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Decays of charmed mesons to *PV* final states

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New data on the decays of the charmed particles D^0, D^+, and D_s to PV final states consisting of a light pseudoscalar meson P and a light vector meson V are analyzed. Following the same methods as in a previous analysis of $D \to PP$ decays, one can test flavor symmetry, extract key amplitudes, and obtain information on relative strong phases. Analyses are performed for Cabibbo-favored decays and then extended to predict properties of singly and doubly Cabibbo-suppressed processes.

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I. INTRODUCTION

In the past few years rich data on charmed particle decays have been contributed by a variety of experiments. Among the decays studied are those involving PV final states, where P and V denote light pseudoscalar and vector mesons, respectively. These decays obey an approximate flavor SU(3) symmetry [1–3], allowing one to investigate such questions as the strong phases of amplitudes in these decays. These strong phases can be important when analyzing D decay Dalitz plots in the context of studies of CP violation in $B \to DX$ decays. We have recently performed a similar analysis of $D \to PP$ decays [4].

The diagrammatic approach to flavor symmetry is reviewed briefly in Sec. II. Cabibbo-favored decays are discussed in Sec. III, singly Cabibbo-suppressed decays in Sec. IV, and doubly Cabibbo-suppressed decays in Sec. V. It is possible to obtain a few of the relevant amplitudes using factorization techniques. We discuss factorization calculations in Sec. VI and conclude in Sec. VII.

II. DIAGRAMMATIC AMPLITUDE EXPANSION

A flavor-topology description of $D \to PV$ decays uses amplitudes defined as in Ref. [3]. Cabibbo-favored amplitudes, proportional to the product $V_{ud}V_{cs}^*$ of Cabibbo-Kobayashi-Maskawa factors, will be denoted by unprimed quantities; singly Cabibbo-suppressed amplitudes proportional to $V_{us}V_{cs}^*$ or $V_{ud}V_{cd}^*$ will be denoted by primed quantities; and doubly Cabibbo-suppressed quantities proportional to $V_{us}V_{cd}^*$ will be denoted by amplitudes with a tilde. These amplitudes are in the ratio $1{:}\lambda{:}-\lambda{:}-\lambda^2$, where $\lambda = \tan\theta_C = 0.2317$ [5], with θ_C the Cabibbo angle.

The relevant amplitudes are labeled as T (‘‘tree’’), C (‘‘-color-suppressed’’), E (‘‘exchange’’), and (‘‘A’’) (annihilation). For PV final states, a subscript on the amplitude denotes the meson (P or V) containing the spectator quark.

The partial width $\Gamma(H \to PV)$ for the decay of a heavy meson H may be expressed in terms of an invariant amplitude $\mathcal{A}$ as

$$\Gamma(H \to PV) = \frac{p^{*3}}{8\pi M_H^2}|\mathcal{A}|^2, \tag{1}$$

where p^* is the center-of-mass 3-momentum of each final particle, and M_H is the mass of the decaying particle.

III. CABIBBO-FAVORED DECAYS

In Table I we summarize predicted and observed amplitudes for Cabibbo-favored decays of charmed mesons to PV. The experimental values are based on those in Ref. [5] unless noted otherwise. Topological amplitudes are then obtained from these processes by algebraic solution. The values of $|T_V|$ and $|E_P|$ are uniquely given by the rates for $D_s \to \pi^+\phi$ and $D^0 \to \bar{K}^0\phi$, respectively. A twofold ambiguity then is found for the amplitude $|C_P|$ and phases of C_P and E_P, as summarized in Table II.

As explained in Ref. [6], the solution ‘‘B’’ with $|C_P| < |T_V|$ is expected for a color-suppressed amplitude. However, on the basis of fits to data from singly Cabibbo-suppressed $D \to PV$ decays, it will turn out that we will prefer the solution ‘‘A’’ with $|C_P| > |T_V|$. In Fig. 1 we plot these two solutions for amplitudes and relative phases of T_V, C_P and E_P.

Using the solutions for T_V, C_P and E_P as inputs, the other amplitudes T_P, C_V and E_V were obtained. The amplitude T_P was assumed real relative to T_V, in accord with the expectation from factorization. Six sets of solutions were obtained for each of the two cases $|T_V| < |C_P|$ (A) and $|T_V| > |C_P|$ (B). These solutions are listed in Table III. The solutions A1 and A2 are found to give the best fit to the data available for singly Cabibbo-suppressed $D \to PV$ decays, and so will be singled out for special consideration. Note the identical magnitudes and phases of T_P, C_V and E_V in solutions A1 and B1.

The magnitudes and phases of solutions A1 and A2 are illustrated in Fig. 2. The amplitudes $T_P + E_V = \mathcal{A}(D^0 \to K^-\rho^+)$, $C_V - E_V = \sqrt{2}\mathcal{A}(D^0 \to \bar{K}^0\rho^0)$, and $T_P + C_V = \mathcal{A}(D^+ \to \bar{K}^0\rho^+)$ form a triangle whose shape is specified by their magnitudes. The amplitudes C_V and E_V form the

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TABLE I. Branching ratios and invariant amplitudes for Cabibbo-favored decays of charmed mesons to one pseudoscalar and one vector meson.

