of a more general chiral Lagrangian for $N_f = 3$. This model contains only one tetraquark field in addition to the scalar quarkonium field and the pions. Although the other mentioned scalar-isoscalar resonances $f_0(980)$, $f_0(1500)$, and $f_0(1710)$ are not included at the present stage, this model has all the essential features to analyze the role of the tetraquark and its mixing with the quarkonium at nonzero T. To this end we employ the Cornwall-Jackiw-Tomboulis (CJT) formalism [15] in the Hartree-Fock approximation [16]. We shall show that, as T increases, the $f_0(1370)$ becomes lighter and its tetraquark admixture grows. Within our model calculation, we also find that, for a large range of parameters, there exists a certain temperature $T_s \leq T_c$ above which the state that is predominantly quarkonium becomes lighter than the state that is mostly tetraquark. At and above T_c, the state which is predominantly quarkonium becomes degenerate with the pion, as expected for chiral symmetry restoration.

II. THE MODEL

We consider the pion triplet $\vec{\pi}$, the bare quarkonium field $\varphi \equiv \bar{n}n$, and the bare tetraquark field $\chi \equiv \frac{1}{2}[u, d][\bar{u}, \bar{d}]$. The potential defining our model emerges as the $SU(2)_r \times SU(2)_l$ limit of a more general $SU(3)_r \times SU(3)_l$ chiral invariant Lagrangian studied in Ref. [6] and reads explicitly

$$V = \frac{\lambda}{4}(\varphi^2 + \vec{\pi}^2 - F^2)^2 - \varepsilon\varphi + \frac{1}{2}M_\chi^2\chi^2 - g\chi(\varphi^2 + \vec{\pi}^2), \tag{1}$$

where ε parametrizes explicit chiral symmetry breaking by nonzero quark masses and g, chosen to be ≥ 0, is the interaction strength of the tetraquark field χ [which is a singlet under $SU(2)_r \times SU(2)_l$] with the quarkonia fields. As we shall see, g also determines the mixing of the scalar fields. When $g \longrightarrow 0$ a simple linear sigma model for φ and $\vec{\pi}$ is left. In fact, the field χ, with mass M_χ, decouples in this limit. The minimum of the potential (1) is, to order $O(\epsilon)$, assumed for

$$\varphi_0 \simeq \frac{F}{\sqrt{1 - 2g^2/(\lambda M_\chi^2)}} + \frac{\varepsilon}{2\lambda F^2}, \qquad \chi_0 = \frac{g}{M_\chi^2}\varphi_0^2, \tag{2}$$

and $\vec{\pi} = 0$. The $\bar{n}n$ condensate φ_0 is identified with the pion decay constant $f_\pi = 92.4$ MeV. Note that the vacuum expectation value (*vev*) χ_0 is proportional to φ_0^2. Thus, the tetraquark condensate χ_0 is induced by the spontaneous symmetry breaking in the quarkonium sector. After shifting the fields $\varphi \longrightarrow \varphi_0 + \varphi$ and $\chi \longrightarrow \chi_0 + \chi$ and expanding the potential around the minimum, we obtain up to second order in the fields

$$V = \frac{1}{2}(\chi, \varphi)\begin{pmatrix} M_\chi^2 & -2g\varphi_0 \\ -2g\varphi_0 & M_\varphi^2 \end{pmatrix}\begin{pmatrix} \chi \\ \varphi \end{pmatrix} + \frac{1}{2}M_\pi^2\vec{\pi}^2 + \ldots, \tag{3}$$

where $M_\varphi^2 = \varphi_0^2(3\lambda - \frac{2g^2}{M_\chi^2}) - \lambda F^2$, $M_\pi^2 = \frac{\varepsilon}{\varphi_0}$. The value $M_\pi = 0.139$ GeV is used. Because of the off diagonal terms in the mass matrix in Eq. (3), the fields φ and χ are not mass eigenstates of the potential V. The latter, denoted by (H, S), are obtained after an $SO(2)$ rotation of the fields (φ, χ)

$$\begin{pmatrix} H \\ S \end{pmatrix} = \begin{pmatrix} \cos\theta_0 & \sin\theta_0 \\ -\sin\theta_0 & \cos\theta_0 \end{pmatrix}\begin{pmatrix} \chi \\ \varphi \end{pmatrix}, \tag{4}$$

where $\theta_0 = \frac{1}{2}\arctan\frac{4g\varphi_0}{M_\varphi^2 - M_\chi^2}$. The tree-level masses of H and S are

$$M_H^2 = M_\chi^2\cos^2\theta_0 + M_\varphi^2\sin^2\theta_0 - 2g\varphi_0\sin(2\theta_0),$$
$$M_S^2 = M_\varphi^2\cos^2\theta_0 + M_\chi^2\sin^2\theta_0 + 2g\varphi_0\sin(2\theta_0).$$

