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Radiative corrections on the $B \to P$ form factors with chiral current in the light-cone sum rules

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Based on the approach of the vector form factor $F^{+}_{B\to\pi,K}(q^2)$ in our previous papers, we extend the calculation of the radiative corrections to the $B \to P$ (P stands π, K, and all light pseudoscalar mesons) scalar and tensor form factors $F^{0,T}_{B\to P}(q^2)$ with chiral current in the light-cone sum rules (LCSRs). The most uncertain twist-3 contributions to the $B \to P$ form factors can be naturally eliminated through a properly designed correlator. We present the next-leading-order formulas of $F^{+,0,T}_{B\to P}(q^2)$ with the b-quark pole mass that is universal. It has been shown that our results are simpler and less uncertain under the same parameter regions since we only need to calculate the next-leading order on the twist-2 part from the obtained LCSR. Second, we obtain $f^{+,0}_{B\to\pi}(0) = 0.260^{+0.059}_{-0.040}$, $f^{T}_{B\to\pi}(0) = 0.276^{+0.052}_{-0.039}$, $f^{+,0}_{B\to K}(0) = 0.334^{+0.094}_{-0.069}$, and $f^{T}_{B\to K}(0) = 0.379^{+0.092}_{-0.077}$ at $q^2 = 0$ and the $SU_f(3)$-breaking effects are discussed too.

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I. INTRODUCTION

The form factors of heavy-to-light transitions at large and intermediate energies are among the most important applications of QCD light-cone sum rule (LCSR), since the validity of the LCSR approach is restricted to the large meson energy ($E_P \gg \Lambda_{\rm QCD}$) via the relation $q^2 = m_B^2 - 2m_B E_P$. In literature there are several approaches to calculate the $B \to$ light meson transition form factors in addition to the QCD LCSR approach, such as the lattice QCD technique and the perturbative QCD (PQCD) approach. These approaches are complementary to each other, since they are adaptable in different energy regions, and by combining the results from these three methods, one may obtain a full understanding of the $B \to$ light meson transition form factors in its whole physical region [1–4]. Since the LCSR is restricted to small and moderate q^2, a better LCSR shall present a better connection to both the PQCD and the lattice QCD results, and then a better understanding of these form factors.

How to "design" a proper correlator for these heavy-to-light form factors is a tricky problem. If the correlator is chosen properly, one can simplify the LCSR greatly. As for the $B \to$ light pseudoscalar mesons, the commonly adopted correlators are usually defined as

$$\Pi^{\pm}_{\mu}(p, q) = i \int d^4x e^{iq\cdot x}\langle P(p)| \times T\{\bar{q}(x)\gamma_\mu b(x), \bar{b}(0) i m_b \gamma_5 q'(0)\}|0\rangle \quad (1)$$

and

$$\Pi^{T}_{\mu}(p, q) = i \int d^4x e^{iq\cdot x}\langle P(p)| \times T\{\bar{q}(x) i\sigma_{\mu\nu} q^\nu b(x), \bar{b}(0) i m_b \gamma_5 q'(0)\}|0\rangle, \quad (2)$$

where $q(x)$ and $q'(0)$ stand for the light quark fields that form the pseudoscalar mesons. By taking such conventional correlation functions, it has been found that the main uncertainties in estimation of the $B \to P$ form factors come from the different twist structures of pion/kaon wave functions, and most importantly, the twist-2 and twist-3 contributions should be treated on equal footing [5–7]. Thus, one has to calculate both the twist-2 and twist-3 contributions up to one-loop accuracy in order to obtain a consistent one-loop estimation of the form factors.

On the other hand, by taking proper chiral currents into the correlator, one can directly eliminate the most uncertain twist-3 terms, and then only needs to calculate the twist-2 contribution to next-to-leading order (NLO) accuracy [8–10]. At the present, the vector form factors $f^{+}_{B\to\pi,K}(q^2)$ have been calculated with the chiral current in the LCSR up to NLO [3,9]. It can be found that the scalar and penguin form factors $f^{0,T}_{B\to\pi,K}(q^2)$ shall be important in due cases, e.g. the penguin form factors shall give sizable contributions to $B \to P l^+ l^-$ or $B \to K^*\gamma$ [11]. So it is interesting to extend the previous study to all the $B \to P$ (P stands for π, K, and all light pseudoscalar mesons) transition form factors $f^{+,0,T}_{B\to P}(q^2)$ with the chiral current in the LCSR up to one-loop accuracy. Furthermore, it may also be interesting to know to what degree the different choices of correlator shall affect the final LCSRs, which is another purpose of the present paper.

The paper is organized as follows. In Sec. II, we present the calculation technology to obtain the LCSRs for the

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$B \to P$ transition form factors $f^{+,0,T}_{B\to P}(q^2)$ with chiral currents, where the $SU_f(3)$-breaking effects for the kaonic case will be explained in due places. Numerical results and discussions are presented in Sec. III, where the uncertainties of form factors under the present LCSRs shall be discussed. The comparison with other approaches will be presented in Sec. IV. Section V is reserved for a summary.

II. CALCULATION TECHNOLOGY FOR THE $B \to P$ TRANSITION FORM FACTORS WITH PROPER CHIRAL CURRENTS

A. A definition of $f^{+,0,T}_{B\to P}$

Based on the previous calculation about the transition form factor $B \to \pi/K$, we present the formulas for the $B \to P$ transition form factors for generality such that these formulas can also be conveniently extended for other light pseudoscalar form factors like $B \to \eta$ and $B \to \eta'$ form factors. With default, we adopt the chiral limit $p_P^2 = m_P^2 = 0$, but point out how to include the $SU_f(3)$-breaking effects for the $B \to K$ form factors in due places, i.e. the dominant $SU_f(3)$-breaking effects will be discussed with the newly obtained K meson distribution amplitudes [12]. The hadronic matrix elements for the $B \to P$ transition form factors are parametrized as

$$\begin{aligned}\langle P(p_P)|\bar{q}'\gamma^\mu b|\bar{B}(p_B)\rangle &= f^+_{B\to P}(q^2)\left(P^\mu - \frac{P\cdot q}{q^2}q^\mu\right) + f^0_{B\to P}(q^2)\frac{P\cdot q}{q^2}q_\mu \\ &= 2f^+_{B\to P}(q^2)p_P^\mu + [f^+_{B\to P}(q^2) + f^-_{B\to P}(q^2)]q_\mu, \langle P(p_P)|\bar{q}' i\sigma_{\mu\nu}q^\nu b|\bar{B}(p_B)\rangle \\ &= \frac{f^T_{B\to P}(q^2)}{m_B + m_P}[P\cdot q q_\mu - q^2 P_\mu] \end{aligned} \tag{3}$$

with P representing the pseudoscalar, $P_\mu = (p_B + p_P)_\mu$, $q_\mu = (p_B - p_P)_\mu$, and $f^+_{B\to P}(q^2)$, $f^0_{B\to P}(q^2)$, $f^T_{B\to P}(q^2)$ stand for the vector, scalar, and tensor form factors, respectively. It can be found that the scalar form factor $f^0_{B\to P}(q^2)$ satisfies the following relation:

$$f^0_{B\to P}(q^2) = f^+_{B\to P}(q^2) + \frac{q^2}{m_B^2 - m_P^2} f^-_{B\to P}(q^2). \tag{4}$$

As for the LCSR calculation, different to the conventional choice of the correlation functions as shown in Eqs. (1) and (2), we choose the following chiral currents in the correlation functions:

$$\begin{aligned}\Pi^\pm_\mu(p,q) &= i\int d^4x e^{iq\cdot x}\langle P(p)|T\{\bar{q}(x)\gamma_\mu(1+\gamma_5)b(x), \bar{b}(0)im_b(1+\gamma_5)q'(0)\}|0\rangle, \\ &= \Pi^+(q^2,(p+q)^2)p_\mu + \Pi^-(q^2,(p+q)^2)q_\mu, \end{aligned} \tag{5}$$

$$\begin{aligned}\Pi^T_\mu(p,q) &= i\int d^4x e^{iq\cdot x}\langle P(p)|T\{\bar{q}(x)i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b(x), \bar{b}(0)im_b(1-\gamma_5)q'(0)\}|0\rangle, \\ &= \Pi^T(q^2,(p+q)^2)[(P\cdot q)q_\mu - q^2P_\mu], \end{aligned} \tag{6}$$

where $P = p + 2q$.

We calculate the form factors $f^{+,0,T}_{B\to P}(q^2)$ following the same calculation technology as described in Refs. [3,9], where the vector form factors $f^+_{B\to\pi,K}(q^2)$ have been calculated. For such purposes, we first give a simple extension to $f^+_{B\to P}(q^2)$ in the large spacelike momentum regions $(p+q)^2 - m_b^2 \ll 0$ and $q^2 \ll m_b^2$ for the momentum transfer, which correspond to the small light-cone distance $x^2 \approx 0$ and are required by the validity of operator product expansion (OPE). Then, we present the newly obtained results for the scalar and tensor form factors.

B. A simple extension to $f^+_{B\to P}$ within LCSR

The vacuum-to-meson matrix elements in terms of the pseudoscalar's LC distribution amplitudes (DAs) of different twist can be expanded by contracting the b-quark fields with the help of the full b-quark propagator within the background field:

$$\langle 0|Tb(x)\bar{b}(0)|0\rangle = i\int \frac{d^4k}{(2\pi)^4} e^{-ikx} \frac{\not{k} + m}{k^2 - m_b^2} - ig_s \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \cdot \int_0^1 dv \left[\frac{1}{2}\frac{m + \not{k}}{(k^2 - m_b^2)^2} G^{\mu\nu}(vx)\sigma_{\mu\nu} - \frac{1}{k^2 - m_b^2} v x_\mu G^{\mu\nu}(vx)\gamma_\nu\right], \tag{7}$$

where only the free propagator and the one-gluon terms are retained, $G_{\mu\nu}$ stands for the background gluonic field strength, and g_s denotes the strong coupling constant. The invariant amplitudes Π^+ can be obtained by substituting the b-quark propagator and the corresponding LC wave functions, and completing the integrations over x and k.

