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Supersymmetric electroweak symmetry breaking

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In the minimal supersymmetric standard model (the MSSM), the electroweak symmetry is restored as supersymmetry-breaking terms are turned off. We describe a generic extension of the MSSM where the electroweak symmetry is broken in the supersymmetric limit. We call this limit the ''sEWSB'' phase, short for supersymmetric electroweak symmetry breaking. We define this phase in an effective field theory that only contains the MSSM degrees of freedom. The sEWSB vacua naturally have an inverted scalar spectrum, where the *heaviest* CP-even Higgs state has standard model-like couplings to the massive vector bosons; experimental constraints in the scalar Higgs sector are more easily satisfied than in the MSSM.

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I. INTRODUCTION

The minimal supersymmetric extension of the standard model (MSSM) provides a framework for understanding the origin of electroweak symmetry breaking (EWSB). The Higgs fields will acquire vacuum expectation values (VEV's) only if their mass parameters live in a window that produces a nontrivial but stable global minimum in the Higgs potential. This window always requires supersymmetry (SUSY) breaking and may occur radiatively [1].

Of the two neutral CP-even states in the MSSM, typically the lightest CP-even state couples to the massive W and Z vector bosons like the standard model Higgs (is ''SM-like''). At tree level, this state has a mass lighter than m_Z because the Higgs potential is stabilized by Kähler terms proportional to the electroweak (EW) gauge couplings. As is well known, large SUSY-breaking effects in the stop-top sector can allow this SM-like Higgs state to escape LEP-II bounds, but only at the cost of tuning the parameters of the theory.

However, if EWSB occurs instead in the supersymmetric limit, it is the non-SM-like Higgs CP-even state whose mass is tied to m_Z, *not* the SM-like Higgs. The SM-like Higgs state is part of a chiral supermultiplet whose mass is not related to the electroweak gauge couplings or to m_Z at tree level. We call any vacuum in which the electroweak symmetry remains broken as SUSY breaking is turned off a ''supersymmetric electroweak symmetry breaking'' vacuum (sEWSB vacuum). Considering again the LEP-II bounds, the most interesting feature of sEWSB vacua is that the CP-even scalar spectrum may be inverted compared to the usual spectrum of the MSSM: the *heavier* CP-even state, not the lighter, is the SM-like Higgs field. In the MSSM, it is possible to have viable inverted CP-even spectra but only with large radiative corrections.

Further, sEWSB will occur with only the mild assumption of a new approximately supersymmetric physics threshold just above the weak scale that couples to the MSSM Higgs fields. We can therefore understand sEWSB most simply by working in an effective theory that only contains the MSSM degrees of freedom and additional nonrenormalizable interactions. Focusing on the Higgs sector of the theory, the most general superpotential that can arise from integrating out a *supersymmetric* threshold at the scale μ_S is

$$W = \mu H_u H_d + \frac{\omega_1}{2\mu_S}(H_u H_d)^2 + \frac{\omega_2}{3\mu_S^3}(H_u H_d)^3 + \cdots, \tag{1}$$

where we have suppressed the $SU(2)_L$ indices and $H_u H_d = H_u^+ H_d^- - H_u^0 H_d^0$. The ellipses represent terms suppressed by higher powers of the scale μ_S, and the ω_i are dimensionless coefficients. Keeping only the first two terms, for simplicity, we see that the F-flatness conditions are satisfied by the origin in field space, and also by a nontrivial VEV,

$$\langle H_u H_d \rangle = -\mu \mu_S / \omega_1. \tag{2}$$

Thus, the EW scale may arise as the geometric mean of the μ-term and the scale of some relatively heavy new physics, and have a purely supersymmetric origin. As we show in Sec. II B, the spectrum of this vacuum is very simple: most of the Higgs fields (scalar and fermion components) are ''eaten'' by the vector superfields and together have masses equal to m_W or m_Z. One neutral Higgs superfield remains, which contains the SM-like Higgs, with mass $2|\mu|$. For $2|\mu| > m_Z$, the scalar spectrum is inverted compared to the decoupling limit of the MSSM: the light CP-even state with mass m_Z is not SM-like, while the heavy CP-even state is at $2|\mu|$ and is SM-like.[1]

Since we are working in a nonrenormalizable theory, it is not enough that sEWSB occurs; we require that the effec-

[1]The requirement that the EFT analysis be reliable (μ_S somewhat above the EW scale and ω_1 not extremely large) implies that the Higgs masses are of order the EW scale. For instance, all the examples we consider satisfy the general bounds derived in [2].

tive field theory (EFT) remain valid in an expansion around this minimum—all ignored operators beyond the first two in Eq. (1) should give only small corrections to our analysis. Supersymmetry plays a prominent role in maintaining the validity of the EFT. Nonrenormalizable operators either in the Kähler potential or in the superpotential are suppressed by

$$\frac{\langle H\rangle^2}{\mu_S^2} \sim \frac{2}{\omega_1}\frac{\mu}{\mu_S}, \qquad (3)$$

and can be self-consistently ignored provided $\mu \ll \mu_S$. SUSY drives this suppression in two ways. First, the separation of scales between μ and μ_S is technically natural in a supersymmetric theory. Second, the sEWSB VEV results from balancing a dimension-6 term in the scalar potential against a dimension-4 term, so that $\langle H^2\rangle$ is proportional to the Higgs quartic times the nonrenormalizable scale, μ_S^2/ω_1^2. However, holomorphicity and gauge invariance in the superpotential only allow a quartic term of order $\omega_1\mu/\mu_S$ along the Higgs D-flat direction in the scalar potential. If any larger quartic term were allowed, the validity of the EFT would be ruined. Ironically, the absence of a large quartic term in the Higgs superpotential is exactly why there is a little hierarchy problem in the MSSM to begin with.

Given the bounds from direct searches on superpartners, SUSY must be broken, and we expect the SUSY limit to be deformed by soft masses of order the electroweak scale. We incorporate the effects of SUSY breaking in Sec. III and show how to consistently identify sEWSB vacua in this limit. Depending on the parameter choice, the sEWSB minimum of Eq. (2) may be the only nontrivial minimum of the theory, or it can be joined by a vacuum which is continuously connected to the usual EWSB vacuum of the MSSM in the limit that the nonrenormalizable operators of Eq. (1) are turned off (MSSM-like vacua). Even with SUSY breaking turned on, we show in Sec. IV that sEWSB vacua can share the qualitative features of the pure SUSY limit: the heavier CP-even state has SM-like Higgs couplings to massive vector bosons.

One of the main phenomenological tensions in this vacuum is the forced separation between μ and μ_S. This ratio should be small, to maintain control of the effective theory, but there is a tension between making μ_S large while keeping the ratio $\mu\mu_S \sim v^2$ fixed. The SUSY limit forces the charginos to have mass m_W. Pushing these states above LEP-II bounds requires keeping μ as large as possible when SUSY is broken. In Sec. IV, we show that charginos and neutralinos near the LEP-II bound are a fairly generic prediction of sEWSB vacua, and that the lightest chargino may be lighter than the lightest neutralino [the gravitino could be the lightest supersymmetric particle (LSP) in this case]. This next-to-lightest supersymmetric particle (NLSP) chargino would lead to an enhanced set of W bosons in cascade decays [3].

In Sec. V we discuss one of the simplest ultraviolet completions that can lead to sEWSB vacua: adding a singlet superfield S to the MSSM, with a supersymmetric mass μ_S and a trilinear SH_uH_d coupling. Unlike the next-to-minimal supersymmetric standard model [4], we do not explain the origin of the μ-term in the MSSM: this UV theory includes an explicit $\mu H_u H_d$ term. It is well known that the LEP-II limit can also be escaped by integrating out a singlet superfield in the *non*-SUSY limit [5]; here we assume μ_S is much larger than the scale of SUSY breaking. The fat Higgs model [6] is another example of a singlet-extended MSSM theory that exhibits EWSB in the SUSY limit, but is not described by our EFT since the field S cannot be decoupled from the spectrum in a supersymmetric limit. The singlet UV completion of our theory belongs to the more general analyses of theories with singlet superfields and the coupling λSH_uH_d [7].

An EFT approach to parametrize extensions to the MSSM up to terms of $\mathcal{O}(H^4)$ in the superpotential has already been used to analyze the effects of the leading, renormalizable, $\mathcal{O}(H^4)$ terms in the scalar potential [8–10]. These analyses are useful for calculating perturbations to MSSM-like vacua. The sEWSB vacua require keeping terms of order $\mathcal{O}(H^4)$ in the superpotential *and the full set* of $\mathcal{O}(H^6)$ terms in the scalar potential that are generated by the superpotential, a case not seriously considered in previous studies.

II. SUPERSYMMETRIC ELECTROWEAK SYMMETRY BREAKING

As we will see, the qualitative physical properties of the sEWSB vacuum can already be understood in the supersymmetric limit. It is therefore useful to study in some detail the physics of EWSB when SUSY is exact, which we do in this section. We consider the effects of SUSY breaking, under the assumption that the heavy threshold μ_S is approximately supersymmetric, in Sec. III.

A. Validity of the effective theory on the sEWSB vacuum

Our main observation is that in the presence of the higher-dimension operators in the superpotential of Eq. (1) there is a nontrivial ground state that can be reliably studied within the EFT framework. The only condition is that there exists a mild hierarchy between μ and the new physics threshold μ_S.

Indeed, assuming that the first nonrenormalizable operator in Eq. (1) is nonvanishing, the F-flatness conditions can be satisfied both at the origin of field space and at a VEV of order $\mu\mu_S/\omega_1$. This solution exists for any sign of the dimensionless coefficient ω_1. It is a solution to the F-flatness conditions where the two leading terms in Eq. (1) approximately cancel, while the remaining operators give contributions that are suppressed by powers of μ/μ_S (times ratios of dimensionless coefficients). Thus, we can capture the physical properties of this vacuum to leading order in μ/μ_S by keeping the first two terms in Eq. (1). This defines the zeroth order approximation. Operators in the superpotential suppressed by $1/\mu_S^{2n+1}$ with $n \geq 1$ give corrections to physical observables that are suppressed by at least $(\mu/\mu_S)^n$, which we refer to as the nth order approximation. Notice that the importance of an operator, whether nonrenormalizable or not, depends on the vacuum state one is expanding field fluctuations about. In general, to estimate the relevance of any operator one should do the power counting after expanding around the VEV of interest.

One might also worry about the effects of higher-dimension operators in the Kähler potential. However, these enter at *next-to-leading* order in the $1/\mu_S$ expansion, e.g.

$$K = H_u^\dagger e^V H_u\left[1 + \frac{1}{\mu_S^2} f_u\right] + H_d^\dagger e^V H_d\left[1 + \frac{1}{\mu_S^2} f_d\right] + \frac{c_1}{\mu_S^2}|H_u H_d|^2 + \cdots, \tag{4}$$

where

$$f_u = \frac{1}{2}a_1^u H_u^\dagger e^V H_u + \frac{1}{2}a_1^{ud} H_d^\dagger e^V H_d + (b_1^u H_u H_d + \text{H.c.}) + \mathcal{O}\left(\frac{1}{\mu_S^2}\right), \tag{5}$$

$$f_d = \frac{1}{2}a_1^d H_d^\dagger e^V H_d + \frac{1}{2}a_1^{ud} H_u^\dagger e^V H_u + (b_1^d H_u H_d + \text{H.c.}) + \mathcal{O}\left(\frac{1}{\mu_S^2}\right). \tag{6}$$

Their effects on the physical properties of the vacuum of Eq. (2) are also suppressed by μ/μ_S and correspond to small corrections to the zeroth order solution described in the previous paragraph.[2] For instance, although the leading order D-terms imply that $\tan\beta = \langle H_u\rangle/\langle H_d\rangle = \pm 1$, the higher-dimension Kähler corrections can lead to $|\tan\beta| \neq 1$ if $a_1^u \neq a_1^d$, or $b_1^u \neq b_1^d$, etc. [see Eqs. (A1)–(A3) in Appendix A for the general expressions of the D-term potential]. However, to the extent that μ/μ_S is small, one finds that $|\tan\beta|$ remains close to 1 in the SUSY limit. Nevertheless, the Kähler terms can have other phenomenologically relevant effects that are pointed out in Sec. II C. There may also be terms containing SUSY covariant derivatives that we do not show explicitly, since they lead to derivative interactions that do not affect the vacuum or spectrum of the theory.