Assuming $-\pi/4 \leq \theta_0 \leq \pi/4$, the state H is then predominantly tetraquark and S is predominantly quarkonium. As discussed above, we shall identify the state H with the resonance $f_0(600)$ and the state S with $f_0(1370)$. A natural choice is then that the pure tetraquark should be lighter than the pure quarkonium, i.e., $M_\chi < M_\varphi$. Using the fact that the trace and the determinant of the mass matrices before and after the $SO(2)$ rotation are equal, we obtain $(M_S^2 - M_H^2)^2 = (M_\varphi^2 - M_\chi^2)^2 + (4g\varphi_0)^2$, implying that for $g > 0$ the masses of the states H and S repel each other: $M_H < M_\chi < M_\varphi < M_S$. We also obtain the constraint $|M_S^2 - M_H^2| \geq 4g\varphi_0$.

As a side remark, we mention that the tree-level decay width of $f_0(600)$ is larger than 300 MeV when the mass lies above 0.6 GeV and when the mixing is large. However, we refrain from a more elaborate study of vacuum properties, since a realistic description of the latter requires the inclusion of other scalar states and of (axial-)vector mesons [17–19] and we concentrate on the behavior at nonzero T.

III. RESULTS

In order to study chiral symmetry restoration at nonzero T we employ the CJT-formalism in the Hartree-Fock approximation; for details see Ref. [16]. As a result, the masses $M_H(T)$, $M_S(T)$, $M_\pi(T)$, and the mixing angle $\theta(T)$ become functions of T. Moreover, both scalar-isoscalar fields attain T-dependent vev's, $\varphi_0 \longrightarrow \varphi(T)$ for the quarkonium and $\chi_0 \longrightarrow \chi(T)$ for the tetraquark state, respectively, with $\varphi(0) = \varphi_0$ and $\chi(0) = \chi_0$; see Eq. (2).

We first study the order of the chiral phase transition and the associated T_c as a function of the model parameters g, $M_H \equiv M_H(0)$, and $M_S \equiv M_S(0)$. Figure 1(a) shows the phase diagram in the g-M_S plane for fixed $M_H = 0.4$ GeV which is close to the value of Ref. [20], while Fig. 1(b) depicts the g-M_S plane for fixed $M_S = 1.2$ GeV which is in the experimentally established range of values for $f_0(1370)$ [1].

One observes in Fig. 1(a) that, in the limit $g \longrightarrow 0$ in which S is a pure quarkonium, the transition is crossover below a value $M_S \simeq 0.948$ GeV and of first order above this value. The fact that a heavy chiral partner of the pion

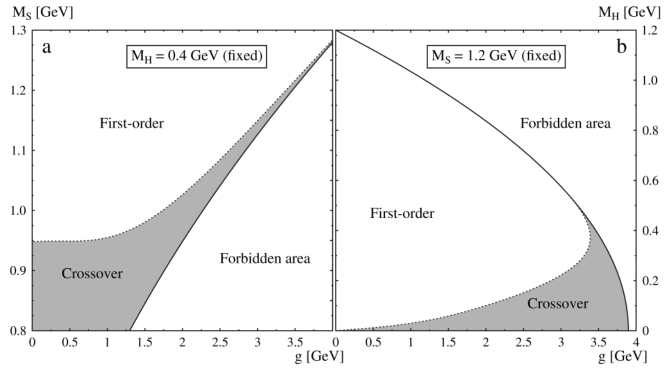

FIG. 1. Order of the phase transition as a function of the parameters of the model. The forbidden area violates the constraint $|M_S^2 - M_H^2| \geq 4g\varphi_0$. On the border line between the first-order and the crossover transitions a second-order phase transition is realized.

