# Associated production of a $\boldsymbol{W}$ boson and one $\boldsymbol{b}$ jet 

J. Campbell, ${ }^{1}$ R. K. Ellis, ${ }^{2}$ F. Febres Cordero, ${ }^{3}$ F. Maltoni, ${ }^{4}$ L. Reina, ${ }^{5}$ D. Wackeroth, ${ }^{6}$ and S. Willenbrock ${ }^{7}$<br>${ }^{1}$ Department of Physics and Astronomy, University of Glasgow, Glasgow G12 8QQ, United Kingdom<br>${ }^{2}$ Theoretical Physics Department, Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, Illinois 60510, USA<br>${ }^{3}$ Department of Physics and Astronomy, University of California Los Angeles, Los Angeles, California 90095-1547, USA<br>${ }^{4}$ Institut de Physique Théorique and Centre for Particle Physics and Phenomenology (CP3), Université Catholique de Louvain, Chemin du Cyclotron 2, B-1348 Louvain-la-Neuve, Belgium<br>${ }^{5}$ Physics Department, Florida State University, Tallahassee, Florida, 32306-4350, USA<br>${ }^{6}$ Department of Physics, State University of New York at Buffalo, Buffalo, New York 14260-1500, USA<br>${ }^{7}$ Department of Physics, University of Illinois at Urbana-Champaign, 1110 West Green Street, Urbana, Illinois 61801, USA

(Received 23 September 2008; published 20 February 2009)


#### Abstract

We calculate the production of a $W$ boson and a single $b$ jet to next-to-leading order in QCD at the Fermilab Tevatron and the CERN Large Hadron Collider. Both exclusive and inclusive cross sections are presented. We separately consider the cross section for jets containing a single $b$ quark and jets containing a $b \bar{b}$ pair. There are a wide variety of processes that contribute, and it is necessary to include them all in order to have a complete description at both colliders.


DOI: 10.1103/PhysRevD.79.034023
PACS numbers: 12.38.Bx, 13.85.Ni, 14.65.Fy, 14.70.Fm

## I. INTRODUCTION

Many signals of new physics at hadron colliders involve a weak vector boson ( $W, Z$ ) plus jets containing heavy quarks $(c, b)$. For example, the top quark was discovered via the signal $W+4 j$, with at least one $b$ jet [1,2]. More recently, evidence for single top production has been presented via the signal $W+2 j$, with at least one $b$ jet [3-6]. The Higgs boson could manifest itself via the same signal, from the production process $q \bar{q} \rightarrow W h$, followed by $h \rightarrow$ $b \bar{b}$ [7-9]. The Higgs boson could also appear in the signal $Z+2 j$ with at least one $b$ jet, via $q \bar{q} \rightarrow Z h[10,11]$.

Most calculations of the background processes that give rise to $W, Z+n j(n=1,2)$, where one or more jets contain heavy quarks $(Q=c, b)$, have been completed at next-to-leading order (NLO) [12,13]. There exist NLO calculations of $Z Q$ [14], $Z Q \bar{Q}[15,16], Z Q j$ [17], $W c$ [18,19], $W b \bar{b}[20,21]$, and $\operatorname{Wbj}[22]$. An obvious omission from this list is $W b$, that is, $W+1 j$ with at least one $b$ jet at NLO. It is the goal of this paper to fill in that gap. This NLO calculation can be used to normalize the cross section from a leading-order event generator such as ALPGEN [23] or MadEvent [24].

It may seem surprising that the NLO calculation of $W b$ has not already been done, given all the other NLO calculations listed above. The reason for this, as we will discuss shortly, is that it is essential to do this calculation with a finite $b$-quark mass. In contrast, most of the abovementioned calculations were done with a vanishing heavy-quark mass, with the justification that the quark mass is negligible at high transverse momentum $\left(p_{T}\right)$. The ability to do NLO calculations with a finite heavyquark mass for this class of processes was only recently demonstrated, for $W b \bar{b}$, in Ref. [21] (and for $Z b \bar{b}$ in Ref. [16]). We will use this calculation, together with the

NLO calculation of $W b j$ [22], to generate the NLO calculation of $W b$, including the effect of the $b$-quark mass.

We discuss the details of the calculation in Sec. II. We then present results in Sec. III and conclusions in Sec. IV.