Meson	Decay mode	Representation	$\mathcal{B}$ [5] (%)	p^* (MeV)	$\|\mathcal{A}\|$ (10^{-6})
D^0	$K^{*-}\pi^+$	$T_V + E_P$	5.91 ± 0.39	710.9	4.80 ± 0.16
	$K^-\rho^+$	$T_P + E_V$	10.8 ± 0.7	675.4	7.01 ± 0.23
	$\bar{K}^{*0}\pi^0$	$\frac{1}{\sqrt{2}}(C_P - E_P)$	2.82 ± 0.35	709.3	3.33 ± 0.21
	$\bar{K}^0\rho^0$	$\frac{1}{\sqrt{2}}(C_V - E_V)$	1.54 ± 0.12	673.7	2.66 ± 0.14
	$\bar{K}^{*0}\eta$	$\frac{1}{\sqrt{3}}(C_P + E_P - E_V)$	0.96 ± 0.3	579.9	2.63 ± 0.41
	$\bar{K}^{*0}\eta'$	$-\frac{1}{\sqrt{6}}(C_P + E_P + 2E_V)$	<0.11	101.9	
	$\bar{K}^0\omega$	$-\frac{1}{\sqrt{2}}(C_V + E_V)$	2.26 ± 0.4	670.0	3.25 ± 0.29
	$\bar{K}^0\phi$	$-E_P$	0.868 ± 0.06	520.6	2.94 ± 0.10
D^+	$\bar{K}^{*0}\pi^+$	$T_V + C_P$	1.83 ± 0.14	711.8	1.68 ± 0.06
	$\bar{K}^0\rho^+$	$T_P + C_V$	9.2 ± 2.0	677.0	4.06 ± 0.44
D_s^+	$\bar{K}^{*0}K^+$	$C_P + A_V$	3.9 ± 0.6	682.4	3.97 ± 0.31
	$\bar{K}^0K^{*+}$	$C_V + A_P$	5.3 ± 1.2	683.2	4.61 ± 0.52
	$\rho^+\pi^0$	$\frac{1}{\sqrt{2}}(A_P - A_V)$		825.2	
	$\rho^+\eta$	$\frac{1}{\sqrt{3}}(T_P - A_P - A_V)$	13.0 ± 2.2	723.8	6.63 ± 0.56
	$\rho^+\eta'$	$\frac{1}{\sqrt{6}}(2T_P + A_P + A_V)$	12.2 ± 2.0	464.8	12.5 ± 1.0
	$\pi^+\rho^0$	$\frac{1}{\sqrt{2}}(A_V - A_P)$		824.7	
	$\pi^+\omega$	$\frac{1}{\sqrt{2}}(A_V + A_P)$	0.25 ± 0.09	821.8	0.76 ± 0.14
	$\pi^+\phi$	T_V	4.38 ± 0.35	711.7	3.95 ± 0.16

TABLE II. Solutions in Cabibbo-favored charmed meson decays to PV final states.

	Solution A		Solution B	
PV amplitude	Magnitude (10^{-6})	Relative strong phase	Magnitude (10^{-6})	Relative strong phase
T_V	3.95 ± 0.07	$\cdots$	3.95 ± 0.07	$\cdots$
C_P	4.88 ± 0.15	$\delta_{C_PT_V} = (-162 \pm 1)^\circ$	2.84 ± 0.09	$\delta_{C_PT_V} = (-158.2^{+2.0}_{-1.9})^\circ$
E_P	2.94 ± 0.09	$\delta_{E_PT_V} = (-93 \pm 3)^\circ$	2.94 ± 0.10	$\delta_{E_PT_V} = (92.8^{+3.6}_{-3.7})^\circ$

sides of a quadrangle whose diagonals are $C_V - E_V = \sqrt{2}\mathcal{A}(D^0 \to \bar{K}^0\rho)$ and $C_V + E_V = -\sqrt{2}\mathcal{A}(D^0 \to \bar{K}^0\omega)$, and whose vertices lie on a circle with midpoint M. Two vertices are fixed, while the other two (A and B in Fig. 2) lie at any two opposite points on the circle. An additional constraint is the magnitude of $C_P + E_P - E_V = \sqrt{3}\mathcal{A}(D^0 \to \bar{K}^{*0}\eta)$. A discrete ambiguity remains, corresponding to the solutions listed in Tables II and III.

Predictions for the branching ratio for $D^0 \to \bar{K}^{*0}\eta'$, listed in the last column of Table IV, in principle allow one to distinguish among various solutions. In addition, we shall see that only solutions A1 and A2 give rise to acceptable fits to singly Cabibbo-suppressed decays.

We now state a relationship between $|T_P|$ and Cabibbo-favored D_s decay amplitudes:

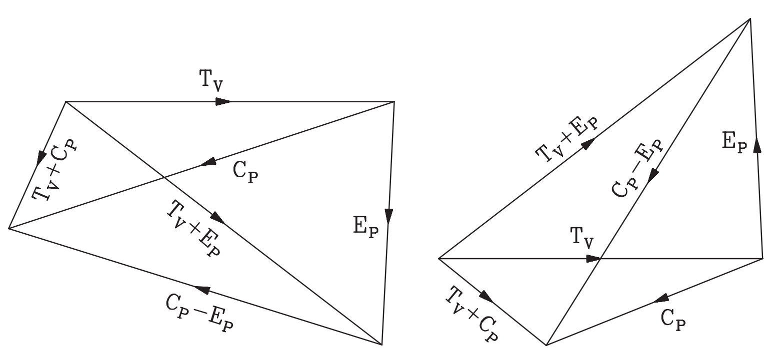

FIG. 1. Magnitudes of and relative phases between T_P, C_V and E_V. Left: Solution (A) with $|C_P| > |T_V|$; right: solution (B) with $|C_P| < |T_V|$.

TABLE III. Alternative solutions for T_P, C_V, and E_V amplitudes in Cabibbo-favored charmed meson decays to PV final states. Solutions A1–A6 correspond to $|T_V| < |C_P|$, while solutions B1–B6 correspond to $|T_V| > |C_P|$.