In summary, it is possible to study the properties of the sEWSB vacuum from Eq. (1) without a complete specification of the physics that gives rise to the tower of higher-dimension operators, so that an EFT analysis is appropriate. In particular, the theory that includes the higher-dimension operators has at least two degenerate SUSY-preserving minima: the origin and a vacuum where EWSB occurs. These supersymmetric vacua are degenerate and separated by a potential barrier as shown schematically in Fig. 1. We can characterize the sEWSB minimum by

$$\langle H_u^0\rangle \approx \langle H_d^0\rangle \approx \sqrt{\mu\mu_S/\omega_1}, \tag{7}$$

which holds up to corrections of order μ/μ_S. Here we have used a combination of $SU(2)_L \times U(1)_Y$ gauge transformations to make both VEV's real, positive, and in the electrically neutral components. We have also made an additional field redefinition to make the quantity $\mu\mu_S/\omega_1$ real and positive. In the following we will refer to the vacuum of Eq. (7) as the "sEWSB vacuum" (short for supersymmetric EWSB vacuum).

One might still wonder if other nontrivial vacua exist when the superpotential has the form of Eq. (1). In general, except for the sEWSB vacuum described above, all other potential solutions to the F-flatness conditions would correspond to VEV's of order μ_S, and are therefore outside the realm of the EFT. In fact, the question of whether such vacua actually exist or not can only be answered within the context of a given UV completion. It is logically possible that additional solutions with VEV's parametrically smaller than μ_S exist, but this can only happen for special choices of the coefficients ω_i. For example, solutions that arise from balancing the μ-term with an ω_n operator [the operator with coefficient ω_n in Eq. (1)] exist only if the coefficients of all ω_i operators with $i < n$ are suppressed by appropriate powers of μ/μ_S. This latter quantity has to be small in order that the ω_i operators with $i > n$ can be neglected. In particular, if the ω_1 operator is generated by the physics at μ_S with a coefficient larger than

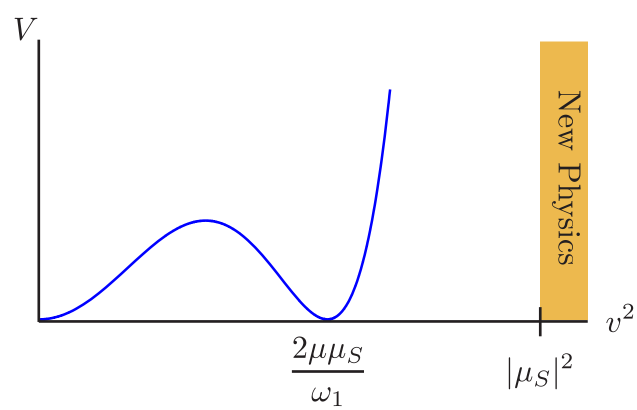

FIG. 1 (color online). The phase structure of the superpotential in Eq. (1), keeping only the leading correction, along the $\tan\beta = 1$ slice. Supersymmetry allows us to reliably calculate around the EWSB minima, since the scale of new physics may be much larger than all other mass scales in the effective theory.

[2] Kähler terms suppressed by $1/\mu_S^{2n}$ give corrections suppressed by at least $(\mu/\mu_S)^n$.

$\mathcal{O}(\mu/\mu_S)^{1/2}$, no such solutions exist. We also assume here that the ω_n are smaller than the naïve dimensional analysis estimate $(16\pi^2)^n$ [11]. If the physics at μ_S is strongly coupled, our analysis cannot reliably establish the existence of nontrivial minima in the SUSY limit. However, notice that due to nonrenormalization theorems, it is possible that all but a finite number of operators in the superpotential vanish.

In this paper, we concentrate on the sEWSB vacuum of Eq. (7) for which we do not need to make strong assumptions regarding the dimensionless coefficients ω_i. We expect that there is a large region of parameter space (hence a large number of UV completions) where the sEWSB vacua are physically relevant.

B. Supersymmetric Higgs spectrum

The spectrum and interactions of the Higgs sector in the sEWSB vacuum are particularly simple due to the constraints imposed by the unbroken supersymmetry: the massive W and Z gauge bosons are components of two separate massive vector superfields, a charged field with mass m_W and a neutral field with mass m_Z. Each massive vector superfield is made up of a massless vector superfield and an eaten chiral superfield. The *complex* massive vector superfield corresponding to the $W^\pm$ gauge bosons eats the superfields H_u^+ and H_d^-. The massive vector superfield that contains the Z boson eats the linear combination that does *not* acquire a VEV, $H \equiv (H_u^0 - H_d^0)/\sqrt{2}$. The orthogonal combination (or "superradial" mode), $h \equiv (H_u^0 + H_d^0)/\sqrt{2}$, remains as an additional degree of freedom and corresponds to the physical Higgs superfield (the fact that $\langle h\rangle = v$ signals that these degrees of freedom are responsible for the unitarization of WW scattering).

The scalar components of the superfields, in unitary gauge, are

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} H^+ \\ \frac{v}{\sqrt{2}} + \frac{1}{2}(H + h + iA^0) \end{pmatrix}, \qquad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix} = \begin{pmatrix} \frac{v}{\sqrt{2}} + \frac{1}{2}(-H + h + iA^0) \\ \frac{1}{\sqrt{2}} H^- \end{pmatrix}. \tag{8}$$

Here, h is exactly the SM-like Higgs and we have decomposed the scalar sector into mass eigenstates. The scalar fields H and $H^\pm$ have masses m_Z and m_W, respectively, and the fields h and A^0—in the zeroth order approximation discussed in the previous subsection—have mass $2|\mu|$.[3] Also, the fermions of each eaten superfield form Dirac partners with the vector superfield gauginos, and have masses equal to their vector partners. The Higgs superpartner is a Majorana fermion. The field content and supermultiplet structure is as follows:

Mass	Scalars	Fermions	Vectors
0	$\cdots$	1 Majorana	A_μ
m_W	$H^\pm$	2 Dirac	$W_\mu^\pm$
m_Z	H	1 Dirac	Z_μ
$2\lvert\mu\rvert$	h, A^0	1 Majorana	$\cdots$

It is remarkable that in the sEWSB vacuum, the mass of the SM-like Higgs (which completely unitarizes WW scattering) is fixed by the μ-term. In particular, the mass of the SM-like Higgs is independent of the SM gauge couplings, contrary to what happens in the MSSM with only renormalizable operators. It should also be noted that this mass can be shifted by order μ/μ_S due to the tower of higher-dimension operators. The H and $H^\pm$ masses remain tied to the corresponding gauge boson masses, in the SUSY limit.

C. Subleading corrections, canonical normalization, and mixing

As mentioned in Sec. II A, the Kähler corrections enter at second order in the $1/\mu_S$ expansion. Such corrections can affect both the spectrum and couplings of various fields, and appear through additional contributions to the scalar potential as well as through corrections to the kinetic terms. It is interesting that the former effects show up as a multiplicative factor in the F-term potential. As a concrete example, when the only nonzero coefficient in the Kähler potential of Eq. (4) is c_1, one finds the simple result

$$V_F = \frac{|H|^2}{1 + \frac{c_1}{\mu_S^2}|H|^2} \left| \mu + \frac{\omega_1}{\mu_S} H_u H_d + \cdots \right|^2, \tag{9}$$

where $|H|^2 \equiv H_u^\dagger H_u + H_d^\dagger H_d$. This case arises precisely when the heavy physics corresponds to an $SU(2)_L \times U(1)$ singlet (with $\kappa = 0$), as discussed in Sec. V. One can show that in the general case the first factor is replaced by a real function $Z(H_u, H_d)$, whose exact form is given in Eq. (A13) of Appendix A. It follows that the Kähler corrections do not affect the vacuum obtained by imposing F flatness as if the Kähler terms were of the minimal form. However, they do affect the spectrum and Higgs self-interactions, though such effects are unlikely to be of immediate phenomenological relevance.

More relevant from a phenomenological point of view are certain corrections to the Higgs kinetic terms, which are of order μ/μ_S. Although in the SUSY limit the properties of the fields involved in the super-Higgs mechanism, $\frac{1}{\sqrt{2}}(H_u^0 - H_d^0)$, H_u^+ and H_d^-, are protected, those of the Higgs superfield itself can receive important corrections. For instance, the operator proportional to c_1 in Eq. (4) contains contributions to the kinetic terms without the corresponding corrections to the gauge interactions [in the sEWSB vacuum of Eq. (7)]:

[3]One can see that the superfield h has mass $2|\mu|$ by using a supersymmetric gauge transformation to completely remove the eaten *superfields* H, H_u^+, H_d^- from the theory. The superpotential then contains the mass term $W \supset \mu h^2$.

$$\int d^2\theta d^2\bar{\theta} \frac{c_1}{\mu_S^2}|H_u H_d|^2 = \frac{c_1 v^2}{\mu_S^2}\Big[\frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\partial_\mu A^0 \partial^\mu A^0 + i\psi^\alpha \sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu \bar{\psi}^{\dot{\alpha}}\Big] + \cdots,$$

where we used the parametrization of Eq. (8) and show only the kinetic terms, including those of the Higgs Majorana partner.

The reason these effects are important is that, although formally of second order in $1/\mu_S$, they correspond to the *leading order corrections* to the Higgs gauge interactions, after a rescaling to restore canonical normalization:

$$(h, A^0, \psi) \to \frac{1}{\sqrt{1+\frac{2c_1\mu}{\omega_1\mu_S}}}(h, A^0, \psi) \approx \Big(1 - \frac{c_1\mu}{\omega_1\mu_S}\Big)(h, A^0, \psi).$$

Physically, these effects correspond to mixing of the light fields with the UV physics at the scale μ_S.

D. Nonrenormalizable operators at the component level

So far we have emphasized the power counting associated with operators in the Kähler potential and superpotential. It is worth noting how the same picture appears at the component level, especially since analyzing the vacuum structure of the theory in the presence of SUSY breaking (as is done in Sec. III D) requires a direct study of the scalar *potential*.

To zeroth order in μ/μ_S, and assuming for simplicity that μ and ω_1 are real, one gets an F-term potential with a quartic interaction, as well as a certain "dimension-6" operator:

$$V_F^{(0)} = \mu^2|H|^2 + \frac{\omega_1\mu}{\mu_S}|H|^2(H_u H_d + \text{H.c.}) + \frac{\omega_1^2}{\mu_S^2}|H|^2|H_u H_d|^2, \tag{10}$$

where $|H|^2$ was defined after Eq. (9). The quartic terms correspond to the λ_6 and λ_7 operators of the two-Higgs doublet model parametrization of Refs. [12,13]. The relevance of the nonrenormalizable term in Eq. (10) depends on the particular vacuum one is studying. One should expand fields in fluctuations around the relevant vacuum to determine which interactions are important. Since the sEWSB vacuum scales like $\mu_S^{1/2}$, the dimension-6 term should not be neglected: it can contribute at the same order as the first two terms in Eq. (10).[4] Thus, although it should be obvious, we stress that the physics we are describing *cannot* be captured by the standard $SU(2)_L \times U(1)_Y$ two-Higgs doublet model parametrization based on renormalizable interactions [8–10].

