Asymmetries involving dihadron fragmentation functions: From DIS to e^+e^- annihilation

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Using a model calculation of dihadron fragmentation functions, we fit the spin asymmetry recently extracted by HERMES for the semi-inclusive pion pair production in deep-inelastic scattering on a transversely polarized proton target. By evolving the obtained dihadron fragmentation functions, we make predictions for the correlation of the angular distributions of two pion pairs produced in electron-positron annihilations at BELLE kinematics. Our study shows that the combination of two-hadron inclusive deep-inelastic scattering and electron-positron annihilation measurements can provide a valid alternative to Collins effect for the extraction of the quark transversity distribution in the nucleon.

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I. INTRODUCTION

Dihadron fragmentation functions (DiFF) describe the hadronization of a quark in two hadrons plus other unobserved fragments. In their simplest form, they represent the probability that at some hard scale a parton hadronizes in two hadrons with fractional energies z_1 and z_2. They were introduced for the first time when studying the $e^+e^- \to h_1h_2X$ process in the context of jet calculus [1]. They are in fact necessary to guarantee the factorization of all collinear singularities for such a process at next-to-leading order in the strong coupling constant [2].

In experiments, often not only the fractional energies of the two hadrons are measured, but also their invariant mass M_h (see, e.g., Refs. [3–5]). Hence, it is useful to introduce extended DiFF (in analogy with extended fracture functions [6]), which are explicitly depending on M_h. Their definition and properties have been analyzed up to subleading twist [7,8]. Their evolution equations are known [9] and presently solved in the leading logarithm approximation (LL), and there are valid arguments to assume that they can be factorized and are universal, similarly to what happens for extended fracture functions [6]. In fact, (extended) DiFF can appear also in two-particle semi-inclusive deep-inelastic scattering (SIDIS) and in hadron-hadron collisions.

DiFF can be used as analyzers of the polarization state of the fragmenting parton [10–12]. Because of this, they have been proposed as tools to investigate the spin structure of the nucleon, in particular, to measure the transversity distribution h_1^q of a parton q in the nucleon N (see Ref. [13] for a review). The h_1^q, together with the momentum f_1^q and helicity g_1^q distributions, fully characterizes the (leading-order) momentum/spin status of q inside N, if quark transverse momentum is integrated over. Transversity is a chiral-odd function and needs to appear in a cross section accompanied by another chiral-odd function. The simplest example is the fully transversely polarized Drell-Yan process, where h_1^q appears multiplied by its antiquark partner $\bar{h}_1^q$ [14]. Although this process is theoretically very clean, it appears to be experimentally very challenging [15], at least at present facilities (the same finding is confirmed for proton-proton collisions leading to prompt photon [16] and semi-inclusive pion production [17]).

An alternative approach, so far the only fruitful one, is to turn to SIDIS and measure the correlation between the transverse polarization of the target and the transverse momentum of the final hadron, which involves a convolution of h_1^q with the chiral-odd Collins fragmentation function $H_1^{\perp q}$ [18]. The resulting asymmetry has already been measured at HERMES [19,20], and at COMPASS [21–23]. The knowledge of the Collins function is required to extract the transversity distribution. This can be obtained through the measurement of azimuthal asymmetries in $e^+e^- \to \pi^+\pi^-X$ with almost back-to-back pions [24]. The BELLE collaboration at KEK has measured this asymmetry [25], making the first-ever extraction of h_1^q possible from the global analysis of SIDIS and e^+e^- data [26].

At present, large uncertainties still affect this analysis and the resulting parametrization of h_1^q. The most crucial issue is the treatment of evolution effects, since the BELLE and the HERMES/COMPASS measurements happened at two very different scales: $Q^2 \sim 100$ and $\langle Q^2\rangle = 2.5$ GeV^2, respectively. Both $h_1 \otimes H_1^\perp$ and $H_1^\perp \otimes \bar{H}_1^\perp$ convolutions involve transverse-momentum dependent functions [27,28] whose behavior upon scale change should be described in the context of Collins-Soper factorization [29,30] (see also Refs. [31,32]). However, the global analysis of Ref. [26] neglects any change of the partonic trans-

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verse momentum with the scale Q^2 leading to a possible overestimation of h_1 [28,33,34]. It would be desirable, therefore, to have an independent way to extract transversity, involving collinear fragmentation functions. Here, we consider the semi-inclusive production of two hadrons inside the same current jet.

As already explained, the fragmentation $q \to (\pi^+\pi^-)X$ is described by an (extended) DiFF. When the quark is transversely polarized, $q^\uparrow$, a correlation can exist between its transverse polarization vector and the normal to the plane containing the two pion momenta. This effect is encoded in the chiral-odd polarized DiFF $H_1^{\sphericalangle q}$ via the dependence on the transverse component of the pion pair relative momentum R_T [7]. The function $H_1^{\sphericalangle q}$ can be interpreted as arising from the interference of $(\pi^+\pi^-)$ being in two states with different angular momenta [35–38]. Since the transverse momentum of the hard parton is integrated out, the cross section can be studied in the context of collinear factorization and its polarized part contains the factorized product $h_1^q H_1^{\sphericalangle q}$ [37,39]. The HERMES Collaboration has recently measured such spin asymmetry using transversely polarized proton targets [40]; the COMPASS Collaboration performed the same measurement on a deuteron target [41] and should soon release data using a proton target.

Similarly to the Collins effect, the unknown $H_1^{\sphericalangle q}$ has to be extracted from electron-positron annihilation, specifically by measuring the angular correlation of planes containing two pion pairs in the $e^+e^- \to (\pi^+\pi^-)_{\mathrm{jet1}} \times (\pi^+\pi^-)_{\mathrm{jet2}} X$ process [12,42]. The BELLE Collaboration is analyzing data for this angular correlation [43,44], also referred to as Artru-Collins asymmetry [42]. Therefore, it seems timely to use available models for extended DiFF to make predictions for the Artru-Collins asymmetry at BELLE kinematics. Since evolution equations for extended DiFF are available at next-to-leading order [9], at variance with the Collins effect the asymmetries with inclusive hadron pairs in SIDIS and e^+e^- can be correctly connected when the scale is ranging over two orders of magnitude. Therefore, the option of using the semi-inclusive production of hadron pairs inside the same jet seems a theoretically clean way to extract transversity [34]. Finally, we point out that pair production in polarized hadron-hadron collisions allows in principle to "self-sufficiently" determine all the unknown DiFF and h_1 [45]. The PHENIX Collaboration has recently presented data on this kind of measurement [46].

The paper is organized as follows. In Sec. II, using the model calculation of DiFF from Ref. [38], we fit the spin asymmetry recently extracted by HERMES for the SIDIS production of $(\pi^+\pi^-)$ pairs on transversely polarized protons [40]. In Sec. III, we describe how we calculate the evolution of the involved extended DiFF starting from the HERMES scale up to the BELLE scale. In Sec. IV, we illustrate the predictions for the correlation of angular distributions of two pion pairs produced in e^+e^- annihilations at BELLE kinematics. Finally, in Sec. V we draw some conclusions.

