Dilepton production from polarized hadron hadron collisions

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In this paper we present a comprehensive formalism for dilepton production from the collision of two polarized spin-$\frac{1}{2}$ hadrons by identifying the general angular distribution of the cross section in combination with a complete set of structure functions. The various structure functions are computed in the parton model approximation where we mainly consider the case when the transverse momentum of the dilepton pair is much smaller than its invariant mass. In this kinematical region dilepton production can be described in terms of transverse momentum dependent parton distributions.

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I. INTRODUCTION

During the past decades dilepton production in high-energy hadron-hadron collisions (the so-called Drell-Yan (DY) process [1,2]) has played an important role in order to pin down parton distributions (PDFs) of hadrons. While the main focus was on PDFs of the nucleon, also information on the partonic structure of the pion was already obtained through Drell-Yan measurements. The crucial tool required for the extraction of PDFs is the QCD-factorization theorem [3–6] which applies if the invariant mass of the dilepton pair is sufficiently large. Experimentally, the Drell-Yan process is quite challenging because of the relatively low counting rates. On the other hand, from the theoretical point of view it is the cleanest hard hadron-hadron scattering process. The fact that no hadron is detected in the final state simplifies the proof of factorization in comparison to hadron-hadron collisions with hadronic final states. This important point is one of the main reasons for the continued interest in the Drell-Yan reaction.

Currently, not less than six programs for future Drell-Yan measurements are pursued. These plans comprise dilepton production in nucleon-nucleon collisions (at RHIC [7], J-PARC (KEK) [8,9], IHEP (Protvino) [10], and at the JINR (Dubna) [11]), in antiproton nucleon collisions (at FAIR (GSI) [12]), as well as in pion nucleon collisions (at COMPASS (CERN) [13]). Past measurements exclusively considered the unpolarized cross section, but all future programs are also aiming at polarization measurements. Including polarization of the incoming hadrons opens up a variety of new opportunities for studying the strong interaction in both the perturbative and the nonperturbative regime. Here we only mention the access to the transversity distribution of the nucleon [14–21], and to transverse momentum dependent parton distributions (TMDs). The TMDs not only depend on the longitudinal momentum of a parton inside a hadron but also on its (intrinsic) transverse momentum, and, in general, describe the strength of various intriguing spin-spin or spin-orbit correlations of the parton-hadron system (see Refs. [22–25] for more information on TMDs).

In order to analyze upcoming data from polarized Drell-Yan measurements it is necessary to have a general and concise formalism at hand. The main motivation for writing the present paper is to provide such a framework. To this end we decompose the hadronic tensor of the polarized Drell-Yan process in terms of 48 basis tensors which are multiplied by structure functions. We limit ourselves to photon exchange and do not consider weak interaction effects. To ensure electromagnetic gauge invariance of the hadronic tensor we make use of a projector method proposed in Ref. [26]. On the basis of the hadronic tensor we then write down the general structure of the angular distribution of the Drell-Yan process. This step is most conveniently done in a dilepton rest frame like the Collins-Soper frame [27]. In addition to our model-independent results we also consider the process in the parton model approximation, where we distinguish between two cases: (1) cross section integrated upon the transverse momentum q_T of the dilepton pair; (2) cross section kept differential in q_T and $q_T \ll q$, where q is the invariant mass of the dilepton pair. While in the former case one ends up with ordinary forward PDFs, in the latter TMDs enter in the parton model description and in a full QCD treatment [28–30].

In addition to our model-independent treatment we also consider the process in the parton model approximation by concentrating on the situation when the cross section is kept differential in the transverse momentum q_T of the dilepton pair. In this case TMDs enter the parton model description as well as a full QCD treatment [28–30].

Part of the results presented here were already given in the literature [14,31–33], and we comment on other work during the course of the paper. However, to the best of our knowledge, a complete formalism for the polarized Drell-Yan process has not been worked out before.

The paper is organized as follows. In Sec. II we fix part of our notation and give the general form of the cross

section in the one-photon exchange approximation. Section III contains the decomposition of the hadronic tensor in terms of basis tensors and structure functions, while in Sec. IV some discussion on reference frames is given. In Sec. V we present the general angular distribution of the polarized Drell-Yan process which can be derived from the results of Sec. III in a straightforward manner. Section VI contains the results for the structure functions in the parton model approximation. We conclude in Sec. VII.

II. CROSS SECTION IN ONE-PHOTON EXCHANGE APPROXIMATION

To be now specific we consider the dilepton production

$$H_a(P_a, S_a) + H_b(P_b, S_b) \rightarrow l^-(l, \lambda) + l^+(l', \lambda') + X, \tag{1}$$

with (P_a, S_a) and (P_b, S_b) denoting the 4-momenta and the spin vectors of the incoming hadrons. One has $P_a^2 = M_a^2$, $P_a \cdot S_a = 0$, $S_a^2 = -1$, and corresponding relations for the second hadron. Throughout this work the mass of the leptons in the final state is neglected. We will sum over the helicities λ, λ' of the leptons.

At large invariant mass q of the dilepton pair the process (1) can approximately be described in the Drell-Yan model [1,2], which corresponds to the parton model approximation. According to this approach a quark from hadron H_a and an antiquark from hadron H_b (and vice versa) annihilate into a timelike virtual photon which subsequently decays into a lepton pair (see Fig. 1).[1] This means the process proceeds according to

$$H_a + H_b \rightarrow \gamma^*(q) + X \rightarrow l^- + l^+ + X, \tag{2}$$

where the 4-momentum of the virtual photon is given by $q = l + l'$.[2] Note that the meaning of (2) remains valid if higher-order QCD corrections are taken into account.

In the one-photon exchange approximation the (frame-independent) cross section of the Drell-Yan process is given by

$$\frac{l^0 l'^0 d\sigma}{d^3\vec{l} d^3\vec{l'}} = \frac{\alpha_{\text{em}}^2}{F q^4} L_{\mu\nu} W^{\mu\nu}, \tag{3}$$

where

$$F = 4\sqrt{(P_a \cdot P_b)^2 - M_a^2 M_b^2} \tag{4}$$

represents the flux of the incoming hadrons. If hadron masses are neglected one can write $F = 2s = 2(P_a + P_b)^2$. The fine structure constant is related to the elementary charge through $\alpha_{\text{em}} = e^2/4\pi$. In Eq. (3) the quantity

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[1]As already mentioned we do not consider weak interaction effects.

[2]In our notation the symbol q describes both the 4-momentum of the virtual photon as well as the invariant mass $\sqrt{q^2}$ of the dilepton pair. This should, however, not lead to any confusion.
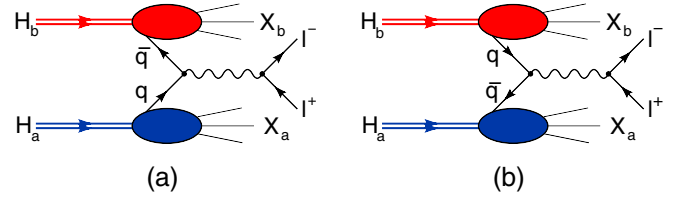

FIG. 1 (color online). Amplitude for dilepton production in parton model approximation. Both diagrams have to be taken into account. The spectator systems X_a and X_b of the two hadrons are not detected.

$L^{\mu\nu}$ denotes the spin-averaged leptonic tensor,

$$L^{\mu\nu} = \sum_{\lambda, \lambda'} (\bar{u}(l, \lambda)\gamma^\mu v(l', \lambda'))(\bar{u}(l, \lambda)\gamma^\nu v(l', \lambda'))^*$$

$$= 4\left(l^\mu l'^\nu + l^\nu l'^\mu - \frac{q^2}{2} g^{\mu\nu}\right), \tag{5}$$

while

$$W^{\mu\nu}(P_a, S_a; P_b, S_b; q) = \frac{1}{(2\pi)^4} \int d^4x e^{iq\cdot x}$$
$$\times \langle P_a, S_a; P_b, S_b | J_{\text{em}}^\mu(0) J_{\text{em}}^\nu(x) |$$
$$\times P_a, S_a; P_b, S_b \rangle \tag{6}$$

is the hadronic tensor, which is determined by the electromagnetic current operator J_{em}^μ.

The tensor $W^{\mu\nu}$ a priori is unknown and contains the information on the hadron structure. It has to fulfill certain constraints due to electromagnetic gauge invariance, parity, and hermiticity. In this order the constraints read

$$q_\mu W^{\mu\nu}(P_a, S_a; P_b, S_b; q) = q_\nu W^{\mu\nu}(P_a, S_a; P_b, S_b; q)$$
$$= 0, \tag{7}$$

$$W^{\mu\nu}(P_a, S_a; P_b, S_b; q) = W_{\mu\nu}(\bar{P}_a, -\bar{S}_a; \bar{P}_b, -\bar{S}_b; \bar{q}), \tag{8}$$

$$W^{\mu\nu}(P_a, S_a; P_b, S_b; q) = [W^{\nu\mu}(P_a, S_a; P_b, S_b; q)]^*, \tag{9}$$

where the notation $\bar{v}^\mu = v_\mu$ for a generic 4-vector v is used. In Sec. III, by imposing the relations (7)–(9), the hadronic tensor is decomposed into a set of 48 basis tensors multiplied by scalar functions (structure functions). In doing so the conditions (7) and (8) considerably reduce the number of allowed basis tensors, while the hermiticity constraint (9) implies that the structure functions are real. Note that time-reversal does not impose any constraint on the hadronic tensor, because this operation converts the two-particle hadronic in-state into a two-particle out-state, and both states are not related. In Sec. VI the hadronic tensor is considered in the parton model approximation.

The angular distribution of the Drell-Yan cross section is most conveniently considered in a dilepton rest frame like the Collins-Soper frame [27] or the Gottfried-Jackson frame [34]. In any dilepton rest frame, one can rewrite Eq.

(3) according to

$$\frac{d\sigma}{d^4q d\Omega} = \frac{\alpha_{em}^2}{2Fq^4} L_{\mu\nu} W^{\mu\nu}, \tag{10}$$

where the solid angle Ω specifies the orientation of the leptons. In Sec. IV we elaborate a bit more on reference frames with the main focus on the center-of-mass frame (cm-frame) and the Collins-Soper frame (CS-frame).

III. HADRONIC TENSOR

The total hadronic tensor can be decomposed into the unpolarized, single polarized (for hadron H_a and hadron H_b), and double-polarized tensor according to

$$W^{\mu\nu} = W_u^{\mu\nu} + W_a^{\mu\nu} + W_b^{\mu\nu} + W_{ab}^{\mu\nu}. \tag{11}$$

In the following we merely have to consider the symmetric part of $W^{\mu\nu}$ because the spin-averaged leptonic tensor in (5) is symmetric under the exchange $\mu \leftrightarrow \nu$.

A. Unpolarized case

Since the unpolarized tensor depends on the 4-vectors q^μ, P_a^μ, and P_b^μ one can immediately write down the tensor basis

$$\begin{aligned}
h_{u,1}^{\mu\nu} &= g^{\mu\nu}, & h_{u,2}^{\mu\nu} &= q^\mu q^\nu, & h_{u,3}^{\mu\nu} &= P_a^\mu P_a^\nu, \\
h_{u,4}^{\mu\nu} &= P_b^\mu P_b^\nu, & h_{u,5}^{\mu\nu} &= q^\mu P_a^\nu + q^\nu P_a^\mu, \\
h_{u,6}^{\mu\nu} &= q^\mu P_b^\nu + q^\nu P_b^\mu, & h_{u,7}^{\mu\nu} &= P_a^\mu P_b^\nu + P_a^\nu P_b^\mu.
\end{aligned} \tag{12}$$

The expressions in (12) constitute a complete list of basis tensors being in accordance with the parity constraint (8). Therefore one can write in a first step

$$W_u^{\mu\nu} = \sum_{i=1}^{7} h_{u,i}^{\mu\nu} \tilde{V}_{u,i}, \tag{13}$$

where the structure functions $\tilde{V}_{u,i}$ depend on the invariants $P_a \cdot q$, $P_b \cdot q$, and q^2.

So far we have not yet used the gauge invariance constraint (7) which, in fact, implies that not all of the $\tilde{V}_{u,i}$ are independent. Contracting the tensor in (13) with the 4-momentum of the virtual photon and imposing (7) one readily finds

$$\begin{aligned}
0 &= \tilde{V}_{u,1} + q^2 \tilde{V}_{u,2} + P_a \cdot q \tilde{V}_{u,5} + P_b \cdot q \tilde{V}_{u,6}, \\
0 &= P_a \cdot q \tilde{V}_{u,3} + q^2 \tilde{V}_{u,5} + P_b \cdot q \tilde{V}_{u,7}, \\
0 &= P_b \cdot q \tilde{V}_{u,4} + q^2 \tilde{V}_{u,6} + P_a \cdot q \tilde{V}_{u,7}.
\end{aligned} \tag{14}$$

These three relations follow because in $W_u^{\mu\nu} q_\nu$ the terms proportional to q^μ, P_a^μ, and P_b^μ must vanish separately. Now one can use (14) to eliminate three structure functions and consequently end up with a hadronic tensor given by just four independent structure functions that are multiplied by four independent basis tensors. The explicit form

of the basis tensors depends of course on which of the structure functions are eliminated.