Similar comments apply at higher orders. For instance, at first order in the μ/μ_S expansion, the operator proportional to c_1 in Eq. (4) leads to additional quartic operators (corresponding to λ_1, λ_2, and λ_3 in the two-Higgs doublet model parametrization of Refs. [12,13]), to an additional dimension-6 operator, and to a particular "dimension-8" operator, as can be derived from Eq. (9)[5]:

$$V_F^{(1)} = -\frac{c_1\mu^2}{\mu_S^2}|H|^4 - \frac{c_1\omega_1\mu}{\mu_S^3}|H|^4(H_u H_d + \text{H.c.}) - \frac{c_1\omega_1^2}{\mu_S^4}|H|^4|H_u H_d|^2.$$

In spite of the different powers of μ_S in the denominators, all of these can contribute to physical observables at first order in the μ/μ_S expansion in the sEWSB vacuum of Eq. (7). Nevertheless, our argument of Sec. II A, performed at the level of the Kähler potential and superpotential, guarantees that the EFT around the sEWSB vacuum has a well-defined expansion parameter and that the infinite tower of operators can be consistently truncated, in spite of the $\mu_S^{1/2}$ scaling of the sEWSB VEV.

In the next section we consider the effects of SUSY breaking at tree level. However, we notice here that although loop effects from supersymmetric partners can—in the presence of SUSY breaking—give contributions to the operators that play a crucial role in the determination of the sEWSB vacuum, these are expected to be subdominant. For instance, the one-loop contributions to the λ_6 and λ_7 quartic couplings are not logarithmically enhanced and are proportional to A_t [14]. If all SUSY-breaking parameters are of order the EW scale, the corresponding one-loop contributions are of order $3y_t^4/(16\pi^2)$ or smaller, which can easily be subdominant compared to the quartic coupling in Eq. (10) for $\mu_S \sim (5\text{–}10)\mu$, as we envision here. We therefore do not consider loop effects any further and restrict ourselves to a tree-level analysis.

III. SUPERSYMMETRY BREAKING

The previous section focused on electroweak symmetry breaking *in the SUSY limit*. Although this limit is not fully realistic, it allows a simple understanding of several properties of the physics when SUSY breaking is taken into account. Here we reconsider the analysis including SUSY-

[4] In fact, it plays an essential role in bounding the potential from below and stabilizing the vacuum of interest; it also induces contributions to the quartic interactions of the physical fluctuations about the sEWSB vacuum.

[5] Note that, for $c_1 > 0$, $V_F^{(1)}$ can be large and negative, which would seem to lead to a potential unbounded from below. However, this occurs at large values of the Higgs fields, where the EFT is not expected to be valid. Indeed, the remaining terms in the expansion of Eq. (9) make the potential positive, as required by SUSY.

breaking effects. SUSY-breaking terms are required, among other reasons, to lift the mass of the photino. They also break the degeneracy between the origin and the nontrivial EWSB minimum.

A. Scalar potential

Our main assumption is that the heavy threshold, μ_S, is very nearly supersymmetric, so that a spurion analysis is appropriate.[6] To order $1/\mu_S$, we must include the effects of the nonrenormalizable operator

$$W \supset \frac{1}{2\mu_S}\tilde{X}(H_u H_d)^2, \tag{11}$$

in addition to the usual soft terms in the MSSM Lagrangian, where $\tilde{X} = \theta^2 m_{\rm soft}$ parametrizes the effective soft SUSY-breaking effects coming from the heavy sector. We write, for convenience, $m_{\rm soft} = \xi\omega_1\mu$, and assume that $|\xi\omega_1| \lesssim \mathcal{O}(1)$. Thus, the relevant SUSY-breaking terms in the scalar potential read

$$V_{\rm SB} = m_{H_u}^2|H_u|^2 + m_{H_d}^2|H_d|^2 + \left[bH_uH_d - \xi\left(\frac{\omega_1\mu}{2\mu_s}\right) \times (H_uH_d)^2 + {\rm H.c.}\right],$$

and the potential to lowest order in the $1/\mu_S$ expansion takes the form

$$V = V_{\rm SB} + V_D + |H|^2\left|\mu + \frac{\omega_1}{\mu_S}H_uH_d\right|^2, \tag{12}$$

where $|H|^2$ was defined after Eq. (9). The D-term potential is as in the MSSM:

$$V_D = \tfrac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2 + |H_u^+| - |H_d^-|)^2 + \tfrac{1}{2}g^2|H_u^+H_d^{0\dagger} + H_d^{-\dagger}H_u^0|^2.$$

We start by considering the minimization of the potential, Eq. (12). Using $SU(2)_L$ transformations, we can take $\langle H_u\rangle = (0, v_u)$, with v_u real, without loss of generality. By redefining the phase of H_d^0 we can then take, as in the previous section, $\mu\mu_S/\omega_1$ real and positive. Note that the phases of b and $\xi\mu^2$ are then physical observables.[7] For simplicity, we will assume in the following analysis that these parameters are real.

We also concentrate in a region of parameter space where no spontaneous CP violation occurs, which can be guaranteed provided either

$$\frac{b}{|\mu|^2} > 0 \quad \text{or} \quad \xi\mu^2 > 0.$$

The first condition ensures that all the solutions to the minimization equations are real, while the second would ensure that any putative complex solution is *not* a minimum of the potential. Although the above are only sufficient conditions to avoid spontaneous CP violation, they will be enough for our purpose. The possibility of spontaneous CP violation in the presence of the higher-dimension operators, although quite interesting, is beyond the scope of this work. Furthermore, we also note that for real solutions to the minimization equations, there are no charge-breaking vacua, provided only that $m_{H_d}^2$ is not too negative. Further details are given in Appendix B.

From here on we restrict ourselves to regions of parameter space where electromagnetism is unbroken and CP is preserved, so that $\langle H_u^0\rangle = v_u$ and $\langle H_d^0\rangle = v_d$ are always real. Notice that, unlike in the MSSM without higher-dimension operators, the sign of $\tan\beta = v_u/v_d$ is physical. However, we still have a remaining $U(1)_Y$ gauge rotation that we use to choose v_d positive, though v_u may be positive or negative. These nontrivial extrema of the potential are described by $v^2 = v_u^2 + v_d^2$ and $-\pi/2 < \beta < \pi/2$, and must satisfy

$$s_{2\beta} = \frac{2b - 4|\mu|^2\rho(\rho s_{2\beta} - 1)}{m_{H_u}^2 + m_{H_d}^2 + 2|\mu|^2(\rho s_{2\beta} - 1)^2 - 2\xi\mu^2\rho}, \tag{13}$$

$$m_Z^2 = \frac{m_{H_u}^2 - m_{H_d}^2}{c_{2\beta}} - [m_{H_u}^2 + m_{H_d}^2 + 2|\mu|^2(\rho s_{2\beta} - 1)^2], \tag{14}$$

with

$$v^2 \equiv \rho\left(\frac{2\mu\mu_S}{\omega_1}\right). \tag{15}$$

Here m_Z^2 should be considered a placeholder for v^2 according to $m_Z^2 = (g^2 + g'^2)v^2/2$. For given ultraviolet parameters ($m_{H_u}^2$, $m_{H_d}^2$, b, μ, μ_s/ω_1, ξ) there may be more than one solution to the above equations where EWSB occurs, in addition to the origin where EWSB does not occur. With our conventions, a valid solution must also have real and positive ρ.

The parameter ρ introduced in Eq. (15) characterizes how close these solutions are to the sEWSB minimum of Sec. II: for vanishing soft parameters, one recovers the SUSY expressions of the previous section, with $\rho \to 1$ and $\tan\beta \to 1$. On the other hand, the MSSM limit corresponds to $\rho \to 0$, or more precisely to the scaling $\rho \to 1/\mu_S$ as $\mu_S \to \infty$ [see Eq. (15)]. This also suggests a definite criterion to distinguish—for finite μ_S—MSSM-like minima from minima that involve the higher-dimension operators in a crucial way. While the VEV in a MSSM-like minimum tends to a constant as μ_S becomes

[6] However, SUSY breaking in the heavy physics sector can be of the same order as in the MSSM Higgs sector. These soft masses, together with the μ-term, are assumed to be parametrically smaller than μ_S, which ensures that the EFT analysis holds.

[7] In the MSSM without higher-dimension operators, it is customary to use the field reparametrization freedom to choose b real and positive. We find it more convenient, when studying the new vacua, to choose $\mu\mu_S/\omega_1$ real and positive.

large, the new vacua are characterized by VEV's that scale like $\sqrt{\mu_S}$ for large μ_S, provided all other microscopic parameters are kept fixed (ρ remains of order one in this limit). This is illustrated in Fig. 2. In other words, the new minima can be described as those that are "brought in from infinity" when the higher-dimension operators are turned on. It is important to notice that, as was argued by an operator analysis in Sec. II A, the EFT gives good control of the physics of such nonstandard vacua provided

$$\frac{v^2}{\mu_S^2} \sim \frac{2\rho}{\omega_1}\frac{\mu}{\mu_S} \ll 1.$$

This approximation becomes even better in the limit described above and leads to the interesting situation in which, although the physics at μ_S is crucial in triggering EWSB, the *details* of that physics actually become unimportant. With a slight abuse of notation we will continue referring to vacua that obey the scaling $v \sim \sqrt{\mu_S}$ in the large μ_S limit as sEWSB vacua, even when SUSY breaking is not negligible. The important property is that they exist only due to the presence of the higher-dimension operators, while being describable within the EFT framework.

B. Higgs spectrum

Besides studying the solutions to Eqs. (13) and (14), which we will do in the next section, it is important to determine their stability properties. Here we work out the Higgs spectrum in any extremum where electromagnetism is unbroken and CP is conserved; the Higgs fields in the unitary gauge are

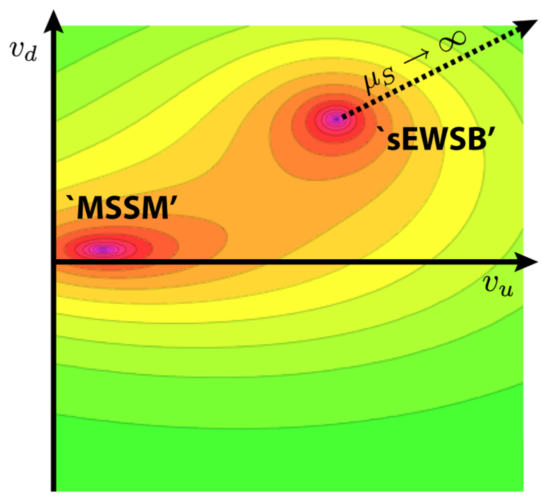

FIG. 2 (color online). An illustration showing the equipotential lines in the $v_u - v_d$ plane for a case with two nontrivial minima. The nature of these minima can be determined by exploring how the physics depends on the UV scale μ_S: the MSSM-like VEV remains near the origin as $\mu_S \to \infty$, while the sEWSB VEV scales like $\sqrt{\mu_S}$ (as indicated by the arrow) for large μ_S. The limit is taken with all other microscopic parameters fixed.

$$H_u = \begin{pmatrix} c_\beta H^+ \\ v s_\beta + \frac{1}{\sqrt{2}}(s_\alpha H^0 + c_\alpha h^0 + i c_\beta A^0) \end{pmatrix},$$
$$H_d = \begin{pmatrix} v c_\beta + \frac{1}{\sqrt{2}}(c_\alpha H^0 - s_\alpha h^0 + i s_\beta A^0) \\ s_\beta H^- \end{pmatrix},$$

where $s_\beta = \sin\beta$, $c_\beta = \cos\beta$, etc. For arbitrary (ρ, β), the charged and CP-odd Higgs masses are then

$$\begin{aligned} m_{A^0}^2 &= \frac{2b}{s_{2\beta}} + \frac{4\rho|\mu|^2}{s_{2\beta}} + 4\rho\xi\mu^2, \\ m_{H^\pm}^2 &= m_W^2 + m_{A^0}^2 - 4\rho^2|\mu|^2 - 2\rho\xi\mu^2, \end{aligned} \tag{16}$$

while the masses for the two CP-even scalars are given by

$$m_{H^0,h^0}^2 = \bar{m}^2 \pm \sqrt{\Delta m^4 + m_{12}^4},$$

where

$$\begin{aligned} \bar{m}^2 &= \frac{1}{2}m_Z^2 + \frac{b}{s_{2\beta}} + \left(\frac{2\rho c_{4\beta}}{s_{2\beta}} + 4\rho^2 s_{2\beta}^2\right)|\mu|^2, \\ \Delta m^2 &= -\frac{1}{2}m_Z^2 s_{2\beta} - b - 2(3\rho - 4\rho^2 s_{2\beta})|\mu|^2 \\ &\quad - 2\rho\xi\mu^2 s_{2\beta}, \\ m_{12}^2 &= -\frac{1}{2}m_Z^2 c_{2\beta} + b\cot_{2\beta} + 2\rho|\mu|^2\cot_{2\beta}. \end{aligned}$$