## II. Wb AT NLO

The leading-order (LO) processes for the production of a $W$ boson and one jet containing a $b$ quark are shown in Fig. 1. In both cases there are two partons in the final state, but we require that only one of them (which contains a $b$ quark) reside at high transverse momentum, with $p_{T j}>$ 15 GeV at the Fermilab Tevatron $(\sqrt{s}=1.96 \mathrm{TeV} p \bar{p})$ and $p_{T j}>25 \mathrm{GeV}$ at the CERN Large Hadron Collider (LHC, $\sqrt{s}=14 \mathrm{TeV} p p$ ). We also require this parton to lie within a pseudorapidity of $\left|\eta_{j}\right|<2$ at the Tevatron and $\left|\eta_{j}\right|<2.5$ at the LHC. Furthermore, we demand that two partons be separated by $|\Delta R|>0.7$; if they are not, then their four-momenta are added and they are considered as occupying a single jet with this four-momentum. These requirements are made to crudely simulate the acceptance and resolution of the detectors. They are listed in Table I,


FIG. 1. Leading-order processes for the production of a $W$ boson and one jet, which contains a $b$ quark.

TABLE I. Cuts used to simulate the acceptance and resolution of the detectors and parameters used in the calculations.

| Tevatron: $p_{T j}>15 \mathrm{GeV}$ | $\left\|\eta_{j}\right\|<2$ |
| :--- | :---: |
| LHC: $p_{T j}>25 \mathrm{GeV}$ | $\left\|\eta_{j}\right\|<2.5$ |
| $\left\|\Delta R_{b \bar{b}}\right\|>0.7$ | $\left\|\Delta R_{b j}\right\|>0.7$ |
| $M_{W}=80.44 \mathrm{GeV}$ | $m_{b}=4.7 \mathrm{GeV}$ |
| LO: CTEQ6L1 | NLO: CTEQ6M [25] |
| $\mu_{F}=M_{W}$ | $\mu_{R}=M_{W}$ |
| $\alpha_{S}^{L O}\left(M_{Z}\right)=0.130$ | $\alpha_{S}^{L O}\left(M_{W}\right)=0.132$ |
| $\alpha_{S}^{N O}\left(M_{Z}\right)=0.118$ | $\alpha_{S}^{N O}\left(M_{W}\right)=0.120$ |
| $g^{2}=8 M_{W}^{2} G_{F} / \sqrt{2}$ | $G_{F}=1.16639 \times 10^{-5} \mathrm{GeV}^{-2}$ |
| $V_{u d}=V_{c s}=0.975$ | $V_{u s}=V_{c d}=0.222$ |

along with the parameters used in the calculations. In all calculations, the light quarks are summed over $q=u, d, s$, $c$, including Cabibbo-Kobayashi-Maskawa mixing.

The LO process shown in Fig. 1(b) contains a $b$ quark in the initial state and requires further discussion [26,27]. To understand the usefulness of this approach, consider the alternative approach, shown in Fig. 2 [28]. In that approach the $b$-quark mass is kept nonzero, so one obtains a finite result even when one of the $b$ quarks is emitted at zero $p_{T}$. However, although the collinear singularity is regulated by the $b$ mass, one obtains an enhancement factor of $\ln \left(M_{W} / m_{b}\right)$. Another power of this factor appears at each order of perturbation theory, degrading the convergence of the series. To ameliorate this, one sums this enhancement factor to all orders into a $b$ distribution function, and uses this function in the LO calculation of Fig. 1(b). The other big advantage of this approach is that the LO process of Fig. 1(b) is simpler than that of Fig. 2, and hence a NLO calculation becomes tractable. This effectively allows one to include a set of higher order corrections to the process of Fig. 2 that would appear only at next-to-next-to-leading order in the fixed order calculation and will probably not be available for quite some time.

There are a variety of processes that must be calculated at NLO:
(1) $q \bar{q}^{\prime} \rightarrow W b \bar{b}$ at tree level [Fig. 1(a)] and one loop $\left(m_{b} \neq 0\right)$,
(2) $q \bar{q}^{\prime} \rightarrow W b \bar{b} g$ at tree level $\left(m_{b} \neq 0\right)$,
(3) $b q \rightarrow W b q^{\prime}$ at tree level [Fig. 1(b)] and one loop ( $m_{b}=0$ ),


FIG. 2. An alternative way of calculating the process in Fig. 1 (b).
(4) $b q \rightarrow W b q^{\prime} g$ at tree level $\left(m_{b}=0\right)$,
(5) $b g \rightarrow W b q^{\prime} \bar{q}$ at tree level $\left(m_{b}=0\right)$, and
(6) $g q \rightarrow W b \bar{b} q^{\prime}$ at tree level (Fig. 2) $\left(m_{b} \neq 0\right)$.

Processes 1 and 2 are calculated with a nonzero $b$-quark mass, using the code developed in Ref. [21]. Processes 3-5, which involve an initial-state $b$ quark, are calculated with $m_{b}=0$, using the code developed in Ref. [22]. Process 6, calculated with a nonzero $b$-quark mass, can be taken from either code. We used this process to cross-check the two codes. Note that the counterterm that subtracts from Process 6 the logarithmic terms already included in Process 3 has been added, in our calculation, to Processes 3-5, since it shares the same final state phase space configuration.