No.	PV ampl.	Magnitude (10^{-6})	Relative phase	$\mathcal{B}(D^0 \to \bar{K}^{*0}\eta')$ (10^{-4})
A1[a]	T_P	7.46 ± 0.21	Assumed 0	
	C_V	3.46 ± 0.18	$\delta_{C_V T_V} = (172 \pm 3)°$	1.52 ± 0.22
	E_V	2.37 ± 0.19	$\delta_{E_V T_V} = (-110 \pm 4)°$	
A2[b]	T_P	6.51 ± 0.23	Assumed 0	
	C_V	2.47 ± 0.22	$\delta_{C_V T_P} = (-174 \pm 4)°$	1.96 ± 0.23
	E_V	3.39 ± 0.16	$\delta_{E_V T_P} = (-96 \pm 3)°$	
A3	T_P	-5.67 ± 0.22	Assumed 0	
	C_V	3.64 ± 0.27	$\delta_{C_V T_P} = (-46 \pm 4)°$	1.42 ± 0.28
	E_V	2.09 ± 0.28	$\delta_{E_V T_P} = (-122^{+5}_{-6})°$	
A4	T_P	-5.60 ± 0.24	Assumed 0	
	C_V	1.68 ± 0.24	$\delta_{C_V T_P} = (-20 \pm 6)°$	2.21 ± 0.25
	E_V	3.85 ± 0.15	$\delta_{E_V T_P} = (-94 \pm 3)°$	
A5	T_P	-3.22 ± 0.21	Assumed 0	
	C_V	1.79 ± 0.32	$\delta_{C_V T_P} = (-104 \pm 5)°$	2.18 ± 0.25
	E_V	3.79 ± 0.13	$\delta_{E_V T_P} = (-180^{+4}_{-5})°$	
A6	T_P	3.21 ± 0.21	Assumed 0	
	C_V	1.78 ± 0.31	$\delta_{C_V T_P} = (105 \pm 5)°$	2.18 ± 0.25
	E_V	3.80 ± 0.13	$\delta_{E_V T_P} = (-180^{+5}_{-4})°$	
B1	T_P	7.46 ± 0.21	Assumed 0	
	C_V	3.46 ± 0.17	$\delta_{C_V T_P} = (172 \pm 3)°$	0.33 ± 0.05
	E_V	2.37 ± 0.19	$\delta_{E_V T_P} = (-110 \pm 4)°$	
B2	T_P	6.43 ± 0.22	Assumed 0	
	C_V	3.95 ± 0.24	$\delta_{C_V T_P} = (-143 \pm 4)°$	$0.052^{+0.020}_{-0.021}$
	E_V	1.40 ± 0.32	$\delta_{E_V T_P} = (-71^{+6}_{-7})°$	
B3	T_P	4.53 ± 0.24	Assumed 0	
	C_V	0.80 ± 0.21	$\delta_{C_V T_P} = (130^{+16}_{-15})°$	1.18 ± 0.10
	E_V	4.12 ± 0.15	$\delta_{E_V T_P} = (72 \pm 3)°$	
B4	T_P	4.97 ± 0.22	Assumed 0	
	C_V	3.28 ± 0.29	$\delta_{C_V T_P} = (126 \pm 4)°$	0.42 ± 0.10
	E_V	2.61 ± 0.25	$\delta_{E_V T_P} = (47 \pm 5)°$	
B5	T_P	-3.33 ± 0.22	Assumed 0	
	C_V	0.75 ± 0.19	$\delta_{C_V T_V} = (164^{+14}_{-15})°$	1.19 ± 0.11
	E_V	4.13 ± 0.17	$\delta_{E_V T_V} = (-140 \pm 2)°$	
B6	T_P	-7.70 ± 0.21	Assumed 0	
	C_V	4.01 ± 0.17	$\delta_{C_V T_V} = (17^{+3}_{-4})°$	0.020 ± 0.011
	E_V	1.24 ± 0.22	$\delta_{E_V T_V} = (-52^{+9}_{-8})°$	

[a]Preferred solution based on fit to singly Cabibbo-suppressed decays.
[b]Alternative solution giving acceptable fit to singly Cabibbo-suppressed decays.

$$|A(D_s \to \rho^+\eta')|^2 = |T_P|^2 + |A(D_s \to \pi^+\omega)|^2 - |A(D_s \to \rho^+\eta)|^2. \qquad (2)$$

Using the value of $|T_P|$ from solution A1 of Table III and the decay amplitudes ($D_s \to \rho^+\eta, \pi^+\omega$) from Table I, we calculate the amplitude $|A(D_s \to \rho^+\eta')| = (3.50 \pm 1.15) \times 10^{-6}$, which deviates from the experimental value (Table I) by a large amount. This problem with the quoted experimental rate for $D_s \to \rho^+\eta'$ was already noted in Ref. [6]. It indicates either the importance of neglected amplitudes involving the flavor-singlet component of η', or an overestimate of the experimental decay rate in this mode.

The remaining parameters A_P and A_V were determined using the amplitudes of $D_s \to (\bar{K}^{*0}K^+, \bar{K}^0K^{*+}, \pi^+\omega)$ and have been listed in Table IV. A direct calculation of the amplitudes for $D_s \to \rho^+(\eta, \eta')$ is now possible using these amplitudes. For the amplitude solutions (A1 and A2) preferred by fits to singly Cabibbo-suppressed decays, we find $\mathcal{B}(D_s \to \rho^+\eta) = (5.6 \pm 1.2, 5.55 \pm 0.60)\%$, to be compared with the experimental value of $(6.63 \pm 0.56)\%$, and $\mathcal{B}(D_s \to \rho^+\eta') = (2.9 \pm 0.3, 1.89 \pm 0.20)\%$, to be compared with the experimental value of $(12.5 \pm 1.0)\%$. The agreement between prediction and experiment for $\mathcal{B}(D_s \to \rho^+\eta)$ is good for the solutions A1, A2, B1, and B2, while no solution gives agreement for $\mathcal{B}(D_s \to \rho^+\eta')$. We await forthcoming CLEO data on this mode.

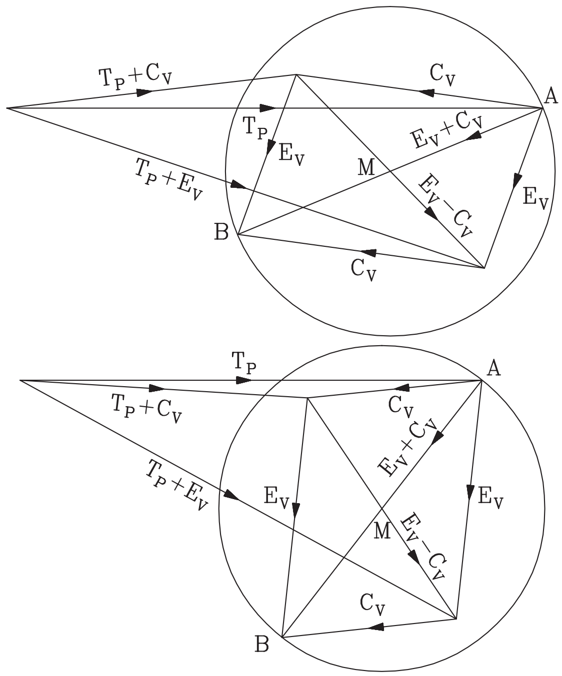

FIG. 2. Amplitudes T_P, C_V, and E_V in solutions A1 (top) and A2 (bottom).

IV. SINGLY CABIBBO-SUPPRESSED DECAYS

The topological amplitude decomposition of singly Cabibbo-suppressed decays of $D^0 \to PV$ is listed in Table V along with the measured branching ratios and amplitudes for the decays. Unlike the $D \to PP$ case [4], here we have neglected the Okubo-Zweig-Iizuka suppressed disconnected diagrams that form the singlet-exchange and singlet-annihilation amplitudes.

We now make use of the amplitudes determined in Sec. III to predict the singly Cabibbo-suppressed decay amplitudes. Here we assume the simple hierarchy of amplitudes explained in Sec. II. Based on the available data we calculated the global χ^2 of singly Cabibbo-suppressed $D \to PV$ decays for solutions A1–A6 and B1–B6. Solutions A1 and A2 have the two lowest values of χ^2 and hence were chosen as the preferred and alternative solutions. Table VI summarizes the global χ^2 values for each of the 12 solutions. It also includes, for each solution, two processes that contribute the most towards a high value of χ^2.