The mass mixing angle α satisfies

$$\tan_{2\alpha} = -\frac{\Delta m^2}{m_{12}^2}.$$

The angle α is defined to agree with the two-Higgs doublet model conventions for $m_{H^0}^2 > m_{h^0}^2$ of [12,13]. The SUSY limit occurs as $\alpha \to \pi/4$. We note also that the Z-Z-H^0 (Z-Z-h^0) coupling is proportional to $c_{\beta-\alpha}$ ($s_{\beta-\alpha}$), where

$$\begin{aligned} c_{\beta-\alpha}^2 = \frac{1}{2(m_{H^0}^2 - m_{h^0}^2)}&[3(m_{H^0}^2 - m_{A^0}^2) - 2m_Z^2 s_{2\beta}^2 \\ &+ (m_{h^0}^2 - m_Z^2) + 8\rho\mu^2 s_{2\beta} + 4\rho\xi\mu^2(c_{2\beta}^2 + 2)]. \end{aligned}$$

C. Charginos and neutralinos

The chargino and neutralino spectra are also shifted from the SUSY limit due to the presence of SUSY breaking, in some cases (the photino) drastically. The shifts can be traced to multiple sources: the presence of the b-ino and W-ino soft masses (M_1, M_2), and the shift of (ρ, $s_{2\beta}$) away from the SUSY limit because of soft breaking in the Higgs scalar sector (see Sec. III A).

The chargino mass matrix in the sEWSB vacuum is

$$\mathcal{L} \supset (\tilde{W}^+, \tilde{H}_u^+)\begin{pmatrix} M_2 & \sqrt{2}m_W c_\beta \\ \sqrt{2}m_W s_\beta & \mu(1-\rho s_{2\beta}) \end{pmatrix}\begin{pmatrix} \tilde{W}^- \\ \tilde{H}_d^- \end{pmatrix}. \tag{17}$$

In the SUSY limit $(\rho, s_{2\beta}) \to (1, 0)$ and the pure Higgsino entry in the chargino mass matrix vanishes; both charginos become degenerate with the W vector-boson. In the more general case with SUSY breaking turned on, the eigenvalues are

$$m^2_{\chi_1,\chi_2} = \frac{1}{2}M_0^2\left\{1 \pm \sqrt{1 - \frac{4[m_W^2 s_{2\beta} - M_2\mu(1-\rho s_{2\beta})]^2}{M_0^4}}\right\},$$

$$M_0^2 \equiv [M_2^2 + 2m_W^2 + \mu^2(1-\rho s_{2\beta})^2].$$

The neutralino mass matrix in the sEWSB vacuum is

$$\mathcal{L} \supset \frac{1}{2}(\tilde{B}, \tilde{W}^3, \tilde{H}_d^0, \tilde{H}_u^0)\begin{pmatrix} M_1 & & -m_Z s_W c_\beta & m_Z s_W s_\beta \\ & M_2 & m_Z c_W c_\beta & -m_Z c_W s_\beta \\ -m_Z s_W c_\beta & m_Z c_W c_\beta & 2\mu\rho s_\beta^2 & -\mu(1-2\rho s_{2\beta}) \\ m_Z s_W s_\beta & -m_Z c_W s_\beta & -\mu(1-2\rho s_{2\beta}) & 2\mu\rho c_\beta^2 \end{pmatrix}\begin{pmatrix} \tilde{B} \\ \tilde{W}^3 \\ \tilde{H}_d^0 \\ \tilde{H}_u^0 \end{pmatrix},$$

where c_W stands for the weak-mixing angle $\cos\theta_W$. A massless neutralino with exactly the couplings of the photino emerges from the spectrum in the SUSY limit.

D. Vacuum structure

The presence of the higher-dimension operators in Eqs. (1) and (11) leads to a rather rich vacuum structure, even when restricted to the Higgs sector of the theory.

Let us start by recalling the situation in the MSSM without higher-dimension operators. The breaking of the EW symmetry can be simply characterized by the behavior of the potential at the origin. One considers the *signs* of the determinant and trace of the matrix of second derivatives (evaluated at the origin):

$$\begin{aligned} \text{det} &= (m^2_{H_u} + |\mu|^2)(m^2_{H_d} + |\mu|^2) - b^2, \\ \text{trace} &= m^2_{H_u} + m^2_{H_d} + 2|\mu|^2, \end{aligned} \tag{18}$$

so that sign(det, trace) $= (+, +)$ indicates that the origin is a local minimum (the mass matrix squared has two positive eigenvalues), while the other cases indicate that the origin is unstable: $(+, -)$ is a maximum with two negative eigenvalues; $(-, +)$ and $(-, -)$ indicate a saddle point with one negative and one positive eigenvalue. In the MSSM, the fact that all the quartic terms arise from the D-terms, which have a flat direction along $|v_u| = |v_d|$, leads to an additional constraint:

$$m^2_{H_u} + m^2_{H_d} + 2|\mu|^2 - 2|b| > 0, \tag{19}$$

which simply states that the *quadratic* terms should be positive along the flat direction. This requirement eliminates the cases $(-, -)$ and $(+, -)$ above [the trace is automatically positive; hence it is not usually considered]. Using the MSSM minimization conditions [Eqs. (13) and (14) with $\rho = 0$], we can eliminate b in favor of β and $(m^2_{H_u} - m^2_{H_d})$ in favor of m_Z^2, so that

$$\text{trace} = -\frac{1}{2}m_Z^2 - \frac{2\sec^2 2\beta}{m_Z^2}\,\text{det}, \tag{20}$$

which shows that "trace" depends linearly on "det" with a β-dependent slope. In addition, due to Eq. (19), for EWSB only the region sign(det, trace) $= (-, +)$ should be considered. We show this triangular region (light color) in Fig. 3.

When the higher-dimension operators are included, the region of parameter space in the (det, trace) plane that leads to EWSB is considerably enlarged. To illustrate this, we also show in Fig. 3 the region that leads to a nontrivial minimum for fixed $\tan\beta = 1$ [which from Eq. (14) corresponds to $m^2_{H_u} = m^2_{H_d}$]. For simplicity, we took $\mu/\mu_S = 1/10$, $\omega_1 = 2$, $\xi = 0$, and scanned over the other parameters, requiring that $|b|, |m^2_{H_{u,d}}| \lesssim (\mu_S/5)^2$ to make sure that the EFT analysis is reliable throughout. We see that not only are the four quadrants $(+, +)$, $(+, -)$, $(-, +)$, and $(-, -)$ accessible, but also that the stability condition (19) is no longer necessary [9].

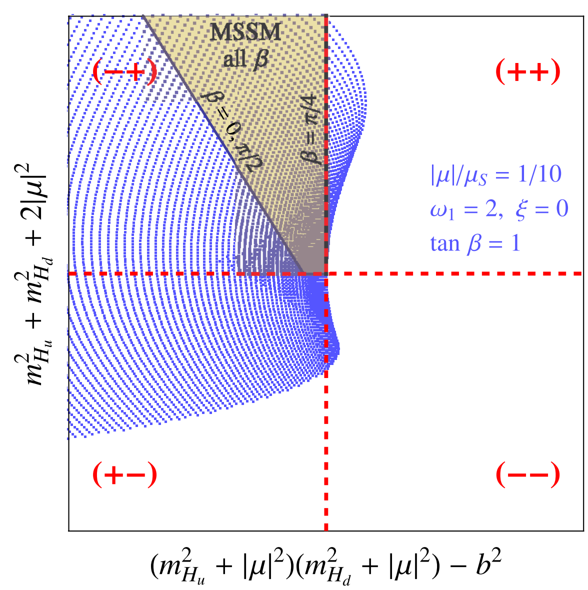

FIG. 3 (color online). Region of parameters in the (det, trace) plane of Eqs. (18), that lead to EWSB. The light-shaded triangular region corresponds to the complete EWSB parameter space in the MSSM (in the absence of higher-dimension operators). The (blue) dots correspond to theories that break the EW symmetry, taking $\omega_1 = 2$, $\xi = 0$, and for fixed $\tan\beta = 1$ $(m^2_{H_u} = m^2_{H_d})$. We scanned over b and $m^2_{H_u}$ with $|b|, |m^2_{H_u}| < (\mu_S/5)^2$. All points have been normalized so that $v = 174$ GeV.

More interestingly, there are regions with multiple physically inequivalent EWSB minima. This should be clear from our discussion of the supersymmetric limit in Sec. II, where we pointed out that two degenerate minima exists (one that breaks the EW symmetry and one that does not). If a small amount of SUSY breaking is turned on, such that the origin is destabilized, the minimum initially at the origin can become nontrivial but remain near the origin, while the originally sEWSB minimum is shifted only slightly. The question then arises as to which of these two is the true global minimum. In the small SUSY-breaking limit, this question is readily answered by working out the shift in the potential energy to leading order in the soft SUSY-breaking terms:

$$V \approx (m_{H_u}^2 + m_{H_d}^2 + 2b)\frac{v^2}{2},$$

where v corresponds to the unperturbed SUSY VEV. For minima near the origin, this result shows that its energy is not shifted at lowest order in SUSY breaking. Furthermore, we learn that the sEWSB minimum with $v \approx (2\mu\mu_S/\omega_1)^{1/2}$ is the global minimum provided $m_{H_u}^2 + m_{H_d}^2 + 2b < 0$, at least when these parameters are small compared to μ.

In the general case, when SUSY breaking is not necessarily small compared to μ (but still assuming it is small compared to μ_S so that the EFT gives a reasonably good description of the physics), we can approach the problem as follows: both Eqs. (13) and (14) are only quadratic in ρ, but fairly complicated in β. We can solve Eq. (13) to characterize all extrema by two branches[8]:

$$\rho_\pm(\beta) = \frac{(1 + \frac{1}{2}\xi s_{2\beta} + s_{2\beta}^2)}{s_{2\beta}(2 + s_{2\beta}^2)}\left[1 \pm \sqrt{1 - \frac{s_{2\beta}(2 + s_{2\beta}^2)}{(1 + \frac{1}{2}\xi s_{2\beta} + s_{2\beta}^2)^2}\left\{s_{2\beta}\left(1 + \frac{m_{H_u}^2 + m_{H_d}^2}{2\mu^2}\right) - \frac{b}{\mu^2}\right\}}\right]. \qquad (21)$$

The sEWSB vacua may be found in either the ρ_+ or the ρ_- branch, while MSSM-type vacua are always in the ρ_- branch and are characterized by $\rho \sim 1/\mu_S$ as $\mu_S \to \infty$.

Just as in the limit of small SUSY-breaking effects, it is possible to find potentials that contain multiple, inequivalent, sEWSB and MSSM-type vacua with potential barriers in between. A complete description of the phase space as a function of input parameters is difficult to obtain, but it is straightforward to find examples of EWSB minima that violate standard MSSM assumptions. For example, the origin can be unstable and outside of the MSSM-required light-triangular region in Fig. 3, but a nontrivial sEWSB vacuum is the stable, global minimum of the theory due to the physics at μ_S. More interestingly, there are potentials with a local MSSM-type minimum that is unstable to decay to a sEWSB global minimum, or vice versa. These structures may have interesting implications for cosmology and the cosmological phase transition to the EWSB vacuum.

IV. SEWSB VACUA: PHENOMENOLOGY

In this section we begin a preliminary analysis of the phenomenology of the sEWSB vacua. As defined in Sec. III A, the sEWSB vacua are distinguished from MSSM-like vacua due to their behavior as $\mu_S \to \infty$, with all other microscopic parameters fixed. The sEWSB vacua exhibit a qualitative difference from MSSM-like vacua in this limit: since the sEWSB vacua depend on the scale μ_S to generate electroweak symmetry breaking, $v^2/\mu\mu_S$ tends toward a constant as $\mu_S \to \infty$, even in the presence of SUSY breaking.