In the formalism that our calculation is based upon [26,27], the $b$-quark mass is set to zero in any process in which the $b$ quark appears in the initial state (Processes 35). This is not a limitation of the formalism, nor is it an approximation. In all other processes, where the $b$ quark appears only in the final state, the $b$-quark mass is kept nonzero, although it is often a good approximation to set it to zero. For the calculation we are performing, it is essential to keep the $b$-quark mass nonzero. This is because we demand that only one $b$ jet be at high $p_{T}$, and we do not restrict the other $b$ quark. Thus there are regions of phase space where the $b \bar{b}$ invariant mass is not much greater than $2 m_{b}$, in which case it is very inaccurate to neglect the $b$-quark mass [22]. This issue arises already at leading order, as evidenced by Fig. 3, where we show the $b \bar{b}$ invariant mass from Process 1 for the $W b$ exclusive cross section at the Tevatron for both $m_{b}=0$ and $m_{b}=4.7 \mathrm{GeV}$ (with $\Delta R_{j j}>0.7$ ). We see that the cross section (the area


FIG. 3 (color online). The differential cross section for $W$ plus one $b$ jet $\left(p_{T j}>15 \mathrm{GeV},\left|\eta_{j}\right|<2\right)$, $v s$ the invariant mass of the $b$ quark and the other $b$ quark (outside the fiducial region and separated by $\Delta R_{j j}>0.7$ ), at the Fermilab Tevatron ( $\sqrt{s}=$ $\left.1.96 \mathrm{TeV} p_{-} \bar{p}\right)$. Only the contribution from the LO subprocess $q \bar{q}^{\prime} \rightarrow W b \bar{b}$ is shown. The (solid, blue) curve includes the $b$-quark mass, while the (dashed, red) curve does not.

TABLE II. Exclusive cross sections (pb) for $W$ boson plus one jet, which contains at least one $b$ quark, at the Tevatron $(\sqrt{s}=$ $1.96 \mathrm{TeV} p \bar{p})$ and LHC $(\sqrt{s}=14 \mathrm{TeV} p p)$. No branching ratios or tagging efficiencies are included. The labels on the columns have the following meaning: $W b=$ exactly one jet, which contains a $b$ quark; $W(b \bar{b})=$ exactly one jet, which contains two $b$ quarks; $W j=$ exactly one jet, including both light quarks ( $u, d, s, c$ ) and gluons. Both the leading-order (in square brackets) and next-to-leading-order cross sections are given. The first number given is from Processes 1-2; the second number is from Processes 3-6 (with Process 6 given separately in parenthesis); and the third number is their sum.

|  | Exclusive cross sections $(\mathrm{pb})$ |  |
| :--- | :---: | ---: |
| Collider | $W b$ | $W(b \bar{b})$ |
| TeV $W^{+}\left(=W^{-}\right)$ | $[5.28+0.75=6.03] 8.02+0.62(-0.05)=8.64$ | $[2.66] 3.73-0.02=3.71$ |
| LHC $W^{+}$ | $[30.2+54.3=84.5] 40.0+48.4(22.6)=88.4$ | $[17.6] 22.7+11.7=34.4$ |
| LHC $W^{-}$ | $[21.6+31.4=53.0] 29.8+29.4(12.6)=59.2$ | $[12.9] 17.2+6.5=23.7$ |
|  |  | $W j$ |
| TeV $W^{+}\left(=W^{-}\right)$ | $[1410] 1790$ |  |
| LHC $W^{+}$ | $[11040] 12040$ |  |
| LHC $W^{-}$ |  |  |

under the curve) is very sensitive to the $b$-quark mass, even with the required separation of the two $b$ quarks.

All NLO calculations are done in the MS scheme. When we refer to the NLO Processes 2, 4-6, it is understood that initial-state collinear singularities are subtracted in this scheme. For massless quarks, the collinear singularities are regulated dimensionally, while for massive $b$ quarks (Processes 2 and 6) they are regulated using a finite $b$ mass [22,26,27]. The calculation of Processes 3-6 was done using the Monte Carlo code MCFM [29,30].