Though this procedure of implementing gauge invariance in principle is straightforward it gets rather cumbersome for single and double polarization because in those cases considerably more structure functions and basis tensors are involved. Therefore we resort to an alternative and very elegant method proposed in Ref. [26] which makes use of projection operators. We define[3]

$$P^{\mu\nu} = g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}, \tag{15}$$

and let this operator act on the basis tensors in (12) according to

$$P^\mu{}_\rho h_{u,i}^{\rho\sigma} P_\sigma{}^\nu. \tag{16}$$

Because of the property

$$q_\mu P^{\mu\nu} = P^{\mu\nu} q_\nu = 0 \tag{17}$$

the tensors in (16) vanish for $i = 2, 5, 6$, while the remaining four nonzero tensors are gauge invariant by construction. This means that one arrives at the following final form of the unpolarized hadronic tensor:

$$W_u^{\mu\nu} = \sum_{i=1}^{4} t_{u,i}^{\mu\nu} V_{u,i}, \tag{18}$$

with the four structure functions, $V_{u,i}$ and the tensor basis

$$\begin{aligned}
t_{u,1}^{\mu\nu} &= g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}, & t_{u,2}^{\mu\nu} &= \tilde{P}_a^\mu \tilde{P}_a^\nu, \\
t_{u,3}^{\mu\nu} &= \tilde{P}_b^\mu \tilde{P}_b^\nu, & t_{u,4}^{\mu\nu} &= \tilde{P}_a^\mu \tilde{P}_b^\nu + \tilde{P}_a^\nu \tilde{P}_b^\mu.
\end{aligned} \tag{19}$$

In Eq. (19) we make use of the vectors

$$\tilde{P}_a^\mu = P_a^\mu - \frac{P_a \cdot q q^\mu}{q^2}, \qquad \tilde{P}_b^\mu = P_b^\mu - \frac{P_b \cdot q q^\mu}{q^2}, \tag{20}$$

which vanish upon contraction with q. Needless to say that the tensor in (18) is frame-independent. The contraction with the leptonic tensor can therefore be performed in any frame. In the context of the parton model calculation in Sec. VI, for instance, this contraction is carried out in the cm-frame. The specific form of the tensor (18) is by no means unique. Other sets of basis tensors can be found in the literature (see, e.g., Refs. [14,35,36] and also [37]), and it is straightforward to write down relations between different sets. Here we refrain from doing so because it does not give much further insight and, in addition, is not needed for the main purpose of this manuscript. We have discussed the unpolarized case in some detail in order to outline the procedure which is used in the following two subsections that are dealing with hadron polarization.

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[3]Note that the projection operator is not unique [26]. One can also define an operator involving the hadron momentum P_a or P_b.

B. Single polarized case

Now we proceed to the case when one of the hadrons in the initial state is polarized. We first consider polarization of the hadron H_a, and then just quote the result for the case when H_b is polarized. In order to construct a tensor basis

we now have also the spin vector S_a^μ at our disposal—in addition to the 4-momenta q^μ, P_a^μ, P_b^μ. Imposing the parity constraint (8) one finds the following list of tensors which are symmetric under the exchange $\mu \leftrightarrow \nu$:

$$
\begin{aligned}
&h_{a,1}^{\mu\nu}, \ldots, h_{a,7}^{\mu\nu} = \varepsilon^{S_a q P_a P_b}\{g^{\mu\nu}, q^\mu q^\nu, P_a^\mu P_a^\nu, P_b^\mu P_b^\nu, q^\mu P_a^\nu + q^\nu P_a^\mu, q^\mu P_b^\nu + q^\nu P_b^\mu, P_a^\mu P_b^\nu + P_a^\nu P_b^\mu\}, \\
&h_{a,8}^{\mu\nu}, h_{a,9}^{\mu\nu} = \{S_a \cdot q, S_a \cdot P_b\}(\varepsilon^{\mu q P_a P_b} q^\nu + \varepsilon^{\nu q P_a P_b} q^\mu), \qquad h_{a,10}^{\mu\nu}, h_{a,11}^{\mu\nu} = \{S_a \cdot q, S_a \cdot P_b\}(\varepsilon^{\mu q P_a P_b} P_a^\nu + \varepsilon^{\nu q P_a P_b} P_a^\mu), \\
&h_{a,12}^{\mu\nu}, h_{a,13}^{\mu\nu} = \{S_a \cdot q, S_a \cdot P_b\}(\varepsilon^{\mu q P_a P_b} P_b^\nu + \varepsilon^{\nu q P_a P_b} P_b^\mu), \qquad h_{a,14}^{\mu\nu} = \varepsilon^{\mu S_a q P_a} q^\nu + \varepsilon^{\nu S_a q P_a} q^\mu, \\
&h_{a,15}^{\mu\nu} = \varepsilon^{\mu S_a q P_b} q^\nu + \varepsilon^{\nu S_a q P_b} q^\mu, \qquad h_{a,16}^{\mu\nu} = \varepsilon^{\mu S_a P_a P_b} q^\nu + \varepsilon^{\nu S_a P_a P_b} q^\mu, \qquad h_{a,17}^{\mu\nu} = \varepsilon^{\mu S_a q P_a} P_a^\nu + \varepsilon^{\nu S_a q P_a} P_a^\mu, \\
&h_{a,18}^{\mu\nu} = \varepsilon^{\mu S_a q P_b} P_a^\nu + \varepsilon^{\nu S_a q P_b} P_a^\mu, \qquad h_{a,19}^{\mu\nu} = \varepsilon^{\mu S_a P_a P_b} P_a^\nu + \varepsilon^{\nu S_a P_a P_b} P_a^\mu, \qquad h_{a,20}^{\mu\nu} = \varepsilon^{\mu S_a q P_a} P_b^\nu + \varepsilon^{\nu S_a q P_a} P_b^\mu, \\
&h_{a,21}^{\mu\nu} = \varepsilon^{\mu S_a q P_b} P_b^\nu + \varepsilon^{\nu S_a q P_b} P_b^\mu, \qquad h_{a,22}^{\mu\nu} = \varepsilon^{\mu S_a P_a P_b} P_b^\nu + \varepsilon^{\nu S_a P_a P_b} P_b^\mu, \qquad h_{a,23}^{\mu\nu} = \varepsilon^{\mu q P_a P_b} S_a^\nu + \varepsilon^{\nu q P_a P_b} S_a^\mu.
\end{aligned}
\tag{21}
$$

To shorten the notation we have used abbreviations like $\varepsilon^{S_a q P_a P_b} = \varepsilon_{\mu\nu\rho\sigma} S_a^\mu q^\nu P_a^\rho P_b^\sigma$. Note that the hadron spin vector can only appear linearly. It turns out that not all of the tensors $h_{a,i}^{\mu\nu}$ in (21) are independent of each other. The identity

$$
g^{\alpha\beta}\varepsilon^{\mu\nu\rho\sigma} = g^{\mu\beta}\varepsilon^{\alpha\nu\rho\sigma} + g^{\nu\beta}\varepsilon^{\mu\alpha\rho\sigma} + g^{\rho\beta}\varepsilon^{\mu\nu\alpha\sigma} + g^{\sigma\beta}\varepsilon^{\mu\nu\rho\alpha}
\tag{22}
$$

allows one to eliminate several out of the 23 tensors. To be explicit one finds 10 linearly independent relations between the tensors in (21) which may be written in the form

$$
\begin{aligned}
&2h_{a,1}^{\mu\nu} = -h_{a,16}^{\mu\nu} + h_{a,18}^{\mu\nu} - h_{a,20}^{\mu\nu} + h_{a,23}^{\mu\nu}, \qquad && 2h_{a,2}^{\mu\nu} = h_{a,8}^{\mu\nu} - P_b \cdot q h_{a,14}^{\mu\nu} + P_a \cdot q h_{a,15}^{\mu\nu} - q^2 h_{a,16}^{\mu\nu}, \\
&2h_{a,3}^{\mu\nu} = -P_a \cdot P_b h_{a,17}^{\mu\nu} + M_a^2 h_{a,18}^{\mu\nu} - P_a \cdot q h_{a,19}^{\mu\nu}, \qquad && 2h_{a,4}^{\mu\nu} = h_{a,13}^{\mu\nu} - M_b^2 h_{a,20}^{\mu\nu} + P_a \cdot P_b h_{a,21}^{\mu\nu} - P_b \cdot q h_{a,22}^{\mu\nu}, \\
&h_{a,5}^{\mu\nu} = h_{a,10}^{\mu\nu} - P_b \cdot q h_{a,17}^{\mu\nu} + P_a \cdot q h_{a,18}^{\mu\nu} - q^2 h_{a,19}^{\mu\nu}, \qquad && h_{a,5}^{\mu\nu} = -P_a \cdot P_b h_{a,14}^{\mu\nu} + M_a^2 h_{a,15}^{\mu\nu} - P_a \cdot q h_{a,16}^{\mu\nu}, \\
&h_{a,6}^{\mu\nu} = h_{a,9}^{\mu\nu} - M_b^2 h_{a,14}^{\mu\nu} + P_a \cdot P_b h_{a,15}^{\mu\nu} - P_b \cdot q h_{a,16}^{\mu\nu}, \qquad && h_{a,6}^{\mu\nu} = h_{a,12}^{\mu\nu} - P_b \cdot q h_{a,20}^{\mu\nu} + P_a \cdot q h_{a,21}^{\mu\nu} - q^2 h_{a,22}^{\mu\nu}, \\
&h_{a,7}^{\mu\nu} = h_{a,11}^{\mu\nu} - M_b^2 h_{a,17}^{\mu\nu} + P_a \cdot P_b h_{a,18}^{\mu\nu} - P_b \cdot q h_{a,19}^{\mu\nu}, \qquad && h_{a,7}^{\mu\nu} = -P_a \cdot P_b h_{a,20}^{\mu\nu} + M_a^2 h_{a,21}^{\mu\nu} - P_a \cdot q h_{a,22}^{\mu\nu}.
\end{aligned}
\tag{23}
$$

On the basis of the relations in (23) we choose to eliminate the tensors $h_{a,14}^{\mu\nu}, \ldots, h_{a,23}^{\mu\nu}$.

Following Eq. (16) the projection operator $P^{\mu\nu}$ is now applied to the remaining tensors in order to implement electromagnetic gauge invariance. This procedure provides, in a straightforward manner, the final form of the hadronic tensor for the case of single hadron polarization. One finds

$$
W_a^{\mu\nu} = \sum_{i=1}^{8} t_{a,i}^{\mu\nu} V_{a,i},
\tag{24}
$$

with the eight structure functions $V_{a,i}$, and the tensor basis

$$
\begin{aligned}
&t_{a,1}^{\mu\nu}, \ldots, t_{a,4}^{\mu\nu} = \varepsilon^{S_a q P_a P_b}\Big\{g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}, \tilde{P}_a^\mu \tilde{P}_a^\nu, \tilde{P}_b^\mu \tilde{P}_b^\nu, \tilde{P}_a^\mu \tilde{P}_b^\nu \\
&\qquad + \tilde{P}_a^\nu \tilde{P}_b^\mu\Big\}, \\
&t_{a,5}^{\mu\nu}, t_{a,6}^{\mu\nu} = \{S_a \cdot q, S_a \cdot P_b\}(\varepsilon^{\mu q P_a P_b} \tilde{P}_a^\nu + \varepsilon^{\nu q P_a P_b} \tilde{P}_a^\mu), \\
&t_{a,7}^{\mu\nu}, t_{a,8}^{\mu\nu} = \{S_a \cdot q, S_a \cdot P_b\}(\varepsilon^{\mu q P_a P_b} \tilde{P}_b^\nu + \varepsilon^{\nu q P_a P_b} \tilde{P}_b^\mu).
\end{aligned}
\tag{25}
$$

Here we used the 4-vectors $\tilde{P}_a^\mu$ and $\tilde{P}_b^\mu$ as given in (20).

Note that the first four tensors in (25) correspond to the four tensors in (19) for the unpolarized case, multiplied by the structure $\varepsilon^{S_a q P_a P_b}$. It is worthwhile pointing out the following: we have chosen to first remove redundant tensors in (21) by means of the identity (22) and then implemented gauge invariance. If one reverses these two steps one can obtain the same final result for the hadronic tensor.