A. Inverted *CP*-even scalars

Collider experiments have put tight constraints on the parameter space of the MSSM. These constraints are mainly due to the LEP-II bound of 114 GeV on the neutral *CP*-even state which has SM-like couplings to massive vector Z bosons. It is much more natural for sEWSB vacua to satisfy the 114 GeV bound on the SM-like Higgs state, since sEWSB vacua naturally have an inverted scalar sector: the heavy *CP*-even state is SM-Higgs-like and is subject to the LEP-II bounds, while the light *CP*-even state is not SM-like, couples more weakly to Z bosons, and is more difficult to observe.

Regions where the light *CP*-even state is not SM-like exist in the MSSM, but are relatively rare and tuned [15]. The inverted hierarchy spectrum is distinct from the usual decoupling limit of the MSSM, where an entire $SU(2)$ doublet of fields (H^+, H^0, A^0) becomes much heavier than the weak-scale, while the lighter *CP*-even state h^0 is increasingly SM-like. In Fig. 4, we qualitatively show in the $m_{A^0} - \tan\beta$ plane the inverted hierarchy region (hatched) where H^0 is more SM-like than h^0 (i.e. $g_{H^0ZZ}^2/g_{h_{SM}ZZ}^2 = c_{\beta-\alpha}^2 > 1/2$). We use a smooth interpolation of LEP-II bounds on the *CP*-even states only [16] to describe regions of parameter space where h^0/H^0 are allowed (blue/yellow regions). We assume that all superpartners are sufficiently heavy that no Higgs decay channels other than the SM ones are open. We take

[8] To simplify this expression, we assume that μ is real, though this is not necessary. The general case is obtained by making $\mu^2 \to |\mu|^2$ and $\xi \to \xi\mu^2/|\mu|^2$.

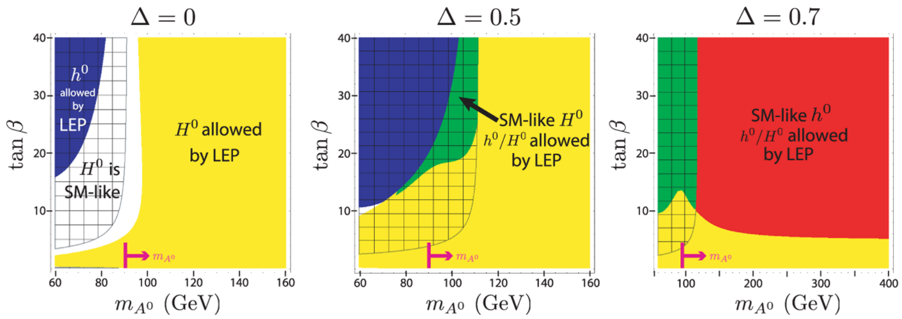

FIG. 4 (color online). Inverted scalar hierarchy region in the MSSM, where the heavier CP-even state H^0 is SM-like (hatched region), together with the LEP-II allowed regions for h^0/H^0 (blue/yellow)—with and without quantum corrections from top-stop loops. There is no viable region with an inverted scalar hierarchy without quantum corrections (leftmost plot). Including a correction of size $\Delta = 0.5$ leads to a viable inverted scalar hierarchy (green region, middle figure). Δ is the size of the quantum correction to the $H_u^0 - H_u^0$ component of the neutral scalar mass matrix, normalized by m_Z^2. Setting $\Delta = 0.7$ (right figure) produces both a viable inverted scalar hierarchy region (green) and a viable standard hierarchy region (red), where h^0 is SM-like. These bounds include quantum corrections only through their effects on the CP-even mixing angle α, and they assume $\mathrm{B}(h^0, H^0 \to b\bar{b}) \sim 0.85$. The purple arrow indicates the approximate LEP bound on m_{A^0} (a possible $\tan\beta$ dependence has not been taken into account).

$\mathrm{B}(h^0, H^0 \to b\bar{b}) \sim 0.85$, which is the tree-level approximation for h^0 and H^0 in the MSSM if the only important decays are to tau and bottom pairs.[9] The LEP bound on m_{A^0} of about 90 GeV [17] is indicated by the purple arrow in the plots. At tree level (leftmost panel in Fig. 4) in the MSSM there is no inverted hierarchy region that is compatible with LEP-II bounds. Crucially in the inverted hierarchy region, H^0 has too large a coupling to Z bosons, while its mass is within 10% of m_Z.

SUSY-breaking effects from top-stop loops create a narrow, viable inverted hierarchy region (green region which is the overlap between blue, yellow, and hatched regions in the middle panel of Fig. 4). We consider only quantum corrections from the stop sector. Inverted hierarchies occur in the MSSM at large $\tan\beta$ whenever $m_{A^0}^2 < m_Z^2(1 + \Delta)$ (where Δ is the size of the quantum correction to the $H_u^0 - H_u^0$ component of the neutral scalar mass matrix, normalized by m_Z^2).[10] As Δ increases, the hatched region of Fig. 4 therefore begins to move to larger m_{A^0}. Meanwhile, $m_{H^0}^2$ grows in the inverted hierarchy region $(\sim m_Z\sqrt{1+\Delta})$ and begins to escape the LEP-II bounds (its Z couplings are relatively unaffected by Δ). The lighter CP-even state is bounded from above by m_{A^0}, and the effect of Δ is to reduce the couplings of h^0 to Z bosons (for fixed m_{A^0} and $\tan\beta$, its mass is unaffected). Therefore, the blue region where h^0 passes LEP constraints also moves to heavier m_{A^0}. This leads to a single region where both experimental constraints overlap with the inverted scalar spectrum (shown in green). Although there is a viable inverted scalar spectrum, $m_{A^0}^2 \sim m_{H_d}^2 - m_{H_u}^2 - m_Z^2$ must be satisfied to a high degree of accuracy in this region [15].

As is well known, if top-stop corrections are sufficiently large, a region where h^0 is SM-like and escapes LEP-II bounds appears. This region is shown in red in the rightmost panel of Fig. 4 for $\Delta = 0.7$. For sufficiently large Δ, this region is much larger than the viable inverted hierarchy region where H^0 is SM-like. It is also possible that explicit CP violation in the third generation squarks leads to a relaxation of the LEP bounds on the MSSM Higgs sector at low and intermediate values of $\tan\beta$ [18].

In sEWSB vacua the scalar Higgs properties can change significantly. When the nonrenormalizable operators of Sec. III are included, the scalar Higgs sector cannot be parametrized by $\tan\beta$ and m_{A^0} alone, even at tree level. As an illustration, we show in Fig. 5 two examples of the $m_{A^0} - \tan\beta$ plane that exhibit the inverted CP-even scalar hierarchy region (hatched), fixing the values of $|\mu|$, the sum $m_{H_u}^2 + m_{H_d}^2$, and the SUSY-breaking parameter ξ [the difference $m_{H_u}^2 - m_{H_d}^2$ is fixed by Eq. (14)].

We see that, unlike in the MSSM, there exists a large, LEP allowed, inverted hierarchy region at low $\tan\beta$. For reference, we also show the regions allowed by the LEP Higgs searches in the CP-even sector, using the same color code as in Fig. 4. We perform a tree-level analysis at leading order in the $1/\mu_S$ expansion, ignoring loop corrections that depend on additional SUSY-breaking parameters (associated with the third generation). All the points we consider are within the domain of validity of the EFT. We do not include in the plots the direct chargino/neutralino exclusion limits, that are expected to impose further

[9]This assumption can hold approximately beyond tree level. For instance, at large $\tan\beta$ these two decay channels are enhanced, and the branching fractions can be close to the values used here even when quantum corrections are included (see, for instance, Ref. [13]). In the low $\tan\beta$ region, decays of H^0 into W pairs can be important, but only when m_{H^0} is above the 114 GeV bound, so that the LEP allowed regions are not expected to change.

[10]For degenerate stops and small stop-mixing, the stop masses must be close to 400 GeV to produce $\Delta \sim 0.5$, or 600 GeV to produce $\Delta \sim 0.7$.

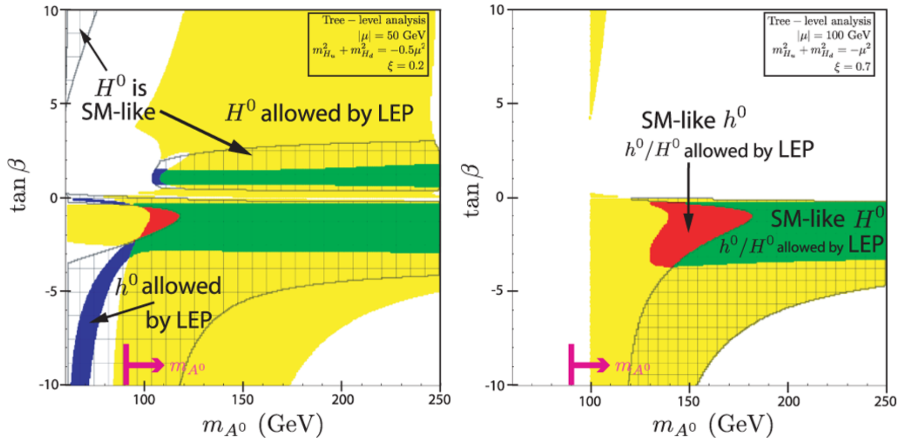

FIG. 5 (color online). Examples illustrating the inverted hierarchy region in the presence of nonrenormalizable operators, as well as the regions allowed by LEP. The color code is the same as in Fig. 4. The leading order tree-level expressions of Sec. III are used, and no loop corrections are included. The charged Higgs direct bounds are satisfied in the LEP allowed regions. The purple arrow indicates the LEP bound on m_{A^0}. Direct limits on the lightest chargino/neutralino are not shown. The two plots correspond to different choices of the parameters of the model other than $\tan\beta$ and m_{A^0}.

constraints (see Sec. IV C); we have checked that they do not change the qualitative picture shown in the plots. These limits depend on the gaugino soft-mass parameters that do not enter in the scalar sector. The neutralinos can be sufficiently heavy for the bounds on the Higgs mass from invisible decays to be satisfied in the regions marked as allowed in the plots. We also assume that the Higgs decays into $b\bar{b}$ are as important as in the MSSM (we only consider SM decays in the Higgs sector). The qualitative lesson is that there are interesting new regions of parameter space that can be consistent with existing limits, even at tree level. Furthermore, this tends to happen for $|\tan\beta| = \mathcal{O}(1)$.

B. sEWSB vacua: The $|\tan\beta| \sim 1$ limit

To better understand the features discussed in the previous subsection, we take a $|\tan\beta| \sim 1$ limit, where the analytic expressions in the scalar sector from Sec. III B are more easily understood. In the formulas of this section we assume, for simplicity, that μ is real.[11] Writing $\tan\beta = \pm 1 + 2\delta\beta$, the extrema conditions of Eqs. (13) and (14) reduce to

$$\rho_\epsilon = \frac{\frac{1}{2}\xi \pm 2}{3}\left\{1 + \epsilon\sqrt{1 - \frac{3}{(\frac{1}{2}\xi \pm 2)^2}\left(1 + \frac{m^2_{H_u} + m^2_{H_d}}{2\mu^2} \mp \frac{b}{\mu^2}\right)}\right\},$$
$$\delta\beta = \pm\frac{m^2_{H_d} - m^2_{H_u}}{2(m^2_Z + m^2_{H_u} + m^2_{H_d} + 2\mu^2(1 \mp \rho_\epsilon)^2)},$$

where the two branches discussed in Sec. III D are labeled by $\epsilon = \pm$.

[11] See footnote 8 if the complex μ expression is needed.