The OPE results for the invariant amplitudes Π^+ can be represented as a sum of LO and NLO parts:

$$\Pi^+(q^2, (p+q)^2) = \Pi_0^+(q^2, (p+q)^2) + \frac{\alpha_s C_F}{4\pi}\Pi_1^+(q^2, (p+q)^2), \tag{8}$$

where $\Pi_0^+(q^2, (p+q)^2)$ and $\Pi_1^+(q^2, (p+q)^2)$ stands for the LO and the NLO contributions, respectively. As for the LO invariant amplitude, we obtain

$$\Pi_0^+(q^2, (p+q)^2) = 2f_P m_b^2 \left[\int_0^1 \frac{du}{u}\frac{\varphi_P(u)}{\Delta} - \int_0^1 \frac{du}{u^3}\frac{m_b^2}{2\Delta^3}\phi_{4P}(u) + \int_0^1 \frac{du}{u\Delta^2} G_{4P}(u) + \int_0^1 dv \int D\alpha_i \frac{2\Psi_{4P}(\alpha_i) + 2\tilde{\Psi}_{4P}(\alpha_i) - \Phi_{4P}(\alpha_i) - \tilde{\Phi}_{4P}(\alpha_i)}{\Delta^2(\alpha_1 + v\alpha_3)^2}\right], \tag{9}$$

where the parameters are defined as $\Delta = s - (p+q)^2$ ($s = [q^2 + (m_b^2 - q^2)/u]$), $G_{4P}(u) = -\int_0^u dv \psi_{4P}(v)$, and $D\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3)$. Here φ_P is the twist-2 LC wave function, and $\phi_P(u)$, $\psi_{4P}(u)$, $\Psi_{4P}(\alpha_i)$, $\tilde{\Psi}_{4P}(\alpha_i)$, $\Phi_{4P}(\alpha_i)$, and $\tilde{\Phi}_{4P}(\alpha_i)$ are twist-4 LC wave functions defined in the same way as the pionic case that have been defined in Ref. [12], whose explicit forms are put in the Appendix. It is found that only the twist-2 and twist-4 contributions are contained in the above expressions, and the twist-3 terms are rightly eliminated by taking the present adopted chiral currents within the correlators.

Since the most uncertain twist-3 contributions are eliminated and the twist-4 contribution itself is quite small, we only need to consider the NLO correction to the twist-2 terms. The NLO invariant amplitude Π_1^+ for the twist-2 contribution can be written in the following factorized form:

$$\Pi_1^+(q^2, (p+q)^2) = -f_P \int_0^1 du T_1^+(q^2, (p+q)^2, u)\varphi_P(u), \tag{10}$$

where by taking m_b to be the b-quark pole mass, the NLO hard scattering amplitudes T_1^+ can be written as

$$T_1^+(r_1, r_2, u) = \frac{6}{1-\rho}\left(2 - \ln\frac{m_b^2}{\mu^2}\right) - \frac{4}{1-\rho}[2G(\rho) - G(r_1) - G(r_2)] + \frac{4}{(r_1 - r_2)^2}\left\{\frac{1 - r_2}{u}[G(\rho) - G(r_1)] + \frac{1 - r_1}{\bar{u}}[G(\rho) - G(r_2)]\right\} + 2\frac{\rho + (1-\rho)\ln(1-\rho)}{\rho^2} - \frac{4}{1-\rho}\frac{(1 - r_2)\ln(1 - r_2)}{r_2} - \frac{4}{\rho - r_2}\left[\frac{(1-\rho)\ln(1-\rho)}{\rho} - \frac{(1 - r_2)\ln(1 - r_2)}{r_2}\right], \tag{11}$$

with

$$\begin{aligned} \bar{u} &= 1 - u, \\ \rho &= [r_1 + u(r_2 - r_1) - u(1-u)M_P^2/m_b^2], \\ \mathrm{Li}_2(x) &= -\int_0^x \frac{dt}{t}\ln(1-t), \\ G(\rho) &= \mathrm{Li}_2(\rho) + \ln^2(1-\rho) - \ln(1-\rho)\left(1 - \ln\frac{m_b^2}{\mu^2}\right), \end{aligned} \tag{12}$$

where the dilogarithm function $\mathrm{Li}_2(x) = -\int_0^x \frac{dt}{t}\ln(1-t)$, $r_1 = q^2/m_b^2$, and $r_2 = (p+q)^2/m_b^2$.

Next, the QCD LCSR for $f_{B\to P}^+(q^2)$ can be schematically written as

$$f_B f_{B\to P}^+(q^2) = \frac{1}{2m_B^2}\int_{m_b^2}^{s_0} e^{(m_B^2 - s)/M^2} \times [\rho_{T2}^+(s, q^2) + \rho_{T4}^+(q^2)]ds, \tag{13}$$

where $\rho_{T2}^+(s, q^2)$ is the contribution from the twist-2 DA and $\rho^+_{T4}(q^2)$ is for twist-4 DA, f_B is the B-meson decay constant. The Borel parameter M^2 and the continuum threshold s_0 are determined such that the resulting form factor does not depend too much on the precise values of these parameters; in addition the continuum contribution, which is the part of the dispersive integral from s_0 to ∞ that has been subtracted from both sides of the equation, should not be too large, e.g. less than 30% of the total dispersive integral.

As for the LO twist-2 and twist-4 contributions, we obtain

$$f_B f^+_{B\to P}(q^2)|_{\rm LO} = \frac{m_b^2 f_P}{m_B^2} e^{m_B^2/M^2}\bigg\{\int_\triangle^1 du e^{-\{[m_b^2-\bar{u}(q^2-um_P^2)]/(uM^2)\}}\bigg[\frac{\varphi_P(u)}{u}+\frac{G_{4P}(u)}{uM^2}-\frac{m_b^2\phi_{4P}(u)}{4u^3M^4}\bigg] + \int_0^1 dv\int D\alpha_i \frac{\theta(\alpha_1+v\alpha_3-\Delta)}{(\alpha_1+v\alpha_3)^2M^2} e^{-\{m_b^2-(1-\alpha_1-v\alpha_3)[q^2-(\alpha_1+v\alpha_3)m_P^2]\}/[M^2(\alpha_1+v\alpha_3)]} \times[2\Psi_{4P}(\alpha_i)+2\tilde{\Psi}_{4P}(\alpha_i)-\Phi_{4P}(\alpha_i)-\tilde{\Phi}_{4P}(\alpha_i)]\bigg\}, \tag{14}$$

where $\triangle = (m_b^2-q^2)/(s_0-q^2)$ for $M_P=0$; $\triangle = [\sqrt{(s_0-q^2-M_P^2)^2+4M_P^2(m_b^2-q^2)}-(s_0-q^2-M_P^2)]/(2M_P^2)$ for $M_P\neq 0$.

As for the NLO twist-2 contribution, it is convenient to write the NLO $\rho^+_{T2}(s,q^2)$ in the following form:

$$\rho^+_{T2}(s,q^2)|_{\rm NLO} = -\frac{f_P}{\pi}\bigg(\frac{\alpha_s C_F}{4\pi}\bigg)\int_0^1 du\phi_P(u,\mu)\,{\rm Im}T_1^+\bigg(\frac{q^2}{m_b^2},\frac{s}{m_b^2},u,\mu\bigg), \tag{15}$$

where

$$\frac{1}{4\pi}{\rm Im}_s T_1^+ = \theta(1-\rho)\bigg[\frac{L_2(r_2)}{\rho-1}\bigg|_+ + \frac{1-r_1}{(r_2-r_1)(r_2-\rho)}L_1(r_2)-\frac{r_2-1}{(r_2-\rho)r_2}\bigg] + \theta(\rho-1)\bigg[\frac{L_2(r_2)-2L_1(\rho)}{\rho-1}\bigg|_+ + \frac{1-r_1}{(r_2-r_1)(r_2-\rho)}L_1(r_2)+\frac{1+\rho-r_1-r_2}{(r_1-\rho)(r_2-\rho)}L_1(\rho)+\frac{1}{2\rho}\bigg(1-\frac{1}{\rho}-\frac{2}{r_2}\bigg)\bigg] + \delta(\rho-1)\bigg[\bigg(\ln\frac{r_2-1}{1-r_1}\bigg)^2-\bigg(\frac{1-r_2}{r_2}+\ln r_2\bigg)\ln\frac{(r_2-1)^2}{1-r_1}-\frac{3}{2}\ln\bigg(\frac{m_b^2}{\mu^2}\bigg)+{\rm Li}_2(r_1)-3{\rm Li}_2(1-r_2)+3-\frac{\pi^2}{2}\bigg], \tag{16}$$

for the case of $r_1<1$ and $r_2>1$. The operation " + " is defined by

$$\int d\rho f(\rho)\frac{1}{1-\rho}\bigg|_+ = \int d\rho[f(\rho)-f(1)]\frac{1}{1-\rho}. \tag{17}$$

The two functions $L_1(x)=\ln[\frac{(x-1)^2}{x}(m_b^2/\mu^2)]-1$ and $L_2(x)=\ln[\frac{(x-1)^2}{x}(m_b^2/\mu^2)]-\frac{1}{x}$ are introduced to make the formulas short. The above formulas are derived in the Feynman gauge and by regularizing both the ultraviolet and collinear divergences by the standard dimensional regularization in the $\overline{MS}$ scheme.