If the hadron H_b is polarized one can now write immediately

$$
W_b^{\mu\nu} = \sum_{i=1}^{8} t_{b,i}^{\mu\nu} V_{b,i},
\tag{26}
$$

with the eight structure functions $V_{b,i}$, and the tensor basis

$$
\begin{aligned}
&t_{b,1}^{\mu\nu}, \ldots, t_{b,4}^{\mu\nu} = \varepsilon^{S_b q P_b P_a}\Big\{g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}, \tilde{P}_a^\mu \tilde{P}_a^\nu, \tilde{P}_b^\mu \tilde{P}_b^\nu, \tilde{P}_a^\mu \tilde{P}_b^\nu \\
&\qquad + \tilde{P}_a^\nu \tilde{P}_b^\mu\Big\}, \\
&t_{b,5}^{\mu\nu}, t_{b,6}^{\mu\nu} = \{S_b \cdot q, S_b \cdot P_a\}(\varepsilon^{\mu q P_b P_a} \tilde{P}_a^\nu + \varepsilon^{\nu q P_b P_a} \tilde{P}_a^\mu), \\
&t_{b,7}^{\mu\nu}, t_{b,8}^{\mu\nu} = \{S_b \cdot q, S_b \cdot P_a\}(\varepsilon^{\mu q P_b P_a} \tilde{P}_b^\nu + \varepsilon^{\nu q P_b P_a} \tilde{P}_b^\mu).
\end{aligned}
\tag{27}
$$

In Ref. [31] the case of single hadron polarization for the Drell-Yan process was already considered. In that paper, however, the focus of the model-independent part was on the angular distribution of the cross section in the CS-frame rather than on the general form of the hadronic tensor. We will discuss the angular distribution of the cross section in Sec. V.

C. Double-polarized case

Eventually, we consider the situation when both hadrons in the initial state are polarized. In that case the basis tensors depend linearly on both S_a and S_b. A full set of tensors respecting the parity constraint (8) and being symmetric under the exchange $\mu \leftrightarrow \nu$ reads

$$
\begin{aligned}
h_{ab,1}^{\mu\nu}, \ldots, h_{ab,7}^{\mu\nu} &= S_a \cdot S_b \{ g^{\mu\nu}, q^\mu q^\nu, P_a^\mu P_a^\nu, P_b^\mu P_b^\nu, q^\mu P_a^\nu + q^\nu P_a^\mu, q^\mu P_b^\nu + q^\nu P_b^\mu, P_a^\mu P_b^\nu + P_a^\nu P_b^\mu \}, \\
h_{ab,8}^{\mu\nu}, \ldots, h_{ab,14}^{\mu\nu} &= S_a \cdot q S_b \cdot q \{ g^{\mu\nu}, q^\mu q^\nu, P_a^\mu P_a^\nu, P_b^\mu P_b^\nu, q^\mu P_a^\nu + q^\nu P_a^\mu, q^\mu P_b^\nu + q^\nu P_b^\mu, P_a^\mu P_b^\nu + P_a^\nu P_b^\mu \}, \\
h_{ab,15}^{\mu\nu}, \ldots, h_{ab,21}^{\mu\nu} &= S_a \cdot q S_b \cdot P_a \{ g^{\mu\nu}, q^\mu q^\nu, P_a^\mu P_a^\nu, P_b^\mu P_b^\nu, q^\mu P_a^\nu + q^\nu P_a^\mu, q^\mu P_b^\nu + q^\nu P_b^\mu, P_a^\mu P_b^\nu + P_a^\nu P_b^\mu \}, \\
h_{ab,22}^{\mu\nu}, \ldots, h_{ab,28}^{\mu\nu} &= S_b \cdot q S_a \cdot P_b \{ g^{\mu\nu}, q^\mu q^\nu, P_a^\mu P_a^\nu, P_b^\mu P_b^\nu, q^\mu P_a^\nu + q^\nu P_a^\mu, q^\mu P_b^\nu + q^\nu P_b^\mu, P_a^\mu P_b^\nu + P_a^\nu P_b^\mu \}, \\
h_{ab,29}^{\mu\nu}, \ldots, h_{ab,35}^{\mu\nu} &= S_a \cdot P_b S_b \cdot P_a \{ g^{\mu\nu}, q^\mu q^\nu, P_a^\mu P_a^\nu, P_b^\mu P_b^\nu, q^\mu P_a^\nu + q^\nu P_a^\mu, q^\mu P_b^\nu + q^\nu P_b^\mu, P_a^\mu P_b^\nu + P_a^\nu P_b^\mu \}, \quad (28) \\
h_{ab,36}^{\mu\nu}, \ldots, h_{ab,38}^{\mu\nu} &= S_a \cdot q \{ S_b^\mu q^\nu + S_b^\nu q^\mu, S_b^\mu P_a^\nu + S_b^\nu P_a^\mu, S_b^\mu P_b^\nu + S_b^\nu P_b^\mu \}, \\
h_{ab,39}^{\mu\nu}, \ldots, h_{ab,41}^{\mu\nu} &= S_b \cdot q \{ S_a^\mu q^\nu + S_a^\nu q^\mu, S_a^\mu P_a^\nu + S_a^\nu P_a^\mu, S_a^\mu P_b^\nu + S_a^\nu P_b^\mu \}, \\
h_{ab,42}^{\mu\nu}, \ldots, h_{ab,44}^{\mu\nu} &= S_a \cdot P_b \{ S_b^\mu q^\nu + S_b^\nu q^\mu, S_b^\mu P_a^\nu + S_b^\nu P_a^\mu, S_b^\mu P_b^\nu + S_b^\nu P_b^\mu \}, \\
h_{ab,45}^{\mu\nu}, \ldots, h_{ab,47}^{\mu\nu} &= S_b \cdot P_a \{ S_a^\mu q^\nu + S_a^\nu q^\mu, S_a^\mu P_a^\nu + S_a^\nu P_a^\mu, S_a^\mu P_b^\nu + S_a^\nu P_b^\mu \}, \qquad h_{ab,48}^{\mu\nu} = S_a^\mu S_b^\nu + S_a^\nu S_b^\mu.
\end{aligned}
$$

Like in the case of single hadron polarization not all 48 tensors in (28) are independent of each other. An explicit relation between a certain subset of the $h_{ab,i}^{\mu\nu}$ can be found by means of the determinant identity [38]

$$
D^{\mu\alpha\beta\gamma\delta;\nu\bar{\alpha}\bar{\beta}\bar{\gamma}\bar{\delta}} =
\begin{vmatrix}
g^{\mu\nu} & g^{\mu\bar{\alpha}} & g^{\mu\bar{\beta}} & g^{\mu\bar{\gamma}} & g^{\mu\bar{\delta}} \\
g^{\alpha\nu} & g^{\alpha\bar{\alpha}} & g^{\alpha\bar{\beta}} & g^{\alpha\bar{\gamma}} & g^{\alpha\bar{\delta}} \\
g^{\beta\nu} & g^{\beta\bar{\alpha}} & g^{\beta\bar{\beta}} & g^{\beta\bar{\gamma}} & g^{\beta\bar{\delta}} \\
g^{\gamma\nu} & g^{\gamma\bar{\alpha}} & g^{\gamma\bar{\beta}} & g^{\gamma\bar{\gamma}} & g^{\gamma\bar{\delta}} \\
g^{\delta\nu} & g^{\delta\bar{\alpha}} & g^{\delta\bar{\beta}} & g^{\delta\bar{\gamma}} & g^{\delta\bar{\delta}}
\end{vmatrix} = 0. \quad (29)
$$

Equation (29) immediately implies

$$
D_{\mu\alpha\beta\gamma\delta;\nu\bar{\alpha}\bar{\beta}\bar{\gamma}\bar{\delta}} (S_a^\alpha S_b^{\bar{\alpha}} + S_a^{\bar{\alpha}} S_b^\alpha) q^\beta q^{\bar{\beta}} P_a^\gamma P_a^{\bar{\gamma}} P_b^\delta P_b^{\bar{\delta}} = 0, \quad (30)
$$

which allows one to eliminate exactly one out of the tensors in (28). For the sake of symmetry we choose to eliminate the tensor $h_{ab,48}^{\mu\nu}$. Equation (30) implies a relation of the type

$$
\{ q^2 [(P_a \cdot P_b)^2 - M_a^2 M_b^2] - 2 P_a \cdot P_b P_a \cdot q P_b \cdot q + M_a^2 (P_b \cdot q)^2 + M_b^2 (P_a \cdot q)^2 \} h_{ab,48}^{\mu\nu} = \ldots, \quad (31)
$$

where the right-hand side of (31) is a linear combination of terms in which most of the $h_{ab,i}^{\mu\nu}$ ($i = 1, \ldots, 47$) enter. We refrain from writing down this (rather lengthy) formula explicitly as it is not needed for the following discussion. We also mention that the determinant identity (29) does not lead to any further relation between the $h_{ab,i}^{\mu\nu}$.

To implement gauge invariance we now apply, according to Eq. (16), the projection operator $P^{\mu\nu}$ to the tensors in (28). This procedure provides, in a straightforward manner, the final form of the hadronic tensor for the case of polarization of both hadrons. One finds

$$
W_{ab}^{\mu\nu} = \sum_{i=1}^{28} t_{ab,i}^{\mu\nu} V_{ab,i}, \quad (32)
$$

with the 28 structure functions $V_{ab,i}$, and the tensor basis

$$t^{\mu\nu}_{ab,1}, \ldots, t^{\mu\nu}_{ab,4} = S_a \cdot S_b \left\{ g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}, \tilde{P}^\mu_a \tilde{P}^\nu_a, \tilde{P}^\mu_b \tilde{P}^\nu_b, \tilde{P}^\mu_a \tilde{P}^\nu_b + \tilde{P}^\nu_a \tilde{P}^\mu_b \right\},$$

$$t^{\mu\nu}_{ab,5}, \ldots, t^{\mu\nu}_{ab,8} = S_a \cdot q S_b \cdot q \left\{ g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}, \tilde{P}^\mu_a \tilde{P}^\nu_a, \tilde{P}^\mu_b \tilde{P}^\nu_b, \tilde{P}^\mu_a \tilde{P}^\nu_b + \tilde{P}^\nu_a \tilde{P}^\mu_b \right\},$$

$$t^{\mu\nu}_{ab,9}, \ldots, t^{\mu\nu}_{ab,12} = S_a \cdot q S_b \cdot P_a \left\{ g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}, \tilde{P}^\mu_a \tilde{P}^\nu_a, \tilde{P}^\mu_b \tilde{P}^\nu_b, \tilde{P}^\mu_a \tilde{P}^\nu_b + \tilde{P}^\nu_a \tilde{P}^\mu_b \right\},$$

$$t^{\mu\nu}_{ab,13}, \ldots, t^{\mu\nu}_{ab,16} = S_b \cdot q S_a \cdot P_b \left\{ g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}, \tilde{P}^\mu_a \tilde{P}^\nu_a, \tilde{P}^\mu_b \tilde{P}^\nu_b, \tilde{P}^\mu_a \tilde{P}^\nu_b + \tilde{P}^\nu_a \tilde{P}^\mu_b \right\}, \tag{33}$$

$$t^{\mu\nu}_{ab,17}, \ldots, t^{\mu\nu}_{ab,20} = S_a \cdot P_b S_b \cdot P_a \left\{ g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}, \tilde{P}^\mu_a \tilde{P}^\nu_a, \tilde{P}^\mu_b \tilde{P}^\nu_b, \tilde{P}^\mu_a \tilde{P}^\nu_b + \tilde{P}^\nu_a \tilde{P}^\mu_b \right\},$$

$$t^{\mu\nu}_{ab,21}, t^{\mu\nu}_{ab,22} = S_a \cdot q \{ \tilde{S}^\mu_b \tilde{P}^\nu_a + \tilde{S}^\nu_b \tilde{P}^\mu_a, \tilde{S}^\mu_b \tilde{P}^\nu_b + \tilde{S}^\nu_b \tilde{P}^\mu_b \},$$

$$t^{\mu\nu}_{ab,23}, t^{\mu\nu}_{ab,24} = S_b \cdot q \{ \tilde{S}^\mu_a \tilde{P}^\nu_a + \tilde{S}^\nu_a \tilde{P}^\mu_a, \tilde{S}^\mu_a \tilde{P}^\nu_b + \tilde{S}^\nu_a \tilde{P}^\mu_b \},$$

$$t^{\mu\nu}_{ab,25}, t^{\mu\nu}_{ab,26} = S_a \cdot P_b \{ \tilde{S}^\mu_b \tilde{P}^\nu_a + \tilde{S}^\nu_b \tilde{P}^\mu_a, \tilde{S}^\mu_b \tilde{P}^\nu_b + \tilde{S}^\nu_b \tilde{P}^\mu_b \},$$

$$t^{\mu\nu}_{ab,27}, t^{\mu\nu}_{ab,28} = S_b \cdot P_a \{ \tilde{S}^\mu_a \tilde{P}^\nu_a + \tilde{S}^\nu_a \tilde{P}^\mu_a, \tilde{S}^\mu_a \tilde{P}^\nu_b + \tilde{S}^\nu_a \tilde{P}^\mu_b \}.$$

Here we used the 4-vectors $\tilde{P}^\mu_a$ and $\tilde{P}^\mu_b$ as given in (20). The vectors $\tilde{S}^\mu_a$ and $\tilde{S}^\mu_b$ are defined accordingly, i.e.,

$$\tilde{S}^\mu_a = S^\mu_a - \frac{S_a \cdot q q^\mu}{q^2}, \qquad \tilde{S}^\mu_b = S^\mu_b - \frac{S_b \cdot q q^\mu}{q^2}. \tag{34}$$

Note that the first 20 tensors in (33) correspond to the four tensors in (19) for the unpolarized case, multiplied by certain scalar products containing the spin vectors of the hadrons. We again emphasize the crucial importance of the relation (30). Without this identity the final form of the hadronic tensor would have 29 rather than 28 basis elements.