The neutral masses reduce to

$$m^2_{A^0} = 4(\pm 1 + \xi)\rho\mu^2 \pm 2b + \mathcal{O}(\delta\beta^2),$$
$$m^2_{H^0} = \tfrac{1}{2}[m^2_Z + m^2_{A^0} + 8\mu^2\rho(\rho \mp 1 - \xi/2) + |D|],$$
$$m^2_{h^0} = \tfrac{1}{2}[m^2_Z + m^2_{A^0} + 8\mu^2\rho(\rho \mp 1 - \xi/2) - |D|],$$
$$D \equiv m^2_Z + m^2_{A^0} - 8\mu^2\rho(2\rho \mp 1),$$

to $\mathcal{O}(\delta\beta^2)$. The mixing angle that determines whether H^0 ($c^2_{\beta-\alpha} > 1/2$) or h^0 is SM-like ($c^2_{\beta-\alpha} < 1/2$) simplifies considerably:

$$c^2_{\beta-\alpha} = \begin{cases} 0 + \mathcal{O}(\delta\beta^2) & D > 0 \\ 1 + \mathcal{O}(\delta\beta^2) & D < 0. \end{cases} \quad (22)$$

It is easy to understand the result for the mixing angle $c^2_{\beta-\alpha}$ (which is the coefficient of the Z-Z-H^0 coupling) for $\tan\beta \sim 1$ by appealing to the SUSY limit of Sec. II. In the SUSY limit, the CP-even field with mass 2μ is always the SM-like Higgs state. When $D < 0$, it is the heavy H^0 field whose mass reduces to the SUSY-limit value of $2|\mu|$ so $\cos^2_{\beta-\alpha} \to 1$. When $D > 0$, it is the light h^0 field whose mass reduces to $2|\mu|$ so $\cos^2_{\beta-\alpha} \to 0$.

Finally, the charged Higgs mass $m^2_{H^+}$ is always very close to the non-SM-like CP-even Higgs mass,

$$m^2_{H^+} = \begin{cases} m^2_{H^0} + (m^2_W - m^2_Z) + \mathcal{O}(\delta\beta^2) & D > 0 \\ m^2_{h^0} + (m^2_W - m^2_Z) + \mathcal{O}(\delta\beta^2) & D < 0. \end{cases}$$

In the $|\tan\beta| \sim 1$ limit with SUSY breaking included, larger ρ always tends to push $D < 0$ so that H^0 becomes the SM-like Higgs state and we have an inverted hierarchy. Up to corrections of order $\delta\beta^2$, we see that the inverted hierarchy spectra is consistent with LEP bounds with only one condition: that the heavy CP-even state H^0 has $m_{H^0} >$ 114 GeV, and *no* condition on the mass of the non-SM-like

CP-even state h^0. Further, when the inverted hierarchy holds, $m_{H^0}^2 = 4\mu^2\rho(3\rho \mp 2 - \xi/2)$ which may easily be larger than 114 GeV for moderate ρ and μ. Recall from the previous subsection that one of the reasons for the rarity of inverted hierarchies in the MSSM is the difficulty of simultaneously satisfying LEP constraints on both CP-even states.

The definition of sEWSB vacua given in Sec. III A allows us to see that sEWSB vacua typically have larger ρ, and hence inverted spectra. This is clear from the $\epsilon = +$ branch in the expression for ρ, but it is also true in the $\epsilon = -$ branch. Working in the EFT makes this clear: we require that $\mu_S^2 \gg \mu^2$, $m_{H_u}^2$, $m_{H_d}^2$, b for the validity of the EFT. Given these input parameters, the only trustworthy vacua where EWSB occurs satisfy two generic relationships: $v^2 \sim \mu_S\mu$ or $v^2 \sim \mu^2$ (with any other soft mass possibly replacing μ), depending on whether the nonrenormalizable terms proportional to μ_S help stabilize the VEV or not. The former case is exactly an sEWSB vacuum by our criteria in Sec. III A, and will have $\rho \sim v^2/(\mu\mu_S) \sim \mathcal{O}(1)$, w hile the latter is an MSSM-like vacua with $\rho \sim v^2/(\mu\mu_S) \sim (\mu/\mu_S)$.

As a complement to the qualitative picture exhibited in Fig. 5, we give a couple of numerical examples (with $|\tan\beta| \sim 1$) that illustrate the inverted hierarchy spectrum, together with the charged Higgs and chargino/neutralino masses. It should be recalled that these numbers are expected to be accurate to approximately $\mathcal{O}(v^2/\mu_S^2)$. To be conservative, we require that charginos are heavier than the kinematic reach at LEP-II, $m_{\chi^+} > 104$ GeV, and that neutralinos are heavier than half of the Z mass: $m_{\chi^0} > 45$ GeV. Depending on the composition of the charginos and neutralinos in terms of the underlying Higgsino and gaugino states, these bounds may be relaxed [19].

We also require that the charged Higgses have mass greater than the direct LEP-II search bound of 80 GeV [19]. There are more stringent constraints from the Tevatron on charged Higgs masses for low $\tan\beta$ when $m_{H^+} < m_t - m_b$. For $\tan\beta \sim 1$, $m_{H^+} \gtrsim 110$ GeV [19]. These searches ignore the possibility that the charged Higgs can decay to a chargino/neutralino, which may alter the limits. Additionally, there are strong indirect constraints, $m_{H^+} > 295$ GeV from the measured rate of $b \to s\gamma$ [20], although additional next-to-next-to-leading-order corrections appear to weaken this bound [21]. These indirect analyses assume no other sources of new physics beyond the charged Higgs itself. However, given that the chargino tends to be light in this theory and is known to interfere with the charged Higgs contribution to $b \to s\gamma$ [22], and the spectrum of squarks (which may also interfere with the charged Higgs contribution) is undetermined, we restrict ourselves to considering only the direct charged Higgs bound.

The following sample points have inverted scalar hierarchies, a wide range of m_{H^0}, and different Z-Z-H^0 couplings.

Point 1:

μ	ω_1	μ/μ_s	b/μ^2	m_u^2/μ^2	$m_{H_d}^2/u^2$	ξ	M_1/μ	M_2/μ
-60	1	0.1	-2.2	-1.7	-0.6	0.2	1.5	1.7

ρ	$\tan\beta$	m_{h^0}	m_{H^0}	$\frac{g_{H^0ZZ}^2}{g_{h_{\rm SM}ZZ}^2}$	m_{A^0}	m_{H^+}	m_{χ^+}	m_{χ^0}
0.47	-1.3	120	150	0.98	100	120	110	90

This is a spectrum where H^0 is SM-like, but its mass is well above the LEP-II limit and well above the mass of h^0.

Point 2:

μ	ω_1	μ/μ_s	b/μ^2	m_u^2/μ^2	$m_{H_d}^2/\mu^2$	ξ	M_1/μ	M_2/μ
-150	2	0.14	-1.1	-0.99	-0.51	0.2	0.36	0.57

ρ	$\tan\beta$	m_{h^0}	m_{H^0}	$\frac{g_{H^0ZZ}^2}{g_{h_{\rm SM}ZZ}^2}$	m_{A^0}	m_{H^+}	m_{χ^+}	m_{χ^0}
0.20	-1.3	190	210	0.77	185	190	105	60

Point 2 is similar to point 1, but all the scalar masses (including m_{H^+}) are closer to 200 GeV. H^0 is not entirely SM-like.

Point 3:

μ	ω_1	μ/μ_s	b/μ^2	m_u^2/μ^2	$m_{H_d}^2/\mu^2$	ξ	M_1/μ	M_2/μ
-70	3.5	0.19	1.95	-0.45	-0.47	0.7	-1.0	0.86

ρ	$\tan\beta$	m_{h^0}	m_{H^0}	$\frac{g_{H^0ZZ}^2}{g_{h_{\rm SM}ZZ}^2}$	m_{A^0}	m_{H^+}	m_{χ^+}	m_{χ^0}
1.8	0.99	100	350	1	300	90	100	48

Point 3 has a very heavy spectrum, due to the large value of ω_1, and—unlike points 1 and 2—it has $\tan\beta > 0$. Note also that m_{h^0} and m_{H^+} are nearly degenerate, and very split from m_{H^0} and m_{A^0}. Corrections to these effective field theory approximations due to unknown physics with $\mathcal{O}(1)$ couplings are expected to be $\sim$20% for point 3, but less than $\sim$10% for points 1 and 2.

Values of $\tan\beta$ near 1 are not usually considered in the MSSM, due to the LEP constraints on the *CP*-even Higgs states. We see here that this region is expected to be viable in a large class of supersymmetric extensions. For $|\tan\beta| \sim 1$ the top Yukawa coupling is $y_t \sim 1/\sin\beta \sim \sqrt{2}$, a sizable enhancement compared to either the SM or the cases normally considered in the MSSM. Since the couplings of the *CP*-even Higgses to top pairs are $g_{ht\bar{t}}/g^{\rm SM}_{ht\bar{t}} \approx \cos\alpha/\sin\beta$ and $g_{Ht\bar{t}}/g^{\rm SM}_{Ht\bar{t}} \approx \sin\alpha/\sin\beta$ (assuming quantum corrections are not particularly large), it is possible that the gluon-fusion Higgs production cross section is enhanced compared to the SM.[12] Also, since a heavy SM-like *CP*-even scalar H^0 can have a sizable branching fraction into W's when its mass is around the WW threshold, the Tevatron may be starting to probe the present scenario [23].

C. Chargino NLSP

In phenomenologically viable sEWSB vacua, it is important that the lightest neutralino and lightest chargino have masses that are significantly different from the SUSY limit. In the SUSY limit, the lightest neutralino is the photino, which is massless, and the lightest chargino is degenerate with the W boson. Adding the soft mass M_1 raises the photino mass without much difficulty. In the SUSY limit, the charged Higgsinos have no mass term, as can be seen from the explicit expression for the chargino mass matrix in Eq. (17). Large $\mu(1-\rho s_{2\beta})$ will help lift the lightest chargino above the LEP-II bound. This tends to favor regions with negative $s_{2\beta} < 0$, and/or $\rho \neq 1$.

It may be the case that the effects of SUSY breaking lift the lightest neutralino above the lightest chargino. In a scenario with a low scale of SUSY breaking, when the gravitino is the LSP, a chargino NLSP may lead to a charged track that eventually decays into an on-shell W boson and missing energy [3]. In the example below, the chargino-neutralino mass difference is only on the order of 5–10 GeV which is approximately the size of additional μ/μ_S contributions from higher-order operators in the $1/\mu_S$ expansion that we have not considered. The precise size of these corrections can only be determined in a given UV completion.

NLSP chargino:

μ	ω_1	μ/μ_s	b/μ^2	m_u^2/μ^2	$m_{H_d}^2/\mu^2$	ξ	M_1/μ	M_2/μ
−70	1	0.11	−1.6	−1.7	0.22	0.2	1.5	1.7
ρ	$\tan\beta$	m_{h^0}	m_{H^0}	$\frac{g^2_{H^0ZZ}}{g^2_{h_{\rm SM}ZZ}}$	m_{A^0}	m_{H^+}	m_{χ^+}	m_{χ^0}
0.34	−1.8	120	140	0.82	110	125	100	110

V. ULTRAVIOLET SCENARIOS

So far we have restricted ourselves to an analysis of the low-energy physics from an EFT point of view. This has the advantage of making more transparent (and also easier to analyze) the effects of the heavy physics on the low-energy degrees of freedom (here the MSSM field content) and has allowed us to focus on the sEWSB vacua.

It is nevertheless worth pointing out that the tower of operators involving only the MSSM Higgs superfields that we have considered [see e.g. Eq. (1)] already arises in one of the simplest extensions of the MSSM: the addition of a SM singlet. To be more precise, consider the renormalizable superpotential

$$W = \mu H_u H_d + \lambda S H_u H_d + \frac{1}{2}\mu_S S^2 + \frac{\kappa}{3} S^3.$$

If the singlet mass μ_S is sufficiently large, we can integrate out S using its supersymmetric equation of motion (we could keep the SUSY covariant derivative terms)

$$S = -\frac{1}{\mu_S}[\lambda H_u H_d + \kappa S^2]. \tag{23}$$

Replacing back in the superpotential and using the above equation of motion iteratively, one gets the effective superpotential

$$W_{\rm eff} = \mu H_u H_d - \frac{\lambda^2}{2\mu_S}(H_u H_d)^2 - \frac{\lambda^3\kappa}{3\mu_S^3}(H_u H_d)^3 + \cdots.$$

The full tower of higher-dimension operators is generated with, in the notation of Eq. (1), $\omega_1 = -\lambda^2$, $\omega_2 = -\lambda^3\kappa$, etc. Note also that for $\kappa = 0$ only the lowest dimension operator, with coefficient ω_1, is generated.