C. Calculation of $f^{0,T}_{B\to P}$ within LCSR

For convenience, we calculate the combined function $f^*_{B\to P}(q^2)=[f^+_{B\to P}(q^2)+f^-_{B\to P}(q^2)]$ first and then derive $f^0_{B\to P}$ with the help of Eq. (4). The OPE results for the needed invariant amplitudes $\Pi^{-,T}$ can be represented as a sum of LO and NLO parts:

$$\Pi^{-,T}(q^2,(p+q)^2) = \Pi_0^{-,T}(q^2,(p+q)^2) + \frac{\alpha_s C_F}{4\pi}\Pi_1^{-,T}(q^2,(p+q)^2), \tag{18}$$

where $\Pi_0^{-,T}(q^2,(p+q)^2)$ and $\Pi_1^{-,T}(q^2,(p+q)^2)$ stand for the LO and NLO contributions, respectively. As for the LO invariant amplitudes, we obtain

$$\Pi_0^-(q^2,(p+q)^2) = 2f_P m_b^2\int_0^1\frac{du}{u^2}\frac{1}{\Delta^2}G_{4P}(u), \tag{19}$$

$$\Pi_0^T(q^2, (p+q)^2) = 2m_b f_P \Big[\int_0^1 \frac{du}{u} \frac{\varphi_P(u)}{\Delta} - \int_0^1 \frac{1}{4u^2\Delta^2}\Big(1 + \frac{2m_b^2}{u\Delta}\Big)\phi_{4P}(u) + \int_0^1 dv \int \mathcal{D}\alpha_i \frac{2\Psi_{4P}(\alpha_i) - (1-2v)\Phi_{4P}(\alpha_i) + 2(1-2v)\tilde{\Psi}_{4P}(\alpha_i) - \tilde{\Phi}_{4P}(\alpha_i)}{\Delta^2(\alpha_1 + v\alpha_3)^2} \Big]. \quad (20)$$

Similar to the case of $\Pi_0^+(q^2, (p+q)^2)$, one may also observe that only the twist-2 and twist-4 contributions are contained in the above expressions, and the twist-3 terms are rightly eliminated by taking the present adopted chiral currents within the correlators. The NLO invariant amplitude $\Pi_1^{-,T}$ for the twist-2 contribution can be written in the following factorized form:

$$\Pi_1^{-,T}(q^2, (p+q)^2) = -f_P \int_0^1 du T_1^{-,T}(q^2, (p+q)^2, u)\varphi_P(u), \quad (21)$$

where by taking m_b to be the b-quark pole mass, we have

$$T_1^-(r_1, r_2, u) = \frac{2(r_1 - r_2)[r_1 + (1-r_1)\ln(1-r_1)]}{r_1^2(1-\rho)} + \frac{2(1-r_1)(r_1+r_2)\ln(1-r_1)}{r_1^2(r_1-\rho)} + \frac{4(1-r_2)\ln(1-r_2)}{r_2(\rho - r_2)} - \frac{2(1-\rho)(r_2+\rho)\ln(1-\rho)}{u(\rho - r_2)\rho^2} + \frac{2(r_1 - r_2)}{r_1\rho} \quad (22)$$

and

$$T_1^T(r_1, r_2, u) = \frac{4}{1-\rho}\Big(3 - 2\ln\frac{m_b^2}{\mu^2}\Big) - \frac{4}{1-\rho}[2G(\rho) - G(r_1) - G(r_2)] - \frac{4}{(r_1-r_2)^2}\Big(\frac{1-r_2}{u}[G(r_1) - G(\rho)] + \frac{1-r_1}{\bar{u}}[G(r_2) - G(\rho)]\Big) - \frac{4}{1-\rho}\Big(\frac{1-r_2}{r_2}\ln(1-r_2) - \frac{1-r_1}{r_1}\ln(1-r_1)\Big) + 4\Big(\frac{1-r_1}{r_1(\rho - r_1)}\Big)\ln\Big(\frac{1-r_1}{1-\rho}\Big) - 4\Big(\frac{1-r_2}{(\rho - r_2)r_2}\Big)\ln\Big(\frac{1-r_2}{1-\rho}\Big) - 2\Big(\frac{\rho + \ln(1-\rho)}{\rho^2}\Big) + \Big(\frac{-4r_1 + 2r_2r_1 + 4r_2}{r_1r_2\rho}\Big)\ln(1-\rho). \quad (23)$$

Schematically, the QCD LCSRs for $f_{B\to P}^{*,T}$ can be written as

$$f_B f_{B\to P}^*(q^2) = \frac{1}{m_B^2}\int_{m_b^2}^{s_0} e^{(m_B^2 - s)/M^2}[\rho_{T2}^*(s, q^2) + \rho_{T4}^*(q^2)]ds, \quad (24)$$

$$f_B f_{B\to P}^T(q^2) = \frac{m_B + m_P}{2m_B^2}\int_{m_b^2}^{s_0} e^{(m_B^2 - s)/M^2}[\rho_{T2}^T(s, q^2) + \rho_{T4}^T(q^2)]ds. \quad (25)$$

As for the LO twist-2 and twist-4 contributions, with the help of Eqs. (19) and (20), we obtain

$$f_B f_{B\to P}^*(q^2)|_{\mathrm{LO}} = \frac{2m_b^2 f_P}{m_B^2} e^{m_B^2/M^2}\int_\triangle^1 du e^{-[m_b^2 - \bar{u}(q^2 - um_P^2)]/(uM^2)}\Big[\frac{G_{4P}(u)}{u^2M^2}\Big], \quad (26)$$

$$f_B f_{B\to P}^T(q^2)|_{\mathrm{LO}} = \frac{(m_B + m_P)m_b f_P}{m_B^2} e^{m_B^2/M^2}\Big\{\int_\triangle^1 du e^{-[m_b^2 - \bar{u}(q^2 - um_P^2)]/(uM^2)}\Big[\frac{\varphi_P(u)}{u} - \frac{\phi_{4P}(u)}{4u^2M^2}\Big(1 + \frac{m_b^2}{uM^2}\Big)\Big] + \int_0^1 dv \int D\alpha_i \frac{\theta(\alpha_1 + v\alpha_3 - \Delta)}{(\alpha_1 + v\alpha_3)^2M^2} e^{-\{m_b^2 - (1-\alpha_1 - v\alpha_3)[q^2 - (\alpha_1 + v\alpha_3)m_P^2]\}/[M^2(\alpha_1 + v\alpha_3)]} \times [2\Psi_{4P}(\alpha_i) - (1-2v)\Phi_{4\pi}(\alpha_i) + 2(1-2v)\tilde{\Psi}_{4P}(\alpha_i) - \tilde{\Phi}_{4P}(\alpha_i)]\Big\}. \quad (27)$$

From the above equations, we immediately obtain the relations among $f_{B\to P}^{\pm,T}(q^2)$ at the LO and up to the twist-3 accuracy, i.e.

$$f^-_{B\to P}(q^2) = -f^+_{B\to P}(q^2) \quad \text{and} \quad f^T_{B\to P}(q^2) = \frac{m_B + m_P}{m_b} f^+_{B\to P}(q^2), \tag{28}$$

which agree with the conclusions drawn in Ref. [13]. Moreover, with the help of Eqs. (4) and (28), we obtain

$$f^0_{B\to P}(q^2) = \left[1 - \frac{q^2}{m_B^2 - m_P^2}\right] f^+_{B\to P}(q^2). \tag{29}$$

As for the NLO twist-2 contribution, the NLO $\rho^{*,T}_{T2}(s, q^2)$ can be written as

$$\rho^{*,T}_{T2}(s, q^2)|_{\rm NLO} = -\frac{f_P}{\pi}\left(\frac{\alpha_s C_F}{4\pi}\right)\int_0^1 du \phi_P(u, \mu) \times {\rm Im} T_1^{-,T}\left(\frac{q^2}{m_b^2}, \frac{s}{m_b^2}, u, \mu\right), \tag{30}$$

where

$$\frac{1}{2\pi}{\rm Im}_s T_1^- = \theta(1-\rho)\left[\frac{2(1-r_2)}{r_2(r_2-\rho)}\right] - \frac{\theta(\rho-1)}{r_1-\rho}\left[\frac{r_1-r_2}{\rho^2} - \frac{(2-r_2)(r_2-r_1)}{r_2\rho} + \frac{2(r_2-1)}{r_2}\right] + \delta(\rho-1)\left[1 - \frac{r_2}{r_1} - \frac{(r_1-1)(r_1-r_2)\ln(1-r_1)}{r_1^2}\right], \tag{31}$$

and

$$\frac{1}{4\pi}{\rm Im}_s T_1^T = \theta(1-\rho)\left[\frac{L_2(r_2)}{\rho-1}\Big|_+ - \frac{1-r_1}{(r_2-r_1)(\rho-r_2)}L_1(r_2) - \frac{r_2-1}{r_2(\rho-r_2)}\right] + \theta(\rho-1)\left[\frac{L_2(r_2)-2L_1(\rho)}{\rho-1}\Big|_+ - \frac{1-r_1}{(r_2-r_1)(\rho-r_2)}L_1(r_2) - \frac{r_1+r_2-\rho-1}{(r_1-\rho)(r_2-\rho)}L_1(\rho) - \frac{r_1-1}{r_1(\rho-r_1)} - \frac{2(r_2-r_1)+r_1r_2}{2r_1r_2\rho} + \frac{1}{2\rho^2}\right] + \delta(\rho-1)\left[\left(\ln\frac{r_2-1}{1-r_1}\right)^2 - \ln\frac{(r_2-1)^2}{1-r_1}\left(\ln r_2 + \frac{1}{r_2} - 1\right) + 3 - \frac{\pi^2}{2} - \left(1-\frac{1}{r_1}\right)\ln(1-r_1) - 2\ln\left(\frac{m_b^2}{\mu^2}\right) + {\rm Li}_2(r_1) - 3{\rm Li}_2(1-r_2)\right], \tag{32}$$

for the case of $r_1 < 1$ and $r_2 > 1$.

As a cross-check of the above NLO formulas for the twist-2 contributions, it can be found that our present results for $f^{+,*,T}_{B\to\pi}$ agree with Ref. [14] by transforming the formulas for the $\overline{MS}$ b-quark mass to be the ones for the b-quark one-loop pole mass, except for an overall factor 2.[1]

Here similar to the treatment of Refs. [3,15,16], we have adopted the b-quark pole mass to do the calculation. References [14,17] have argued to use the b-quark $\overline{MS}$ running mass other than the pole mass. Numerically, we shall show in due places that, if properly choosing the possible ranges for the undetermined parameters, these two treatments are in fact equivalent to each other within reasonable uncertainties. We prefer to take the pole quark mass, since the pole quark mass is universal that can be determined through proper potential model analysis or through lattice QCD calculation, while the running quark mass is process dependent, i.e. depends on the renormalization scheme and the renormalization scale of a particular process.

[1]The overall factor 2 comes from the different choices of correlation function.