To the best of our knowledge the general structure of the hadronic tensor for the double-polarized Drell-Yan process is a new result. Though the double-polarized case was already investigated in Ref. [14], this was only done for the specific cases $q_T = 0$ and cross section integrated upon q_T. In those cases seven basis tensors can be identified.

D. Identical hadrons

If both hadrons in the initial state are identical—as is the case, e.g., for proton-proton —the total hadronic tensor in

Eq. (11) has to satisfy the symmetry relation

$$W^{\mu\nu}(P_a, S_a; P_b, S_b; q) = W^{\mu\nu}(P_b, S_b; P_a, S_a; q). \tag{35}$$

This immediately implies that eight out of the 48 structure functions are symmetric when exchanging the momenta P_a and P_b,

$$V_{u,1}(b, a) = V_{u,1}(a, b), \qquad V_{u,4}(b, a) = V_{u,4}(a, b),$$
$$V_{ab,1}(b, a) = V_{ab,1}(a, b), \qquad V_{ab,4}(b, a) = V_{ab,4}(a, b),$$
$$V_{ab,5}(b, a) = V_{ab,5}(a, b), \qquad V_{ab,8}(b, a) = V_{ab,8}(a, b),$$
$$V_{ab,17}(b, a) = V_{ab,17}(a, b), \qquad V_{ab,20}(b, a) = V_{ab,20}(a, b), \tag{36}$$

where, e.g., the first relation in (36) is a shorthand of

$$V_{u,1}(P_b \cdot q, P_a \cdot q, q^2) = V_{u,1}(P_a \cdot q, P_b \cdot q, q^2). \tag{37}$$

Because of the symmetry property it is sufficient to know the structure functions in (36) for just half of the allowed parameter space. The remaining 40 structure functions fulfil the relations

$$V_{u,3}(b, a) = V_{u,2}(a, b), \qquad V_{b,1}(b, a) = V_{a,1}(a, b), \qquad V_{b,2}(b, a) = V_{a,3}(a, b), \qquad V_{b,3}(b, a) = V_{a,2}(a, b),$$
$$V_{b,4}(b, a) = V_{a,4}(a, b), \qquad V_{b,5}(b, a) = V_{a,7}(a, b), \qquad V_{b,6}(b, a) = V_{a,8}(a, b), \qquad V_{b,7}(b, a) = V_{a,5}(a, b),$$
$$V_{b,8}(b, a) = V_{a,6}(a, b), \qquad V_{ab,3}(b, a) = V_{ab,2}(a, b), \qquad V_{ab,7}(b, a) = V_{ab,6}(a, b), \qquad V_{ab,13}(b, a) = V_{ab,9}(a, b),$$
$$V_{ab,14}(b, a) = V_{ab,11}(a, b), \qquad V_{ab,15}(b, a) = V_{ab,10}(a, b), \qquad V_{ab,16}(b, a) = V_{ab,12}(a, b), \qquad V_{ab,19}(b, a) = V_{ab,18}(a, b),$$
$$V_{ab,23}(b, a) = V_{ab,22}(a, b), \qquad V_{ab,24}(b, a) = V_{ab,21}(a, b), \qquad V_{ab,27}(b, a) = V_{ab,26}(a, b), \qquad V_{ab,28}(b, a) = V_{ab,25}(a, b). \tag{38}$$

For instance the first relation in (38) implies that if one knows the structure function $V_{u,2}$ for the entire parameter space one also knows $V_{u,3}$.

IV. REFERENCE FRAMES

So far our treatment is frame-independent. If, however, one wants to write down the general form of the angular distribution of the cross section—as we are going to do in Sec. V—one has to specify the reference frame. Moreover, the parton model calculation of the hadronic tensor, carried out in Sec. VI, is naturally performed in the cm-frame. Therefore, in the following we will consider both the cm-frame and the CS-frame [27], which is a particular dilepton rest frame. In general, the angular distribution of the cross section is most conveniently given in a dilepton rest frame.

In the cm-frame the 4-momenta P_a^μ, P_b^μ, and q^μ take the form

$$P_{a,\text{CM}}^\mu = (P_{a,\text{CM}}^0, 0, 0, P_{a,\text{CM}}^3) \approx \frac{\sqrt{s}}{2}(1, 0, 0, 1), \quad (39)$$

$$P_{b,\text{CM}}^\mu = (P_{b,\text{CM}}^0, 0, 0, P_{b,\text{CM}}^3) \approx \frac{\sqrt{s}}{2}(1, 0, 0, -1), \quad (40)$$

$$q_{\text{CM}}^\mu = (q_{0,\text{CM}}, q_{T,\text{CM}}, 0, q_{L,\text{CM}}), \quad (41)$$

where the simple relation between the hadron momenta and $\sqrt{s}$ holds if the hadron masses are neglected. Note that without loss of generality the transverse part of the photon momentum is pointing into the x-direction. To shorten the notation we will use $q_T \equiv q_{T,\text{CM}}$ in the following. Equations (39)–(41) fix the axes of the cm-frame according to

$$\hat{e}_{x,\text{CM}} = \frac{\vec{q}_T}{q_T}, \quad \hat{e}_{y,\text{CM}} = \hat{e}_{z,\text{CM}} \times \hat{e}_{x,\text{CM}}, \quad \hat{e}_{z,\text{CM}} = \frac{\vec{P}_{a,\text{CM}}}{|\vec{P}_{a,\text{CM}}|}. \quad (42)$$

To make the transition from the cm-frame to the CS-frame one can apply two subsequent Lorentz boosts [27]. In a first step one boosts along the z-axis such that the virtual photon no longer has a longitudinal momentum component. In a second step one boosts along the x-axis such that also the transverse momentum of the virtual photon disappears. This leads to the following transformation matrix between the two frames:

$$B^\mu{}_\nu = \frac{1}{q}\begin{pmatrix} q_{0,\text{CM}} & -\rho q & 0 & -q_{L,\text{CM}} \\ -\sin\alpha\, q_{0,\text{CM}} & (\cos\alpha)^{-1}q & 0 & \sin\alpha\, q_{L,\text{CM}} \\ 0 & 0 & q & 0 \\ -\cos\alpha\, q_{L,\text{CM}} & 0 & 0 & \cos\alpha\, q_{0,\text{CM}} \end{pmatrix}, \quad (43)$$

with

$$\rho = \frac{q_T}{q}, \quad \sin\alpha = \frac{\rho}{\sqrt{1+\rho^2}}, \quad \cos\alpha = \frac{1}{\sqrt{1+\rho^2}}. \quad (44)$$

Applying the transformation matrix in (43) to the 4-momenta $P_{a,\text{CM}}^\mu$, $P_{b,\text{CM}}^\mu$, q_{CM}^μ one finds, in particular, that the hadron momenta span the xz-plane. The results are

$$P_{a,\text{CS}}^\mu = (P_{a,\text{CS}}^0, -\sin\bar{\alpha}|\vec{P}_{a,\text{CS}}|, 0, \cos\bar{\alpha}|\vec{P}_{a,\text{CS}}|)$$
$$\approx P_{a,\text{CS}}^0(1, -\sin\alpha, 0, \cos\alpha), \quad (45)$$

$$P_{b,\text{CS}}^\mu = (P_{b,\text{CS}}^0, -\sin\bar{\alpha}|\vec{P}_{b,\text{CS}}|, 0, -\cos\bar{\alpha}|\vec{P}_{b,\text{CS}}|)$$
$$\approx P_{b,\text{CS}}^0(1, -\sin\alpha, 0, -\cos\alpha), \quad (46)$$

$$q_{\text{CS}}^\mu = (q, 0, 0, 0), \quad (47)$$

where the energies of the hadrons in the CS-frame are given by

$$P_{a,\text{CS}}^0 = \frac{P_a \cdot q}{q} \approx \frac{\sqrt{s}}{2q}(q_{0,\text{CM}} - q_{L,\text{CM}}),$$
$$P_{b,\text{CS}}^0 = \frac{P_b \cdot q}{q} \approx \frac{\sqrt{s}}{2q}(q_{0,\text{CM}} + q_{L,\text{CM}}). \quad (48)$$

The approximate expressions in (45) and (46) again hold if the hadron masses are neglected. Note that in this case one has $\alpha = \bar{\alpha}$. Equations (45) and (46), imply that the axes in the CS-frame are fixed by the hadron momenta according to

$$\hat{e}_{x,\text{CS}} = -\frac{1}{2\sin\bar{\alpha}}\left(\frac{\vec{P}_{a,\text{CS}}}{|\vec{P}_{a,\text{CS}}|} + \frac{\vec{P}_{b,\text{CS}}}{|\vec{P}_{b,\text{CS}}|}\right),$$
$$\hat{e}_{y,\text{CS}} = \hat{e}_{z,\text{CS}} \times \hat{e}_{x,\text{CS}}, \quad (49)$$
$$\hat{e}_{z,\text{CS}} = \frac{1}{2\cos\bar{\alpha}}\left(\frac{\vec{P}_{a,\text{CS}}}{|\vec{P}_{a,\text{CS}}|} - \frac{\vec{P}_{b,\text{CS}}}{|\vec{P}_{b,\text{CS}}|}\right).$$

In principle there are infinitely many dilepton rest frames. Any other dilepton rest frame is related to the CS-frame through a 3-dimensional rotation. For instance, the frequently used Gottfried-Jackson frame [34], in which the momentum of one of the hadrons is pointing into the z-direction, is connected to the CS-frame by a rotation about the y-axis.

One can readily invert the Lorentz transformation in (43) and find

$$(B^{-1})^\mu{}_\nu = \frac{1}{q}\begin{pmatrix} q_{0,\mathrm{CM}} & \sin\alpha\, q_{0,\mathrm{CM}} & 0 & \cos\alpha\, q_{L,\mathrm{CM}} \\ \rho q & (\cos\alpha)^{-1} q & 0 & 0 \\ 0 & 0 & q & 0 \\ q_{L,\mathrm{CM}} & \sin\alpha\, q_{L,\mathrm{CM}} & 0 & \cos\alpha\, q_{0,\mathrm{CM}} \end{pmatrix}. \quad (50)$$

This inverse transformation is now applied to the 4-momenta of the outgoing leptons, which in the CS-frame take the simple form

$$l_{\mathrm{CS}}^\mu = \frac{q}{2}(1, \sin\theta_{\mathrm{CS}}\cos\phi_{\mathrm{CS}}, \sin\theta_{\mathrm{CS}}\sin\phi_{\mathrm{CS}}, \cos\theta_{\mathrm{CS}}), \quad (51)$$

$$l'^\mu_{\mathrm{CS}} = \frac{q}{2}(1, -\sin\theta_{\mathrm{CS}}\cos\phi_{\mathrm{CS}}, -\sin\theta_{\mathrm{CS}}\sin\phi_{\mathrm{CS}}, -\cos\theta_{\mathrm{CS}}), \quad (52)$$

i.e., the directions of both leptons are specified by the same two angles θ_{CS} and ϕ_{CS}. This feature, of course, holds in any other dilepton rest frame as well. In the cm-frame the lepton momenta are given by

$$l_{\mathrm{CM}}^\mu = \frac{1}{2}\begin{pmatrix} (1 + \sin\alpha\,\sin\theta_{\mathrm{CS}}\cos\phi_{\mathrm{CS}})q_{0,CM} + \cos\alpha\,\cos\theta_{\mathrm{CS}}q_{L,\mathrm{CM}} \\ q_T + (\cos\alpha)^{-1}\sin\theta_{\mathrm{CS}}\cos\phi_{\mathrm{CS}}q \\ \sin\theta_{\mathrm{CS}}\sin\phi_{\mathrm{CS}}q \\ (1 + \sin\alpha\,\sin\theta_{\mathrm{CS}}\cos\phi_{\mathrm{CS}})q_{L,\mathrm{CM}} + \cos\alpha\,\cos\theta_{\mathrm{CS}}q_{0,\mathrm{CM}} \end{pmatrix}, \quad (53)$$

$$l'^\mu_{\mathrm{CM}} = \frac{1}{2}\begin{pmatrix} (1 - \sin\alpha\,\sin\theta_{\mathrm{CS}}\cos\phi_{\mathrm{CS}})q_{0,\mathrm{CM}} - \cos\alpha\,\cos\theta_{\mathrm{CS}}q_{L,\mathrm{CM}} \\ q_T - (\cos\alpha)^{-1}\sin\theta_{\mathrm{CS}}\cos\phi_{\mathrm{CS}}q \\ -\sin\theta_{\mathrm{CS}}\sin\phi_{\mathrm{CS}}q \\ (1 - \sin\alpha\,\sin\theta_{\mathrm{CS}}\cos\phi_{\mathrm{CS}})q_{L,\mathrm{CM}} - \cos\alpha\,\cos\theta_{\mathrm{CS}}q_{0,\mathrm{CM}} \end{pmatrix}. \quad (54)$$

By means of these momenta one can carry out the contraction of the leptonic and the hadronic tensor in the cm-frame. This is particularly convenient in connection with the parton model calculation in Sec. VI.