II. FIT TO DEEP-INELASTIC SCATTERING DATA

We consider the SIDIS process $e(l) + N^\uparrow(P) \to e(l') + \pi^+(P_1) + \pi^-(P_2) + X$, where P is the momentum of the nucleon target with mass M, l, l' are the lepton momenta before and after the scattering and $q = l - l'$ is the spacelike momentum transferred to the target. The final pions, with mass $m_\pi = 0.14$ GeV and momenta P_1 and P_2, have invariant mass M_h (which we consider as much smaller than the hard scale $Q^2 = -q^2 \geq 0$ of the SIDIS process). We introduce the pair total momentum $P_h = P_1 + P_2$ and relative momentum $R = (P_1 - P_2)/2$. Using the traditional Sudakov representation of a 4-momentum a in terms of its light-cone components $a^\pm = (a^0 \pm a^3)/\sqrt{2}$ and transverse spatial components $\boldsymbol{a}_T$, we define the light-cone fractions $x = p^+/P^+$ and $z = P_h^-/k^-$, where p and $k = p + q$ are the momenta of the parton before and after the hard vertex, respectively.

In this process, the following asymmetry can be measured (for the precise definition we refer to Refs. [38,40]):

$$A_{UT}^{\sin(\phi_R+\phi_S)\sin\theta}(x, y, z, M_h^2) = -\frac{\frac{1-y-y^2\gamma^2/4}{xy^2(1+\gamma^2)}\left(1+\frac{\gamma^2}{2x}\right)}{\frac{1-y+y^2/2+y^2\gamma^2/4}{xy^2(1+\gamma^2)}\left(1+\frac{\gamma^2}{2x}\right)} \times \frac{|\boldsymbol{R}|}{M_h} \times \frac{\sum_q e_q^2 h_1^q(x) H_{1,q}^{\sphericalangle sp}(z, M_h^2)}{\sum_q e_q^2 f_1^q(x) D_{1,q}(z, M_h^2)}, \tag{1}$$

where $y = P\cdot q/P\cdot l$ is related to the fraction of beam energy transferred to the hadronic system, $\gamma = 2Mx/Q$, f_1^q and h_1^q are the unpolarized and transversely polarized parton distributions, respectively, and

$$|\boldsymbol{R}| = \frac{M_h}{2}\sqrt{1 - \frac{4m_\pi^2}{M_h^2}}. \tag{2}$$

The spin asymmetry (1) is related to an asymmetric modulation of pion pairs in the angles ϕ_S and ϕ_R, which represent the azimuthal orientation with respect to the scattering plane of the target transverse polarization and of the plane containing the pion pair momenta, respectively (see Ref. [38] for a precise definition, which is consistent with the Trento conventions [47]).

The polar angle θ describes the orientation of P_1, in the center-of-mass frame of the two pions, with respect to the direction of P_h in the lab frame. DiFF depend upon $\cos\theta$ via the light-cone fraction $\zeta = 2R^-/P_h^- = 2\cos\theta|\boldsymbol{R}|/M_h$, which describes how the total momentum of the pair is split between the two pions [38]. DiFF can be expanded in terms of Legendre polynomials of $\cos\theta$ and the expansion can be reasonably truncated to include only the s and p relative partial waves of the pion pair, since their invariant mass is small (typically $M_h \lesssim 1$ GeV). At leading twist and leading order in α_s, the spin asymmetry of Eq. (1) contains only $D_{1,q} = D^s_{1,q} + D^p_{1,q}$, which includes the diagonal pure s- and p-wave contributions, and $H^{\sphericalangle sp}_{1,q}$, which originates from the interference between them [38,39]. Subleading-twist terms have different azimuthal dependences [8].

The polarized DiFF $H^{\sphericalangle sp}_{1,q}$ represents the chiral-odd partner to isolate the transversity distribution h^q_1 in the spin asymmetry of Eq. (1). It describes the interference between the fragmentations of transversely polarized quarks into pairs of pions in relative s and p waves [39]. Together with $D_{1,q}$, it was analytically calculated in a spectator-model framework for the first time in Ref. [37], and later in a refined version [38]. The analytical expressions for $D_{1,q} \equiv D_{1,oo}$ and $H^{\sphericalangle sp}_{1,q} \equiv H^{\sphericalangle}_{1,ot}$ can be found in Eqs. (23) and (26) of Ref. [38], respectively. The model parameters were fixed by adjusting $D_{1,q}$ to the output of the PYTHIA event generator tuned to the SIDIS kinematics at HERMES [40]; their values are listed in Eqs. (32–35) of Ref. [38]. Note that the spectator model by construction gives $D_{1,u} = D_{1,d} = D_{1,\bar{u}} = D_{1,\bar{d}}$ and $H^{\sphericalangle sp}_{1,u} = -H^{\sphericalangle sp}_{1,d} = -H^{\sphericalangle sp}_{1,\bar{u}} = H^{\sphericalangle sp}_{1,\bar{d}}$.

The calculated spin asymmetry follows the same trend of the data. In particular, the shape of the invariant mass dependence is dominated by a resonance peak at $M_h \approx m_\rho$, which is due to the interference between a background production of pion pairs in s wave and the p-wave component dominated by the decay $\rho \to \pi\pi$ of the ρ resonance. Similarly, the model displays also another broad peak at $M_h \approx 0.5$ GeV due to the $\omega \to \pi\pi\pi$ resonant channel. Both predictions and data show no sign change in $A_{UT}^{\sin(\phi_R+\phi_S)\sin\theta}$ as a function of M_h around $M_h \approx m_\rho$, contrary to what was predicted in Ref. [36]. However, the results of Ref. [38] systematically overpredict the experimental data at least by a factor of 2 [40].

To correctly reproduce the size of the asymmetry, we multiply the model prediction of $H^{\sphericalangle sp}_{1,q}$ by an extra parameter α, while we use the model prediction for $D_{1,q}$ without further changes, since its parameters have been already fitted to reproduce the unpolarized cross section, as predicted by PHYTIA. We use also the GRV98 LO parametrization for f^q_1 at the HERMES scale $Q^2 = 2.5$ GeV^2. For h^q_1, we use the recently extracted parametrization from Ref. [48], whose central value is basically the same as the former one from Ref. [26] in the region $x \lesssim 0.2$ of interest here. This parametrization is obtained from a global fit of BELLE, HERMES, and COMPASS data for single-hadron fragmentations. It represents at the moment the only available transversity extraction. However, we remark that it was obtained adopting strong simplifying assumptions on the transverse-momentum dependent evolution with the factorization scale. According to Refs. [28,33,34], this fact might lead to an overestimation of h_1 itself and, consequently, to an underestimation of the parameter α (or, equivalently, of the predicted Artru-Collins asymmetry at BELLE). The asymmetry of Eq. (1) is calculated by averaging the numerator and denominator of $A_{UT}^{\sin(\phi_R+\phi_S)\sin\theta}$ in each experimental bin, while integrating in turns all remaining variables in the ranges

$$0.023 < x < 0.4, \qquad 0 < z < 0.99, \qquad 0.5 \text{ GeV} < M_h < 1 \text{ GeV}. \tag{3}$$

The variable y is always integrated in the range

$$y_{\min} = \max[0.1, Q^2_{\min}/(x(s - M^2)), (W^2_{\min} - M^2)/((1-x)(s - M^2))], \qquad y_{\max} = 0.85, \tag{4}$$

with $s = 56.2$ GeV^2 and $W^2_{\min} = 4$ GeV^2.