One notes in Table VI that the main processes contributing to high global χ^2 for all solutions are $D^0 \to \phi\pi^0$ and $D^0 \to \rho^0\pi^0$. The solutions B1–B6, which correspond to $|C_P| < |T_V|$, yield high χ^2 for the process $D^0 \to \phi\pi^0$. The amplitude of this process depends only on C'_P. This shows

TABLE IV. Solution for annihilation amplitudes in Cabibbo-favored charmed meson decays to PV final states.

No.	PV amplitude	Magnitude (10^{-6})	Relative phase	Prediction (%)
A1[a]	A_P	$1.36^{+1.16}_{-1.04}$	$\delta_{A_P} = (-151^{+83}_{-74})°$	$\mathcal{B}(D_s^+ \to \eta\rho^+) = 5.6 \pm 1.2$
	A_V	$1.25^{+0.34}_{-0.31}$	$\delta_{A_V} = (-19^{+10}_{-9})°$	$\mathcal{B}(D_s^+ \to \eta'\rho^+) = 2.9 \pm 0.3$
A2[b]	A_P	$2.15^{+0.22}_{-0.18}$	$\delta_{A_P} = (-179^{+32}_{-9})°$	$\mathcal{B}(D_s^+ \to \eta\rho^+) = 5.55 \pm 0.60$
	A_V	$1.23^{+0.31}_{-0.19}$	$\delta_{A_V} = (-19^{+34}_{-14})°$	$\mathcal{B}(D_s^+ \to \eta'\rho^+) = 1.89 \pm 0.20$
A3	A_P	$1.24^{+0.34}_{-0.24}$	$\delta_{A_P} = (-89^{+10}_{-14})°$	$\mathcal{B}(D_s^+ \to \eta\rho^+) = 4.20 \pm 0.81$
	A_V	$0.96^{+0.27}_{-0.22}$	$\delta_{A_V} = (34^{+21}_{-14})°$	$\mathcal{B}(D_s^+ \to \eta'\rho^+) = 1.45 \pm 0.28$
A4	A_P	$4.27^{+0.42}_{-0.21}$	$\delta_{A_P} = (-109^{+14}_{-5})°$	$\mathcal{B}(D_s^+ \to \eta\rho^+) = 2.77 \pm 0.27$
	A_V	$3.20^{+0.23}_{-0.19}$	$\delta_{A_V} = (+72^{+6}_{-4})°$	$\mathcal{B}(D_s^+ \to \eta'\rho^+) = 1.77 \pm 0.18$
A5	A_P	$2.88^{+0.35}_{-0.24}$	$\delta_{A_P} = (-123^{+6}_{-4})°$	$\mathcal{B}(D_s^+ \to \eta\rho^+) = 0.58 \pm 0.06$
	A_V	$1.93^{+1.21}_{-0.27}$	$\delta_{A_V} = (69^{+15}_{-5})°$	$\mathcal{B}(D_s^+ \to \eta'\rho^+) = 0.70 \pm 0.07$
A6	A_P	$2.88^{+0.22}_{-0.31}$	$\delta_{A_P} = (+122^{+5}_{-6})°$	$\mathcal{B}(D_s^+ \to \eta\rho^+) = 1.61 \pm 0.17$
	A_V	$2.85^{+0.21}_{-0.26}$	$\delta_{A_V} = (-36 \pm 7)°$	$\mathcal{B}(D_s^+ \to \eta'\rho^+) = 0.43 \pm 0.04$
B1	A_P	$1.57^{+0.82}_{-0.32}$	$\delta_{A_P} = (+121^{+19}_{-9})°$	$\mathcal{B}(D_s^+ \to \eta\rho^+) = 7.08 \pm 1.03$
	A_V	$1.74^{+0.44}_{-0.28}$	$\delta_{A_V} = (-96^{+7}_{-6})°$	$\mathcal{B}(D_s^+ \to \eta'\rho^+) = 2.53 \pm 0.37$
B2	A_P	$1.35^{+0.51}_{-0.27}$	$\delta_{A_P} = (-74^{+12}_{-9})°$	$\mathcal{B}(D_s^+ \to \eta\rho^+) = 5.38^{+2.03}_{-2.11}$
	A_V	$1.52^{+0.70}_{-0.21}$	$\delta_{A_V} = (+150^{+44}_{-10})°$	$\mathcal{B}(D_s^+ \to \eta'\rho^+) = 1.86^{+0.70}_{-0.73}$
B3	A_P	$3.85^{+0.39}_{-0.24}$	$\delta_{A_P} = (+111^{+14}_{-5})°$	$\mathcal{B}(D_s^+ \to \eta\rho^+) = 2.42 \pm 0.16$
	A_V	$2.78^{+0.37}_{-0.22}$	$\delta_{A_V} = (-68^{+17}_{-7})°$	$\mathcal{B}(D_s^+ \to \eta'\rho^+) = 1.01 \pm 0.07$
B4	A_P	$1.74^{+0.34}_{-0.23}$	$\delta_{A_P} = (+77^{+41}_{-10})°$	$\mathcal{B}(D_s^+ \to \eta\rho^+) = 3.04 \pm 0.70$
	A_V	$1.16^{+0.27}_{-0.23}$	$\delta_{A_V} = (-140 \pm 12)°$	$\mathcal{B}(D_s^+ \to \eta'\rho^+) = 1.18 \pm 0.27$
B5	A_P	$4.12^{+0.24}_{-0.31}$	$\delta_{A_P} = (+111^{+6}_{-9})°$	$\mathcal{B}(D_s^+ \to \eta\rho^+) = 1.30 \pm 0.10$
	A_V	$3.22^{+0.29}_{-0.38}$	$\delta_{A_V} = (-60^{+8}_{-11})°$	$\mathcal{B}(D_s^+ \to \eta'\rho^+) = 0.571^{+0.045}_{-0.044}$
B6	A_P	$0.67^{+0.26}_{-0.29}$	$\delta_{A_P} = (+45^{+22}_{-25})°$	$\mathcal{B}(D_s^+ \to \eta\rho^+) = 4.80 \pm 2.54$
	A_V	$1.28^{+0.23}_{-0.20}$	$\delta_{A_V} = (+168^{+11}_{-15})°$	$\mathcal{B}(D_s^+ \to \eta'\rho^+) = 3.42 \pm 1.81$

[a]Preferred solution based on fit to singly Cabibbo-suppressed decays.
[b]Alternative solution giving acceptable fit to singly Cabibbo-suppressed decays.