We close this section with a brief discussion on the hadron spin vectors. In the cm-frame one can write

$$S_{a,\mathrm{CM}}^\mu = \left(S_{aL,\mathrm{CM}}\frac{|\vec{P}_{a,\mathrm{CM}}|}{M_a}, |\vec{S}_{aT,\mathrm{CM}}|\cos\phi_{a,\mathrm{CM}}, |\vec{S}_{aT,\mathrm{CM}}|\sin\phi_{a,\mathrm{CM}}, S_{aL,\mathrm{CM}}\frac{P^0_{a,\mathrm{CM}}}{M_a}\right), \quad (55)$$

$$S_{b,\mathrm{CM}}^\mu = \left(S_{bL,\mathrm{CM}}\frac{|\vec{P}_{b,\mathrm{CM}}|}{M_b}, |\vec{S}_{bT,\mathrm{CM}}|\cos\phi_{b,\mathrm{CM}}, |\vec{S}_{bT,\mathrm{CM}}|\sin\phi_{b,\mathrm{CM}}, -S_{bL,\mathrm{CM}}\frac{P^0_{b,\mathrm{CM}}}{M_b}\right), \quad (56)$$

with the longitudinal components $S_{aL,\mathrm{CM}}$, $S_{bL,\mathrm{CM}}$, and the transverse components $\vec{S}_{aT,\mathrm{CM}}$, $\vec{S}_{bT,\mathrm{CM}}$. The condition $S_a^2 = -1$ implies $(S_{aL,\mathrm{CM}})^2 + (\vec{S}_{aT,\mathrm{CM}})^2 = 1$ (and analogously for the hadron H_b). One can also write down, e.g., S_a^μ in the CS-frame in terms of longitudinal and transverse components.[4] Mainly for the following reason we prefer, however, to work with components of the spin vectors in the cm-frame. If one has a pure transverse polarization in the cm-frame (in the xz-plane), this implies also a longitudinal polarization component in the CS-frame. Therefore, longitudinal and transverse polarization components can get mixed up when switching between both frames. Since an experimental setup and also the parton model approximation have a closer connection to the cm-frame than to the

CS-frame it is preferable to work with cm-frame components of the hadron spin vectors.

V. ANGULAR DISTRIBUTION OF THE CROSS SECTION

By means of the general form of the hadronic tensor as derived in Sec. III one can now write down the full angular distribution of the DY cross section. Since the hadronic tensor is frame-independent this can be done, in principle, for any reference frame. We focus here on a dilepton rest frame because in that case the angular distribution takes the most compact and transparent form. Expressing the orientation of the leptons through the CS-angles θ_{CS} and ϕ_{CS} (see Eqs. (51)–(54)) and contracting the leptonic tensor in (5) with the hadronic tensor one finds the following general form of the cross section in Eq. (10):

---

[4]The resulting expression looks a bit more complicated because $\vec{P}_{a,\mathrm{CS}}$ is not pointing in the z-direction.

$$\frac{d\sigma}{d^4qd\Omega} = \frac{\alpha_{\rm em}^2}{Fq^2}\{((1+\cos^2\theta)F_{UU}^1 + (1-\cos^2\theta)F_{UU}^2 + \sin2\theta\cos\phi F_{UU}^{\cos\phi} + \sin^2\theta\cos2\phi F_{UU}^{\cos2\phi})$$

$$+ S_{aL}(\sin2\theta\sin\phi F_{LU}^{\sin\phi} + \sin^2\theta\sin2\phi F_{LU}^{\sin2\phi}) + S_{bL}(\sin2\theta\sin\phi F_{UL}^{\sin\phi} + \sin^2\theta\sin2\phi F_{UL}^{\sin2\phi})$$

$$+ |\vec{S}_{aT}|[\sin\phi_a((1+\cos^2\theta)F_{TU}^1 + (1-\cos^2\theta)F_{TU}^2 + \sin2\theta\cos\phi F_{TU}^{\cos\phi} + \sin^2\theta\cos2\phi F_{TU}^{\cos2\phi})$$

$$+ \cos\phi_a(\sin2\theta\sin\phi F_{TU}^{\sin\phi} + \sin^2\theta\sin2\phi F_{TU}^{\sin2\phi})] + |\vec{S}_{bT}|[\sin\phi_b((1+\cos^2\theta)F_{UT}^1 + (1-\cos^2\theta)F_{UT}^2$$

$$+ \sin2\theta\cos\phi F_{UT}^{\cos\phi} + \sin^2\theta\cos2\phi F_{UT}^{\cos2\phi}) + \cos\phi_b(\sin2\theta\sin\phi F_{UT}^{\sin\phi} + \sin^2\theta\sin2\phi F_{UT}^{\sin2\phi})]$$

$$+ S_{aL}S_{bL}((1+\cos^2\theta)F_{LL}^1 + (1-\cos^2\theta)F_{LL}^2 + \sin2\theta\cos\phi F_{LL}^{\cos\phi} + \sin^2\theta\cos2\phi F_{LL}^{\cos2\phi})$$

$$+ S_{aL}|\vec{S}_{bT}|[\cos\phi_b((1+\cos^2\theta)F_{LT}^1 + (1-\cos^2\theta)F_{LT}^2 + \sin2\theta\cos\phi F_{LT}^{\cos\phi} + \sin^2\theta\cos2\phi F_{LT}^{\cos2\phi})$$

$$+ \sin\phi_b(\sin2\theta\sin\phi F_{LT}^{\sin\phi} + \sin^2\theta\sin2\phi F_{LT}^{\sin2\phi})] + |\vec{S}_{aT}|S_{bL}[\cos\phi_a((1+\cos^2\theta)F_{TL}^1 + (1-\cos^2\theta)F_{TL}^2$$

$$+ \sin2\theta\cos\phi F_{TL}^{\cos\phi} + \sin^2\theta\cos2\phi F_{TL}^{\cos2\phi}) + \sin\phi_a(\sin2\theta\sin\phi F_{TL}^{\sin\phi} + \sin^2\theta\sin2\phi F_{TL}^{\sin2\phi})]$$

$$+ |\vec{S}_{aT}||\vec{S}_{bT}|[\cos(\phi_a+\phi_b)((1+\cos^2\theta)F_{TT}^1 + (1-\cos^2\theta)F_{TT}^2 + \sin2\theta\cos\phi F_{TT}^{\cos\phi} + \sin^2\theta\cos2\phi F_{TT}^{\cos2\phi})$$

$$+ \cos(\phi_a-\phi_b)((1+\cos^2\theta)\bar{F}_{TT}^1 + (1-\cos^2\theta)\bar{F}_{TT}^2 + \sin2\theta\cos\phi \bar{F}_{TT}^{\cos\phi} + \sin^2\theta\cos2\phi \bar{F}_{TT}^{\cos2\phi})$$

$$+ \sin(\phi_a+\phi_b)(\sin2\theta\sin\phi F_{TT}^{\sin\phi} + \sin^2\theta\sin2\phi F_{TT}^{\sin2\phi})$$

$$+ \sin(\phi_a-\phi_b)(\sin2\theta\sin\phi \bar{F}_{TT}^{\sin\phi} + \sin^2\theta\sin2\phi \bar{F}_{TT}^{\sin2\phi})]\}. \tag{57}$$

In Eq. (57) 48 structure functions show up which exactly matches with the number of the V_i defined in Sec. III. The structure functions again depend on the three variables $P_a \cdot q$, $P_b \cdot q$, and q^2, i.e., $F_{UU}^1 = F_{UU}^1(P_a \cdot q, P_b \cdot q, q^2)$ and so on. We refrain from giving the explicit relations between the structure functions in (57) and the V_i because these lengthy formulae are not needed for the following discussion. In order to shorten the notation in (57) we left out indices for the angles which characterize the lepton momenta and the transverse spin vectors of the hadrons. There is yet another reason for omitting those indices: the form of the angular distribution in (57) holds for any dilepton rest frame and not just the CS-frame. The numerical values of the structure functions of course change when going from one frame to another. Furthermore, note that the components of the spin vectors can be understood in different frames like the rest frame of one of the hadrons, the cm-frame, or a dilepton rest frame.

In particular for the angular distribution of the unpolarized cross section different notations can be found in the literature (see, e.g., [37] and references therein). Here we just quote the frequently used formula

$$\frac{dN}{d\Omega} \equiv \frac{d\sigma}{d^4qd\Omega}\bigg/\frac{d\sigma}{d^4q}$$

$$= \frac{3}{4\pi}\frac{1}{\lambda+3}\bigg(1 + \lambda\cos^2\theta + \mu\sin2\theta\cos\phi$$

$$+ \frac{\nu}{2}\sin^2\theta\cos2\phi\bigg). \tag{58}$$

One readily finds

$$\lambda = \frac{F_{UU}^1 - F_{UU}^2}{F_{UU}^1 + F_{UU}^2},$$

$$\mu = \frac{F_{UU}^{\cos\phi}}{F_{UU}^1 + F_{UU}^2}, \tag{59}$$

$$\nu = \frac{2F_{UU}^{\cos2\phi}}{F_{UU}^1 + F_{UU}^2}.$$

The so-called Lam-Tung relation [35,36,39]

$$\lambda + 2\nu = 1, \tag{60}$$

which in terms of the structure functions defined in (57) reads

$$F_{UU}^2 = 2F_{UU}^{\cos2\phi}, \tag{61}$$

has attracted considerable attention in the past. This relation is exact if one computes the DY process to $\mathcal{O}(\alpha_s)$ in the standard collinear perturbative QCD framework. Even at $\mathcal{O}(\alpha_s^2)$ the numerical violation of (60) is small [40]. On the other hand data for $\pi^-N \to \mu^-\mu^+X$ taken at CERN [41,42] and at Fermilab [43] are in disagreement with the Lam-Tung relation. In particular, an unexpectedly large $\cos2\phi$ modulation of the cross section was observed, and in the meantime different explanations for this phenomenon have been put forward in the literature [44–50]. In Ref. [33] it was pointed out that intrinsic transverse motion of initial state partons might be responsible for the observed violation of the Lam-Tung relation. In the following section we will briefly return to this point in connection with the parton model calculation. It is also worthwhile to mention that more recent Fermilab data on proton-deuteron Drell-Yan do agree with the Lam-Tung relation [51].

The hadronic tensor given in Sec. III also allows one to find the angular distribution of the cross section for the specific kinematical point $q_T = 0$. Altogether, in that case one has nine independent angular dependences and structure functions,

$$
\begin{aligned}
\frac{d\sigma}{d^4 q d\Omega}\bigg|_{q_T=0} = \frac{\alpha_{\text{em}}^2}{Fq^2}\bigg\{ &((1+\cos^2\theta)F_{UU}^1 + (1-\cos^2\theta)F_{UU}^2) + S_{aL}S_{bL}((1+\cos^2\theta)F_{LL}^1 + (1-\cos^2\theta)F_{LL}^2) \\
&+ S_{aL}|\vec{S}_{bT}|\bigg(\sin2\theta\cos(\phi-\phi_b)\frac{1}{2}(F_{LT}^{\cos\phi} + F_{LT}^{\sin\phi})\bigg) + |\vec{S}_{aT}|S_{bL}\bigg(\sin2\theta\cos(\phi-\phi_a)\frac{1}{2}(F_{TL}^{\cos\phi} + F_{TL}^{\sin\phi})\bigg) \\
&+ |\vec{S}_{aT}||\vec{S}_{bT}|\bigg[\sin^2\theta\cos(2\phi-\phi_a-\phi_b)\frac{1}{2}(F_{TT}^{\cos2\phi} + F_{TT}^{\sin2\phi}) \\
&+ \cos(\phi_a-\phi_b)((1+\cos^2\theta)\bar{F}_{TT}^1 + (1-\cos^2\theta)\bar{F}_{TT}^2)\bigg]\bigg\}.
\end{aligned}
\tag{62}
$$

This result was already given in Ref. [14] using a different notation. We note that in the double-polarized sector the following relations hold:

$$
F_{LT}^{\cos\phi}|_{q_T=0} = F_{LT}^{\sin\phi}|_{q_T=0}, \qquad F_{TL}^{\cos\phi}|_{q_T=0} = F_{TL}^{\sin\phi}|_{q_T=0},
$$
$$
F_{TT}^{\cos2\phi}|_{q_T=0} = F_{TT}^{\sin2\phi}|_{q_T=0}.
\tag{63}
$$

All the structure functions that show up in (57) but not in (62) have a kinematical zero at $q_T = 0$. Notice that the angular distribution of the q_T-integrated cross section agrees with the one in (62), but the corresponding structure functions differ numerically (see also [14]).

We have pointed out that our hadronic tensor provides the angular distribution of the cross section at $q_T = 0$. In fact, this statement is not totally correct. The hadronic tensor in Eq. (33) does not generate the term proportional to $\cos(2\phi-\phi_a-\phi_b)$ in (62). This can be understood by taking a close look at Eq. (31): there the prefactor in front of $h_{ab,48}^{\mu\nu}$ vanishes for $q_T = 0$ implying that at this particular kinematical point one is actually not allowed to elimi-

nate this tensor which does generate the required $\cos(2\phi-\phi_a-\phi_b)$ term for $q_T = 0$. One may instead eliminate, for instance, the tensor $h_{ab,47}^{\mu\nu}$. Since the rather specific case $q_T = 0$ was already worked out in the literature [14] we have proposed the tensor in (33) for the sake of symmetry.