## III. RESULTS

In Table II, we give the exclusive cross section for $W+$ 1 jet, where the jet contains a $b$ quark, and there are no other jets present within the acceptance (listed in Table I). We list the cross section for the case where there is only one $b$ in the jet (denoted $W b$ ), and when there are two $b$ 's in the jet [denoted $W(b \bar{b})$ ]. The tagging probability for a jet with two $b$ 's differs from that of a jet with one $b$ [31]. We notice that in the $W(b \bar{b})$ case the two $b$ quarks can be collinear and give origin to large logarithms of the form $\ln \left(p_{T}^{b} / m_{b}\right)$. Our results do not contain a resummation of these logarithms and are therefore subject to some degree of uncertainty. We could have rejected the $W(b \bar{b})$ configuration in our $W+b$-jet NLO results or used a different jet algorithm as suggested, for instance, in Ref. [32]. We prefer to keep it and quote it separately in order to make it available to different kinds of experimental analyses. Table II gives both the LO cross section (in square brackets) ${ }^{1}$ and the NLO cross section. The first number given is from Processes 1-2; the second from Processes 3-6; and the third is their sum. In the case of $W b$ we also quote, in parentheses, the contribution of Process 6 by itself. Indeed

[^0]Process 6 is part of both the fixed order $(1+2+6)$ and resummed $(3+4+5+6)$ calculations, ${ }^{2}$ and results could be quoted grouping processes either way. In order to assess the impact of resumming initial-state collinear logarithms, however, one needs to compare the sum of Processes 1, 2, and 6 to the sum of Processes 3, 4, 5, and 6. This can be easily deduced from our tables knowing Process 6 independently. In the case of $W(b \bar{b})$, only Process 1 contributes at LO, and only Processes 1,2 , and 6 at NLO. Thus we do not need to quote Process 6 separately. The NLO results correspond to a pure fixed order calculation. We see that at the Tevatron, Processes 1-2 make a much larger contribution to $W b$ than Processes 3-6, while at the LHC the two sets of processes make comparable contributions. This mirrors the results for the $W b j$ final state [22], first noted in Ref. [28]. The fixed order calculation (Processes $1+$ $2+6$ ) dominates at both the Tevatron and the LHC, but the corrections included in Processes $3+4+5$ are as much as $25 \%$ at the LHC and therefore very relevant. We also notice that for both $W b$ and $W(b \bar{b})$, Processes 1-2 dominate at the Tevatron, while at the LHC Process 6 is nearly half the size of Processes 1-2. Therefore it is very important to include higher order corrections to this process, which is achieved by adding to the fixed order result Processes 3, 4, and 5 (see discussion in Sec. II). Finally, we notice that for $W(b \bar{b})$, the NLO cross section is significantly larger than the LO cross section, while the NLO correction is modest for $W b$.

Also given in Table II is the exclusive cross section for $W j(j=u, d, s, c, g)$, both at LO and NLO [33,34]. These numbers are useful to compute the fraction of $W+1 j$ events in which the jet contains a $b$ quark. This fraction is around $0.7 \%$ at the Tevatron and $0.8 \%$ at the LHC.

[^1]TABLE III. Inclusive cross sections (pb) for $W$ boson plus one jet, which contains at least one $b$ quark, at the Tevatron $(\sqrt{s}=$ $1.96 \mathrm{TeV} p \bar{p})$ and LHC $(\sqrt{s}=14 \mathrm{TeV} p p)$. No branching ratios or tagging efficiencies are included. The labels on the columns have the following meaning: $W b+X=$ one or more jets, at least one of which contains a $b$ quark; $W(b \bar{b})+X=$ one or more jets, one of which contains two $b$ quarks; $W j+X=$ one or more jets, including both light quarks ( $u, d, s, c$ ) and gluons. Both the leading-order (in square brackets) and next-to-leading-order cross sections are given. The first number given is from Processes 1-2; the second number is from Processes 3-6 (with Process 6 given separately in parentheses); and the third number is their sum.

|  |  |  |
| :--- | :---: | :---: |
| Collider | $W b+X$ | Inclusive cross sections $(\mathrm{pb})$ |
| $\mathrm{TeV} W^{+}\left(=W^{-}\right)$ | $[7.56+1.81=9.37] 11.77+2.40(0.77)=14.17$ | $W(b \bar{b})+X$ |
| LHC $W^{+}$ | $[39.3+106.0=145.3] 53.6+136.1(68.9)=189.7$ | $[2.66] 4.17+0.39=4.56$ |
| LHC $W^{-}$ | $[27.9+67.0=94.9] 39.3+88.2(44.6)=127.5$ | $W j+X$ |
|  | $[14.6] 25.1+35.9=61.0$ |  |
| TeV $W^{+}\left(=W^{-}\right)$ | $[14240] 2030$ |  |
| LHC $W^{+}$ | $[11040] 15220$ |  |
| LHC $W^{-}$ |  |  |