TABLE V. Branching ratios and invariant amplitudes for singly Cabibbo-suppressed decays of charmed mesons to one pseudoscalar and one vector meson.

Meson	Decay mode	Representation	$\mathcal{B}$ [5] (%)	p^* (MeV)	$\lvert\mathcal{A}\rvert$ (10^{-6})
D^0	$\pi^+\rho^-$	$-(T'_V + E'_P)$	0.497 ± 0.023	763.8	1.25 ± 0.03
	$\pi^-\rho^+$	$-(T'_P + E'_V)$	0.980 ± 0.040	763.8	1.76 ± 0.04
	$\pi^0\rho^0$	$\frac{1}{2}(E'_P + E'_V - C'_P - C'_V)$	0.373 ± 0.022	764.2	1.08 ± 0.03
	K^+K^{*-}	$T'_V + E'_P$	0.153 ± 0.015	609.8	0.97 ± 0.05
	K^-K^{*+}	$T'_P + E'_V$	0.441 ± 0.021	609.8	1.65 ± 0.04
	$K^0\bar{K}^{*0}$	$E'_V - E'_P$	<0.18	605.3	
	$\bar{K}^0K^{*0}$	$E'_P - E'_V$	<0.09	605.3	
	$\pi^0\phi$	$\frac{1}{\sqrt{2}}C'_P$	0.124 ± 0.012	644.7	0.81 ± 0.04
	$\pi^0\omega$	$\frac{1}{2}(E'_P + E'_V - C'_P + C'_V)$		761.2	
	$\eta\rho^0$	$\frac{1}{\sqrt{6}}(2C'_V - C'_P - E'_P - E'_V)$		652.0	
	$\eta\omega$	$-\frac{1}{\sqrt{6}}(2C'_V + C'_P + E'_P + E'_V)$		488.8	
	$\eta\phi$	$\frac{1}{\sqrt{3}}(C'_P - E'_P - E'_V)$		648.1	
	$\eta'\rho^0$	$\frac{1}{2\sqrt{3}}(E'_P + E'_V + C'_P + C'_V)$		342.5	
	$\eta'\omega$	$\frac{1}{2\sqrt{3}}(E'_P + E'_V + C'_P - C'_V)$		333.5	
D^+	$\rho^0\pi^+$	$\frac{1}{\sqrt{2}}(A'_P - A'_V - C'_P - T'_V)$	0.082 ± 0.015	767	0.32 ± 0.03
	$\omega\pi^+$	$-\frac{1}{\sqrt{2}}(A'_P + A'_V + C'_P + T'_V)$	<0.034	764	
	$\phi\pi^+$	C'_P	0.620 ± 0.070	647	1.13 ± 0.06
	$\bar{K}^{*0}K^+$	$(T'_V - A'_V)$	0.435 ± 0.048	611	1.03 ± 0.06
	$\pi^0\rho^+$	$\frac{1}{\sqrt{2}}(A'_V - A'_P - C'_V - T'_P)$		767	
	$\eta\rho^+$	$\frac{1}{\sqrt{6}}(A'_V + A'_P + 2C'_V + T'_P)$	<0.7	656	
	$\eta'\rho^+$	$\frac{1}{\sqrt{6}}(C'_V - A'_V - A'_P - T'_P)$	<0.5	349	
	$\bar{K}^0K^{*+}$	$(T'_P - A'_P)$	3.18 ± 1.38	612	2.78 ± 0.60
D_s^+	π^+K^{*0}	$(A'_V - T'_V)$	0.225 ± 0.039	773	0.79 ± 0.07
	π^0K^{*+}	$-\frac{1}{\sqrt{2}}(C'_V + A'_V)$		775	
	ηK^{*+}	$\frac{1}{\sqrt{3}}(T'_P + 2C'_V + A'_P - A'_V)$		661	
	$\eta' K^{*+}$	$\frac{1}{\sqrt{6}}(2T'_P + C'_V + 2A'_P + A'_V)$		337	
	$K^0\rho^+$	$(A'_P - T'_P)$		743	
	$K^+\rho^0$	$-\frac{1}{\sqrt{2}}(C'_P + A'_P)$	0.27 ± 0.05	745	0.92 ± 0.09
	$K^+\omega$	$-\frac{1}{\sqrt{2}}(C'_P - A'_P)$		741	
	$K^+\phi$	$T'_V + C'_P + A'_V$	<0.057	607	

that $|C_P| < |T_V|$ is not favored by the process $D^0 \to \phi\pi^0$. The processes $D^0 \to \rho^0\pi^0$ and $D^+ \to \rho^0\pi^+$ contribute to high χ^2 for the solutions A3–A6.

The predicted and experimental D^0 branching ratios are in qualitative agreement but with some notable exceptions. The predictions for $D^0 \to \pi\rho$ fall slightly short of experiment for all charge states, most prominently for $\pi^0\rho^0$. Recall that the predicted branching ratio for $D^0 \to \pi^+\pi^-$ lies significantly *above* the experimental value [4]. The predictions for $D^0 \to K^+K^{*-}$ and $D^0 \to K^-K^{*+}$ are not badly obeyed, while those for $D^0 \to K^0\bar{K}^{*0}$ and $D^0 \to \bar{K}^0K^0$ are far below the current experimental upper limits. The predicted branching ratio for $D^0 \to \pi^0\phi$ is approximately the same as the observed value. The value of χ^2 for solutions A1 and A2 are, respectively, 61.8 and 65.9 (Table VI), where we have used the 18 data points for which the branching ratios are available.

In Table VII we present our predictions for branching ratios of singly Cabibbo-suppressed $D \to PV$ modes corresponding to the two solutions A1 and A2 having the lowest value of global χ^2 for these modes. There is little one can do to distinguish between them given the available data on branching ratios. Both solutions yield fairly similar central values for most of the singly Cabibbo-suppressed $D \to PV$ modes. A slight distinction may be made in a few cases. For example, the predicted central values of $\mathcal{B}(D^0 \to (K^0\bar{K}^{*0}, \bar{K}^0K^{*0}))$ are larger for solution A1 than for A2, though differing only by 1.5σ. Another example is the process $D^0 \to \pi^0\omega$, for which the central value of the branching ratio in solution A2 is nearly 3 times its value in A1. Still another example is the process $D^+ \to \eta'\rho^+$, for which the predicted (very small) branching ratio in A1 is twice its value in A2. Measurements of the branching ratios for both Cabibbo-favored and singly Cabibbo-suppressed decays with higher precision will be necessary in order to distinguish between the two solutions.