In Sec. III D we have considered the case of identical hadrons in the initial state and the resultant constraints for the structure functions V_i. One can do a corresponding analysis for the structure functions defined in Eq. (57). The key ingredient of such an analysis is that the cross section remains the same if the hadrons are exchanged. Note that the exchange $H_a \leftrightarrow H_b$ also leads to the reversal of the z-direction which, in particular, implies

$$
\phi_a \leftrightarrow -\phi_b, \qquad \phi \to -\phi, \qquad \theta \to \pi - \theta. \tag{64}
$$

Twenty structure functions are either symmetric or antisymmetric under the exchange $P_a \leftrightarrow P_b$. Using the shorthand notation of Eqs. (36) and (38), one finds

$$
\begin{aligned}
&F_{UU}^1(b,a) = F_{UU}^1(a,b), && F_{UU}^2(b,a) = F_{UU}^2(a,b), && F_{UU}^{\cos\phi}(b,a) = -F_{UU}^{\cos\phi}(a,b), && F_{UU}^{\cos2\phi}(b,a) = F_{UU}^{\cos2\phi}(a,b), \\
&F_{LL}^1(b,a) = F_{LL}^1(a,b), && F_{LL}^2(b,a) = F_{LL}^2(a,b), && F_{LL}^{\cos\phi}(b,a) = -F_{LL}^{\cos\phi}(a,b), && F_{LL}^{\cos2\phi}(b,a) = F_{LL}^{\cos2\phi}(a,b), \\
&F_{TT}^1(b,a) = F_{TT}^1(a,b), && F_{TT}^2(b,a) = F_{TT}^2(a,b), && F_{TT}^{\cos\phi}(b,a) = -F_{TT}^{\cos\phi}(a,b), && F_{TT}^{\cos2\phi}(b,a) = F_{TT}^{\cos2\phi}(a,b), \\
&\bar{F}_{TT}^1(b,a) = \bar{F}_{TT}^1(a,b), && \bar{F}_{TT}^2(b,a) = \bar{F}_{TT}^2(a,b), && \bar{F}_{TT}^{\cos\phi}(b,a) = -\bar{F}_{TT}^{\cos\phi}(a,b), && \bar{F}_{TT}^{\cos2\phi}(b,a) = \bar{F}_{TT}^{\cos2\phi}(a,b), \\
&F_{TT}^{\sin\phi}(b,a) = -F_{TT}^{\sin\phi}(a,b), && F_{TT}^{\sin2\phi}(b,a) = F_{TT}^{\sin2\phi}(a,b), && \bar{F}_{TT}^{\sin\phi}(b,a) = \bar{F}_{TT}^{\sin\phi}(a,b), && \bar{F}_{TT}^{\sin2\phi}(b,a) = -\bar{F}_{TT}^{\sin2\phi}(a,b).
\end{aligned}
\tag{65}
$$

The remaining structure functions fulfil the relations

$$
\begin{aligned}
&F_{UL}^{\sin\phi}(b,a) = F_{LU}^{\sin\phi}(a,b), && F_{UL}^{\sin2\phi}(b,a) = -F_{LU}^{\sin2\phi}(a,b), && F_{UT}^1(b,a) = -F_{TU}^1(a,b), && F_{UT}^2(b,a) = -F_{TU}^2(a,b), \\
&&F_{UT}^{\cos\phi}(b,a) = F_{TU}^{\cos\phi}(a,b), && F_{UT}^{\cos2\phi}(b,a) = -F_{TU}^{\cos2\phi}(a,b), && F_{UT}^{\sin\phi}(b,a) = F_{TU}^{\sin\phi}(a,b), \\
&F_{UT}^{\sin2\phi}(b,a) = -F_{TU}^{\sin2\phi}(a,b), && F_{TL}^1(b,a) = F_{LT}^1(a,b), && F_{TL}^2(b,a) = F_{LT}^2(a,b), && F_{TL}^{\cos\phi}(b,a) = -F_{LT}^{\cos\phi}(a,b), \\
&&F_{TL}^{\cos2\phi}(b,a) = F_{LT}^{\cos2\phi}(a,b), && F_{TL}^{\sin\phi}(b,a) = -F_{LT}^{\sin\phi}(a,b), && F_{TL}^{\sin2\phi}(b,a) = F_{LT}^{\sin2\phi}(a,b).
\end{aligned}
\tag{66}
$$

It is of course intuitively clear that for identical hadrons relations as given in (66) have to exist. But one has to keep in mind

that relative signs between the corresponding structure functions can show up. Eventually, we mention that (65) and (66) can also be derived from (36) and (38) and the relations between the two sets of structure functions.

VI. PARTON MODEL APPROXIMATION

This section deals with the parton model description of the structure functions in Eq. (57). Up to this point we did not specify the external kinematics of the process. In the following we will consider the kinematical regime where the transverse photon momentum q_T is of the order of a typical hadronic mass scale which means, in particular, that it is much smaller than the hard scale q. This is the region where TMDs enter the description of the DY process in a natural way.

Our treatment is restricted to leading twist, i.e., to the leading order of an expansion in powers of $1/q$. Mainly because of the potential problems of subleading twist TMD-factorization pointed out in Refs. [52,53] we refrain here from including the twist-3 case. Moreover, we neither take into account higher-order hard scattering corrections nor effects associated with soft gluon radiation. For three of the structure functions such contributions were considered in [54].

A. Hadronic tensor

The parton model description of the Drell-Yan process can be represented by the diagrams shown in Fig. 1, where, e.g., the scattering amplitude for diagram (a) reads

$$iM_{(a)} = \sum_q \sum_{c=1}^{N_c} \frac{ie_q e^2}{q^2} \langle X_a | \psi_i^{c,q}(0) | P_a, S_a \rangle$$
$$\times \langle X_b | \bar{\psi}_j^{c,q}(0) | P_b, S_b \rangle [(\gamma^\mu)_{ji} \bar{u}(l, \lambda) \gamma_\mu v(l', \lambda')]. \tag{67}$$

A sum over color c and the quark flavors q is implemented explicitly in this expression. The electromagnetic charge of the quark, in units of the elementary charge e, is denoted by e_q. A corresponding formula holds for the amplitude $M_{(b)}$ of the graph (b). The differential cross section (10) in a dilepton rest frame is then given by

$$\frac{d\sigma}{d^4 q d\Omega} = \frac{1}{8(2\pi)^2 F} \sum_{\lambda, \lambda'} \sum\!\!\!\!\!\!\!\int_{X_a, X_b} (|M_{(a)}|^2 + |M_{(b)}|^2)$$
$$\times \delta^{(4)}(P_{X_a} + P_{X_b} + q - P_a - P_b). \tag{68}$$

Note that there is no interference between the two diagrams in Fig. 1. One can modify this formula by introducing the momenta of the active partons, k_a and k_b. This allows one to sum over a complete set of intermediate states and to rewrite the hadronic part of the cross section in terms of fully unintegrated quark-quark correlators (see, e.g., Refs. [23,24,32,55–58]). In doing so one finds the hadronic tensor

$$W^{\mu\nu} = \frac{1}{N_c} \sum_q e_q^2 \int d^4 k_a d^4 k_b \delta^{(4)}(q - k_a - k_b)$$
$$\times \text{Tr}[\gamma^\mu \Phi^q(k_a, P_a, S_a | n_a) \gamma^\nu \bar{\Phi}^q(k_b, P_b, S_b | n_b)]$$
$$+ \{\Phi \leftrightarrow \bar{\Phi}\}, \tag{69}$$

where the quark-quark correlators, which depend on the full 4-momentum of the quarks, are defined as

$$\Phi_{ij}^q(k_a, P_a, S_a | n_a) = \int \frac{d^4 z}{(2\pi)^4} e^{ik_a \cdot z} \langle P_a, S_a | \bar{\psi}_j^q(0)$$
$$\times \mathcal{W}_{\text{DY}}[0, z | n_a] \psi_i^q(z) | P_a, S_a \rangle, \tag{70}$$

$$\bar{\Phi}_{ij}^q(k_b, P_b, S_b | n_b) = \int \frac{d^4 z}{(2\pi)^4} e^{ik_b \cdot z} \langle P_b, S_b | \psi_i^q(0)$$
$$\times \mathcal{W}_{\text{DY}}[0, z | n_b] \bar{\psi}_j^q(z) | P_b, S_b \rangle. \tag{71}$$

The object $\mathcal{W}$ denotes a gauge link operator (Wilson line) which ensures color gauge invariance of the correlators. We note that actually the Wilson lines cannot be derived from the diagrams in Fig. 1. They are generated, however, if in addition collinear gluon exchanges between the active partons and the remnants of the incoming hadrons are taken into account (see, e.g., Refs. [59–62]). In general, the Wilson lines entering unintegrated parton correlators are process-dependent. For the DY process we will specify them below but already emphasize here their dependence on a light-cone vector n_a or n_b. Note that in Eqs. (70) and (71) a color sum is implicit, leading to the factor $1/N_c = 1/3$ in (69). The term $\{\Phi \leftrightarrow \bar{\Phi}\}$ in Eq. (69) represents the contribution of the diagram in Fig. 1(b) and is obtained from the first term by interchanging the correlators.

In the parton model initial state partons are assumed to move quasicollinearly with respect to their parent hadron. Consequently, the components of the parton momenta behave like the corresponding components of the hadron momenta. The following estimates for the parton momenta in the DY process are valid in frames where the hadron H_a has a large light-cone plus-momentum and the hadron H_b a large minus-momentum (this applies, in particular, to the cm-frame—see also the discussion in Sec. IV):

$$k_a^+ \sim \mathcal{O}(q), \qquad k_a^- \sim \mathcal{O}(1/q), \qquad k_{aT} \sim \mathcal{O}(q^0),$$
$$k_b^+ \sim \mathcal{O}(1/q), \qquad k_b^- \sim \mathcal{O}(q), \qquad k_{bT} \sim \mathcal{O}(q^0), \tag{72}$$

where we use the light-cone components $v^\pm = (v^0 \pm v^3)/\sqrt{2}$ for a generic 4-vector v. From the standpoint of factorization this means that Φ and $\bar{\Phi}$ are treated as nonperturbative objects because the kinematical invariants $k_a \cdot P_a$, k_a^2, $k_b \cdot P_b$, k_b^2 on which the correlators depend are much smaller than q^2. According to (72) the momentum components k_a^- and k_b^+ are small and hence can be neglected in the δ-function in Eq. (69). This also automatically implies $q^+ \approx k_a^+$ and $q^- \approx k_b^-$. The hadronic tensor then reduces to

$$W^{\mu\nu} = \frac{1}{N_c} \sum_q e_q^2 \int d^2\vec{k}_{aT} d^2\vec{k}_{bT} \delta^{(2)}(\vec{q}_T - \vec{k}_{aT} - \vec{k}_{bT})$$

$$\times \text{Tr}[\gamma^\mu \Phi^q(x_a, \vec{k}_{aT}, S_a|n_a)\gamma^\nu \bar{\Phi}^q(x_b, \vec{k}_{bT}, S_b|n_b)]$$

$$+ \{\Phi \leftrightarrow \bar{\Phi}\}, \tag{73}$$

where we used the common DY variables

$$x_a = \frac{q^2}{2P_a \cdot q} \approx \frac{k_a^+}{P_a^+}, \qquad x_b = \frac{q^2}{2P_b \cdot q} \approx \frac{k_b^-}{P_b^-}. \tag{74}$$

The transverse momentum dependent quark-quark correlators in (73) are defined according to

$$\Phi_{ij}^q(x_a, \vec{k}_{aT}, S_a|n_a) = \int \frac{dz^- d^2\vec{z}_T}{(2\pi)^3} e^{ik_a \cdot z} \langle P_a, S_a|\bar{\psi}_j^q(0)$$

$$\times \mathcal{W}_{\text{DY}}[0, z|n_a]\psi_i^q(z)|P_a, S_a\rangle|_{z^+=0}, \tag{75}$$

$$\bar{\Phi}_{ij}^q(x_b, \vec{k}_{bT}, S_b|n_b) = \int \frac{dz^+ d^2\vec{z}_T}{(2\pi)^3} e^{ik_b \cdot z} \langle P_b, S_b|\psi_i^q(0)$$

$$\times \mathcal{W}_{\text{DY}}[0, z|n_b]\bar{\psi}_j^q(z)|P_b, S_b\rangle|_{z^-=0}, \tag{76}$$

and they are obtained from the correlators in (70) and (71) by integrating out the respective small light-cone momentum of the parton. We now specify the Wilson lines in the quark-quark correlators. The appropriate choice for the DY process is [59–61]

$$\mathcal{W}_{\text{DY}}[0, z|n_a]|_{z^+=0} = [0; -\infty n_a] \times [-\infty n_a; -\infty n_a + z_T]$$

$$\times [-\infty n_a + z_T; z^- n_a + z_T], \tag{77}$$

$$\mathcal{W}_{\text{DY}}[0, z|n_b]|_{z^-=0} = [0; -\infty n_b] \times [-\infty n_b; -\infty n_b + z_T]$$

$$\times [-\infty n_b + z_T; z^- n_b + z_T], \tag{78}$$

with $[a; b]$ denoting a straight gauge link between the positions a and b, and $z_T^\mu \equiv (0, \vec{z}_T, 0)$. The light-cone vectors in (77) and (78) are given by

$$n_a^\mu = \frac{1}{\sqrt{2}}(1, 0, 0, -1), \qquad n_b^\mu = \frac{1}{\sqrt{2}}(1, 0, 0, 1). \tag{79}$$

Note that the diagram in Fig. 1(b) generates, e.g., the correlator $\Phi^q(k_b, P_b, S_b|n_b)$ which can be related to $\Phi^q(k_a, P_a, S_a|n_a)$ in Eq. (70) by means of the parity transformation.