We give in Table III the inclusive cross sections for $W+$ $1 j+X$, where the jet contains a $b$ quark, and where there may be other jets present (up to two additional jets at NLO). The relative importance of Processes 3-6 is significantly increased compared with the exclusive cross sections, especially at the LHC, due, in particular, to Process 6. This is expected since Process 6 is more effectively cut in the exclusive cross section where the NLO cross section is required to have the same number of jets as the LO cross section. The fixed order cross section (Processes $1+2+6$ ) dominates at both the Tevatron and LHC, but corrections coming from Processes $3+4+$ 5 are large and of the order of $50 \%$ at the LHC. Moreover, given the relevance of Process 6, having included part of the NLO corrections to Process 6 by calculating Process 3 at NLO in QCD increases the stability and therefore the validity of the theoretical prediction. The NLO cross sections are also increased by a larger factor than in the exclusive cross sections. The fraction of $W+1 j+X$ events in which the jet contains a $b$ quark is around $0.9 \%$ at the Tevatron and $1.2 \%$ at the LHC.

We estimate the theoretical uncertainty in the exclusive and inclusive LO and NLO cross sections by varying the
renormalization $\left(\mu_{R}\right)$ and factorization $\left(\mu_{F}\right)$ scales independently. We vary $\mu_{R}$ in the $(0.5-2) M_{W}$ range while keeping $\mu_{F}$ fixed at its central value, $\mu_{F}=M_{W}$. Similarly we vary $\mu_{F}$ in the $(0.5-2) M_{W}$ range keeping $\mu_{R}=M_{W}$. The results are illustrated in Figs. 4 and 5, where the plots on the left-hand side correspond to $W$ production with a $b$ jet $(W b)$ while the plots on the righthand side correspond to $W$ production with a double- $b$ jet $[W(b \bar{b})]$. The upper plots refer to $W^{+} b / W^{+}(b \bar{b})=$ $W^{-} b / W^{-}(b \bar{b})$ production at the Tevatron, the middle plots to $W^{+} b / W^{+}(b \bar{b})$ production at the LHC, and the lower plots to $W^{-} b / W^{-}(b \bar{b})$ production at the LHC. The cross sections in Figs. 4 and 5 have been normalized to their $\mu_{R}=\mu_{F}=M_{W}$ value. The horizontal axis represents the variation of either $\mu_{R}$ or $\mu_{F}$, depending on the curve (see figure captions), normalized to the central value $\mu_{0}=M_{W}$. In Tables IV and $V$ we quantitatively give the variation with $\mu_{R}$ and $\mu_{F}$ as (asymmetric) uncertainties on the central value, corresponding to the choice $\mu_{R}=\mu_{F}=M_{W}$ used to obtain the results of Tables II and III. We have not included in the theoretical uncertainties reported in this paper the uncertainty coming from the parton distribution functions.

TABLE IV. Exclusive cross sections (pb) for $W$ boson plus one jet, which contains at least one $b$ quark, with the theoretical uncertainty due to renormalization (first uncertainty) and factorization (second uncertainty) scale dependence. The uncertainty due to the renormalization scale $\left(\mu_{R}\right)$ dependence is estimated by varying $\mu_{R}$ by a factor of 2 with respect to its central value $\mu_{R}=M_{W}$, while keeping the factorization scale $\left(\mu_{F}\right)$ fixed at its central value $\mu_{F}=M_{W}$. The uncertainty due to the factorization scale is estimated analogously, i.e. varying $\mu_{F}$ by a factor of 2 about its central value $\mu_{F}=M_{W}$, while keeping $\mu_{R}$ fixed at $\mu_{R}=M_{W}$. The central values are extracted from Table II. The labeling of columns and rows are as described in the caption of Table II. Results within brackets are LO, results with no brackets are NLO.

| Collider | Exclusive cross sections (pb) |  |
| :---: | :---: | :---: |
|  | Wb | $W(b \bar{b})$ |
| $\mathrm{TeV} W^{+}\left(=W^{-}\right)$ | $\left[6.03 \times\left(1_{-0.19}^{+0.27+0.03}\right)\right] 8.64 \times\left(1_{-0.12}^{+0.13+0.003}\right)$ | $\left[2.66 \times\left(1_{-0.19}^{+0.27+0.04}\right)\right] 3.71 \times\left(1_{-0.11}^{0.12+0.01}{ }^{0.01}\right)$ |
| LHC $W^{+}$ | $\left[84.5 \times\left(1_{-0.19-0.14}^{+0.27+0.11}\right)\right] 88.4 \times\left(1_{-0.11-0.10}^{+0.11+0.08}\right)$ | $\left[17.6 \times\left(1_{-0.19-0.10}^{+0.27}\right)\right] 34.4 \times\left(1_{-0.16-0.04}^{+0.23+0.03}\right)$ |
| LHC $W^{-}$ | $\left[53.0 \times\left(1_{-0.19-0.14}^{+0.27+0.12}\right)\right] 59.2 \times\left(1_{-0.11-0.10}^{+0.12+0.08}\right)$ | $\left[12.9 \times\left(1_{-0.19-0.11}^{+0.27+0.09}\right)\right] 23.7 \times\left(1_{-0.15-0.04}^{+0.21-0.03}\right)$ |