V. DOUBLY CABIBBO-SUPPRESSED DECAYS

We now characterize the doubly Cabibbo-suppressed or wrong-sign (WS) decays of $D \to PV$. A detailed list of possible decays and the corresponding topological ampli-

TABLE VI. Global χ^2 values for fits to singly Cabibbo-suppressed $D \to PV$ decays. Also included are the processes that contribute the most to a high χ^2 value.

		Worst Processes (High $\Delta\chi^2$ value)			
No.	Global χ^2	Decay Channel	$\mathcal{B}_{\rm th}(\%)$	$\mathcal{B}_{\rm expt}(\%)$	$\Delta\chi^2$
A1[a]	61.8	$D^+ \to \bar{K}^{*0}K^+$	0.17 ± 0.04	0.435 ± 0.048	16.1
		$D^+ \to \omega\pi^+$	0.16 ± 0.04	<0.034	11.6
A2[b]	65.9	$D^+ \to \bar{K}^{*0}K^+$	0.17 ± 0.03	0.435 ± 0.048	21.4
		$D^0 \to \rho^0\pi^0$	0.27 ± 0.02	0.373 ± 0.022	10.1
A3	341.4	$D^0 \to \rho^0\pi^0$	$(4.3 \pm 3.1) \times 10^{-3}$	0.373 ± 0.022	275.2
		$D^+ \to \rho^0\pi^+$	$(1.5 \pm 4.0) \times 10^{-3}$	0.082 ± 0.015	25.1
A4	167.1	$D^0 \to \rho^0\pi^0$	0.12 ± 0.01	0.373 ± 0.022	95.4
		$D^+ \to \rho^0\pi^+$	0.73 ± 0.12	0.082 ± 0.015	31.4
A5	324.1	$D^0 \to \rho^0\pi^0$	$(6.1 \pm 3.1) \times 10^{-3}$	0.373 ± 0.022	272.6
		$D^+ \to K^{*0}\bar{K}^*$	0.19 ± 0.02	<0.09	11.9
A6	149.8	$D^+ \to \rho^0\pi^+$	0.91 ± 0.09	0.082 ± 0.015	51.1
		$D^+ \to \bar{K}^{*0}K^+$	0.12 ± 0.03	0.435 ± 0.048	32.1
B1	244.0	$D^0 \to \rho^0\pi^0$	0.12 ± 0.01	0.373 ± 0.022	95.3
		$D^0 \to \phi\pi^0$	0.042 ± 0.003	0.124 ± 0.012	45.3
B2	155.7	$D^0 \to \phi\pi^0$	0.042 ± 0.003	0.124 ± 0.012	45.3
		$D^+ \to \phi\pi^+$	0.21 ± 0.01	0.62 ± 0.07	32.9
B3	165.7	$D^0 \to \phi\pi^0$	0.042 ± 0.003	0.124 ± 0.012	45.3
		$D^+ \to \phi\pi^+$	0.21 ± 0.01	0.62 ± 0.07	32.9
B4	151.7	$D^0 \to \phi\pi^0$	0.042 ± 0.002	0.124 ± 0.012	45.3
		$D^+ \to \rho^0\pi^+$	1.44 ± 0.23	0.082 ± 0.015	34.4
B5	518.8	$D^0 \to \rho^0\pi^0$	$(5.4 \pm 2.8) \times 10^{-3}$	0.373 ± 0.022	274.8
		$D^+ \to \rho^0\pi^+$	1.71 ± 0.21	0.082 ± 0.015	59.3
B6	401.3	$D^0 \to \rho^0\pi^0$	0.015 ± 0.006	0.373 ± 0.022	245.9
		$D^0 \to \phi\pi^0$	0.042 ± 0.003	0.124 ± 0.012	45.3

[a]Preferred solution.
[b]Alternative solution.

tude decompositions are given in Table VIII. We used the Cabibbo-favored amplitudes calculated in Sec. III to predict the WS amplitudes, using the simple hierarchy of amplitudes as explained in Sec. II. The predicted amplitudes have been included in Table VIII for the preferred (A1) and alternative (A2) solutions.

The experimental values for the following decays are available in the literature [5]:

$$\mathcal{B}(D^0 \to K^{*+}\pi^-) = (3.0^{+3.9}_{-1.2}) \times 10^{-4}, \quad (3)$$

$$\mathcal{B}(D^+ \to K^{*0}\pi^+) = (4.35 \pm 0.9) \times 10^{-4}. \quad (4)$$

The predicted values for these branching ratios (Table VIII) are in satisfactory agreement with the experimental values quoted above. An interesting point to note is that both solutions A1 and A2 give the same predicted central values for these branching ratios, but A2 has a larger error bar on both of them. Several other branching ratios in Table VIII predicted to exceed 10^{-4} may help to distinguish between solutions A1 and A2. These include $\mathcal{B}(D^0 \to K^{*0}\pi^0)$, $\mathcal{B}(D^+ \to K^{*+}\pi^0)$, and $\mathcal{B}(D^+ \to K^+\rho^0)$. Reduction in errors on predictions will be needed in order that these distinctions exceed 2–2.5σ. Some of the doubly Cabibbo-suppressed decays in Table VIII may be observable in Dalitz plots of D decays to three pseudoscalars through interference with Cabibbo-favored PV decays. For example, $D^0 \to K_S\pi^+\pi^-$ might be able to provide new information about the decay process $D^0 \to K^{*+}\pi^-$, while $D^+ \to K_S\pi^+\pi^0$ could provide information about $D^+ \to K^{*+}\pi^0$.

VI. FACTORIZATION COMPARISONS

In the current section we compare our results for the amplitudes of T_P and T_V with the values extracted from explicit evaluation of the tree diagram assuming factorization [7]. In order to calculate T_P we use the decay $D^0 \to K^-\rho^+$. In this scenario the spectator $\bar{u}$ quark goes from D^0 to the pseudoscalar K^- and so we use the standard form of the $(D \to P)$ current [8]:

$$H_\mu = f_+(q^2)(p_D + p_K)_\mu - f_-(q^2)(p_D - p_K)_\mu, \quad (5)$$

where f_+ and f_- are the relevant form factors. The current we use for the vector meson is [9]

$$\rho^\mu = \epsilon^\mu m_\rho f_\rho, \quad (6)$$

where ϵ^μ represents the polarization of the vector meson, m_ρ is its mass and f_ρ is the associated decay constant. The

TABLE VII. Comparison between predicted amplitudes based on Cabibbo-favored decays and the experimental values for singly Cabibbo-suppressed decays of D^0 to a pseudoscalar and a vector meson. Predictions are listed for preferred (A1) and alternative (A2) solutions.