We also mention that so-called light-cone divergences, which are caused by the lightlike Wilson lines in (77) and (78), can be avoided if near-light-cone directions for the Wilson lines are chosen instead. For a discussion of such divergences and other nontrivial issues concerning the precise definition of unintegrated parton correlation functions we refer to the recent contribution [63] as well as references therein.

B. Transverse momentum dependent parton distributions

The quark-quark correlators in Eqs. (75) and (76) can be parametrized through TMDs [14,23,24,32,55,56]. A common and rather convenient procedure for performing such a parametrization is by specifying the traces of the correlators with the Dirac-matrices $\Gamma = \gamma^\mu, \gamma^\mu\gamma_5, i\sigma^{\mu\nu}\gamma_5, 1, i\gamma_5$,

$$\Phi^{q[\Gamma]} \equiv \frac{1}{2} \text{Tr}[\Phi^q\Gamma]. \tag{80}$$

In the cm-frame, where the hadron H_a has a large plus-momentum, the leading (twist) traces are $\Phi^{[\gamma^+]}$, $\Phi^{[\gamma^+\gamma_5]}$, and $\Phi^{[i\sigma^{i+}\gamma_5]}$ ($i = \{1, 2\}$), while all the other traces are suppressed in the cross section by at least one power of the large light-cone momentum (and consequently by one power of q). These traces then have the following expressions in terms of leading twist quark TMDs (see, e.g., [24,56][5]):

$$\Phi^{q[\gamma^+]} = f_1^q(x_a, \vec{k}_{aT}^2) - \frac{\varepsilon_T^{ij} k_{aT}^i S_{aT}^j}{M_a} f_{1T}^{\perp q}(x_a, \vec{k}_{aT}^2), \tag{81}$$

$$\Phi^{q[\gamma^+\gamma_5]} = S_{aL} g_{1L}^q(x_a, \vec{k}_{aT}^2) + \frac{\vec{k}_{aT} \cdot \vec{S}_{aT}}{M_a} g_{1T}^q(x_a, \vec{k}_{aT}^2), \tag{82}$$

$$\Phi^{q[i\sigma^{i+}\gamma_5]} = S_{aT}^i h_1^q(x_a, \vec{k}_{aT}^2)$$

$$+ \frac{k_{aT}^i(\vec{k}_{aT} \cdot \vec{S}_{aT}) - \frac{1}{2}\vec{k}_{aT}^2 S_{aT}^i}{M_a^2} h_{1T}^{\perp q}(x_a, \vec{k}_{aT}^2)$$

$$+ S_{aL}\frac{k_{aT}^i}{M_a}h_{1L}^{\perp q}(x_a, \vec{k}_{aT}^2) + \frac{\varepsilon_T^{ij} k_{aT}^j}{M_a}h_1^{\perp q}(x_a, \vec{k}_{aT}^2). \tag{83}$$

For brevity we omitted the arguments of the correlator Φ. Note that the components of the nucleon spin vector in (81)–(83) are understood in the cm-frame. The object ε_T^{ij} represents a short form of the transverse epsilon tensor ε^{-+ij}, where we use the convention $\varepsilon^{-+12} = 1$. The transverse momentum dependent unpolarized quark distribution, helicity distribution, and transversity distribution are denoted by f_1, g_{1L}, and h_1, respectively. Of particular importance are also the time-reversal odd (T-odd) Sivers function $f_{1T}^\perp$ [64,65] and Boer-Mulders function $h_1^\perp$ [55] as they can give rise to quite interesting single spin and/or azimuthal asymmetries in hard semi-inclusive reactions.

The correlator $\bar{\Phi}^q$ in Eq. (76) is related to the correlator $\Phi^{\bar{q}}$ which defines, precisely in analogy to the Eqs. (81)–(83), antiquark distributions. For the different Dirac traces the relation reads [32]

---

[5]Note that the left-hand side in Eq. 16 of [56] should read $\Phi^{[i\sigma^{i+}\gamma_5]}$.

$$\bar{\Phi}^{q[\Gamma]} = \pm \Phi^{\bar{q}[\Gamma]}, \qquad \begin{cases} +\text{for } \gamma^\mu, i\sigma^{\mu\nu}\gamma_5 \\ -\text{for } \gamma^\mu\gamma_5, 1, i\gamma_5 \end{cases}. \qquad (84)$$

Since the correlator $\bar{\Phi}$ in (76) is associated with the hadron H_b having a large minus-momentum in the cm-frame, the leading traces are now $\bar{\Phi}^{[\gamma^-]}$, $\bar{\Phi}^{[\gamma^-\gamma_5]}$, and $\bar{\Phi}^{[i\sigma^{i-}\gamma_5]}$. Taking (84) into account the parametrizations can be directly obtained from (81)–(83),

$$\bar{\Phi}^{q[\gamma^-]} = f_1^{\bar{q}}(x_b, \vec{k}_{bT}^2) + \frac{\varepsilon_T^{ij}k_{bT}^i S_{bT}^j}{M_b} f_{1T}^{\perp\bar{q}}(x_b, \vec{k}_{bT}^2), \quad (85)$$

$$\bar{\Phi}^{q[\gamma^-\gamma_5]} = -S_{bL}g_{1L}^{\bar{q}}(x_b, \vec{k}_{bT}^2) - \frac{\vec{k}_{bT}\cdot\vec{S}_{bT}}{M_b}g_{1T}^{\bar{q}}(x_b, \vec{k}_{bT}^2), \tag{86}$$

$$\bar{\Phi}^{q[i\sigma^{i-}\gamma_5]} = S_{bT}^i h_1^{\bar{q}}(x_b, \vec{k}_{bT}^2)$$
$$+ \frac{k_{bT}^i(\vec{k}_{bT}\cdot\vec{S}_{bT}) - \frac{1}{2}\vec{k}_{bT}^2 S_{bT}^i}{M_b^2} h_{1T}^{\perp\bar{q}}(x_b, \vec{k}_{bT}^2)$$
$$+ S_{bL}\frac{k_{bT}^i}{M_b} h_{1L}^{\perp\bar{q}}(x_b, \vec{k}_{bT}^2)$$
$$- \frac{\varepsilon_T^{ij}k_{bT}^j}{M_b} h_1^{\perp\bar{q}}(x_b, \vec{k}_{bT}^2). \tag{87}$$

Note the respective sign change in front of the epsilon tensor ε_T^{ij} which is due to the interchange of plus-momenta and minus-momenta.

C. Leading spin observables

Now we are in a position to calculate all the leading twist observables for $q_T \ll q$ by inserting the traces (81)–(83) and (85)–(87) into the hadronic tensor (73). We mention again that the contraction of the hadronic and the leptonic tensor is performed in the cm-frame where, in order to get

the leptonic tensor in that frame, use is made of Eqs. (53) and (54). Since the lepton momenta contain angles in the CS-frame our final result for the cross section is of the form (57). Here one has to keep in mind that the leading twist calculation of course merely provides nonzero results for part of the structure functions in (57). Carrying out the contraction of the tensors and keeping only the leading contribution in $1/q$ one finds

$$\frac{d\sigma}{d^4q d\Omega} = \frac{\alpha_{\text{em}}^2 x_a x_b}{2q^4} \frac{1}{N_c} \sum_q e_q^2 \int d^2\vec{k}_{aT}d^2\vec{k}_{bT}$$
$$\times \delta^{(2)}(\vec{q}_T - \vec{k}_{aT} - \vec{k}_{bT})[(1+\cos^2\theta)(\Phi^{q[\gamma^+]}\bar{\Phi}^{q[\gamma^-]}$$
$$+ \Phi^{q[\gamma^+\gamma_5]}\bar{\Phi}^{q[\gamma^-\gamma_5]})$$
$$+ \sin^2\theta(\cos2\phi(\delta^{i1}\delta^{j1} - \delta^{i2}\delta^{j2})$$
$$+ \sin2\phi(\delta^{i1}\delta^{j2} + \delta^{i2}\delta^{j1}))\Phi^{q[i\sigma^{i+}\gamma_5]}\bar{\Phi}^{q[i\sigma^{j-}\gamma_5]}]$$
$$+ \{\Phi \leftrightarrow \bar{\Phi}\} + \mathcal{O}(1/q). \tag{88}$$

To present the leading twist spin observables we will make use of the following notation for the convolution of TMDs in the transverse momentum space:

$$\mathcal{C}[w(\vec{k}_{aT}, \vec{k}_{bT})f_1\bar{f}_2] \equiv \frac{1}{N_c}\sum_q e_q^2 \int d^2\vec{k}_{aT}d^2\vec{k}_{bT}$$
$$\times \delta^{(2)}(\vec{q}_T - \vec{k}_{aT} - \vec{k}_{bT})w(\vec{k}_{aT}, \vec{k}_{bT})$$
$$\times [f_1^q(x_a, \vec{k}_{aT}^2)f_2^{\bar{q}}(x_b, \vec{k}_{bT}^2)$$
$$+ f_1^{\bar{q}}(x_a, \vec{k}_{aT}^2)f_2^q(x_b, \vec{k}_{bT}^2)]. \tag{89}$$

The two terms on the right-hand side of (89) are generated by the two diagrams in Fig. 1. For the parton model calculation it is convenient to introduce a number of linear combinations of various structure functions given in Eq. (57):

$$F_{TU}^{\sin(2\phi-\phi_a)} \equiv -\frac{1}{2}(F_{TU}^{\cos2\phi} - F_{TU}^{\sin2\phi}), \qquad F_{TU}^{\sin(2\phi+\phi_a)} \equiv \frac{1}{2}(F_{TU}^{\cos2\phi} + F_{TU}^{\sin2\phi}), \qquad F_{UT}^{\sin(2\phi-\phi_b)} \equiv -\frac{1}{2}(F_{UT}^{\cos2\phi} - F_{UT}^{\sin2\phi}),$$

$$F_{UT}^{\sin(2\phi+\phi_b)} \equiv \frac{1}{2}(F_{UT}^{\cos2\phi} + F_{UT}^{\sin2\phi}), \qquad F_{LT}^{\cos(2\phi-\phi_b)} \equiv \frac{1}{2}(F_{LT}^{\cos2\phi} + F_{LT}^{\sin2\phi}), \qquad F_{LT}^{\cos(2\phi+\phi_b)} \equiv \frac{1}{2}(F_{LT}^{\cos2\phi} - F_{LT}^{\sin2\phi}),$$

$$F_{TL}^{\cos(2\phi-\phi_a)} \equiv \frac{1}{2}(F_{TL}^{\cos2\phi} + F_{TL}^{\sin2\phi}), \qquad F_{TL}^{\cos(2\phi+\phi_a)} \equiv \frac{1}{2}(F_{TL}^{\cos2\phi} - F_{TL}^{\sin2\phi}), \qquad F_{TT}^{\cos(2\phi-\phi_a-\phi_b)} \equiv \frac{1}{2}(F_{TT}^{\cos2\phi} + F_{TT}^{\sin2\phi}),$$

$$F_{TT}^{\cos(2\phi-\phi_a+\phi_b)} \equiv \frac{1}{2}(\bar{F}_{TT}^{\cos2\phi} + \bar{F}_{TT}^{\sin2\phi}), \qquad F_{TT}^{\cos(2\phi+\phi_a-\phi_b)} \equiv \frac{1}{2}(\bar{F}_{TT}^{\cos2\phi} - \bar{F}_{TT}^{\sin2\phi}), \qquad F_{TT}^{\cos(2\phi+\phi_a+\phi_b)} \equiv \frac{1}{2}(F_{TT}^{\cos2\phi} - F_{TT}^{\sin2\phi}).$$
$$\tag{90}$$