TABLE V. Inclusive cross sections (pb) for $W$ boson plus one jet, which contains at least one $b$ quark, with the theoretical uncertainty due to renormalization (first uncertainty) and factorization (second uncertainty) scale dependence. The uncertainty due to the renormalization scale $\left(\mu_{R}\right)$ dependence is estimated by varying $\mu_{R}$ by a factor of 2 with respect to its central value $\mu_{R}=M_{W}$, while keeping the factorization scale $\left(\mu_{F}\right)$ fixed at its central value $\mu_{F}=M_{W}$. The uncertainty due to the factorization scale is estimated analogously, i.e. varying $\mu_{F}$ by a factor of 2 about its central value $\mu_{F}=M_{W}$, while keeping $\mu_{R}$ fixed at $\mu_{R}=M_{W}$. The central values are extracted from Table III. The labeling of columns and rows are as described in the caption of Table III. Results within brackets are LO, results with no brackets are NLO.

| Collider | Inclusive cross sections (pb) Wb | $W(b \bar{b})$ |
| :---: | :---: | :---: |
| $\mathrm{TeV} W^{+}\left(=W^{-}\right)$ | $\left[9.37 \times\left(1_{-0.19}^{+0.27+0.02}\right)\right] 14.17 \times\left(1_{-0.13}^{+0.15+0.001}{ }^{+0.0002}\right)$ | $\left[2.66 \times\left(1_{-0.19}^{+0.27+0.04}{ }^{+0.04}\right)\right] 4.56 \times\left(1_{-0.14-0.02}^{+0.17+0.03}\right)$ |
| LHC $W^{+}$ | $\left[145.3 \times\left(1_{-0.19-0.14}^{+0.27+0.12}\right)\right] 189.7 \times\left(1_{-0.13-0.10}^{+0.16+0.07}\right)$ | $\left[17.6 \times\left(1_{-0.19-0.10}^{+0.27+0.09}\right)\right] 61.0 \times\left(1_{-0.21-0.02}^{+0.33+0.02}\right)$ |
| LHC $W^{-}$ | $\left[94.9 \times\left(1_{-0.19}^{+0.27+0.15}\right)\right] 127.5 \times\left(1_{-0.13-0.10}^{+0.16+0.08}\right)$ | $\left[12.9 \times\left(1_{-0.19-0.11}^{+0.27}+0.09\right)\right] 42.5 \times\left(1_{-0.21-0.03}^{+0.32+0.02}\right)$ |

From Figs. 4 (and Fig. 5 and Tables IV and V) we see that the theoretical uncertainty due to the dependence on the renormalization scale is larger than the corresponding uncertainty from the factorization-scale dependence. The decrease in the factorization-scale dependence in going from LO to NLO is mild, while the decrease in the renormalization-scale dependence is significant. An exception is the inclusive $W(b \bar{b})$ cross sections at the LHC, where the renormalization-scale dependence slightly increases at NLO. Even in the exclusive case, the improvement in the renormalization-scale dependence in going
from LO to NLO for $W(b \bar{b})$ is mild at the LHC. These exceptions can be explained by the fact that only Processes 1, 2, and 6 contribute to $W(b \bar{b})$ production at NLO and, among those, Process 6 opens a new initial state, namely $q g$, and is effectively a LO process. The effect is larger at the LHC because, due to the large gluon density, the $q g$ channel is more relevant. The effect is also larger for inclusive rather than exclusive cross sections because the exclusive final state suppresses the contribution of Process 6 (which is a $2 \rightarrow 4$ process), as evidenced by the numerical results in Tables II and III.


FIG. 4 (color online). Renormalization- and factorization-scale dependence of $W b$ (left-hand side) and $W(b \bar{b})$ (right-hand side) exclusive production. The upper plots correspond to both $W^{+} b / W^{+}(b \bar{b})$ and $W^{-} b / W^{-}(b \bar{b})$ production at the Tevatron. The middle plots correspond to $W^{+} b / W^{+}(b \bar{b})$ production at the LHC and the lower plots correspond to $W^{-} b / W^{-}(b \bar{b})$ production at the LHC. The (black) dashed (LO) and (red) solid (NLO) curves represent the dependence on the renormalization scale ( $\mu_{R}$ ) when $\mu_{R}$ is varied with respect to its central value $\mu_{0}=M_{W}$, while the factorization scale ( $\mu_{F}$ ) is fixed at $\mu_{F}=M_{W}$. In a similar way, the (magenta) dotted (LO) and (blue) dotted-dashed (NLO) curves represent the dependence on $\mu_{F}$ when $\mu_{F}$ is varied with respect to its central value $\mu_{0}=M_{W}$, while $\mu_{R}$ is fixed at $\mu_{R}=M_{W}$. The cross sections have been normalized to their $\mu_{R}=\mu_{F}=M_{W}$ value.