III. NUMERICAL RESULTS FOR $f^{+,0,T}_{B\to\pi,K}(q^2)$ WITHIN THE QCD LCSR WITH CHIRAL CURRENT

A. Parameters and distribution amplitudes of the light mesons

First, we specify the input parameters used in the LCSRs for $B\to\pi$ and $B\to K$ transition form factors. For the needed meson masses and the light mesons' decay constants, we adopt the center values as listed by the Particle Data Group [18]:

$$f_\pi = 130.4\ {\rm MeV}, \qquad f_K = 155.5\ {\rm MeV},$$
$$M_B = 5.279\ {\rm GeV}, \qquad M_\pi = 139.570\ {\rm MeV},$$
$$M_K = 493.667\ {\rm MeV}.$$

As has been argued in the last section, we shall adopt the b-quark pole mass to do numerical calculation throughout the paper. As for the value of f_B, to be consistent with the present calculation technology, they should be determined by using the two-point sum rule with proper chiral currents up to NLO. Such a calculation has been done in Ref. [3], the interesting reader may turn to Ref. [3] for more calculation detail, and here we only quote some typical results as

TABLE I. The value of f_B (in units GeV) within the LCSRs with chiral currents up to NLO; the corresponding formulas can be found in Ref. [3], where m_b is taken to be the b-quark pole mass.

-	s_0 (GeV^2)	M^2	f_B (GeV)
$m_b = 4.75$ (GeV)	33.0	2.48	0.192
$m_b = 4.80$ (GeV)	32.6	2.28	0.169
$m_b = 4.85$ (GeV)	32.3	2.10	0.146

shown in Table I, where the one-loop pole mass m_b is taken to be (4.80 ± 0.05) GeV [19].

Naively, the leading twist-2 DAs ϕ_π and ϕ_K can be expanded as Gegenbauer polynomials as shown in the Appendix. The first two Gegenbauer moments, e.g. a_2^π and a_4^π for pion and a_1^K and a_2^K for kaon, have been studied with various processes. We adopt two constraints for $a_2^\pi(1\ \mathrm{GeV})$ and $a_4^\pi(1\ \mathrm{GeV})$, e.g. $a_2^\pi(1\ \mathrm{GeV}) + a_4^\pi(1\ \mathrm{GeV}) = 0.1 \pm 0.1$ [20] and $-\frac{9}{4}a_2^\pi(1\ \mathrm{GeV}) + \frac{45}{16}a_4^\pi(1\ \mathrm{GeV}) + \frac{3}{2} = 1.2 \pm 0.3$ [5,21], such that the allowed values of a_2^π and a_4^π are correlated and given by the rhomboid shown in Fig. 1. Note here we do not adopt the wider range of $a_2^\pi = 0.25 \pm 0.15$ as suggested by Ref. [12], since we prefer a more asymptoticlike pion DA as favored by a very recent QCD LCSR analysis of the $B \to \pi$ vector form factor [22]. The first Gegenbauer moment a_1^K has been studied by several references, e.g. Refs. [12,23–27], etc. For convenience, we quote the values for the twist-2 Gegenbauer moments of kaon as obtained from the average of those obtained in literature to do the discussion, $a_1^K(1\ \mathrm{GeV}) = 0.06 \pm 0.03$ and $a_2^K(1\ \mathrm{GeV}) = 0.25 \pm 0.15$ [12].

Furthermore, for the twist-2 DAs, we do not adopt the Gegenbauer expansion (A1), since its higher Gegenbauer moments are still determined with large errors whose contributions may not be too small, i.e. their contributions are comparable to that of higher twist structures [3]. As a compensation, we adopt the suggestion of deriving the pion and kaon DAs from their corresponding wave functions (WFs) by integrating over the transverse momentum [3]. The twist-2 pion and kaon WFs can be constructed on their first two Gegenbauer moments and on the Brodsky-Huang-Lepage prescription [28], i.e.,

$$\Psi_\pi(x, \mathbf{k}_\perp) = [1 + B_\pi C_2^{3/2}(2x-1) + C_\pi C_4^{3/2}(2x-1)] \times \frac{A_\pi}{x(1-x)} \exp\left[-\beta_\pi^2\left(\frac{\mathbf{k}_\perp^2 + m_q^2}{x(1-x)}\right)\right], \quad (33)$$

and

$$\Psi_K(x, \mathbf{k}_\perp) = [1 + B_K C_1^{3/2}(2x-1) + C_K C_2^{3/2}(2x-1)] \times \frac{A_K}{x(1-x)} \times \exp\left[-\beta_K^2\left(\frac{\mathbf{k}_\perp^2 + m_q^2}{x} + \frac{\mathbf{k}_\perp^2 + m_s^2}{1-x}\right)\right], \quad (34)$$

where $q = u$, d, $C_{1,2}^{3/2}(1-2x)$ are Gegenbauer polynomials. The constitute quark masses are set to be $m_q = 0.30$ GeV and $m_s = 0.45$ GeV. After doing the integration over the transverse momentum dependence, we obtain the twist-2 kaon DA, e.g. $\phi_K(x, \mu_b) = \int_{k_\perp^2 < \mu_b^2} \frac{d^2\mathbf{k}_\perp}{16\pi^3} \Psi_K(x, \mathbf{k}_\perp)$, where $\mu_b = 2.2$ GeV for the present case. The Gegenbauer moments $a_n^{\pi,K}(\mu_b)$ are defined as

$$a_n^{\pi,K}(\mu_b) = \frac{\int_0^1 dx \phi_{\pi,K}(1-x, \mu_b) C_n^{3/2}(2x-1)}{\int_0^1 dx 6x(1-x)[C_n^{3/2}(2x-1)]^2}. \quad (35)$$

The four unknown parameters can be determined by the first two Gegenbauer moments, the normalization condition $\int_0^1 dx \int_{k_\perp^2 < \mu_b^2} \frac{d^2\mathbf{k}_\perp}{16\pi^3} \Psi_{\pi,K}(x, \mathbf{k}_\perp) = 1$, and the constraint $\langle \mathbf{k}_\perp^2 \rangle_K^{1/2} \approx \langle \mathbf{k}_\perp^2 \rangle_\pi^{1/2} = 0.350$ GeV [29], where the average value of the transverse momentum square is defined as

$$\langle \mathbf{k}_\perp^2 \rangle_{\pi,K}^{1/2} = \frac{\int dx d^2\mathbf{k}_\perp |\mathbf{k}_\perp^2| |\Psi_{\pi,K}(x, \mathbf{k}_\perp)|^2}{\int dx d^2\mathbf{k}_\perp |\Psi_{\pi,K}(x, \mathbf{k}_\perp)|^2}.$$

Some typical parameters for the pion and kaon WFs are presented in Tables II and III. A comparison with the conventional Gegenbauer expansion DAs is presented in Fig. 2. The remaining parameters for the twist-4 DA's $(\delta_{\pi,K}^2, \epsilon_{\pi,K})$ are presented in Table IV, which are taken from [12].

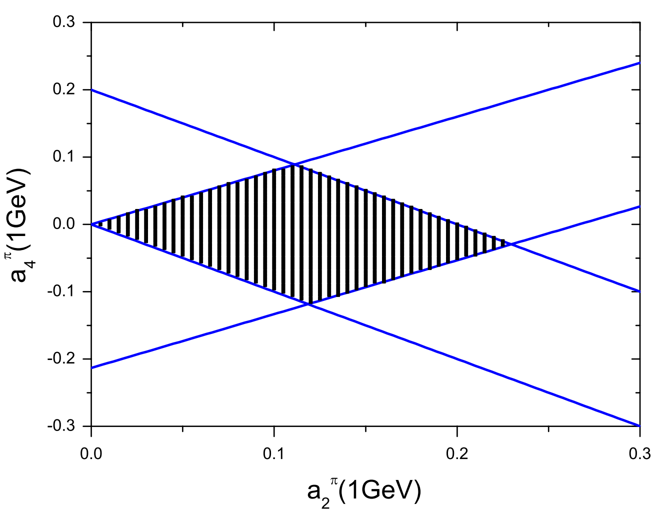

FIG. 1 (color online). $a_2^\pi(1\ \mathrm{GeV})$ and $a_4^\pi(1\ \mathrm{GeV})$ as determined from the two constraints adopted in the body of the text, where the rhomboid stands for the allowable range.

B. Properties of $f_{B\to\pi,K}^{+,0,T}(q^2)$ within QCD LCSR with chiral current

Taking the above-mentioned parameters, we discuss the properties of $f_{B\to\pi,K}^{+,0,T}(q^2)$ within QCD LCSRs with chiral

TABLE II. Pion twist-2 wave function parameters for some typical Gegenbauer moments, where $\mu_0 = 1$ GeV. Note the obtained WF parameters are for $\mu = 2.2$ GeV.

$a_2^\pi(\mu_0)$	0.00		0.115		0.230
$a_4^\pi(\mu_0)$	0.00	0.092	−0.015	−0.120	−0.030
A_π (GeV^{-2})	226.0	196.7	199.4	199.1	173.8
B_π	−0.079	−0.024	−0.018	−0.014	0.043
C_π	0.027	0.073	0.012	−0.050	−0.006 56
β_π (GeV)	0.902	0.862	0.868	0.870	0.832

TABLE III. Kaon twist-2 wave function parameters for some typical Gegenbauer moments, where $\mu_0 = 1$ GeV. Note the obtained WF parameters are for $\mu = 2.2$ GeV.