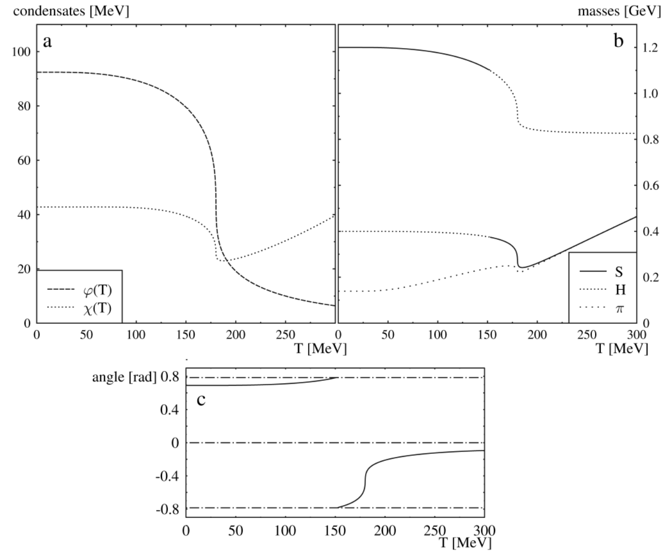

FIG. 2. Condensates (panel a), masses (panel b), and mixing angle (panel c) as functions of T.

induces a first-order chiral phase transition has been discussed previously; see e.g. Ref. [18]. This means that, in a linear sigma model without tetraquark degrees of freedom, a heavy (i.e., mass larger 1 GeV) chiral partner of the pion is excluded by lattice QCD calculations [14], which indicate a crossover transition. Including tetraquarks dramatically changes this conclusion: as one observes in Fig. 1(a), for increasing g the region of crossover transitions extends towards larger values of M_S. Thus, the scenario outlined in the Introduction, in which $H \equiv f_0(600)$ and $S \equiv f_0(1370)$, can accommodate for a crossover transition if g, and thus the mixing between quarkonium and tetraquark, is sufficiently large. Note that the range of g values for which the transition is crossover, narrows substantially when M_S increases. Along the line of second-order phase transitions in Fig. 1(a), T_c sizably decreases for increasing g, for instance from $T_c \simeq 241$ MeV at $g = 0$ to $T_c \simeq 186$ MeV at $g = 3$ GeV and $T_c \simeq 173$ MeV at $g = 4$ GeV. We observe in Fig. 1(b) that a crossover transition occurs only for small values of M_H. The crossover region widens when g increases. In order to accommodate a value $\sim$0.4 GeV [20], a large value of g is required. Along the line of second-order transitions, T_c first decreases, and then increases for increasing g. The minimum $T_c \simeq 145$ MeV occurs for $g \simeq 2$ GeV.

We now study the T-dependence of masses, condensates [21], and the mixing angle in more detail in the case of $M_H = 0.4$ GeV and $M_S = 1.2$ GeV (in the range quoted by Refs. [1,9,20]; a mass $M_H \sim 0.4$ GeV, although leading to a too small tree-level decay width due to lack of phase space, allows for a nice illustrative description of the qualitative features of the nonzero T). Also, we set the coupling strength $g = 3.4$ GeV, in order to obtain a crossover phase transition in agreement with lattice QCD calculations [14]. These parameter values lead (together with values $\varphi_0 = f_\pi = 92.4$ MeV and $M_\pi = 0.139$ GeV) to $M_\chi = 0.82$ GeV, $M_\varphi = 0.96$ GeV, and $\lambda = 52.85$.

The condensates $\varphi(T)$ and $\chi(T)$ are shown in Fig. 2(a). The quark condensate $\varphi(T)$ drops at T_c and then approaches zero, signaling the restoration of chiral symmetry. The tetraquark condensate $\chi(T)$ first drops together with $\varphi(T)$, but increases above T_c: this is due to the fact that in the equation determining $\chi(T)$, the growth of the S and H tadpole contributions with T has to be balanced by an increase of $\chi(T)$; this could be different if we include additional terms $\sim\chi^4$ in the potential (1). In any case, the increase of $\chi(T)$ affects the behavior of the masses or of other physical quantities only slightly. Note that the field χ is a singlet under chiral transformations in the $N_f = 2$ case and therefore the nonzero value of the condensate does not imply a breaking of chiral symmetry at high T.