Using the unit vector $\vec{h} \equiv \vec{q}_T/q_T$ one eventually finds the following leading-order structure functions in the CS-frame:

$$F_{UU}^1 = \mathcal{C}[f_1\bar{f}_1], \tag{91}$$

$$F_{UU}^{\cos2\phi} = \mathcal{C}\left[\frac{2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}}{M_aM_b}h_1^\perp\bar{h}_1^\perp\right], \tag{92}$$

$$F_{LU}^{\sin2\phi} = \mathcal{C}\left[\frac{2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}}{M_a M_b}h_{1L}^{\perp}\bar{h}_1^{\perp}\right], \tag{93}$$

$$F_{UL}^{\sin2\phi} = -\mathcal{C}\left[\frac{2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}}{M_a M_b}h_1^{\perp}\bar{h}_{1L}^{\perp}\right], \tag{94}$$

$$F_{TU}^1 = -\mathcal{C}\left[\frac{\vec{h}\cdot\vec{k}_{aT}}{M_a}f_{1T}^{\perp}\bar{f}_1\right], \tag{95}$$

$$F_{TU}^{\sin(2\phi-\phi_a)} = \mathcal{C}\left[\frac{\vec{h}\cdot\vec{k}_{bT}}{M_b}h_1\bar{h}_1^{\perp}\right], \tag{96}$$

$$F_{TU}^{\sin(2\phi+\phi_a)} = \mathcal{C}\left[\frac{2(\vec{h}\cdot\vec{k}_{aT})[2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}] - \vec{k}_{aT}^2(\vec{h}\cdot\vec{k}_{bT})}{2M_a^2 M_b}h_{1T}^{\perp}\bar{h}_1^{\perp}\right], \tag{97}$$

$$F_{UT}^1 = \mathcal{C}\left[\frac{\vec{h}\cdot\vec{k}_{bT}}{M_b}f_1\bar{f}_{1T}^{\perp}\right], \tag{98}$$

$$F_{UT}^{\sin(2\phi-\phi_b)} = -\mathcal{C}\left[\frac{\vec{h}\cdot\vec{k}_{aT}}{M_a}h_1^{\perp}\bar{h}_1\right], \tag{99}$$

$$F_{UT}^{\sin(2\phi+\phi_b)} = -\mathcal{C}\left[\frac{2(\vec{h}\cdot\vec{k}_{bT})[2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}] - \vec{k}_{bT}^2(\vec{h}\cdot\vec{k}_{aT})}{2M_a M_b^2}h_1^{\perp}\bar{h}_{1T}^{\perp}\right], \tag{100}$$

$$F_{LL}^1 = -\mathcal{C}[g_{1L}\bar{g}_{1L}], \tag{101}$$

$$F_{LL}^{\cos2\phi} = \mathcal{C}\left[\frac{2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}}{M_a M_b}h_{1L}^{\perp}\bar{h}_{1L}^{\perp}\right], \tag{102}$$

$$F_{LT}^1 = -\mathcal{C}\left[\frac{\vec{h}\cdot\vec{k}_{bT}}{M_b}g_{1L}\bar{g}_{1T}\right], \tag{103}$$

$$F_{LT}^{\cos(2\phi-\phi_b)} = \mathcal{C}\left[\frac{\vec{h}\cdot\vec{k}_{aT}}{M_a}h_{1L}^{\perp}\bar{h}_1\right], \tag{104}$$

$$F_{LT}^{\cos(2\phi+\phi_b)} = \mathcal{C}\left[\frac{2(\vec{h}\cdot\vec{k}_{bT})[2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}] - \vec{k}_{bT}^2(\vec{h}\cdot\vec{k}_{aT})}{2M_a M_b^2}h_{1L}^{\perp}\bar{h}_{1T}^{\perp}\right], \tag{105}$$

$$F_{TL}^1 = -\mathcal{C}\left[\frac{\vec{h}\cdot\vec{k}_{aT}}{M_a}g_{1T}\bar{g}_{1L}\right], \tag{106}$$

$$F_{TL}^{\cos(2\phi-\phi_a)} = \mathcal{C}\left[\frac{\vec{h}\cdot\vec{k}_{bT}}{M_b}h_1\bar{h}_{1L}^{\perp}\right], \tag{107}$$

$$F_{TL}^{\cos(2\phi+\phi_a)} = \mathcal{C}\left[\frac{2(\vec{h}\cdot\vec{k}_{aT})[2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}] - \vec{k}_{aT}^2(\vec{h}\cdot\vec{k}_{bT})}{2M_a^2 M_b}h_{1T}^{\perp}\bar{h}_{1L}^{\perp}\right], \tag{108}$$

$$F_{TT}^1 = \mathcal{C}\left[\frac{2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}}{2M_a M_b}(f_{1T}^{\perp}\bar{f}_{1T}^{\perp} - g_{1T}\bar{g}_{1T})\right], \tag{109}$$

$$\bar{F}_{TT}^{1} = -\mathcal{C}\left[\frac{\vec{k}_{aT} \cdot \vec{k}_{bT}}{2M_a M_b}(f_{1T}^{\perp}\bar{f}_{1T}^{\perp} + g_{1T}\bar{g}_{1T})\right], \tag{110}$$

$$F_{TT}^{\cos(2\phi-\phi_a-\phi_b)} = \mathcal{C}[h_1\bar{h}_1], \tag{111}$$

$$F_{TT}^{\cos(2\phi-\phi_a+\phi_b)} = \mathcal{C}\left[\frac{2(\vec{h} \cdot \vec{k}_{bT})^2 - \vec{k}_{bT}^2}{2M_b^2} h_1\bar{h}_{1T}^{\perp}\right], \tag{112}$$

$$F_{TT}^{\cos(2\phi+\phi_a-\phi_b)} = \mathcal{C}\left[\frac{2(\vec{h} \cdot \vec{k}_{aT})^2 - \vec{k}_{aT}^2}{2M_a^2} h_{1T}^{\perp}\bar{h}_1\right], \tag{113}$$

$$F_{TT}^{\cos(2\phi+\phi_a+\phi_b)} = \mathcal{C}\left[\left(\frac{4(\vec{h} \cdot \vec{k}_{aT})(\vec{h} \cdot \vec{k}_{bT})[2(\vec{h} \cdot \vec{k}_{aT})(\vec{h} \cdot \vec{k}_{bT}) - \vec{k}_{aT} \cdot \vec{k}_{bT}]}{4M_a^2 M_b^2}\right.\right.$$
$$\left.\left.+ \frac{\vec{k}_{aT}^2\vec{k}_{bT}^2 - 2\vec{k}_{aT}^2(\vec{h} \cdot \vec{k}_{bT})^2 - 2\vec{k}_{bT}^2(\vec{h} \cdot \vec{k}_{aT})^2}{4M_a^2 M_b^2}\right)h_{1T}^{\perp}\bar{h}_{1T}^{\perp}\right]. \tag{114}$$

We close this section with a number of comments.

(i) The structure functions depend on the variables (x_a, x_b, q_T). Instead of using q_T one may also work with the transverse momentum of one of the hadrons in the CS-frame.

(ii) One finds nonzero contributions for 24 out of the 48 structure functions defined in Eq. (57). This also means that exactly half of the structure functions are of subleading twist for the kinematical region $q_T \ll q$ we are interested in here.

(iii) The leading twist parton model calculation containing T-even effects was first carried out in Ref. [32], while T-odd effects were investigated in [33]. We obtain the same number of nonzero structures identified in those articles, though we do not agree with certain angular dependences given in [33].

(iv) Our results are for the structure functions in Eq. (57) with the lepton angles understood in the CS-frame, and the components of the hadron spin vectors in the cm-frame. Note that the expressions would be exactly the same for structure functions defined in the Gottfried-Jackson frame, because differences be-

tween those two dilepton rest frames are only of $\mathcal{O}(q_T/q)$.

(v) For identical hadrons in the initial state the results in Eqs. (91)–(114) satisfy the model-independent constraints listed in (65) and (66). In particular, we point out that the parton model result

$$F_{TT}^{\cos(2\phi+\phi_a-\phi_b)}(x_b, x_a) = F_{TT}^{\cos(2\phi-\phi_a+\phi_b)}(x_a, x_b) \tag{115}$$

has a model-independent status. It is worthwhile to mention that, by means of charge conjugation, in the case of proton-antiproton DY one also finds symmetries for structure functions (like $F_{UU}(x_a, x_b, q_T) = F_{UU}(x_b, x_a, q_T)$), and relations between various structure functions. In particular, when studying single spin effects one can obtain the same information by either polarizing the proton or the antiproton.

(vi) If the cross section is integrated upon q_T only three structure functions (F_{UU}^1, F_{LL}^1, $F_{TT}^{\cos(2\phi-\phi_a-\phi_b)}$) survive. Neglecting hadron masses one obtains

$$\frac{d\sigma}{dx_a dx_b d\Omega} = \frac{s}{2}\frac{d\sigma}{dq^+ dq^- d\Omega}$$

$$= \frac{\alpha_{\text{em}}^2}{12q^2}\left\{(1 + \cos^2\theta)\sum_q e_q^2(f_1^q(x_a)f_1^{\bar{q}}(x_b) + f_1^{\bar{q}}(x_a)f_1^q(x_b)) - S_{aL}S_{bL}(1 + \cos^2\theta)\sum_q e_q^2(g_1^q(x_a)g_1^{\bar{q}}(x_b)\right.$$

$$\left.+ g_1^{\bar{q}}(x_a)g_1^q(x_b)) + |\vec{S}_{aT}||\vec{S}_{bT}|\sin^2\theta\cos(2\phi - \phi_a - \phi_b)\sum_q e_q^2(h_1^q(x_a)h_1^{\bar{q}}(x_b) + h_1^{\bar{q}}(x_a)h_1^q(x_b))\right\}. \tag{116}$$

Further integration upon the solid angle Ω provides

$$\frac{d\sigma}{dx_a dx_b} = \frac{4\pi\alpha_{\text{em}}^2}{9q^2}\Bigg\{\sum_q e_q^2(f_1^q(x_a)f_1^{\bar{q}}(x_b) + f_1^{\bar{q}}(x_a)f_1^q(x_b))$$

$$- S_{aL}S_{bL}\sum_q e_q^2(g_1^q(x_a)g_1^{\bar{q}}(x_b)$$

$$+ g_1^{\bar{q}}(x_a)g_1^q(x_b))\Bigg\}. \tag{117}$$

Note that the term containing the transversity dropped out.

(vii) For the q_T-dependent cross section all chiral-odd parton distributions disappear after integrating out the azimuthal angle ϕ. On the other hand, all the chiral-even effects survive this integration.

(viii) The large number of independent structure functions—for instance 16 for identical hadrons in the initial state—indicates the high potential of the polarized DY process for studying TMDs. Therefore, this process has also a certain advantage over semi-inclusive DIS (if in that reaction polarization of the initial state lepton and hadron are exploited) where eight leading twist structure functions exist [23,24], being just sufficient to map out, in principle, all the eight leading twist TMDs.

(ix) As already pointed out in Sec. V data on $\pi^- N \rightarrow \mu^-\mu^+ X$ [41–43] show a rather large $\cos 2\phi$ dependence of the unpolarized cross section which cannot be explained by collinear perturbative QCD. However, if intrinsic transverse parton motion in the initial state is taken into account the Boer-Mulders function $h_1^\perp$ contributes to the $\cos 2\phi$ term according to (92) which may explain the observed violation of the Lam-Tung relation [33]. This finding stimulated a lot of phenomenological work on this subject [11,33,66–80].

(x) Of particular interest is also the transverse single spin effect given by F_{TU}^1 in Eq. (95) or F_{UT}^1 in (98). Both structure functions contain the Sivers parton distribution which was predicted to have the opposite sign in DY as compared to semi-inclusive DIS [59,81,82]. As the sign reversal is at the core of our present understanding of transverse single spin asymmetries in hard scattering processes an experimental check of this prediction is of utmost importance. Theoretical work on the Sivers effect in DY can be found in [11,83–89].

(xi) The expected sign reversal of T-odd TMDs can also be investigated through the structure functions $F_{TU}^{\sin(2\phi-\phi_a)}$ in (96) or $F_{UT}^{\sin(2\phi-\phi_b)}$ in (99) in which the Boer-Mulders function enters (see also Refs. [11,71,73]).

(xii) A phenomenological study of the structure functions in (109) and (110) was carried out in [90].

VII. SUMMARY

We have presented a formalism for dilepton production from the collision of two polarized spin-$\frac{1}{2}$ particles. To this end we have derived in a first step a general expression for the hadronic DY tensor. This tensor consists of 48 basis elements, and each basis tensor is multiplied by a scalar function (structure function). In order to ensure electromagnetic gauge invariance of the hadronic tensor we have made use of an elegant projection method proposed in [26]. In general, our treatment completes earlier work [14,31]. The double-polarized case, which is the most challenging part, was studied before only for the specific kinematical case $q_T = 0$ [14].

The result for the hadronic tensor allows one to obtain the general angular distribution of the cross section for any reference frame. In this work we have focussed on a dilepton rest frame where the angular distribution takes the most compact form and shows a high degree of symmetry. We repeat here that the angular distribution as given in Eq. (57), which represents a central result of our work, holds for any dilepton rest frame.

Our analysis is supplemented by a parton model calculation of the polarized DY reaction (see also [32,33]). For this part of the work we concentrated on the kinematical situation where the transverse momentum of the dilepton pair is much smaller than its invariant mass. This region is the realm of TMDs which are currently under intense investigation both from the experimental and the theoretical side.

We reemphasize that the polarized DY process has a high potential for studying TMDs which contain important information on the nonperturbative structure of hadrons. Moreover, polarized dilepton measurements can provide us with a crucial and highly nontrivial check of QCD-factorization [59]. In addition, one can systematically study different resummation techniques [91–93] in an unprecedented way. Consequently, there is sufficient reason for looking forward to the first polarized DY data.

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