$a_1^K(\mu_0)$		0.09			0.06			0.03	
$a_2^K(\mu_0)$	0.40	0.25	0.10	0.40	0.25	0.10	0.40	0.25	0.10
A_K (GeV^{-2})	171.2	209.3	253.8	172.6	211.8	255.9	173.9	213.5	258.1
B_K	0.0845	0.0732	0.0588	0.107	0.0966	0.0825	0.130	0.119	0.106
C_K	0.203	0.122	0.0371	0.207	0.127	0.0422	0.211	0.132	0.0471
β_K (GeV)	0.774	0.821	0.869	0.775	0.823	0.870	0.776	0.824	0.871

current. At the maximum recoil region, $q^2 = 0$, by varying the parameters within their reasonable regions, we obtain

$$f^{+,0}_{B\to\pi}(0) = 0.260^{+0.059}_{-0.040}, \qquad f^{T}_{B\to\pi}(0) = 0.276^{+0.052}_{-0.039} \tag{36}$$

and

$$f^{+,0}_{B\to K}(0) = 0.334^{+0.094}_{-0.069}, \qquad f^{T}_{B\to K}(0) = 0.379^{+0.092}_{-0.077}. \tag{37}$$

By comparing $B \to K$ form factors with the $B \to \pi$ form factors, we find the following $SU_f(3)$-breaking effects among the $B \to$ light form factors:

$$\frac{f^{+,0}_{B\to K}(0)}{f^{+,0}_{B\to\pi}(0)} = 1.28^{+0.06}_{-0.08}, \qquad \frac{f^{T}_{B\to K}(0)}{f^{T}_{B\to\pi}(0)} = 1.37^{+0.07}_{-0.02}. \tag{38}$$

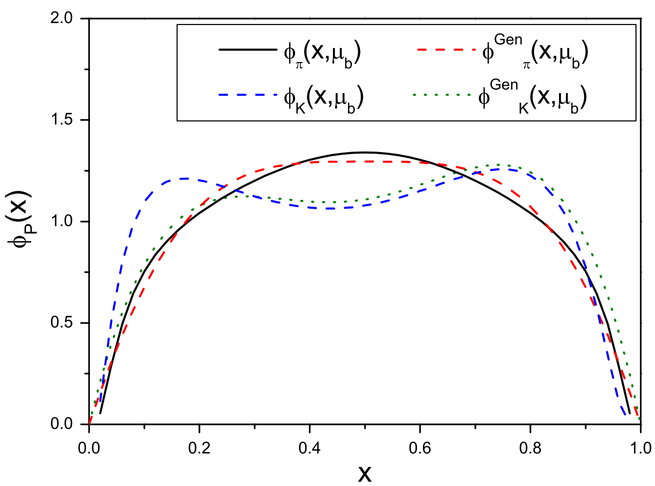

FIG. 2 (color online). Typical distribution amplitudes $\phi_P(x)$ at $\mu_b = 2.2$ GeV, where $\phi_\pi(x)$ and $\phi_K(x)$ are for $a_2^\pi(1\text{ GeV}) = 0.115$ and $a_4^\pi(1\text{ GeV}) = -0.015$, $a_1^K(1\text{ GeV}) = 0.06$, and $a_2^K(1\text{ GeV}) = 0.25$, respectively, and $\phi_\pi^{\text{gen}}(x)$ and $\phi_K^{\text{gen}}(x)$ are for Gegenbauer expansion (A1) with the same Gegenbauer moments.

It is found that this larger $SU_f(3)$-breaking effect is obtained by taking a larger $a_2^K(1\text{ GeV}) \in [0.10, 0.40]$; if taking a smaller $a_2^K(1\text{ GeV})$, then one can obtain a smaller $SU_f(3)$-breaking effect, e.g. $[f^{+,0}_{B\to K}(0)]/[f^{+,0}_{B\to\pi}(0)] = 1.13 \pm 0.03$ for $a_2^K(1\text{ GeV}) \in [0.05, 0.10]$ [3] and $[f^{+,0}_{B\to K}(0)]/[f^{+,0}_{B\to\pi}(0)] = 1.08^{+0.19}_{-0.17}$ for $a_2^K(1\text{ GeV}) \in [-0.11, 0.27]$ [30]. Note that this larger $SU_f(3)$-breaking effect is consistent with the some other LCSR calculation as Refs. [17,31] and a recently relativistic treatment that is based on the study of the Dyson-Schwinger equation in QCD, i.e. $[f^{+,0}_{B\to K}(0)]/[f^{+,0}_{B\to\pi}(0)] = 1.23$ [32]. So a better determination of $a_2^K(1\text{ GeV})$ will be helpful to obtain a better understanding of the $SU_f(3)$-breaking effect.

We show the $B \to \pi$ vector, scalar, and tensor form factors with their corresponding errors in Fig. 3, where the center dashed line is for $m_b = 4.80$ GeV, $a_2^\pi(1\text{ GeV}) = 0.115$, $a_4^\pi(1\text{ GeV}) = -0.015$, $\delta_\pi^2 = 0.18\text{ GeV}^2$, and $\epsilon_\pi = 0.525$. For $f^{+,0}_{B\to\pi}(q^2)$, the lower edge of the shaded band is obtained by setting $m_b = 4.75$ GeV, $a_2^\pi(1\text{ GeV}) = 0.0$, $a_4^\pi(1\text{ GeV}) = 0.0$, $\delta_\pi^2 = 0.12\text{ GeV}^2$, and $\epsilon_\pi = 0.2625$ and the upper edge is ob-

TABLE IV. Input parameters for the pion and kaon twist-4 DAs' [12].

Twist	π	$\mu = 1$ GeV	K	$\mu = 1$ GeV
4	δ_π^2	$0.18 \pm 0.06\text{ GeV}^2$	δ_K^2	$0.20 \pm 0.06\text{ GeV}^2$
	ϵ_π	$\frac{21}{8}(0.2 \pm 0.1)$	ϵ_K	$\frac{21}{8}(0.2 \pm 0.1)$

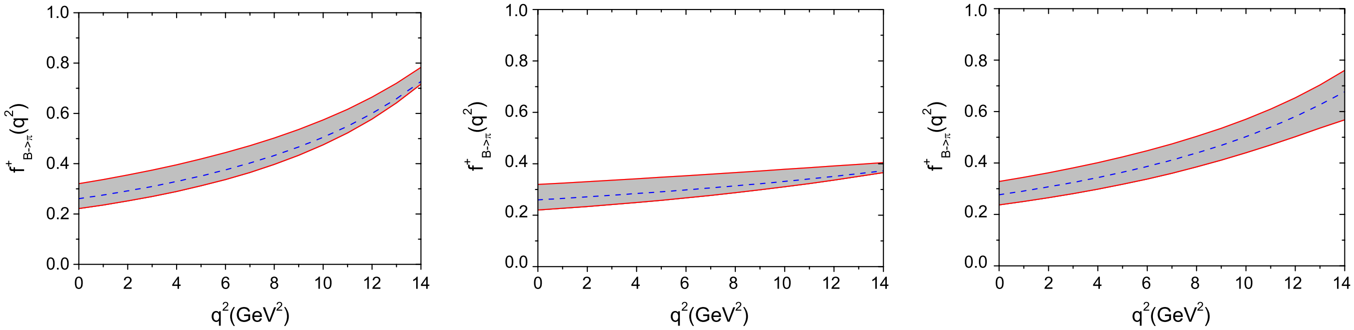

FIG. 3 (color online). Uncertainties of the $B \to \pi$ form factors $f^{+,0,T}_{B\to\pi}(q^2)$ within the allowable regions for the undetermined parameters. The center dashed line is for $m_b = 4.80$ GeV, $a_2^\pi(1\text{ GeV}) = 0.115$, $a_4^\pi(1\text{ GeV}) = -0.015$, $\delta_\pi^2 = 0.18\text{ GeV}^2$, and $\epsilon_\pi = 0.525$.

tained by setting $m_b = 4.85$ GeV, $a_2^\pi(1\text{ GeV}) = 0.230$, $a_4^\pi(1\text{ GeV}) = -0.030$, $\delta_\pi^2 = 0.24\text{ GeV}^2$, and $\epsilon_\pi = 0.7875$. For $f^T_{B\to\pi}(q^2)$, the lower edge of the shaded band is obtained by setting $m_b = 4.75$ GeV, $a_2^\pi(1\text{ GeV}) = 0.0$, $a_4^\pi(1\text{ GeV}) = 0$, $\delta_\pi^2 = 0.24\text{ GeV}^2$, and $\epsilon_\pi = 0.7875$ and the upper edge is obtained by setting $m_b = 4.85$ GeV, $a_2^\pi(1\text{ GeV}) = 0.230$, $a_4^\pi(1\text{ GeV}) = -0.030$, $\delta_\pi^2 = 0.12\text{ GeV}^2$, and $\epsilon_\pi = 0.2625$. This difference is caused by the fact that the twist-4 structures lead to positive and negative contributions to the $f^{+,0}_{B\to\pi}(q^2)$ and $f^T_{B\to\pi}(q^2)$, respectively. The main uncertainties of the form factors are caused by the value of m_b and a_2^π, and it can be found that all the $B \to \pi$ form factors shall increase with the increment of m_b and a_2^π. Furthermore, we obtain $f^T_{B\to\pi}(0)/f^+_{B\to\pi}(0) \in [1.03, 1.08]$. This shows that the NLO correction shall affect the usual simple relation (28), e.g. $[f^T_{B\to\pi}(0)/f^+_{B\to\pi}(0)] = (m_B + m_\pi)/m_b \in [1.12, 1.14]$, to a certain degree. Furthermore, we show contributions to the $B \to \pi$ form factors $f^{+,0,T}_{B\to\pi}(q^2)$ from the different parts in Fig. 4, where all the parameters are taken to be their center values. For $f^{+,0}_{B\to\pi}(q^2)$, it can be found that the LO twist-2, the NLO twist-2, and the LO twist-4 contributions are positive, more specifically at $q^2 = 0$, they are about 68%, 26%, and 6%, respectively. $f^0_{B\to\pi}(q^2)$ is very close to the asymptotic LO result derived from Eq. (29), which is caused by the fact that the LO twist-2 gives zero contribution to the sum of the form factor $[f^+_{B\to\pi} + f^-_{B\to\pi}]$ and then $[f^+_{B\to\pi} + f^-_{B\to\pi}]$ gives a negligible contribution to $f^0_{B\to\pi}(q^2)$. For $f^T_{B\to\pi}(q^2)$, the LO twist-2 and the NLO twist-2 give a positive contribution while the LO twist-4 gives a negative contribution, more specifically at $q^2 = 0$, they are about 72%, 30%, and -2% respectively.