The best value for the parameter α is found by means of a χ^2 fit; the results are shown in Fig. 1. We took into consideration the experimental errors by adding in quadrature the statistical and systematic errors (the error bands in Fig. 1 represent such a sum). We did not include the small theoretical errors coming from the uncertainty on the other model parameters of the fragmentation functions, from the uncertainties of the parton distribution functions, and from the choice of factorization scale. The best-fit value of our reduction parameter turns out to be $\alpha = 0.32 \pm 0.06$ corresponding to $\chi^2/\text{d.o.f.} = 1.24$. In summary, the model calculation of the $H^{\sphericalangle sp}_{1,q}$ function has to be reduced by a factor 3 to reproduce the HERMES data, if the transversity from Ref. [48] is used.

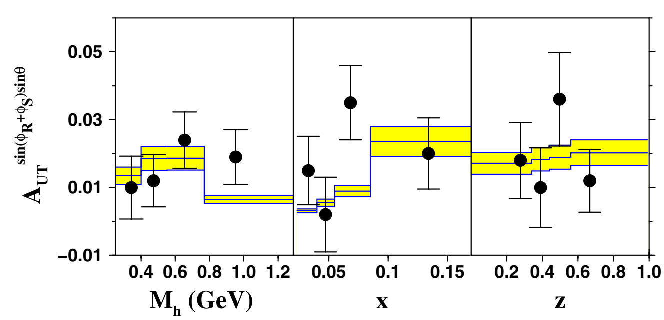

FIG. 1 (color online). The spin asymmetry for the semi-inclusive production of a pion pair in deep-inelastic scattering on a transversely polarized proton, as a function of the invariant mass M_h of the pion pair, of the light-cone momentum fraction x of the initial parton, of the energy fraction z carried by the pion pair with respect to the fragmenting parton. Data from Ref. [40]. The uncertainty band is a fit to the data based on the DiFF spectator model of Ref. [38] and on the h_1 parametrization of Ref. [48].

III. EVOLUTION OF DIHADRON FRAGMENTATION FUNCTIONS

In order to predict the azimuthal asymmetry in the distribution of two pion pairs produced in e^+e^- annihilation, we need to evolve the DiFF $D_{1,q}$ and $H_{1,q}^{\sphericalangle sp}$ from the HERMES scale to the BELLE scale.

DiFF usually depend on z, $\zeta = 2\cos\theta|\boldsymbol{R}|/M_h$ [or, alternatively, on $z_1 = z(1+\zeta)/2$, $z_2 = z(1-\zeta)/2$], and on R_T^2, which is connected to the pair invariant mass by [9]

$$R_T^2 = \frac{(P_{1T} - P_{2T})^2}{4} = \frac{z_1 z_2}{z_1 + z_2}\left[\frac{M_h^2}{z_1 + z_2} - \frac{M_1^2}{z_1} - \frac{M_2^2}{z_2}\right]. \tag{5}$$

The further dependence on the scale Q^2 of the process is described by usual DGLAP (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi) evolution equations; at LL, they read [9]

$$\frac{d}{d\log Q^2} D_q(z_1, z_2, R_T^2, Q^2) = \frac{\alpha_s(Q^2)}{2\pi}\int_{z_1+z_2}^1 \frac{du}{u^2} D_{q'}\left(\frac{z_1}{u}, \frac{z_2}{u}, R_T^2, Q^2\right) \times P_{q'q}(u), \tag{6}$$

where $P(u)$ are the usual leading-order splitting functions [49]. A similar equation holds for $H_q^{\sphericalangle}$ involving the splitting functions $\delta P(u)$ for transversely polarized partons [50,51] (see also the Appendix of Ref. [9], for convenience).

The same strategy can be applied to study evolution of single components of extended DiFF in the expansion in relative partial waves of the pion pair. In fact, Eq. (6) can be rewritten as

$$\frac{d}{d\log Q^2} D_q(z, \zeta, M_h^2, Q^2) = \frac{\alpha_s(Q^2)}{2\pi}\int_z^1 \frac{du}{u} D_{q'}\left(\frac{z}{u}, \zeta, M_h^2, Q^2\right) \times P_{q'q}(u). \tag{7}$$

Note that the evolution kernel affects only the dependence on z, leaving untouched the dependence on ζ. That is, it affects the dependence on the fractional momentum of the pion pair with respect to the hard fragmenting parton, but not the dependence on the nonperturbative processes that make the fractional momentum split inside the pair itself. The net effect is that extended DiFF display evolution equations very similar to the single-hadron fragmentation case. Using the above identity $\zeta = 2\cos\theta|\boldsymbol{R}|/M_h$, we can again expand both sides of Eq. (7) in terms of Legendre functions of $\cos\theta$ and apply the evolution kernel to each member of the expansion. By integrating in $d\cos\theta$ both sides we come to the final result

$$\frac{d}{d\log Q^2} D_{1,q}(z, M_h^2, Q^2) = \frac{\alpha_s(Q^2)}{2\pi}\int_z^1 \frac{du}{u} D_{1,q'}\left(\frac{z}{u}, M_h^2, Q^2\right) P_{q'q}(u), \tag{8}$$

that involves the DGLAP evolution of the single diagonal component $D_{1,q} = D_{1,q}^s + D_{1,q}^p$ related to the pure s and p relative partial waves of the pion pair. Analogously, we can get an evolution equation similar to Eq. (8) for $H_{1,q}^{\sphericalangle sp}$ provided that $P(u)$ is replaced by $\delta P(u)$.

Equation (8) shows that also the dependence on the pair invariant mass M_h is not affected by the evolution kernel, as is reasonable, since M_h is a scale much lower than Q^2. However, in order to get the M_h dependence at a different scale $Q'^2 \neq Q^2$ it is important to completely integrate away the z dependence. Usually, experimental phase spaces are limited by the geometry of the apparatus and, in this case, the integration in dz is performed in the interval $[z_{\min}, 1]$ with $z_{\min} \neq 0$. In Fig. 2, we show $D_{1,u}(M_h)$ for the up quark at the HERMES scale $Q^2 = 2.5\ \text{GeV}^2$ (dotted-dashed line) and at the BELLE scale of $Q^2 = 100\ \text{GeV}^2$ (solid line). In the left panel, results are obtained using $z_{\min} = 0.02$, in the right panel with $z_{\min} = 0.2$ as in the BELLE setup. Since the DGLAP evolution shifts the strength at lower z for increasing Q^2, cutting the z phase space from below makes the final result miss most of the strength and, consequently, display a reduced $D_{1,q}(M_h)$. The apparent (and contradictory) effect of perturbative evolution on the dependence upon the nonperturbative scale M_h in the right panel is actually spurious, and it disappears as soon as the phase space for z integration is properly enlarged to include the lowest z for $z_{\min} \to 0$, as shown in the left panel. When extracting azimuthal (spin) asymmetries, it is, therefore, crucial to keep in mind these features to estimate the effect of evolution.