[12] Such a large value of the top Yukawa coupling can lead to the loss of perturbativity at high energies. However, this would happen above the new physics threshold at μ_S, and it is a UV-dependent issue that we do not address here (see further comments in Sec. V).

Similarly, replacing Eq. (23) in the minimal kinetic term for the singlet $S^\dagger S$, one generates the operator in Eq. (4) proportional to c_1, with $c_1 = |\lambda|^2$, as well as other higher-dimension operators whose coefficients are proportional to κ.

The soft SUSY-breaking operator considered in the EFT of the previous sections can be generated from the following terms in the superpotential:

$$W \supset -\alpha_1 X S H_u H_d - \tfrac{1}{2}\alpha_2 \mu_S X S^2,$$

where α_1 and α_2 are dimensionless coefficients, and X is a spurion that parametrizes SUSY breaking in the singlet sector. If these SUSY-breaking effects are sufficiently small so that the threshold at μ_S is approximately supersymmetric, we can simply use Eq. (23) to obtain the operator of Eq. (11), with the identification $\tilde{X} = \lambda(2\alpha_1 - \alpha_2\lambda)X$.

As illustrated in the sample points discussed in Sec. IV B, we envision a case where $\omega_1 \sim 1 -$ few. This is a result of the fact that the weak scale in the sEWSB vacua arises as the geometric mean between μ and μ_S, that for phenomenological reasons μ cannot be too small, and from the requirement that the EFT description be valid [see Eqs. (2) and (3)]. In the singlet UV completion discussed in this section, we see that $\omega_1 \sim 1 -$ few corresponds to $\lambda \sim$ 1–2. Thus, the fact that the lightest Higgs scalar is heavier than in the MSSM can be understood as arising from a moderately large coupling. In addition, the interesting new phenomenologically viable regions, with $\tan\beta \sim 1$, also have a top Yukawa coupling y_t slightly larger than 1. For $\lambda = y_t = \sqrt{2}$ and $\kappa = 0$, the renormalization group equations for the singlet theory above the scale μ_S lead to a Landau pole around 100 TeV. The presence of such a Landau pole (as well as the issue of gauge coupling unification) is a UV-dependent question. Note, however, that we are not required to assume strong coupling at the scale μ_S.

Finally, we emphasize here that the EFT approach allows one to consider more general scenarios than the addition of one singlet, even if at the lowest order the singlet theory already induces all operators considered in the detailed analysis of Secs. III and IV. The point is that the next-to-leading order corrections can be different in other UV completions that also generate the same lowest order operators. In general, the coefficients of operators of higher dimension need not obey the correlations that follow from the identification between the EFT and singlet theory coefficients discussed above.

VI. CONCLUSIONS

Supersymmetric electroweak symmetry breaking divorces LEP-II constraints from the spectrum of CP-even masses in the most direct route: the SM-like Higgs mass is not related to weak SM gauge couplings.

We showed explicitly that sEWSB happens in the most general effective theory describing the MSSM Higgs degrees of freedom. We argued that the sEWSB vacua can be consistently defined and captured within the EFT, even in the presence of soft terms that perturb the SUSY limit. In particular, we showed that although higher-dimension operators play a key role in the appearance of the sEWSB vacua, the physics is under perturbative control and can be studied without the specification of a UV completion. This EFT captures any UV theory that has the following properties: (i) a nearly supersymmetric threshold just above the weak scale, (ii) physics beyond the MSSM that couples to the MSSM Higgs superfields, and (iii) the MSSM low-energy field content. The vacuum structure of the theory is quite rich and may have interesting cosmological consequences.

The EFT approach we use greatly simplifies the analysis of sEWSB phenomenology. We derived expressions for the low-energy spectrum that generalize those of the MSSM with only renormalizable operators. The sEWSB vacua naturally have an inverted scalar spectrum which is more easily compatible with the LEP-II experimental constraints: it is the heavier CP-even Higgs state that is SM-like, not the lighter. We also find that typically $\tan\beta \sim \mathcal{O}(1)$ in the sEWSB vacua. In the fermion sector, charginos may be lighter than neutralinos, leading to NLSP chargino scenarios. Further phenomenological studies are needed to understand the full range of collider signatures.

The most important open question deals with the coincidence of scales in the theory. Although the three important scales of the theory, μ_S, μ, m_S, are separately technically natural, the clustering of these scales suggests a common origin. Only in the context of an ultraviolet theory can one address whether there is a reason for μ_S to be slightly above both the μ and soft-supersymmetry-breaking scales.

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APPENDIX A: EXACT SCALAR POTENTIAL FOR AN ARBITRARY KÄHLER METRIC

In this appendix, we consider the most general Kähler potential, without SUSY covariant derivatives, in a theory with two $SU(2)_L$ doublets, H_u and H_d, with $U(1)_Y$ charges $-1/2$ and $+1/2$, respectively. This must be a real function of the $SU(2)_L \times U(1)_Y$ invariants $H_u^\dagger e^V H_u$, $H_d^\dagger e^V H_d$, $H_u \epsilon H_d$, and $H_u^\dagger \epsilon H_d^\dagger$, where the e^V factors ensure gauge invariance, and ϵ is the $SU(2)_L$ antisymmetric two-index tensor, which we restore explicitly in this appendix. Notice that we employ a matrix notation and always write $H_u \epsilon H_d$ and $H_u^\dagger \epsilon H_d^\dagger$ with H_u and $H_u^\dagger$ to the left. This makes it easier to keep track of signs associated with these $SU(2)_L$ contractions. In an expansion in a tower of operators suppressed by a large scale μ_S, the Kähler potential takes the form given in Eqs. (4)–(6). The nonminimal character of the Kähler potential has to be taken into account when deriving the scalar potential, which in the supersymmetric limit takes the form $V = V_D + V_F$, where the first term arises from integrating out the D-terms, while the second arises from the F-terms. Assuming that the gauge sector is described by the minimal SUSY kinetic terms, $\int d^2\theta W^\alpha W_\alpha + \text{H.c.}$, the D-term potential takes the form

$$V_D = \tfrac{1}{2} D_2^a D_2^a + \tfrac{1}{2} D_1^2, \tag{A1}$$

where

$$\begin{aligned} D_2^a &= \left(\frac{\partial K}{\partial (H_u^\dagger e^V H_u)} H_u^\dagger \tau^a H_u + \frac{\partial K}{\partial (H_d^\dagger e^V H_d)} H_d^\dagger \tau^a H_d \right)_{V=0} D_1 \\ &= \frac{1}{2} \left(\frac{\partial K}{\partial (H_u^\dagger e^V H_u)} H_u^\dagger H_u - \frac{\partial K}{\partial (H_d^\dagger e^V H_d)} H_d^\dagger H_d \right) \Bigg|_{V=0} \end{aligned} \tag{A2}$$

with τ^a the $SU(2)_L$ generators, and

$$\begin{aligned} \frac{\partial K}{\partial (H_u^\dagger e^V H_u)} \Bigg|_{V=0} &= 1 + \frac{a_1^u}{\mu_S^2} H_u^\dagger H_u + \frac{a_1^{ud}}{\mu_S^2} H_d^\dagger H_d + \left(\frac{b_1^u}{\mu_S^2} H_u \epsilon H_d + \text{H.c.} \right) + \cdots, \\ \frac{\partial K}{\partial (H_d^\dagger e^V H_d)} \Bigg|_{V=0} &= 1 + \frac{a_1^d}{\mu_S^2} H_d^\dagger H_d + \frac{a_1^{ud}}{\mu_S^2} H_u^\dagger H_u + \left(\frac{b_1^d}{\mu_S^2} H_u \epsilon H_d + \text{H.c.} \right) + \cdots. \end{aligned} \tag{A3}$$

In order to derive V_F we need to invert the Kähler metric, whose components take the form

$$\begin{aligned} g_{H_u^\dagger}{}^{H_u} &\equiv \partial_{H_u^\dagger} \partial_{H_u} K = A_0 + A_1 H_u H_u^\dagger + A_2 H_u (\epsilon H_d) + A_3 (\epsilon H_d^\dagger)(\epsilon H_d) + A_4 (\epsilon H_d^\dagger) H_u^\dagger, \\ g_{H_u^\dagger}{}^{H_d} &\equiv \partial_{H_u^\dagger} \partial_{H_d} K = B_1 H_u (H_u \epsilon) + B_2 H_u H_d^\dagger + B_3 (\epsilon H_d^\dagger) H_d^\dagger + B_4 (\epsilon H_d^\dagger)(H_u \epsilon), \\ g_{H_d^\dagger}{}^{H_u} &\equiv \partial_{H_d^\dagger} \partial_{H_u} K = C_1 H_d (\epsilon H_d) + C_2 H_d H_u^\dagger + C_3 (H_u^\dagger \epsilon) H_u^\dagger + C_4 (H_u^\dagger \epsilon)(\epsilon H_d), \\ g_{H_d^\dagger}{}^{H_d} &\equiv \partial_{H_d^\dagger} \partial_{H_d} K = D_0 + D_1 H_d H_d^\dagger + D_2 H_d (H_u \epsilon) + D_3 (H_u^\dagger \epsilon)(H_u \epsilon) + D_4 (H_u^\dagger \epsilon) H_d^\dagger, \end{aligned} \tag{A4}$$

where the coefficients A_i, B_i, C_i, and D_i are, in general, field-dependent gauge invariant functions. Notice also that the Hermiticity of the Kähler metric implies that $A_4 = A_2^*$, $D_4 = D_2^*$, $C_1 = B_3^*$, $C_2 = B_2^*$, $C_3 = B_1^*$, and $C_4 = B_4^*$, while A_0, A_1, A_3, D_0, D_1, and D_3 are real. In the above, we use a dyad notation such that, for example, $H_u(\epsilon H_d)$ is a 2×2 matrix with components $M^\alpha{}_\beta = H_u^\alpha (\epsilon H_d)_\beta = H_u^\alpha \epsilon_{\beta\gamma} H_d^\gamma$, where α, β, γ are $SU(2)_L$ indices. The inverse metric $\tilde{g}$ can be similarly expanded in terms of gauge covariant quantities as

$$\begin{aligned} \tilde{g}_{H_u^\dagger}{}^{H_u} &= \tilde{A}_0 + \tilde{A}_1 H_u H_u^\dagger + \tilde{A}_2 H_u (\epsilon H_d) + \tilde{A}_3 (\epsilon H_d^\dagger)(\epsilon H_d) + \tilde{A}_4 (\epsilon H_d^\dagger) H_u^\dagger, \\ \tilde{g}_{H_u^\dagger}{}^{H_d} &= \tilde{B}_1 H_u (H_u \epsilon) + \tilde{B}_2 H_u H_d^\dagger + \tilde{B}_3 (\epsilon H_d^\dagger) H_d^\dagger + \tilde{B}_4 (\epsilon H_d^\dagger)(H_u \epsilon), \\ \tilde{g}_{H_d^\dagger}{}^{H_u} &= \tilde{C}_1 H_d (\epsilon H_d) + \tilde{C}_2 H_d H_u^\dagger + \tilde{C}_3 (H_u^\dagger \epsilon) H_u^\dagger + \tilde{C}_4 (H_u^\dagger \epsilon)(\epsilon H_d), \\ \tilde{g}_{H_d^\dagger}{}^{H_d} &= \tilde{D}_0 + \tilde{D}_1 H_d H_d^\dagger + \tilde{D}_2 H_d (H_u \epsilon) + \tilde{D}_3 (H_u^\dagger \epsilon)(H_u \epsilon) + \tilde{D}_4 (H_u^\dagger \epsilon) H_d^\dagger. \end{aligned} \tag{A5}$$

The coefficients $\tilde{A}_i$, $\tilde{B}_i$, $\tilde{C}_i$, and $\tilde{D}_i$ are found in a straightforward computation from

$$\sum_{j=u,d} \tilde{g}_{H_i^\dagger}{}^{H_j} g_{H_j^\dagger}{}^{H_k} = \delta_{ik}. \tag{A6}$$

The terms proportional to the identity give $\tilde{A}_0 = 1/A_0$ and $\tilde{D}_0 = 1/D_0$. Further requiring that the coefficients of the nontrivial $SU(2)_L$ invariants vanish, and using $(H_u^\dagger \epsilon)^\alpha \times (H_u \epsilon)_\alpha = H_u^\dagger H_u$ and $(\epsilon H_d^\dagger)^\alpha (\epsilon H_d)_\alpha = H_d^\dagger H_d$, gives four groups of four equations each that can be solved for $(\tilde{A}_1, \tilde{A}_2, \tilde{B}_1, \tilde{B}_2)$, $(\tilde{A}_3, \tilde{A}_4, \tilde{B}_3, \tilde{B}_4)$, $(\tilde{C}_1, \tilde{C}_2, \tilde{D}_1, \tilde{D}_2)$, and $(\tilde{C}_3, \tilde{C}_4, \tilde{D}_3, \tilde{D}_4)$.