Second, we show the $B \to K$ form factors with their corresponding errors in Fig. 5, where the center dashed line is for $m_b = 4.80$ GeV, $a_1^K(1\text{ GeV}) = 0.06$, $a_2^K(1\text{ GeV}) = 0.25$, $\delta_K^2 = 0.20\text{ GeV}^2$, and $\epsilon_K = 0.525$. For $f^{+,0}_{B\to K}(q^2)$, the lower edge of the shaded band is obtained by setting $m_b = 4.75$ GeV, $a_1^K(1\text{ GeV}) = 0.09$, $a_2^K(1\text{ GeV}) = 0.10$, $\delta_K^2 = 0.14\text{ GeV}^2$, and $\epsilon_K = 0.2625$, and the upper edge is obtained by setting $m_b = 4.85$ GeV, $a_1^K(1\text{ GeV}) = 0.03$, $a_2^K(1\text{ GeV}) = 0.40$, $\delta_K^2 = 0.26\text{ GeV}^2$, and $\epsilon_K = 0.7875$. For $f^T_{B\to K}(q^2)$, the lower edge of the shaded band is obtained by setting $m_b = 4.75$ GeV, $a_1^K(1\text{ GeV}) = 0.09$, $a_2^K(1\text{ GeV}) = 0.10$, $\delta_K^2 = 0.26\text{ GeV}^2$, and $\epsilon_K = 0.7875$, and the upper edge is obtained by setting $m_b = 4.85$ GeV, $a_1^K(1\text{ GeV}) = 0.03$, $a_2^K(1\text{ GeV}) = 0.40$, $\delta_\pi^2 = 0.14\text{ GeV}^2$, and $\epsilon_K = 0.2625$. The main uncertainties are caused by the value of m_b, a_1^K, and a_2^K, and it can be found that all the $B \to K$ form factors shall increase with the increment of m_b and a_2^K, and decrease with the increment of a_1^K. As for the LO results,

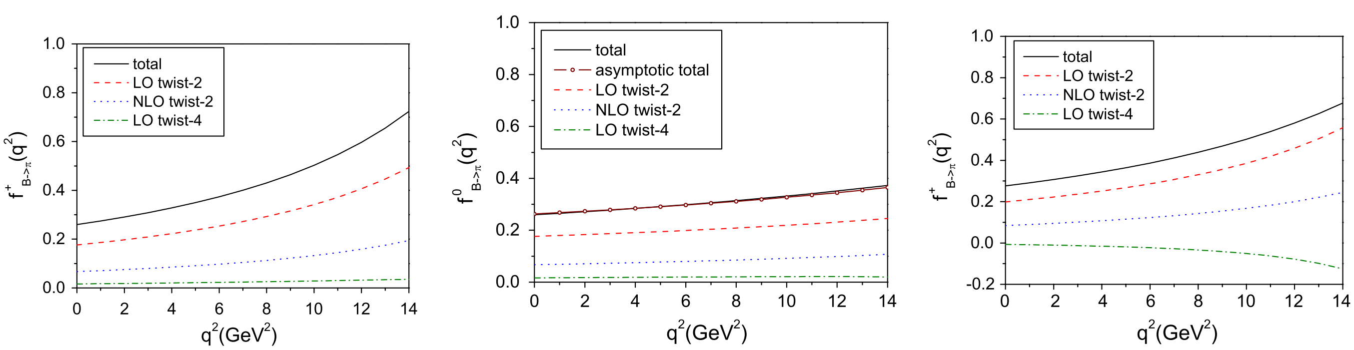

FIG. 4 (color online). Different parts' contributions to the $B \to \pi$ form factors $f^{+,0,T}_{B\to\pi}(q^2)$ for all the parameters taken to be their center values. The curve of asymptotic total in the middle figure stands for the LO $f^0_{B\to\pi}(q^2)$ up to twist-3 that is derived from Eq. (29).

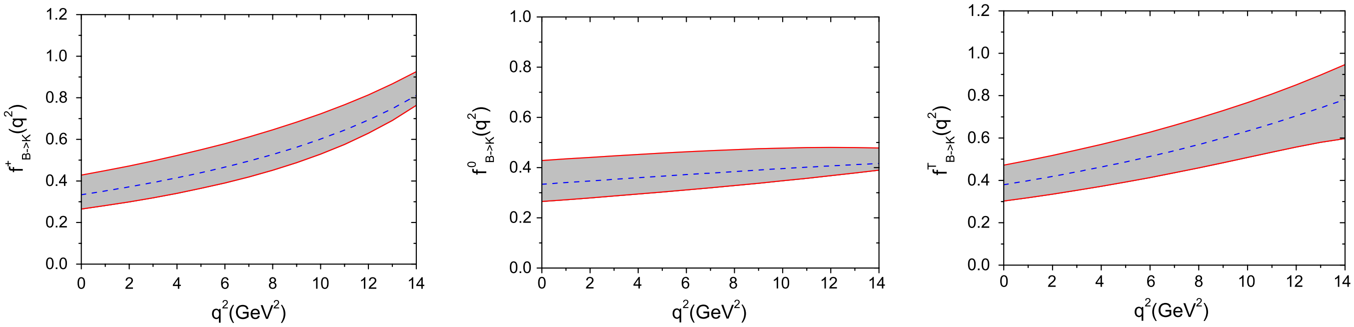

FIG. 5 (color online). Uncertainties of the $B \to K$ form factors $f^{+,0,T}_{B\to\pi}(q^2)$ within the allowable regions for the undetermined parameters. The center dashed line is for $m_b = 4.80$ GeV, $a_1^K(1\text{ GeV}) = 0.06$, $a_2^K(1\text{ GeV}) = 0.25$, $\delta_K^2 = 0.20\text{ GeV}^2$, and $\epsilon_K = 0.525$.

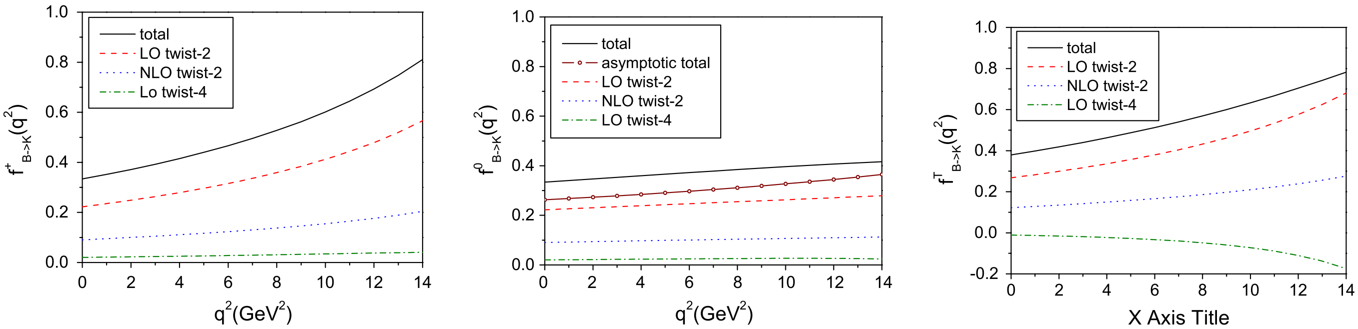

FIG. 6 (color online). Different parts' contributions to the $B \to K$ form factors $f^{+,0,T}_{B\to K}(q^2)$ for all the parameters taken to be their center values. The curve of asymptotic total in the middle figure stands for the LO $f^0_{B\to K}(q^2)$ up to twist-3 that is derived from Eq. (29).

we obtain $[f^T_{B\to K}(0)/f^+_{B\to K}(0)]_{\rm LO} \in [1.19, 1.22]$; while for the NLO results, we obtain $[f^T_{B\to K}(0)/f^+_{B\to K}(0)]_{\rm NLO} \in [1.09, 1.15]$. Furthermore, we show the different parts' contributions to the $B \to K$ form factors $f^{+,0,T}_{B\to K}(q^2)$ in Fig. 6, where all the parameters are taken to be their center values. For $f^{+,0}_{B\to K}(q^2)$, it can be found that the LO twist-2, the NLO twist-2, and the LO twist-4 contributions are positive, more specifically at $q^2 = 0$, they are about 67%, 27%, and 6%, respectively. Even though the LO twist-2 gives zero contribution to the sum of the form factor $[f^+_{B\to K} + f^-_{B\to K}]$ but due to $SU_f(3)$-breaking effect they shall give sizable contribution to $f^0_{B\to K}(q^2)$, so $f^0_{B\to K}(q^2)$ is higher than the LO result derived from Eq. (29) as shown in Fig. 6. For $f^T_{B\to K}(q^2)$, the LO twist-2 and the NLO twist-2 give a positive contribution while the LO twist-4 gives negative contribution, more specifically at $q^2 = 0$, they are about 70%, 32% and -2% respectively.

IV. COMPARATIVE STUDIES OF $f^{+,0,T}_{B\to\pi,K}(q^2)$ WITH OTHER APPROACHES IN QCD LCSRS

A. A striking advantage of the present approach with the chiral current

The adopted chiral current approach has a striking advantage that the twist-3 LC functions which are not known as well as the twist-2 light-cone functions are eliminated, and then it is considered to provide results with less uncertainties. On the other hand, by using the standard weak current in the correlator as shown by Eqs. (1) and (2), it has been pointed out that the twist-3 contributions can contribute $\sim$30%–40% to the total contribution [33]. So to obtain a more accurate result, one has to calculate the above correlator by including one-loop radiative corrections to both the twist-2 and the twist-3 contributions. Such a calculation together with the updated pion and kaon twist-3 wave functions has been done by Ref. [5].

It may be interesting to do a comparison of their results with our present ones so as to show whether these two treatments are consistent with each other or not. For such a purpose, we adopt the following convenient form for the QCD sum rules obtained by Ref. [5], which splits the $B \to P$ form factors into contributions from different Gegenbauer moments:

$$F^{+,0,T}_{B\to P}(q^2) = f^{as}(q^2) + a_1^P(\mu_0) f^{a_1^P}(q^2) + a_2^P(\mu_0) f^{a_2^P}(q^2) + a_4^P(\mu_0) f^{a_4^P}(q^2), \qquad (39)$$

where f^{as} contains the contributions to the form factor from the asymptotic DA and all higher-twist effects from three-particle quark-quark-gluon matrix elements, $f^{a_1^P, a_2^P, a_4^P}$ contains the contribution from the higher Gegenbauer term of DA that is proportional to a_1^P, a_2^P, and a_4^P, respectively. The explicit expressions of $f^{as, a_1^P, a_2^P, a_4^P}$ for all the mentioned form factors can be found in Tables V IX of Ref. [5]. In

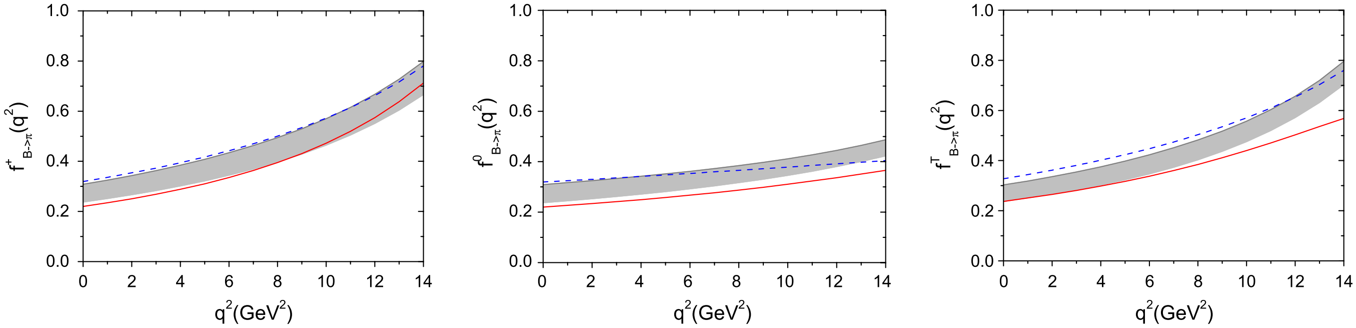

FIG. 7 (color online). $f^{+,0,T}_{B\to\pi}(q^2)$ with the allowed values of a_2^π and a_4^π being correlated and given by the rhomboid shown in Fig. 1. The solid line is obtained with $a_2^\pi(1\text{ GeV}) = 0.0$ and $a_4^\pi(1GeV) = 0.0$ and the dashed line is obtained with $a_2^\pi(1\text{ GeV}) = 0.23$ and $a_4^\pi(1\text{ GeV}) = -0.030$, which set the upper and the lower ranges of $f^{+,0,T}_{B\to\pi}(q^2)$, respectively. As a comparison, the shaded band shows the results of Ref. [5] together with its 12% theoretical uncertainty.

doing the comparison, we take the same DA moments for both methods.