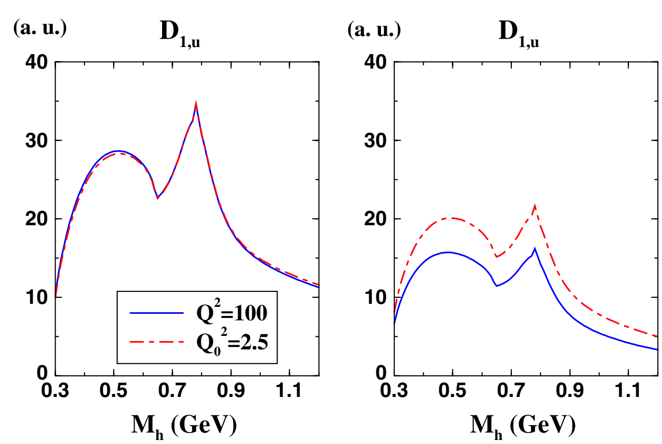

FIG. 2 (color online). The unpolarized extended DiFF $D_{1,u}(M_h, Q^2)$ in arbitrary units, after integrating the z dependence away in the interval [0.02, 1] (left panel) and [0.2, 1] (right panel). Dotted-dashed line for $Q^2 = 2.5\ \text{GeV}^2$ at HERMES, solid line for $Q^2 = 100\ \text{GeV}^2$ at BELLE (see text).

As a last general comment, we stress that the analysis of evolution is facilitated by the fact that azimuthal asymmetries based on the mechanism of dihadron fragmentation can be studied using collinear factorization. This feature makes them a cleaner observable than the Collins effect, from the theoretical point of view.

In the next section, we compute azimuthal asymmetries for two pion pairs production in e^+e^- annihilations using evolved extended DiFF. The goal is to make model predictions at BELLE kinematics, and, at the same time, to estimate the evolution effects, both the pure one on the z dependence and the spurious one on the M_h dependence, due to the limited experimental phase space.

IV. PREDICTIONS FOR ELECTRON-POSITRON ANNIHILATION

For the process $e^+e^- \to (\pi^+\pi^-)_{\mathrm{jet1}}(\pi^+\pi^-)_{\mathrm{jet2}}X$, the momentum transfer $q = l + l'$ is timelike, i.e. $q^2 = Q^2 \geq 0$, with l, l', the momenta of the two annihilating fermions. We have now two pairs of pions, one originating from a fragmenting parton and the other one from the related antiparton. Therefore, we will use in analogy to Sec. II the variables ϕ_R, θ, P_1, P_2, P_h, R, M_h, z, ζ, for one pair, adding the variables $\bar{\phi}_R$, $\bar{\theta}$, $\bar{P}_1$, $\bar{P}_2$, $\bar{P}_h$, $\bar{R}$, $\bar{M}_h$, $\bar{z}$, $\bar{\zeta}$, for the other pair. Since we assume that the two pairs belong to two back-to-back jets, we have $P_h \cdot \bar{P}_h \approx Q^2$. The momenta and angles involved in the description of the process are depicted in Fig. 3. The azimuthal angles ϕ_R and $\bar{\phi}_R$ are defined by

$$\phi_R = \frac{(\boldsymbol{l}_{e^+} \times \boldsymbol{P}_h)\cdot \boldsymbol{R}_T}{|(\boldsymbol{l}_{e^+} \times \boldsymbol{P}_h)\cdot \boldsymbol{R}_T|} \arccos\left(\frac{\boldsymbol{l}_{e^+} \times \boldsymbol{P}_h}{|\boldsymbol{l}_{e^+} \times \boldsymbol{P}_h|} \cdot \frac{\boldsymbol{R}_T \times \boldsymbol{P}_h}{|\boldsymbol{R}_T \times \boldsymbol{P}_h|}\right),$$
$$\bar{\phi}_R = \frac{(\boldsymbol{l}_{e^+} \times \boldsymbol{P}_h)\cdot \bar{\boldsymbol{R}}_T}{|(\boldsymbol{l}_{e^+} \times \boldsymbol{P}_h)\cdot \bar{\boldsymbol{R}}_T|} \arccos\left(\frac{(\boldsymbol{l}_{e^+} \times \boldsymbol{P}_h)}{|\boldsymbol{l}_{e^+} \times \boldsymbol{P}_h|} \cdot \frac{(\bar{\boldsymbol{R}}_T \times \boldsymbol{P}_h)}{|\bar{\boldsymbol{R}}_T \times \boldsymbol{P}_h|}\right), \tag{9}$$

where l_{e^+} is the momentum of the positron, and $\boldsymbol{R}_T$, $\bar{\boldsymbol{R}}_T$ indicate the transverse component of $\boldsymbol{R}$, $\bar{\boldsymbol{R}}$ with respect to $\boldsymbol{P}_h$, $\bar{\boldsymbol{P}}_h$, respectively. They are measured in the plane identified by $\boldsymbol{l}_{e^+} \times \boldsymbol{P}_h$ and $(\boldsymbol{l}_{e^+} \times \boldsymbol{P}_h) \times \hat{z}$, with $\hat{z} \parallel -\boldsymbol{P}_h$ in analogy to the Trento conventions [47]; this plane is perpendicular to the lepton plane identified by $\boldsymbol{l}_{e^+}$ and $\hat{P}_h$ (see Fig. 3). Note that the difference between $\bar{\phi}_R$, as defined in Eq. (9), and the azimuthal angle of $\bar{\boldsymbol{R}}_T$, as measured around $\bar{\boldsymbol{P}}_h$, is a higher-twist effect. The invariant $y = P_h \cdot l / P_h \cdot q$ is now related, in the lepton center-of-mass frame, to the angle $\theta_2 = \arccos(\boldsymbol{l}_{e^+} \cdot \boldsymbol{P}_h / (|\boldsymbol{l}_{e^+}||\boldsymbol{P}_h|))$ by $y = (1 + \cos\theta_2)/2$.

Starting from Eq. (21) of Ref. [42], we define the so-called Artru-Collins azimuthal asymmetry $A(\cos\theta_2, z, \bar{z}, M_h^2, \bar{M}_h^2)$ as the ratio between weighted leading-twist cross sections for the $e^+e^- \to (\pi^+\pi^-)_{\mathrm{jet1}} \times (\pi^+\pi^-)_{\mathrm{jet2}}X$ process, once integrated upon all variables but $\cos\theta_2$, z, $\bar{z}$, M_h^2, $\bar{M}_h^2$. We define the weighted cross section as

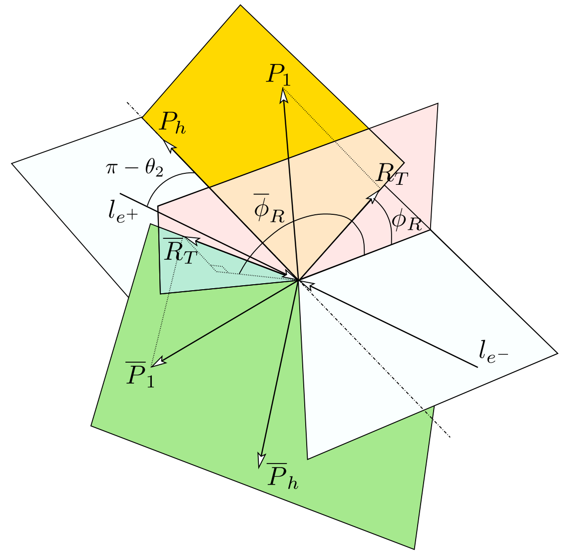

FIG. 3 (color online). Momenta and angles involved in the process $e^+e^- \to (\pi^+\pi^-)_{\mathrm{jet1}}(\pi^+\pi^-)_{\mathrm{jet2}}X$.