			Predicted $\mathcal{B}(\%)$	
Meson	*PV* decay mode	Experimental $\mathcal{B}(\%)$	Solution A1	Solution A2
D^0	$\pi^+\rho^-$	0.497 ± 0.023	0.39 ± 0.03	0.39 ± 0.03
	$\pi^-\rho^+$	0.980 ± 0.040	0.84 ± 0.06	0.84 ± 0.06
	$\pi^0\rho^0$	0.373 ± 0.022	0.29 ± 0.02	0.27 ± 0.02
	K^+K^{*-}	0.153 ± 0.015	0.20 ± 0.01	0.20 ± 0.01
	K^-K^{*+}	0.441 ± 0.021	0.43 ± 0.03	0.43 ± 0.03
	$K^0\bar{K}^{*0}$	<0.18	0.0080 ± 0.0036	0.0020 ± 0.0016
	$\bar{K}^0K^{*0}$	<0.09	0.0080 ± 0.0036	0.0020 ± 0.0016
	$\pi^0\phi$	0.124 ± 0.012	0.122 ± 0.007	0.122 ± 0.007
	$\pi^0\omega$		0.043 ± 0.008	0.119 ± 0.012
	$\eta\rho^0$		0.106 ± 0.013	0.095 ± 0.010
	$\eta\omega$		0.140 ± 0.009	0.127 ± 0.009
	$\eta\phi$		0.093 ± 0.009	0.14 ± 0.01
	$\eta'\rho^0$		0.0154 ± 0.0009	0.0158 ± 0.0009
	$\eta'\omega$		0.0066 ± 0.0005	0.0077 ± 0.0005
D^+	$\rho^0\pi^+$	0.082 ± 0.015	0.097 ± 0.048	0.23 ± 0.12
	$\omega\pi^+$	<0.034	0.15 ± 0.04	0.14 ± 0.12
	$\phi\pi^+$	0.620 ± 0.070	0.62 ± 0.04	0.62 ± 0.04
	$\bar{K}^{*0}K^+$	0.435 ± 0.048	0.17 ± 0.04	0.17 ± 0.03
	$\pi^0\rho^+$		0.062 ± 0.047	0.012 ± 0.015
	$\eta\rho^+$	<0.7	0.0017 ± 0.0040	0.0057 ± 0.013
	$\eta'\rho^+$	<0.5	0.083 ± 0.010	0.044 ± 0.005
	$\bar{K}^0K^{*+}$	3.18 ± 1.38	1.66 ± 0.20	1.66 ± 0.12
D_s^+	π^+K^{*0}	0.225 ± 0.039	0.15 ± 0.04	0.15 ± 0.03
	π^0K^{*+}		0.049 ± 0.012	0.020 ± 0.008
	ηK^{*+}		0.014 ± 0.011	0.012 ± 0.008
	$\eta'K^{*+}$		0.029 ± 0.006	0.015 ± 0.003
	$K^0\rho^+$		1.29 ± 0.15	1.29 ± 0.09
	$K^+\rho^0$	0.27 ± 0.05	0.33 ± 0.05	0.42 ± 0.05
	$K^+\omega$		0.108 ± 0.029	0.072 ± 0.033
	$K^+\phi$	<0.057	0.038 ± 0.009	0.037 ± 0.028

TABLE VIII. Branching ratios and invariant amplitudes for doubly Cabibbo-suppressed decays of charmed mesons to one pseudoscalar and one vector meson. Predictions are shown for favored (A1) and alternative (A2) solutions.

				Predicted $\mathcal{B}(10^{-4})$	
Meson	Decay mode	Representation	p^* (MeV)	Solution A1	Solution A2
D^0	$K^{*+}\pi^-$	$T''_P + E''_V$	711	3.63 ± 0.26	3.63 ± 0.27
	$K^{*0}\pi^0$	$(C''_P - E''_V)/\sqrt{2}$	709	0.55 ± 0.06	0.80 ± 0.08
	$K^{*0}\eta$	$(C''_P - E''_P + E''_V)/\sqrt{3}$	580	0.38 ± 0.04	0.37 ± 0.04
	$K^{*0}\eta'$	$-(C''_P + 2E''_P + E''_V)/\sqrt{6}$	102	0.0046 ± 0.0004	0.0052 ± 0.0004
	$K^+\rho^-$	$T''_V + E''_P$	675	1.46 ± 0.10	1.46 ± 0.10
	$K^0\rho^0$	$(C''_V - E''_P)/\sqrt{2}$	674	0.70 ± 0.07	0.39 ± 0.05
	$K^0\omega$	$-(C''_V + E''_P)/\sqrt{2}$	670	0.58 ± 0.06	0.52 ± 0.06
	$K^0\phi$	$-E''_V$	521	0.16 ± 0.03	0.33 ± 0.03
D^+	$K^{*0}\pi^+$	$C''_P + A''_V$	712	2.94 ± 0.52	2.94 ± 0.65
	$K^{*+}\pi^0$	$(T''_P - A''_V)/\sqrt{2}$	714	3.74 ± 0.49	2.71 ± 0.30
	$K^{*+}\eta$	$-(T''_P - A''_P + A''_V)/\sqrt{3}$	586	3.37 ± 0.43	3.37 ± 0.25
	$K^{*+}\eta'$	$(T''_P + 2A''_P + A''_V)/\sqrt{6}$	137	0.0095 ± 0.0029	0.0026 ± 0.0010
	$K^0\rho^+$	$C''_V + A''_P$	677	3.43 ± 0.75	3.43 ± 0.47
	$K^+\rho^0$	$(T''_V - A''_P)/\sqrt{2}$	679	2.17 ± 0.40	3.01 ± 0.24
	$K^+\omega$	$(T''_V + A''_P)/\sqrt{2}$	675	0.64 ± 0.21	0.26 ± 0.07
	$K^+\phi$	A''_V	527	0.12 ± 0.06	0.12 ± 0.06
D_s^+	$K^{*0}K^+$	$T''_V + C''_P$	682	0.20 ± 0.02	0.20 ± 0.02
	$K^{*+}K^0$	$T''_P + C''_V$	683	1.18 ± 0.16	1.18 ± 0.18

invariant amplitude and the decay rate for the process $D^0 \to K^- \rho^+$ via the tree diagram may then be written as

$$\mathcal{M}(D^0 \to K^- \rho^+)_{T_P} = -i\frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* H_\mu \rho^\mu, \quad (7)$$

$$\Gamma(D^0 \to K^- \rho^+)_{T_P} = \frac{p^*}{8\pi M_{D^0}^2} \times \sum_{\epsilon_\mu q^\mu = 0} |\mathcal{M}(D^0 \to K^- \rho^+)_{T_P}|^2. \quad (8)$$