We record the solution when only the operators explicitly shown in Eqs. (4)–(6) are included, assuming that all their coefficients are real, and specializing, for simplicity, to the case where $a_1^u = a_1^d = a_1^{ud} \equiv a_1$ and $b_1^u = b_1^d \equiv b_1$:

$$\tilde{A}_1 = -\frac{1}{D}\left[\frac{a_1}{\mu_S^2} + \frac{a_1 c_1 - b_1^2}{\mu_S^4 A_0}|H|^2\right], \quad \tilde{A}_3 = -\frac{1}{D}\left[\frac{c_1}{\mu_S^2} + \frac{a_1 c_1 - b_1^2}{\mu_S^4 A_0}|H|^2\right], \quad \tilde{A}_4 = -\frac{1}{D}\left[\frac{b_1}{\mu_S^2} - 2\frac{a_1 c_1 - b_1^2}{\mu_S^4 A_0}H_u\epsilon H_d\right],$$
$$\tilde{D}_0 = \tilde{A}_0 = 1/A_0, \quad \tilde{B}_2 = \tilde{C}_2 = \tilde{D}_1 = \tilde{A}_1, \quad \tilde{B}_4 = \tilde{C}_4 = \tilde{D}_3 = \tilde{A}_3, \quad \tilde{A}_2^* = \tilde{B}_1^* = \tilde{B}_3 = \tilde{C}_1^* = \tilde{C}_3 = \tilde{D}_2^* = \tilde{D}_4 = \tilde{A}_4, \tag{A7}$$

where

$$D = 3A_0^2 - A_0\left[2 + \frac{a_1 - c_1}{\mu_S^2}|H|^2\right] + \frac{a_1 c_1 - b_1^2}{\mu_S^4}(|H|^4 - 4|H_u\epsilon H_d|^2) = 1 + \frac{3a_1 + c_1}{\mu_S^2}|H|^2 + 4\frac{b_1}{\mu_S^2}(H_u\epsilon H_d + H_u^\dagger\epsilon H_d^\dagger) + \mathcal{O}(H^4/\mu_S^4), \tag{A8}$$

$$A_0 = 1 + \frac{a_1}{\mu_S^2}|H|^2 + \frac{b_1}{\mu_S^2}(H_u\epsilon H_d + H_u^\dagger\epsilon H_d^\dagger), \tag{A9}$$

and we used the shorthand notation $|H|^2 = H_u^\dagger H_u + H_d^\dagger H_d$. The F-term potential can then be derived from the superpotential, W, and the inverse metric, Eqs. (A5) and (A6), according to

$$V_F = \sum_{i,j=u,d} \frac{\partial W}{\partial H_i} \tilde{g}_{H_i^\dagger}{}^{H_j} \frac{\partial W^\dagger}{\partial H_j^\dagger}. \tag{A10}$$

In general, the fields in the above potential are not canonically normalized as a result of the nonminimal Kähler terms, and this should be taken into account when reading off physical properties such as the spectrum. However, the minima of the potential are not affected by this.

In the same spirit as above, the superpotential can be expanded as a power series in the holomorphic gauge invariant $H_u\epsilon H_d$ as

$$W = \mu H_u\epsilon H_d + \sum_{n=1}^{\infty}\frac{1}{n+1}\frac{\omega_n}{\mu_S^{2n-1}}(H_u\epsilon H_d)^{n+1}, \tag{A11}$$

which leads to the F-term potential (still with noncanonically normalized kinetic terms)

$$V_F = Z(H_u, H_d)\left|\mu + \sum_{n=1}^{\infty}\frac{\omega_n}{\mu_S^{2n-1}}(H_u\epsilon H_d)^n\right|^2, \tag{A12}$$

where $Z(H_u, H_d)$ is the real function,

$$Z(H_u, H_d) = |H_d|^2\{\tilde{A}_0 + \tilde{A}_3|H_d|^2 + [(\tilde{A}_2 + \tilde{C}_1)H_u\epsilon H_d + \text{H.c.}]\} + |H_u|^2\{\tilde{D}_0 + \tilde{D}_3|H_u|^2 + [(\tilde{B}_1 + \tilde{D}_2)H_u\epsilon H_d + \text{H.c.}]\} + 2(\text{Re}\tilde{B}_4)|H_u|^2|H_d|^2 + [\tilde{A}_1 + \tilde{D}_1 + 2\,\text{Re}\tilde{B}_2]|H_u\epsilon H_d|^2, \tag{A13}$$

and we used the relations among the $\tilde{A}_i$, $\tilde{B}_i$, $\tilde{C}_i$, $\tilde{D}_i$ that follow from the Hermiticity of the inverse metric $\tilde{g}$ [see comment after Eq. (A4)]. In the special case considered in Eq. (A7), we have

$$Z(H_u, H_d) = \frac{1}{D}\left\{|H|^2\left[1 + \frac{b_1}{\mu_S^2}(H_u\epsilon H_d + H_u^\dagger\epsilon H_d^\dagger)\right] + 2\frac{a_1}{\mu_S^2}(|H|^4 - 2|H_u\epsilon H_d|^2)\right\}, \tag{A14}$$

where D is given in Eq. (A8). Setting $a_1 = b_1 = 0$ leads to Eq. (9) in the main text.

It is also straightforward to include SUSY-breaking effects that can be parametrized by a spurion chiral superfield $X = \theta^2 F_X$. The contributions to the scalar potential can be written in terms of the inverse Kähler metric derived above. Consider a Kähler potential of the form

$$K(H_i, H_j^\dagger) + X^\dagger K_1(H_i, H_j^\dagger) + XK_1^\dagger(H_i, H_j^\dagger) + X^\dagger X K_2(H_i, H_j^\dagger), \tag{A15}$$

where K, K_1, and K_2 are arbitrary functions (except K and K_2 are real). By using the F-term equations of motion, one easily finds an F-term potential

$$V_F = (\partial_{H_i}W)\tilde{g}_{H_i^\dagger}{}^{H_j}(\partial_{H_j^\dagger}W^\dagger) + [F_X(\partial_{H_i}W)\tilde{g}_{H_i^\dagger}{}^{H_j}(\partial_{H_j^\dagger}K_1^\dagger) + \text{H.c.}] + F_X^\dagger F_X(\partial_{H_i}K_1)\tilde{g}_{H_i^\dagger}{}^{H_j}(\partial_{H_j^\dagger}K_1^\dagger)$$

that generalizes Eq. (A10) [sums over i, $j = u$, d are implicit]. The inverse metric $\tilde{g}$ is given in Eq. (A5). The contribution to the potential from the last term in Eq. (A15) is simply $F_X^\dagger F_X K_2(H_i, H_j^\dagger)$ with the fields H_i interpreted as the scalar components. There are no new contributions to the D-term potential.

APPENDIX B: *CP* VIOLATION AND CHARGE-BREAKING MINIMA

Consider the potential of Eq. (12) and look for minima of the form $\langle H_u\rangle = (0, v_u)$, $\langle H_d\rangle = (v_{\text{CB}}, v_d e^{i\delta})$, where v_u, v_d, and v_{CB} are real. We can choose this form for $\langle H_u\rangle$ by performing an appropriate $SU(2)_L$ rotation. It is also clear from the form of the potential that, having set $H_u^+ = 0$, it depends only on $|H_d^-| \equiv v_{\text{CB}}$. Furthermore, as discussed in the main text, we can assume that $\mu\mu_S/\omega_1$ is real and positive, while the phases of b and $\xi\mu^2$ are physically observable. However, we will assume, for simplicity, that these two phases vanish and establish simple conditions such that spontaneous *CP* violation or charge-breaking minima do not occur.

The δ-dependent part of the potential takes the form

$$V \supset x\cos\delta + y\cos^2\delta, \tag{B1}$$

with

$$x = -v^2 s_{2\beta}\left[b + 2\rho|\mu|^2\frac{v^2 + v_{\rm CB}^2}{v^2}\right], \quad y = -v^2 s_{2\beta}^2 \rho\xi\mu^2, \tag{B2}$$

where $\rho > 0$ was defined in Eq. (15). Hence, the derivative with respect to δ vanishes either for $\sin\delta = 0$, or when

$$\cos\delta = -\frac{x}{2y} = -\frac{|\mu|^2}{\xi\mu^2 s_{2\beta}}\left[\frac{v^2 + v_{\rm CB}^2}{v^2} + \frac{1}{2\rho}\frac{b}{|\mu|^2}\right]. \tag{B3}$$

Since $|\cos\delta| \leq 1$, this solution is not always physical. In particular, it does not exist provided $b/|\mu|^2 \geq 0$ and $\xi \lesssim \mathcal{O}(1)$ [for $\omega_1 \sim \mathcal{O}(1)$, we are already assuming this latter condition to ensure that the heavy physics corresponds to an approximately supersymmetric threshold]. On the other hand, the solution may be allowed if there is some degree of cancellation between the two terms in the parentheses. In this case, one should still check whether the extremum corresponds to a minimum of the potential or not. In particular, the second derivative with respect to δ, evaluated on Eq. (B3), is

$$\frac{\partial^2 V}{\partial\delta^2} = 2y[1 - \cos^2\delta], \tag{B4}$$

which has the sign of y, and hence the sign of $-\xi\mu^2$. Therefore, if $\xi\mu^2 > 0$ this solution cannot be a minimum, and the minima must be described by real VEV's. We always assume one of these two simple, sufficient conditions, $b/|\mu|^2 \geq 0$ or $\xi\mu^2 > 0$, in the main text.

With these conditions for real VEV's, we can address the issue of dangerous charge-breaking minima, i.e. solutions with $v_{\rm CB} \neq 0$. Setting $\delta = 0$, and considering $\partial V/\partial v_{\rm CB} = 0$, one can see that any solution with $v_{\rm CB} \neq 0$ must satisfy

$$v_{\rm CB}^2 = -\frac{1}{(g^2 + g'^2)}\{4m_{H_d}^2 + v^2(g^2 + g'^2 c_{2\beta}) + 4|\mu|^2(\rho s_{2\beta} - 1)^2\}. \tag{B5}$$

Except for $m_{H_d}^2$, all the terms in the braces are explicitly positive (recall $g' < g$). Since $v_{\rm CB}^2$ must be positive, $m_{H_d}^2 \geq 0$ (or not too negative) is a sufficient condition to ensure that charge-breaking extrema do not exist. However, we note that even if (B5) is positive, one must check that it is compatible with the remaining extremization conditions, that any such solution is indeed a minimum, and whether it is a global as opposed to a local minimum.

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