$$\langle w \rangle = \int d\zeta d\bar{\zeta} \int d\phi_R d\bar{\phi}_R \int d\boldsymbol{q}_T w \times \frac{d\sigma}{d\cos\theta_2 dz d\bar{z} d\zeta d\bar{\zeta} dM_h^2 d\bar{M}_h^2 d\phi_R d\bar{\phi}_R d\boldsymbol{q}_T}. \tag{10}$$

The weight of the numerator in the asymmetry is $\cos(\phi_R + \bar{\phi}_R)$, for the denominator it is just 1. Elaborating upon the work of Ref. [42],[1] we recall the change of variables $\zeta = 2\cos\theta|\boldsymbol{R}|/M_h$ (and similarly for $\bar{\zeta}$) and we perform an expansion in terms of Legendre functions of $\cos\theta$ (and $\cos\bar{\theta}$) by keeping only the s- and p-wave components of the relative partial waves of the pion pair. By further integrating upon $d\cos\theta$ and $d\cos\bar{\theta}$, we deduce the analogue of Eq. (32) of Ref. [42] for the specific contribution of s and p partial waves to the Artru-Collins azimuthal asymmetry, namely,

$$A(\cos\theta_2, z, M_h^2, \bar{z}, \bar{M}_h^2) \equiv \frac{\langle\cos(\phi_R + \bar{\phi}_R)\rangle}{\langle 1 \rangle} = \frac{\sin^2\theta_2}{1+\cos^2\theta_2} \frac{\pi^2}{32} \frac{|\boldsymbol{R}||\bar{\boldsymbol{R}}|}{M_h \bar{M}_h} \times \frac{\sum_q e_q^2 H_{1,q}^{\sphericalangle sp}(z, M_h^2) \bar{H}_{1,q}^{\sphericalangle sp}(\bar{z}, \bar{M}_h^2)}{\sum_q e_q^2 D_{1,q}(z, M_h^2) \bar{D}_{1,q}(\bar{z}, \bar{M}_h^2)}. \tag{11}$$

[1] With respect to Ref. [42], we use the modified definition $\zeta = 2\xi - 1$, the integration measure of the parton transverse momentum reads $z^2 d\boldsymbol{k}_T$, and the scaling factor $1/(M_1 + M_2)$ of the chiral-odd projections of the parton-parton correlator is replaced by $1/M_h$ (and similarly for the antiparton correlator), in agreement with Refs. [38,39].

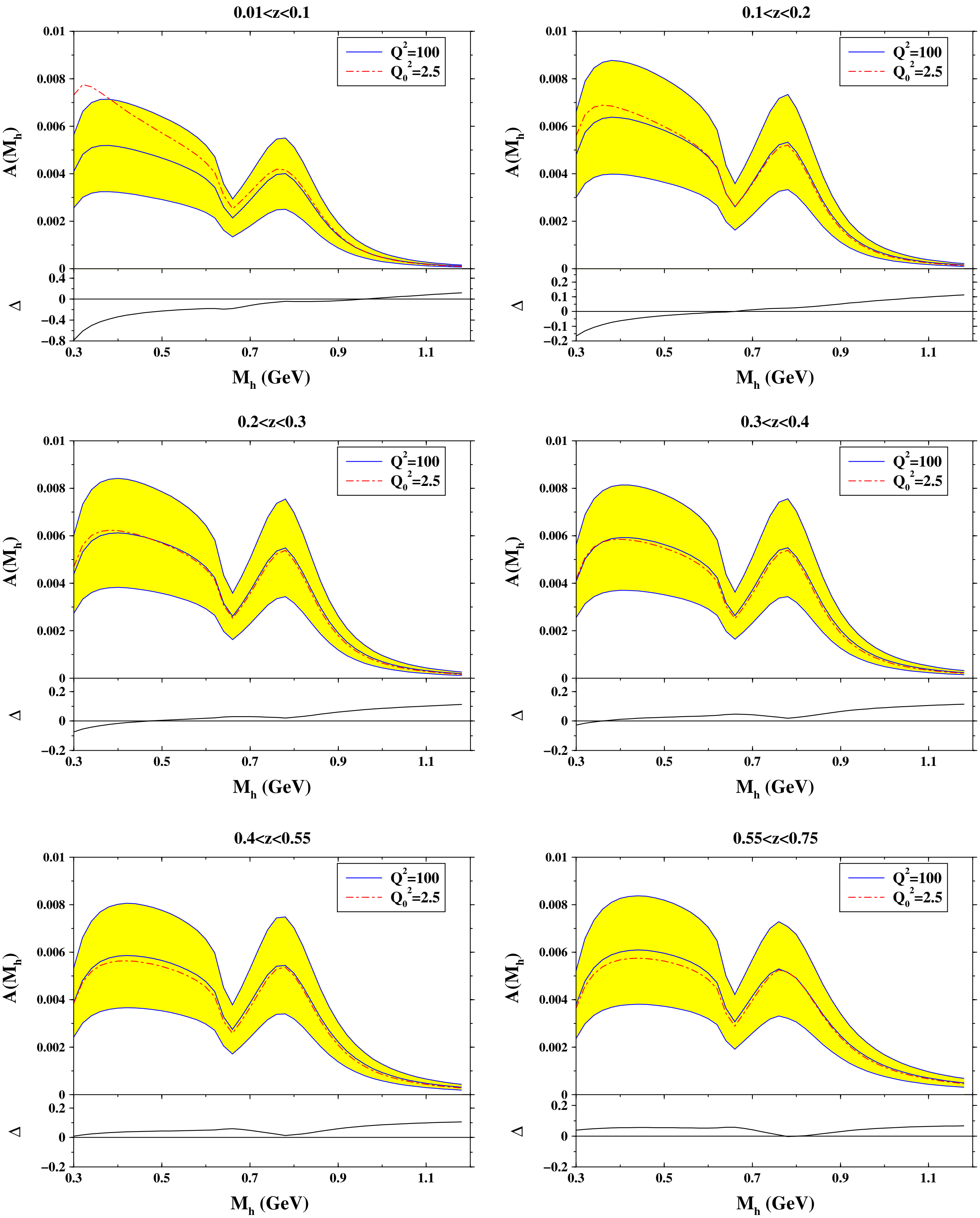

FIG. 4 (color online). The azimuthal asymmetry for two pion pairs production in e^+e^- annihilation as a function of the invariant mass M_h of one pair for the indicated bins in its momentum fraction z. Notations as in Fig. 2. The uncertainty band around the solid line originates from the fit error of Fig. 1 through error propagation. For each panel, the lower plot shows the modification factor of the final result because of DGLAP evolution (see text).