After summing over the ρ polarization and taking the modulus squared of the invariant amplitude one obtains the final form for $|T_P|$:

$$|T_P| = \frac{G_F}{\sqrt{2}} \frac{|V_{ud}||V_{cs}||f_+(m_\rho^2)|f_\rho}{p^*} \quad (9)$$

$$\times \sqrt{(m_D^2 - m_K^2)^2 - m_\rho^2(m_D^2 + m_K^2 + 2m_D\sqrt{m_K^2 + p^{*2}})} \quad (10)$$

$$= (5.45 \pm 0.07) \times 10^{-6}, \quad (11)$$

which is to be compared with the values quoted in Table III, and favors solution A2 over A1.

In obtaining the result stated above we used $|f_+(m_\rho^2)||V_{cs}| = 0.869 \pm 0.009$ [10]. The particle masses and the quantity $|V_{ud}|$ were taken from [5]. p^* is as quoted in Table I. We calculated the value of f_ρ using the following formula:

$$f_\rho = f_\pi \left[\frac{\mathcal{B}(\tau^- \to \nu_\tau \rho^-)}{\mathcal{B}(\tau^- \to \nu_\tau \pi^-)}\right]^{1/2} \frac{m_\tau^2 - m_\pi^2}{m_\tau^2 - m_\rho^2} \frac{m_\tau}{\sqrt{m_\tau^2 + 2m_\rho^2}} \quad (12)$$

$$= (209 \pm 1.6)\ \text{MeV}, \quad (13)$$

where once again the particle masses and branching fractions were taken from [5].

A similar approach may be taken in order to evaluate $|T_V|$ by looking at the decay $D^0 \to K^{*-}\pi^+$ via the tree diagram. In this case the spectator $\bar{u}$ quark goes from D^0 to the vector meson K^{*-}, so we use the standard forms of the $(D \to V)$ vector and axial-vector currents [8] and the pion current [9]:

$$V_\mu = ig\epsilon_{\mu\rho\tau\sigma}\epsilon^{*\rho}(p_D + p_{K^*})^\sigma(p_D - p_{K^*})^\tau, \quad (14)$$

$$A_\mu = f\epsilon_\mu^* + a_+(\epsilon^* \cdot p_D)(p_D + p_{K^*})_\mu + a_-(\epsilon^* \cdot p_D)(p_D - p_{K^*})_\mu, \quad (15)$$

$$\pi^\mu = if_\pi q^\mu. \quad (16)$$

We obtain for the amplitude $|T_V|$ the following expression:

$$|T_V| = \frac{G_F}{\sqrt{2}}|V_{cs}||V_{ud}|f_\pi \frac{m_D}{m_{K^*}}|f + a_+(m_D^2 - m_{K^*}^2) + a_- m_\pi^2|. \quad (17)$$

In principle this can be used to calculate T_V once the form factors are given. However, we may adopt a simplification using a result from Ref. [6], based on the earlier discussion in Ref. [9]. In the heavy-quark limit one expects $\Gamma(D \to \bar{K}^*\pi^+)_T = \Gamma(D \to \bar{K}\pi^+)_T$ and hence

$$p_{K\pi}^*|T|_{K\pi}^2 = (p_{K^*\pi}^*)^3|T|_{K^*\pi}^2, \quad (18)$$

where $T_{K^*\pi} = T_V$. In Ref. [4] we found in a fit to $D \to PP$ amplitudes that $|T|_{K\pi} = (2.78 \pm 0.13) \times 10^{-6}$ GeV. With $p_{K\pi}^* = 0.861$ GeV and $p_{K^*\pi}^* = 0.711$ GeV we then obtain the result

$$T_V = (4.3 \pm 0.2) \times 10^{-6}, \quad (19)$$

in reasonable agreement with the value of $(3.95 \pm 0.07) \times 10^{-6}$ quoted in Table II, especially considering the uncertainties associated with QCD corrections and with the use of the heavy-quark limit for the final strange quark.

VII. CONCLUSIONS

We have used the flavor-topology description to study the validity of flavor SU(3) for describing $D \to PV$ decays, to obtain relative phases and magnitudes of various contributing amplitudes, and to predict rates for as-yet-unseen singly and doubly Cabibbo-suppressed decays. We assumed flavor SU(3) to be an exact symmetry for the tree level diagrams. We found that singly Cabibbo-suppressed decays favor a ratio of color-suppressed to tree amplitudes $|C_P/T_V| > 1$, where the subscript denotes the meson (P or V) containing the spectator quark. The present data for the Cabibbo-favored decays are compatible with 12 distinct sets of solutions for the amplitudes T_P, C_P, E_P, A_P, T_V, C_V, E_V, and A_V (up to discrete ambiguities). However, on the basis of experimental branching ratios for singly Cabibbo-suppressed decays we were able to choose two sets of solutions giving substantially lower values for χ^2 than the other ten.

Our predictions of the branching ratios for singly Cabibbo-suppressed decays deviate from the available experimental data in several cases, such as those in the first four lines of Table VI. This shows that flavor SU(3) is not an exact symmetry. However flavor SU(3) breaking, though present, is no worse in $D \to PV$ decays than in the $D \to PP$ decays discussed in Ref. [4].

Our prediction for the $D_s^+ \to \eta'\rho^+$ branching ratio is much lower than the available experimental value. Either there are additional contributions to η' production which we have neglected, or the experimental situation needs to be reevaluated.

Our analysis of the singly Cabibbo-suppressed decays shows that processes such as $D^0 \to \pi^0\omega$ can be used to distinguish between the two most likely amplitude solutions. The mean values predicted for the branching ratios of these processes differ by nearly a factor of 3 in the two solutions, but experimental data are not yet available to resolve this problem.

The branching ratios predicted for doubly Cabibbo-suppressed decays are close to the experimental values in the two cases for which data are available. A precise measurement of a few of the other branching ratios may help select one of the two most-favored amplitude solutions.

Finally, factorization computations of the tree amplitudes agree with results obtained in direct analyses. However, a more precise calculation of the amplitudes using the factorization assumption could be done if data on the relevant form factors were available.

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