We show a comparison of our results of $F^{+,0,T}_{B\to\pi,K}(q^2)$ with those of Eq. (39) in Figs. 7 and 8, respectively. Figure 7 shows $f^{+,0,T}_{B\to\pi}(q^2)$ with a_2^π and a_4^π being correlated and given by the rhomboid shown in Fig. 1, where the solid line is obtained with $a_2^\pi(1\text{ GeV}) = 0.0$ and $a_4^\pi(1\text{ GeV}) = 0.0$ and the dashed line is obtained with $a_2^\pi(1\text{ GeV}) = 0.23$ and $a_4^\pi(1\text{ GeV}) = -0.030$, which set the upper and the lower ranges of $f^{+,0,T}_{B\to\pi}(q^2)$, respectively. Figure 8 shows $f^{+,0,T}_{B\to K}(q^2)$ with $a_1^K(1\text{ GeV}) \in [0.03, 0.09]$ and $a_2^K(1\text{ GeV}) \in [0.10, 0.40]$, where the solid line is obtained with $a_1^K(1\text{ GeV}) = 0.03$ and $a_2^K(1\text{ GeV}) = 0.10$ and the dashed line is obtained with $a_1^K(1\text{ GeV}) = 0.09$ and $a_2^K(1\text{ GeV}) = 0.40$, which set the upper and the lower ranges of $f^{+,0,T}_{B\to K}(q^2)$, respectively. As a comparison, the shaded bands in these figures show the results of Eq. (39) within the same a_1^K and a_2^K region and with their estimated $[12\% + 3\%]$ theoretical uncertainty, where the extra 3% uncertainty is from a_1^K uncertainty [5].

More explicitly, we show the comparison in detail:

(i) At the large recoil region $q^2 = 0$, Ref. [5] gives $f^{+,0}_{B\to\pi}(0) = 0.258 \pm 0.031$, $f^T_{B\to\pi}(0) = 0.253 \pm 0.028$, $f^{+,0}_{B\to K}(0) = 0.304 \pm 0.076$, and $f^T_{B\to K}(0) = 0.332 \pm 0.080$. It can be found that our results as shown by Eqs. (36) and (37) are consistent with those of Ref. [5], especially in the lower q^2 region.

(ii) With the increment of q^2, the form factors of Ref. [5] increase faster than ours. We can see this clearly from the scalar and tensor form factors $f^{0,T}_{B\to\pi,K}(q^2)$. These differences, especially in the larger q^2 region, are mainly caused by the treatment of the twist-3 contribution and by the different treatment of the uncertainty. The twist-3 contribution can affect the shape of the form factors. For example, as shown by Fig. 4, the present obtained $f^0_{B\to\pi,K}(q^2)$ are close to the LO result derived from Eq. (29); while Ref. [5] gives a larger $f^0_{B\to\pi,K}(q^2)$ at higher q^2 region due to the fact that the twist-3 contribution to $[f^+_{B\to\pi,K} + f^-_{B\to\pi,K}]$ is dominant over the leading twist contribution at large momentum transfer. In Ref. [5], the total uncertainty is

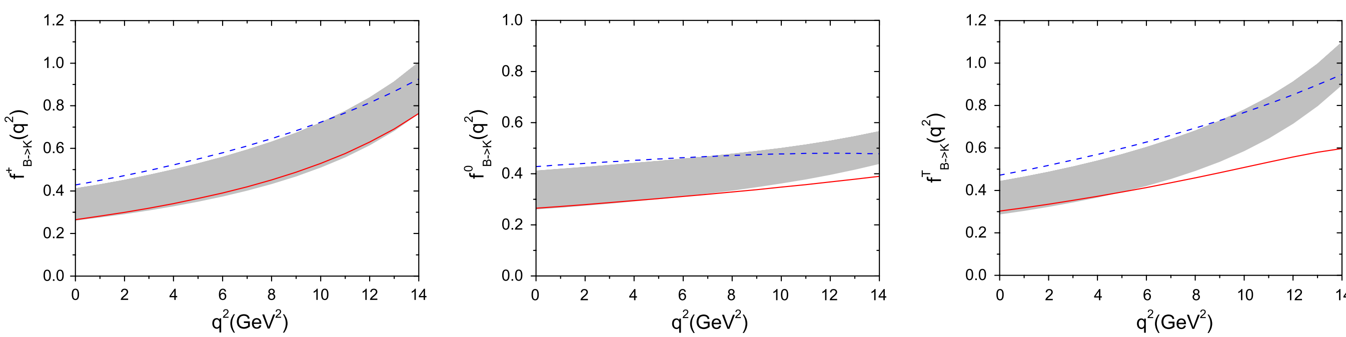

FIG. 8 (color online). $f^{+,0,T}_{B\to K}(q^2)$ for $a_1^K(1\text{ GeV}) \in [0.03, 0.09]$ and $a_2^K(1\text{ GeV}) \in [0.10, 0.40]$. The solid line is obtained with $a_1^K(1\text{ GeV}) = 0.09$ and $a_2^K(1\text{ GeV}) = 0.10$ and the dashed line is obtained with $a_1^K(1\text{ GeV}) = 0.03$ and $a_2^K(1\text{ GeV}) = 0.40$, which set the upper and the lower ranges of $f^{+,0,T}_{B\to K}(q^2)$, respectively. As a comparison, the shaded band shows the results of Ref. [5] together with its 15% theoretical uncertainty.

obtained by adding up the uncertainties caused by each parameter in quadrature; while at the present, we vary the parameters within their possible regions and adopt the minimum and the maximum ones as the uncertainty boundary. Moreover, we have adopted a simple overall uncertainty 12% or 15% for the form factors within all q^2 for the LCSRs of Ref. [5], which in fact should be varied according to different q^2, e.g. we have found that such uncertainty may be up to 5% for the mentioned form factors with the region of $q^2 \in [0, 14]$ GeV^2.

(iii) One may observe that in the lower q^2 region, different from Ref. [5] where $F^{B\to K}_{+,0,T}(q^2)$ increases with the increment of both a_1^K and a_2^K, our present predicted $F^{B\to K}_{+,0,T}(q^2)$ will increase with the increment of a_2^K but with the decrement of a_1^K. This difference is caused by the fact that we adopt the pion and kaon DAs derived from their wave functions to do our discussion, whose parameters are determined by the combined effects of a_1^K and a_2^K; while in Ref. [5], a_1^K and a_2^K are varied independently and then their contributions are changed separately.

B. A comparison of the choosing of the pole or the $\overline{MS}$ *b*-quark mass

References [14,17] have argued to use $\overline{MS}$ *b*-quark mass instead of the pole quark mass. The $\overline{MS}$ *b*-quark running mass ($\bar{m}_b$) is related to the one-loop *b*-quark pole mass (m_b^*) through the following well-known relation:

$$\bar{m}_b(\mu) = m_b^*\left\{1 + \frac{\alpha_S(\mu)C_F}{4\pi}\left(-4 + 3\ln\frac{m_b^{*2}}{\mu^2}\right)\right\}. \quad (40)$$

With the help of the relation (40), one can conveniently transform the form factor expressions among these two choices of *b*-quark mass. One only need to be careful to use all the parameters calculated under the same choice; e.g. the value of f_b should be calculated by using the same currents in the correlator and under the same choice of *b*-quark mass. By calculating the ordinary correlators (1) and (2) up to NLO and by varying the $\overline{MS}$ *b*-quark mass within the region of $\bar{m}_b(\bar{m}_b) = 4.164 \pm 0.025$ GeV, Refs. [14,17] obtain $f^{+,0}_{B\to\pi}(0) = 0.26^{+0.04}_{-0.03}$, $f^T_{B\to\pi}(0) = 0.255 \pm 0.035$, $f^{+,0}_{B\to K}(0) = 0.36^{+0.05}_{-0.04}$, $f^T_{B\to K}(0) = 0.38 \pm 0.05$, and

$$\frac{f^{+,0}_{B\to K}(0)}{f^{+,0}_{B\to\pi}(0)} = 1.38^{+0.11}_{-0.10}, \qquad \frac{f^T_{B\to K}(0)}{f^T_{B\to\pi}(0)} = 1.49^{+0.18}_{-0.06}. \quad (41)$$

These results are consistent with ours and also with those of Ref. [5] within reasonable errors, which is also calculated by taking the pole quark mass. This shows that these two choices of *b*-quark mass are equivalent to each other.