The extended DiFF $D_{1,q}$ and $H_{1,q}^{\sphericalangle sp}$ are the same universal functions appearing in the SIDIS spin asymmetry of Eq. (1). Hence, tuning model predictions for them at BELLE kinematics would help in reducing the uncertainty in the extraction of the transversity h_1 at the HERMES scale. In this strategy, a crucial role is played by evolution. At variance with the Collins effect, the dihadron fragmentation mechanism is fully collinear, since only $\boldsymbol{R}_T$ matters and $\boldsymbol{P}_{h\perp}$ can be integrated. Hence, evolution equations for extended DiFF are easily under control, presently at LL level [9], and are represented by Eq. (8) and its analogue for $H_{1,q}^{\sphericalangle sp}$.

In Fig. 4, the azimuthal asymmetry of Eq. (11) is displayed as a function of M_h for the z bins [0.01, 0.1], [0.1, 0.2], [0.2, 0.3], [0.3, 0.4], [0.4, 0.55], and [0.55, 0.75], after integrating upon the other variables $0.4 \le \bar{M}_h \le 1.2$ GeV, $0.2 \le \bar{z} \le 0.9$, and $-0.6 \le \cos\theta_2 \le 0.9$, according to the BELLE experimental phase space. In particular, according to Ref. [25] for each bin we have assumed the following coefficient

$$\frac{\langle \sin^2\theta_2 \rangle}{\langle 1 + \cos^2\theta_2 \rangle} \approx 0.7. \tag{12}$$

In each panel, the upper plot shows the $A(z_{\text{bin}}, M_h^2)$ at the HERMES scale $Q^2 = 2.5\ \text{GeV}^2$ (dotted-dashed line) and at the BELLE scale $Q^2 = 100\ \text{GeV}^2$ (solid line), the latter being supplemented by the uncertainty band propagated from the SIDIS fit error of Fig. 1. The lower plot shows

$$\Delta = \frac{A(z_{\text{bin}}, M_h^2, Q^2 = 100) - A(z_{\text{bin}}, M_h^2, Q^2 = 2.5)}{A(z_{\text{bin}}, M_h^2, Q^2 = 100)}, \tag{13}$$

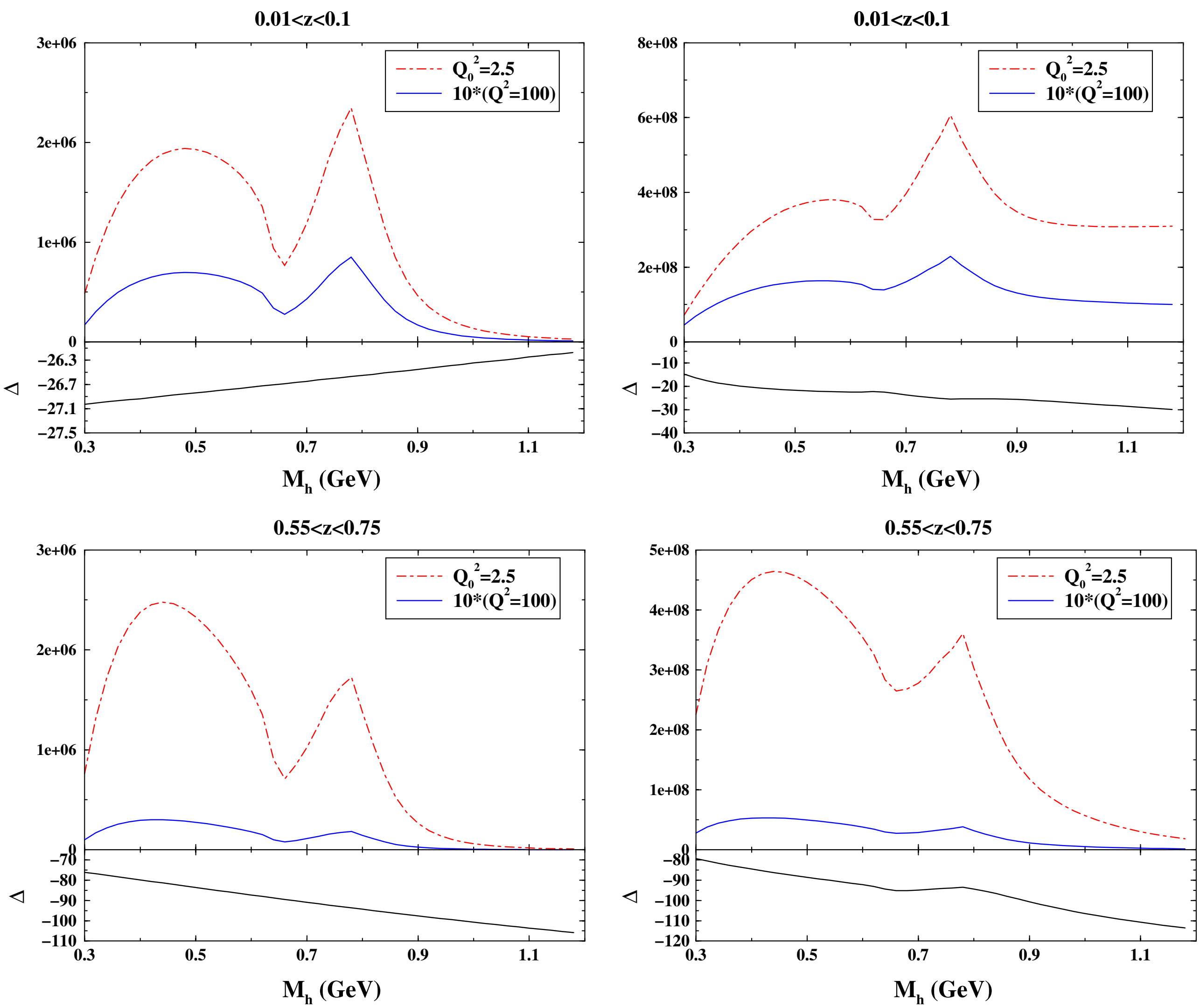

FIG. 5 (color online). Numerator (left panels) and denominator (right panels) of the azimuthal asymmetry in the same conditions and with the same notations as in Fig. 4, for the indicated boundary bins in z. The result at $Q^2 = 100\ \text{GeV}^2$ (solid line in the upper plot of each panel) is emphasized by the factor 10.

i.e. the modification factor of the final result due to the evolution starting at the HERMES scale.

Some comments are in order about Fig. 4. First of all, the absolute size of the Artru-Collins asymmetry is small, reaching at most the percent level (see Fig. 6). However, it should be within the reach of the BELLE experimental capabilities, if compared with the corresponding Collins effect for two separated single-hadron fragmentations [25].