The T-dependent masses $M_H(T)$, $M_S(T)$, $M_\pi(T)$ of the particles H, S, and π are shown in Fig. 2(b). The solid line corresponds to $M_S(T)$, the mass of the state which is predominantly quarkonium [$|\theta(T)| < \pi/4$], and the dotted line to $M_H(T)$, the mass of the state which is predominantly tetraquark. At $T_s \simeq 160$ MeV, both masses behave discontinuously and the states interchange their roles: for $T < T_s$, the state S is the heavier scalar and H the lighter one, and for $T > T_s$, the state S is lighter than H. Above T_s the state S becomes degenerate with the pions as in the sigma model without tetraquark. Note that, before becoming degenerate with the pion, the thermal mass of the lightest state (identified with H for $T \leq T_s$ and with S above it) slightly decreases; see Ref. [19] for comparison.

In Fig. 2(c), the mixing angle $\theta(T)$ is plotted. At T_s, $\theta(T)$ is discontinuous, which leads to the discontinuity in the masses noted above: it jumps suddenly from $\pi/4$ to -$\pi/4$, $\lim_{T\to T_s^\pm}\theta(T) = \mp\frac{\pi}{4}$. At T_s the mixing is maximal: the two physical states H and S have the same amount (50%) of quarkonium and tetraquark. Note that, at T_s, the field φ and χ are degenerate in mass, which explains the maximal value of the mixing angle. As a last remark we note that the relative magnitude of T_s and T_c ($T_s < T_c$ as in our example or vice versa) depends on the choice of the parameters. When increasing the mixing strength g the temperature T_s decreases faster than T_c, thus realizing the ordering $T_s < T_c$ in which the jump occurs at smaller temperatures than chiral symmetry restoration.

IV. CONCLUSIONS

In this paper, we proposed a novel scenario for chiral symmetry restoration at nonzero T, in which two scalar-isoscalar states, a tetraquark and bare quarkonium field, are considered. The mixing of the latter two generates two physical states which can be associated with the resonances $f_0(600)$ and $f_0(1370)$. When the tetraquark mass is smaller than the quarkonium mass, as supported by various spectroscopic studies in the vacuum, the state $f_0(600)$ is predominantly tetraquark and $f_0(1370)$ is predominantly quarkonium. This scenario has been studied by employing a simple model which includes only these two scalar resonances and the pion triplet.

A remarkable aspect of our results is that the tetraquark-quarkonium mixing generates a softer first-order phase transition or, depending on the coupling strength, even a crossover transition. While in the standard linear sigma model ($g = 0$) a heavy chiral partner of the pion (with mass exceeding 1 GeV) always leads to a first-order phase transition, this is not necessarily the case when tetraquark-quarkonium mixing is considered: for sufficiently large coupling strength g, the chiral transition is crossover, just like in lattice QCD studies [14]. We also demonstrated that the mixing between quarkonium and tetraquark states increases with increasing temperature, and, in most cases, reaches its maximal value of 45° at a temperature T_s where the physical states consist of an equal amount of quarkonium and tetraquark. For $T > T_s$ the physical states interchange their roles: the lighter state is predominantly quarkonium and the heavier predominantly tetraquark. Further increasing T leads to the standard scenario of chiral symmetry restoration, where the scalar quarkonium becomes degenerate in mass with its chiral partner, the pion. Thus, our approach can possibly solve an inconsistency between low-energy spectroscopy, where a nonquarkonium structure for $f_0(600)$ is favored, and studies at nonzero temperature, where the scalar partner of the pion should be sufficiently light (~ 0.6 GeV) in order for the chiral symmetry restoring transition to be crossover.

Since the present work is a first explorative study on the relevance of the tetraquark at nonzero T, we omitted other scalar-isoscalar states such as $f_0(980)$, $f_0(1500)$, and $f_0(1710)$, which would naturally appear in an $SU(3)$-symmetric model with two scalar nonets and a glueball state. Also (axial-)vector mesons should be considered [17,18]. All these fields are important in a more realistic framework which aims to describe at the same time vacuum phenomenology and nonzero T properties. We regard the results of this paper as a motivation to undertake this more ambitious step in the near future.

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