C. Extrapolations of the LCSR results to the higher q^2 region

In order to allow a simple implementation of our results, we present a parametrization that includes the main features of the analytical properties of the form factors and is valid in the full physical regime $0 \le q^2 \le (m_B - m_P)^2$. Following the same argument of Ref. [5], we fit the LCSR results to the following parametrizations that are based on the procedure advocated by Becirevic and Kaidalov [34], where we take the LCSR results with all the parameters taken to be their center values to do the extrapolation, i.e. the *b*-quark one-loop pole mass $m_b = 4.8$ GeV, $a_2^\pi(1\text{ GeV}) = 0.115$, $a_4^\pi(1\text{ GeV}) = -0.015$, $a_1^K(1\text{ GeV}) = 0.06$, and $a_2^K(1\text{ GeV}) = 0.25$. To measure the quality of the fit, we introduce the parameter Δ that is defined as

$$\Delta = 100\max_t\left|\frac{f(t) - f^{\rm fit}(t)}{f(t)}\right|, \quad t \in \left\{0, \frac{1}{2}, \ldots, \frac{23}{2}, 12\right\}\text{ GeV}^2, \quad (42)$$

i.e. it gives, in percent, the maximum deviation of the fitted form factors from the original LCSR result for $q^2 < 12$ GeV^2.

(i) For $f^\pi_{+,T}$,

$$f(q^2) = \frac{r_1}{1 - q^2/(m_1^\pi)^2} + \frac{r_2}{1 - q^2/m^2_{\rm fit}}, \quad (43)$$

where $m_1^\pi = 5.325$ GeV [18] is the mass of $B^*(1^-)$. For f_+^π, the fit parameters are $r_1 = 0.7411$, $r_2 = -0.4815$, and $m^2_{\rm fit} = 40.01$ GeV^2 for $\Delta \simeq 0.05$. For f_T^π, the fit parameters are $r_1 = 0.7742$, $r_2 = -0.4952$, and $m^2_{\rm fit} = 34.71$ GeV^2 for $\Delta \simeq 0.9$.

(ii) For $f^K_{+,T}$,

$$f(q^2) = \frac{r_1}{1 - q^2/(m_1^K)^2} + \frac{r_2}{1 - q^2/m^2_{\rm fit}}, \quad (44)$$

where $m_1^K = 5.413$ GeV [18] is the mass of the $B_s^*(1^-)$. For f_+^K, the fit parameters are $r_1 = 0.8182$, $r_2 = -0.4862$, and $m^2_{\rm fit} = 41.61$ GeV^2 for $\Delta \simeq 1.3$. For f_T^K, the fit parameters are $r_1 = 0.893$, $r_2 = -0.5073$, and $m^2_{\rm fit} = 33.13$ GeV^2 for $\Delta \simeq 1.8$.

(iii) For $f_0^{\pi,K}$,

$$f_0(q^2) = \frac{r_2}{1 - q^2/m^2_{\rm fit}}. \quad (45)$$

For the case of pion, we obtain $r_2 = 0.2596$ and $m^2_{\rm fit} = 46.09$ GeV^2 for $\Delta = 0.07$. For the case of kaon, we obtain $r_2 = 0.332$ and $m^2_{\rm fit} = 61.64$ GeV^2 for $\Delta = 1.3$.

A comparison of the lattice QCD results can be found in Fig. 9. There are many lattice results in the literature for $B \to \pi$, e.g. [35–41], etc.; for convenience, we have taken the unquenched lattice QCD result [37] and the quenched lattice QCD result [36] of $B \to \pi$ form factors. For the

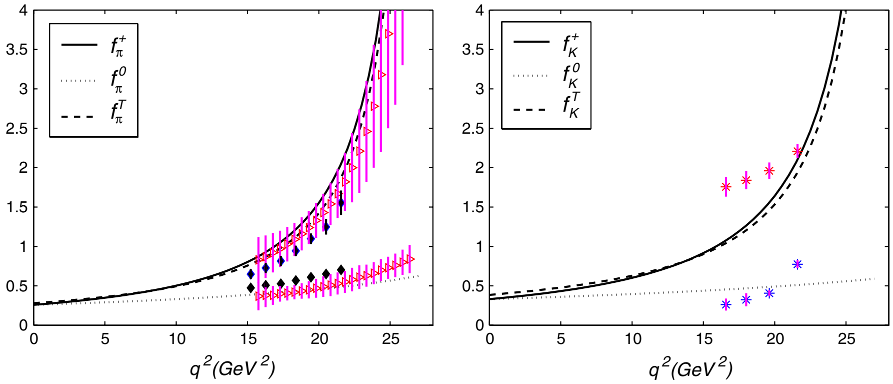

FIG. 9 (color online). Extrapolations of the LCSR results of $B \to \pi$ and K form factors for higher q^2. For comparison, the left diagram shows the unquenched lattice QCD result [37] (diamond) and the quenched lattice QCD result [36] (triangle) with their corresponding errors for the vector and scalar $B \to \pi$ form factors; the right diagram shows the lattice QCD results [41] for the vector and scalar $B \to K$ form factors (asterisk).

$B \to K$ form factors there are little lattice QCD results, and we adopt the preliminary results derived by Ref. [41] for our discussion.

V. SUMMARY

We have calculated all the $B \to P$ transition form factors $F^{+,0,T}_{B\to P}(q^2)$ with chiral current in the LCSR up to NLO, in which the most uncertain twist-3 contributions have been eliminated naturally, with the b-quark pole mass that is universal. The $SU_f(3)$-breaking effects in $B \to K$ form factors have been carefully discussed and their values depend on the moment of the kaon distribution amplitude, $a_2^K(1\ \text{GeV})$. It is found that $[f^{+,0}_{B\to K}(0)]/[f^{+,0}_{B\to\pi}(0)] = 1.28^{+0.06}_{-0.08}$ and $[f^{T}_{B\to K}(0)]/[f^{T}_{B\to\pi}(0)] = 1.37^{+0.07}_{-0.02}$ for $a_1^K(1\ \text{GeV}) \in [0.03, 0.09]$ and $a_2^K(1\ \text{GeV}) \in [0.10, 0.40]$. Based on the LCSR with chiral current, we have made a comparative study on the properties of transition form factors with those obtained in literature [5,14,17], in which the radiative corrections on both the twist-2 and twist-3 parts should be treated in equal footing. It has been found that the present results are less uncertain under the same parameter regions to consider the radiative corrections since the twist-3 contributions have been eliminated naturally in the adopted method, so our results are simpler and consistent with those in literature that has been derived with the usual correlators. These form factors are important ingredients in the analysis of semileptonic B decays, our present results may be especially helpful to clarify the present conditions for the $B \to \eta^{(\prime)}(\ell^- \bar\nu_\ell, \ell^+\ell^-)$ decays and then a better understanding of the η and η' mixing [42].

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APPENDIX: PION AND KAON DISTRIBUTION AMPLITUDES

Generally, the pion and kaon twist-2 and twist-4 DAs can be written in the following forms:

(i) Twist-2 DAs:

$$\begin{aligned}\varphi_P(u,\mu) = 6u\bar{u}[1 &+ a_1^P(\mu)C_1^{3/2}(2u-1) \\ &+ a_2^P(\mu)C_2^{3/2}(2u-1) \\ &+ a_4^P(\mu)C_4^{3/2}(2u-1) + \cdots],\end{aligned} \tag{A1}$$

where P stands for π or K, respectively, $\cdots$ stands for even higher Gegenbauer terms.

(ii) Twist-4 DA's [3]:

$$\begin{aligned}\Psi_{4P}(\alpha_i) &= 30\alpha_3^2(\alpha_2-\alpha_1)[h_{00} + h_{01}\alpha_3 \\ &\quad + \tfrac{1}{2}h_{10}(5\alpha_3 - 3)], \\ \tilde\Psi_{4P}(\alpha_i) &= -30\alpha_3^2[h_{00}(1-\alpha_3) \\ &\quad + h_{01}[\alpha_3(1-\alpha_3) - 6\alpha_1\alpha_2] \\ &\quad + h_{10}[\alpha_3(1-\alpha_3) - \tfrac{3}{2}(\alpha_1^2+\alpha_2^2)]], \\ \Phi_{4P}(\alpha_i) &= 120\alpha_1\alpha_2\alpha_3[a_{10}(\alpha_1-\alpha_2)], \\ \tilde\Phi_{4P}(\alpha_i) &= 120\alpha_1\alpha_2\alpha_3[v_{00} + v_{10}(3\alpha_3 - 1)],\end{aligned} \tag{A2}$$

where

$$h_{00} = v_{00} = -\frac{M_P^2}{3}\eta_{4P} = -\frac{\delta_P^2}{3},$$

$$a_{10} = \frac{21M_P^2}{8}\eta_{4P}\omega_{4P} - \frac{9}{20}a_2^P M_P^2 = \delta_P^2\epsilon_P - \frac{9}{20}a_2^P M_P^2,$$

$$v_{10} = \frac{21M_P^2}{8}\eta_{4P}\omega_{4P} = \delta_P^2\epsilon_P,$$

$$h_{01} = \frac{7M_P^2}{4}\eta_{4P}\omega_{4P} - \frac{3}{20}a_2^P M_P^2 = \frac{2}{3}\delta_P^2\epsilon_P - \frac{3}{20}a_2^P M_P^2$$

and

$$h_{10} = \frac{7M_P^2}{2}\eta_{4P}\omega_{4P} + \frac{3}{20}a_2^P M_P^2 = \frac{4}{3}\delta_P^2\epsilon_P + \frac{3}{20}a_2^P M_P^2,$$

with $\eta_{4P} = \delta_P^2/M_P^2$, $\omega_{4P} = 8\epsilon_P/21$. Taking the leading meson-mass effect into consideration, the remaining two-particle DA's of twist-4 can be written as [3]

$$\phi_{4P}(u) = \frac{4u\bar{u}}{3}\{-5u\bar{u}[30h_{00} + 4h_{01}(3 + u\bar{u}) + 5h_{10}(-3 + 2u\bar{u})] + 2a_{10}[6 + u\bar{u}(9 + 40u\bar{u})]\} + 8a_{10}\{2u^3(10 - 15u + 6u^2)\ln u + 2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2)\ln\bar{u}\}, \quad \text{(A3)}$$

$$\psi_{4P}(u) = 5[-4h_{00} - 2h_{01} + h_{10} + 4(-4a_{10} + 6h_{00} + 4h_{01} + h_{10})u - 6(4h_{00} + 6h_{01} + 9h_{10} - 16a_{10})u^2 + 20(2h_{01} + 5h_{10} - 8a_{10})u^3 + 10(8a_{10} - 2h_{01} - 5h_{10})u^4]. \quad \text{(A4)}$$

Setting $M_P \to 0$ and $M_P \to M_K$, one can obtain the pionic and the kaonic twist-4 DAs, respectively.

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