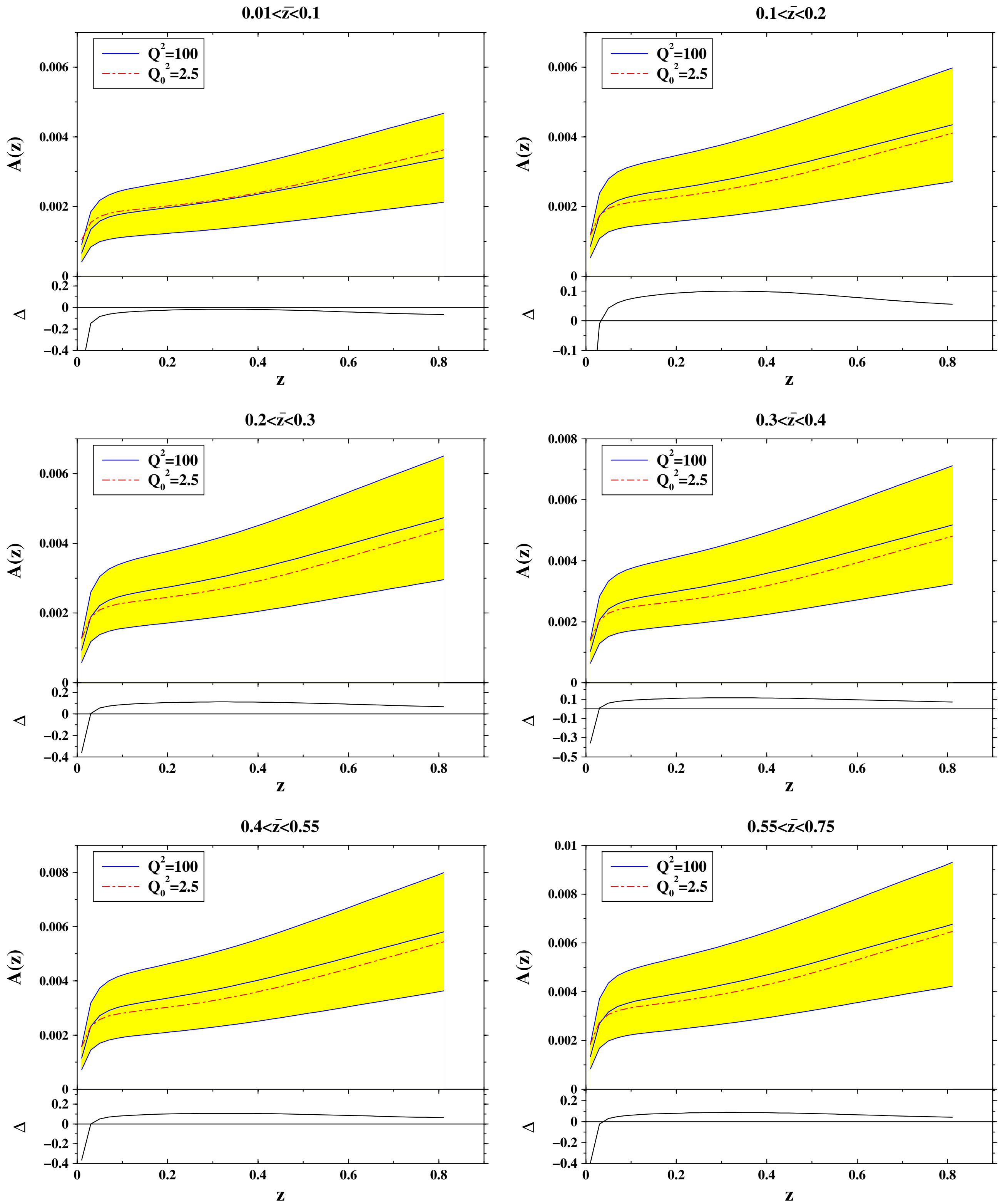

FIG. 6 (color online). Same as in Fig. 4 but as a function of z for the indicated $\bar{z}$ bins.

The error band originates from the uncertainty in the size of DiFF due to the fit of the SIDIS data for the spin asymmetry of Eq. (1). This error band is always much larger than the effects due to evolution.

After the comments about Fig. 2, one would be tempted to attribute this sensitivity of the M_h dependence to the hard scale Q^2 as coming from a spurious effect; indeed, in each panel of Fig. 4 the asymmetry is integrated in the indicated z bin, which is obviously just a small fraction of the available phase space. However, the asymmetry of Eq. (11) is the ratio of two objects that behave very differently under evolution because of their kernels $P(u)$ and $\delta P(u)$, respectively. Hence, there is no fundamental reason to expect the pure M_h dependence of the asymmetry be preserved by DGLAP evolution, even after integrating upon the whole phase space of the other variables. Moreover, the moderate sensitivity of $A(z_{\text{bin}}, M_h^2)$ to the hard scale of the process is the result of a compensation of two very large sensitivities both in the numerator and in the denominator, as is clear from Fig. 5.

In Fig. 5, the numerator (left panels) and denominator (right panels) of the asymmetry (11) are shown with the same notations as in Fig. 4 for the $0.01 \leq z \leq 0.1$ (upper panels) and $0.55 \leq z \leq 0.75$ bins (lower panels). The solid line, corresponding to the result at $Q^2 = 100\ \text{GeV}^2$, is amplified by a factor 10. Therefore, the effect of DGLAP evolution is enormous, both in the numerator and in the denominator, where, in particular, it can reach a reduction factor of more than 2 orders of magnitude. Also the shape of the M_h dependence is altered, making the more or less stable trend of $A(z_{\text{bin}}, M_h^2)$ at different Q^2 just a fortuitous case.

In summary, even if DGLAP evolution of extended DiFF seems to mildly affect the predictions for the azimuthal asymmetry at BELLE, this small sensitivity arises from a dramatic compensation between big modifications in the numerator and in the denominator of the asymmetry. Therefore, it is wise to carefully consider such effect, because it could provide more sizeable modifications in other portions of the phase space.

For sake of completeness, in Fig. 6 the azimuthal asymmetry (11) is displayed as a function of z for the $\bar{z}$ bins [0.01, 0.1], [0.1, 0.2], [0.2, 0.3], [0.3, 0.4], [0.4, 0.55], and [0.55, 0.75], with the same notations as in Fig. 4 and again after integrating upon $0.4 \leq M_h,\ \bar{M}_h \leq 1.2$ GeV, and $-0.6 \leq \cos\theta_2 \leq 0.9$. It shows a rising trend for increasing both z and $\bar{z}$. The effect of DGLAP evolution is small and within 10%, except for the lowest z values.

V. CONCLUSIONS

In this paper, using the transversity distribution function extracted in Ref. [48] and the model calculation of extended dihadron fragmentation functions (DiFF) from Ref. [38] multiplied by a reduction parameter, we fitted the spin asymmetry recently extracted by the HERMES Collaboration for the semi-inclusive deep-inelastic scattering production of $(\pi^+\pi^-)$ pairs on transversely polarized protons [40]. Then, using the results of Ref. [9] we calculated the evolution of extended DiFF at leading logarithm level, starting from the HERMES scale up to the scale of the process $e^+e^- \to (\pi^+\pi^-)_{\text{jet1}}(\pi^+\pi^-)_{\text{jet2}}X$ at BELLE kinematics. Finally, we made our predictions for the so-called Artru-Collins asymmetry, describing the correlation of angular distributions of the involved two pion pairs. The BELLE Collaboration plans to measure this asymmetry in the near future. We remark that the transversity distribution function extracted in Ref. [48] was obtained adopting simplifying assumptions on the evolution of transverse-momentum dependent parton densities that could lead to its overestimation and consequent underestimation of the accompanying chiral-odd extended DiFF. However, most of the features described in our analysis are not influenced by this issue.