FIG. 5 (color online). Renormalization- and factorization-scale dependence of $W b$ (left-hand side) and $W(b \bar{b})$ (right-hand side) inclusive production. The upper plots correspond to both $W^{+} b / W^{+}(b \bar{b})$ and $W^{-} b / W^{-}(b \bar{b})$ production at the Tevatron. The middle plots correspond to $W^{+} b / W^{+}(b \bar{b})$ production at the LHC and the lower plots correspond to $W^{-} b / W^{-}(b \bar{b})$ production at the LHC. The (black) dashed (LO) and (red) solid (NLO) curves represent the dependence on the renormalization scale ( $\mu_{R}$ ) when $\mu_{R}$ is varied with respect to its central value $\mu_{0}=M_{W}$, while the factorization scale ( $\mu_{F}$ ) is fixed at $\mu_{F}=M_{W}$. In a similar way, the (magenta) dotted (LO) and (blue) dotted-dashed (NLO) curves represent the dependence on $\mu_{F}$ when $\mu_{F}$ is varied with respect to its central value $\mu_{0}=M_{W}$, while $\mu_{R}$ is fixed at $\mu_{R}=M_{W}$. The cross sections have been normalized to their $\mu_{R}=\mu_{F}=M_{W}$ value.


FIG. 6 (color online). $\quad p_{T}(b)$ (upper plots) and $p_{T}(W)$ (lower plots) distributions for $W^{+} b$ exclusive production, at the Tevatron (lefthand side plots) and at the LHC (right-hand side plots).





FIG. 7 (color online). $\quad p_{T}(b)$ (upper plots) and $p_{T}(W)$ (lower plots) distributions for $W^{+}(b \bar{b})$ exclusive production, at the Tevatron (left-hand side plots) and at the LHC (right-hand side plots).


FIG. 8 (color online). $\quad p_{T}(b)$ (upper plots) and $p_{T}(W)$ (lower plots) distributions for $W^{+} b$ inclusive production, at the Tevatron (lefthand side plots) and at the LHC (right-hand side plots).


FIG. 9 (color online). $\quad p_{T}(b)$ (upper plots) and $p_{T}(W)$ (lower plots) distributions for $W^{+}(b \bar{b})$ inclusive production, at the Tevatron (left-hand side plots) and at the LHC (right-hand side plots).

In Figs. 6-9 we show the differential cross sections with respect to the transverse momentum of the $b$ jet and of the $W$ boson, for $W^{+} b$ inclusive/exclusive and $W^{+}(b \bar{b})$ inclusive/exclusive production. If there is more than one $b$ jet in the final state, the $p_{T}$ of the highest- $p_{T} b$ jet is chosen. We do not show distributions for $W^{-} b$ and $W^{-}(b \bar{b})$ production at the LHC, since they resemble the ones for $W^{+} b$ and $W^{+}(b \bar{b})$ production at the LHC illustrated here. For all final states, the NLO QCD corrections modify the shape of both the $b$-jet and $W$-boson transverse momentum distributions.

## IV. CONCLUSIONS

In this paper we report on a NLO calculation of the production of a $W$ boson with one $b$ jet. We present both inclusive and exclusive cross sections, as well as cross sections where the jet contains one or two $b$ quarks. We show that it is essential to keep the $b$-quark mass finite throughout the calculation, and we are able to overcome this technical hurdle. The calculation is performed by combining two previous NLO calculations of $W b \bar{b}$ [21]
and Wbj [22], taking care to treat their overlap consistently. The calculation that we present thus represents the state of the art prediction for $W$ plus one $b$ jet production at NLO in QCD.

These calculations can be compared with the large amount of data on $W$ plus $b$ jets already gathered at the Tevatron, and soon to be produced at the LHC. They can also be compared with the inclusive samples obtained by merging matrix-element calculations with parton showers.