The absolute size of the Artru-Collins asymmetry turns out to be small, but it should be within reach of the BELLE experimental capabilities, if compared with the corresponding Collins effect for two separated single-hadron fragmentations [25]. The theoretical error band, originating from the uncertainty in the fit of the SIDIS data, is always larger than the effects produced by the evolution of DiFF. The latter seems to mildly affect the predictions for the azimuthal asymmetry at BELLE. Nevertheless, this small sensitivity arises from a dramatic compensation between big modifications in the numerator and in the denominator of the asymmetry. Therefore, it is wise to carefully consider such effect, because it could provide more sizeable modifications in other portions of the phase space.

We stress that azimuthal asymmetries based on the mechanism of dihadron fragmentation can be studied using collinear factorization, which facilitates the analysis of, e.g., evolution. From the theoretical point of view, this feature makes them a cleaner observable than the Collins effect in single-hadron fragmentation, where transverse-momentum dependent functions are involved, whose evolution is yet not taken into account. All this procedure would not be plagued by theoretical uncertainties about factorization and evolution of transverse-momentum dependent parton densities, which currently affect the analysis of single-hadron fragmentation. As a consequence, the option of using the semi-inclusive production of hadron pairs inside the same jet seems theoretically the cleanest way to extract the transversity distribution, at present [34].

When BELLE data will be available, it will be possible to constrain extended DiFF on e^+e^- data, to evolve them back to the HERMES scale, and to use them in the formula for the SIDIS spin asymmetry to directly extract the transversity distribution.

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[1] K. Konishi, A. Ukawa, and G. Veneziano, Phys. Lett. **78B**, 243 (1978).
[2] D. de Florian and L. Vanni, Phys. Lett. B **578**, 139 (2004).
[3] P. D. Acton *et al.* (OPAL Collaboration), Z. Phys. C **56**, 521 (1992).
[4] P. Abreu *et al.* (DELPHI Collaboration), Phys. Lett. B **298**, 236 (1993).
[5] D. Buskulic *et al.* (ALEPH Collaboration), Z. Phys. C **69**, 379 (1996).
[6] M. Grazzini, L. Trentadue, and G. Veneziano, Nucl. Phys. **B519**, 394 (1998).
[7] A. Bianconi, S. Boffi, R. Jakob, and M. Radici, Phys. Rev. D **62**, 034008 (2000).
[8] A. Bacchetta and M. Radici, Phys. Rev. D **69**, 074026 (2004).
[9] F. A. Ceccopieri, M. Radici, and A. Bacchetta, Phys. Lett. B **650**, 81 (2007).
[10] A. V. Efremov, L. Mankiewicz, and N. A. Tornqvist, Phys. Lett. B **284**, 394 (1992).
[11] J. C. Collins, S. F. Heppelmann, and G. A. Ladinsky, Nucl. Phys. **B420**, 565 (1994).
[12] X. Artru and J. C. Collins, Z. Phys. C **69**, 277 (1996).
[13] V. Barone and P. G. Ratcliffe, *Transverse Spin Physics* (World Scientific, River Edge, NJ, 2003).
[14] J. P. Ralston and D. E. Soper, Nucl. Phys. **B152**, 109 (1979).
[15] O. Martin, A. Schafer, M. Stratmann, and W. Vogelsang, Phys. Rev. D **60**, 117502 (1999).
[16] A. Mukherjee, M. Stratmann, and W. Vogelsang, Phys. Rev. D **67**, 114006 (2003).
[17] A. Mukherjee, M. Stratmann, and W. Vogelsang, Phys. Rev. D **72**, 034011 (2005).
[18] J. C. Collins, Nucl. Phys. **B396**, 161 (1993).
[19] A. Airapetian *et al.* (HERMES Collaboration), Phys. Rev. Lett. **94**, 012002 (2005).
[20] M. Diefenthaler (HERMES Collaboration), *Proceedings of the 15th International Workshop on Deep Inelastic Scattering (DIS 2007), Munich, Germany, 2007*, arXiv:0706.2242.
[21] V. Y. Alexakhin *et al.* (COMPASS Collaboration), Phys. Rev. Lett. **94**, 202002 (2005).
[22] E. S. Ageev *et al.* (COMPASS Collaboration), Nucl. Phys. **B765**, 31 (2007).
[23] S. Levorato (COMPASS Collaboration), arXiv:0808.0086.
[24] D. Boer, R. Jakob, and P. J. Mulders, Nucl. Phys. **B504**, 345 (1997).
[25] R. Seidl *et al.* (Belle Collaboration), Phys. Rev. D **78**, 032011 (2008).
[26] M. Anselmino *et al.*, Phys. Rev. D **75**, 054032 (2007).
[27] D. Boer and P. J. Mulders, Phys. Rev. D **57**, 5780 (1998).
[28] D. Boer, Nucl. Phys. **B806**, 23 (2009).
[29] J. C. Collins and D. E. Soper, Nucl. Phys. **B193**, 381 (1981).
[30] X. Ji, J.-P. Ma, and F. Yuan, Phys. Rev. D **71**, 034005 (2005).
[31] F. A. Ceccopieri and L. Trentadue, Phys. Lett. B **636**, 310 (2006).
[32] A. Bacchetta, L. P. Gamberg, G. R. Goldstein, and A. Mukherjee, Phys. Lett. B **659**, 234 (2008).
[33] D. Boer, Nucl. Phys. **B603**, 195 (2001).
[34] D. Boer, arXiv:0808.2886.
[35] J. C. Collins and G. A. Ladinsky, arXiv:hep-ph/9411444.
[36] R. L. Jaffe, X. Jin, and J. Tang, Phys. Rev. Lett. **80**, 1166 (1998).
[37] M. Radici, R. Jakob, and A. Bianconi, Phys. Rev. D **65**, 074031 (2002).
[38] A. Bacchetta and M. Radici, Phys. Rev. D **74**, 114007 (2006).
[39] A. Bacchetta and M. Radici, Phys. Rev. D **67**, 094002 (2003).
[40] A. Airapetian *et al.* (HERMES Collaboration), J. High Energy Phys. 06 (2008) 017.
[41] A. Martin (COMPASS Collaboration), Czech. J. Phys. **56**, 33 (2006).
[42] D. Boer, R. Jakob, and M. Radici, Phys. Rev. D **67**, 094003 (2003).
[43] K. Abe *et al.* (BELLE Collaboration), Phys. Rev. Lett. **96**, 232002 (2006).
[44] K. Hasuko, M. Grosse Perdekamp, A. Ogawa, J. S. Lange, and V. Siegle, AIP Conf. Proc. **675**, 454 (2003).
[45] A. Bacchetta and M. Radici, Phys. Rev. D **70**, 094032 (2004).
[46] R. Yang (PHENIX Collaboration), PKU-RBRC Workshop on Transverse Spin Physics, Beijing, 2008, http://rchep.pku.edu.cn/workshop/0806/20080627-iff.pdf.
[47] A. Bacchetta, U. D'Alesio, M. Diehl, and C. A. Miller, Phys. Rev. D **70**, 117504 (2004).
[48] M. Anselmino *et al.*, arXiv:0807.0173.
[49] G. Altarelli and G. Parisi, Nucl. Phys. **B126**, 298 (1977).
[50] M. Stratmann and W. Vogelsang, Phys. Rev. D **65**, 057502 (2002).
[51] X. Artru and M. Mekhfi, Z. Phys. C **45**, 669 (1990).