## ACKNOWLEDGMENTS

We are grateful for conversations and correspondence with Ann Heinson and Tony Liss. F. M. and L. R. thank the Aspen Center for Physics for hospitality while this work was being completed. This work was supported in part by the U. S. Department of Energy under Contracts No. DE-AC02-76CH03000, No. DE-FG02-91ER40662, No. DE-FG02-91ER40677, and No. DE-FG02-97IR4102. The work of D.W. is supported in part by the National Science Foundation under Grants No. NSF-PHY-0456681 and No. NSF-PHY-0547564.
[1] F. Abe et al. (CDF Collaboration), Phys. Rev. Lett. 74, 2626 (1995).
[2] S. Abachi et al. (D0 Collaboration), Phys. Rev. Lett. 74, 2632 (1995).
[3] V. M. Abazov et al. (D0 Collaboration), Phys. Rev. Lett. 98, 181802 (2007).
[4] V.M. Abazov et al. (D0 Collaboration), Phys. Rev. D 78, 012005 (2008).
[5] D. Acosta et al. (CDF Collaboration), Phys. Rev. D 71, 012005 (2005).
[6] T. Aaltonen et al. (CDF Collaboration), Phys. Rev. Lett. 101, 252001 (2008); http://www-cdf.fnal.gov/physics/ new/top/public_singletop.html.
[7] T. Aaltonen et al. (CDF Collaboration), Phys. Rev. Lett. 100, 041801 (2008).
[8] T. Aaltonen et al. (CDF Collaboration), Phys. Rev. D 78, 032008 (2008).
[9] V. M. Abazov et al. (D0 Collaboration), Phys. Lett. B 663, 26 (2008).
[10] V. M. Abazov et al. (D0 Collaboration), Phys. Lett. B 655, 209 (2007).
[11] T. Aaltonen et al. (CDF Collaboration), Phys. Rev. Lett. 100, 211801 (2008).
[12] Z. Bern, L. J. Dixon, D. A.Kosower, and S. Weinzierl, Nucl. Phys. B489, 3 (1997).
[13] Z. Bern, L. J.Dixon, and D. A. Kosower, Nucl. Phys. B513, 3 (1998).
[14] J. Campbell, R. K. Ellis, F. Maltoni, and S. Willenbrock, Phys. Rev. D 69, 074021 (2004).
[15] J. M. Campbell and R. K. Ellis, Phys. Rev. D 62, 114012 (2000).
[16] F. Febres Cordero, L. Reina, and D. Wackeroth, Phys. Rev. D 78, 074014 (2008).
[17] J. Campbell, R. K. Ellis, F. Maltoni, and S. Willenbrock, Phys. Rev. D 73, 054007 (2006).
[18] W. T. Giele, S. Keller, and E. Laenen, Phys. Lett. B 372, 141 (1996).
[19] J. Campbell, R. K.Ellis, and R. Mahbubani (unpublished).
[20] R. K. Ellis and S. Veseli, Phys. Rev. D 60, 011501 (1999).
[21] F. Febres Cordero, L. Reina, and D. Wackeroth, Phys. Rev. D 74, 034007 (2006).
[22] J. Campbell, R. K. Ellis, F. Maltoni, and S. Willenbrock, Phys. Rev. D 75, 054015 (2007).
[23] M. L. Mangano, M. Moretti, F. Piccinini, R. Pittau, and A. D. Polosa, J. High Energy Phys. 07 (2003) 001.
[24] F. Maltoni and T. Stelzer, J. High Energy Phys. 02 (2003) 027.
[25] J. Pumplin, D. R. Stump, J. Huston, H. L. Lai, P. Nadolsky, and W. K. Tung, J. High Energy Phys. 07 (2002) 012.
[26] M.A. G. Aivazis, J.C. Collins, F.I. Olness, and W. K. Tung, Phys. Rev. D 50, 3102 (1994).
[27] J. C. Collins, Phys. Rev. D 58, 094002 (1998).
[28] M. L. Mangano, M. Moretti, and R. Pittau, Nucl. Phys. B632, 343 (2002).
[29] R. K. Ellis et al. (QCD Tools Working Group), arXiv:hepph/0011122.
[30] J. Campbell and R. K. Ellis, MCFM-Monte Carlo for FeMtobarn processes, http://mcfm.fnal.gov/.
[31] D. Acosta et al. (CDF Collaboration), Phys. Rev. D 71, 092001 (2005).
[32] A. Banfi, G. P. Salam, and G. Zanderighi, J. High Energy Phys. 07 (2007) 026.
[33] W.T. Giele, E. W. N. Glover, and D. A. Kosower, Nucl. Phys. B403, 633 (1993).
[34] J. Campbell and R. K. Ellis, Phys. Rev. D 65, 113007 (2002).


[^0]:    ${ }^{1}$ The contribution from Process 1 to the $W b$ cross section at the Tevatron corresponds, at LO , to the area under the (solid, blue) curve in Fig. 3.

[^1]:    ${ }^{2}$ As explained in Sec. II, Process 6 is the same in both the resummed and fixed order calculations, i.e. it does not include any counterterm to subtract the logarithmic terms that have been resummed in Process 3. Such counterterm has been included with Process 3 since it shares the same final state phase space.

