Fluctuations of the baryonic flux-tube junction from effective string theory

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In quenched QCD, where the dynamic creation of quark-antiquark pairs out of the vacuum is neglected, a confined baryonic system composed of three static quarks exhibits stringlike behavior at large interquark separation, with the formation of flux tubes characterized by the geometry of the so-called Y ansatz. We study the fluctuations of the junction of the three flux tubes, assuming the dynamics to be governed by an effective bosonic string model. We show that the asymptotic behavior of the effective width of the junction grows logarithmically with the distance between the sources, with the coefficient depending on the number of joining strings, on the dimension of spacetime and on the string tension.

I. INTRODUCTION

One of the crucial features characterizing quantum chromodynamics (QCD) is confinement: the fundamental constituents of strongly interacting matter (quarks and gluons) are not observed as asymptotic states, and the low-energy hadronic spectrum consists of colorless states only.

Because of its nonperturbative nature, a formal proof of confinement from first principles has so far been elusive, and even the degrees of freedom responsible for this phenomenon are subject to debate. However, some low-energy properties of hadrons and QCD forces (see, e.g. Ref. [1] for a review) can be accurately modeled in terms of an effective string picture [2–6], which describes the infrared properties of hadrons in terms of a fluctuating, thin (almost unidimensional) flux tube joining the color sources. At sufficiently large interquark separations the lowest-energy excitations of the confined system are associated with collective degrees of freedom corresponding to transverse stringlike vibrations of the flux tube, whereas the excitation spectrum of the gauge degrees of freedom inside the tube is much higher lying.

While it is unproven that low-energy aspects of confining quantum field theories can indeed be explained in terms of an effective string theory with universal features, strong evidence in favor of this conjecture is provided by recent comparisons between results from lattice simulations and bosonic string predictions (surveyed below). The observation of a linearly rising potential between color sources separated at distances R does not automatically imply the existence of an effective string description; however, the universality of the subleading $1/R$ coefficient found in several gauge models [6–18] provides such a nontrivial test. At present not many quantitative predictions that exceed the classical limit are available to compare lattice data to. In this article we increase the number of such nontrivial predictions by calculating the width of ''baryonic'' flux-tube junctions for general geometries.

Note that while we label our configurations as baryonic, we expect our predictions only to apply to pure Yang-Mills theories with static external charges. The bosonic string model is unlikely to be a good approximation to the baryons of real QCD with sea quarks, which are likely to decay into baryon-meson pairs, before the string limit of large distances can be reached.

For the simplest physical scenario, i.e. a (''mesonlike'') pair of static, infinitely heavy, confined color sources, the effective model has been developed since the early 1980s [2–5]. Later on this description was reformulated in terms of an expansion about the long-string vacuum [19]. More recent theoretical developments include Refs. [20–22], and are reviewed in Ref. [23]. The large-distance string behavior has been observed in numerical lattice simulations of the torelon spectrum (corresponding to closed strings) and static potentials (corresponding to mesonic open strings) of SU(3) lattice gauge theory [6–9] as well as of various other gauge models [10–18].

The effective string picture also predicts the width of the flux tube to grow logarithmically as a function of the interquark distance as well as the coefficient of the logarithm [4]; this was addressed and confirmed in various numerical lattice studies [15,24–30].

Lattice simulations of the baryonic setup [1,31–34] indicate that the flux-tube profile interpolates between the so-called Δ geometry at short distances (where the effective one-gluon exchange dominates), and the Y ansatz for separations between the sources of the order of or larger than approximately 0.8 fm. This Y ansatz that is relevant in the infrared region is characterized by a junction where the flux tubes meet (see Fig. 1). The corresponding leading order string corrections to the baryonic potential have been worked out in Ref. [35].

Building upon the procedure used in this reference, in this article we study the width of the junction, assuming that this is generated by string fluctuations of an effective string theory with the lowest dimensional term given by the
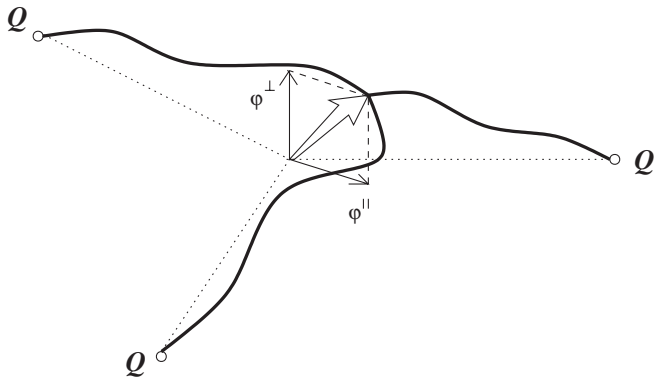
FIG. 1.   A snapshot of the fluctuating flux tubes (idealized as strings) of three static color sources Q, joined at a common junction; φ denotes the position of this junction, relative to its classical location minimizing the total string length. The components of φ parallel and normal to the plane containing the static color sources are also displayed.

Nambu-Goto (NG) action [36,37], with string tension σ. We consider the leading order nontrivial behavior in the limit of large separations between the static color sources.

II. CALCULATION SETUP

The calculation is performed in D-dimensional Euclidean spacetime with $D - 1$ spatial dimensions of infinite extent and a periodic time coordinate $t \in [0, T)$. We consider the general case of $2 \leq n \leq D$ static "quarks" spanning a $(n - 1)$-dimensional hyperplane. We assume the action to be minimal when the n strings

meet in a common junction. For $n = 3$ this is indeed the case, unless one of the angles of the triangle defined by the three sources exceeds the critical value $2\pi/3$. In this latter case the junction will be fixed to the position of the corresponding source and the system can be decomposed into two mesonic strings. Note that also for many $n > 3$ geometries the classical configuration will be characterized by different geometries, with two or more distinct junctions (see also Ref. [38]). In principle our calculation can be extended to these cases.

During their time evolution, the strings span n different world sheets (see Fig. 2); each of these blades is bounded by the (straight) worldline of a static quark on one side, and a generic worldline spanned by the fluctuating junction on the other side.

Classically, the ground state fulfills the constraint of the minimal area of the string world sheets. Therefore the position of the junction is determined by the requirement of minimal total string length. Furthermore, the balance of tensions means that $\sigma \sum_{a=1}^{n} \boldsymbol{e}_a = 0$ which implies equal angles between the strings at any time. $\boldsymbol{e}_a$ denote unit vectors along the direction between the junction and the quark a. Assuming that the string dynamics is described by the NG action S_{NG} means that the quantum weight of a generic configuration of a string world sheet is proportional to the exponential of its total area. The NG action reads

$$S_{\mathrm{NG}} = \sigma \int d^2\zeta \sqrt{-g}, \qquad (2.1)$$

where the position on the surface of the string world sheet is parametrized by the coordinates ζ_i, $i \in \{1, 2\}$. Although
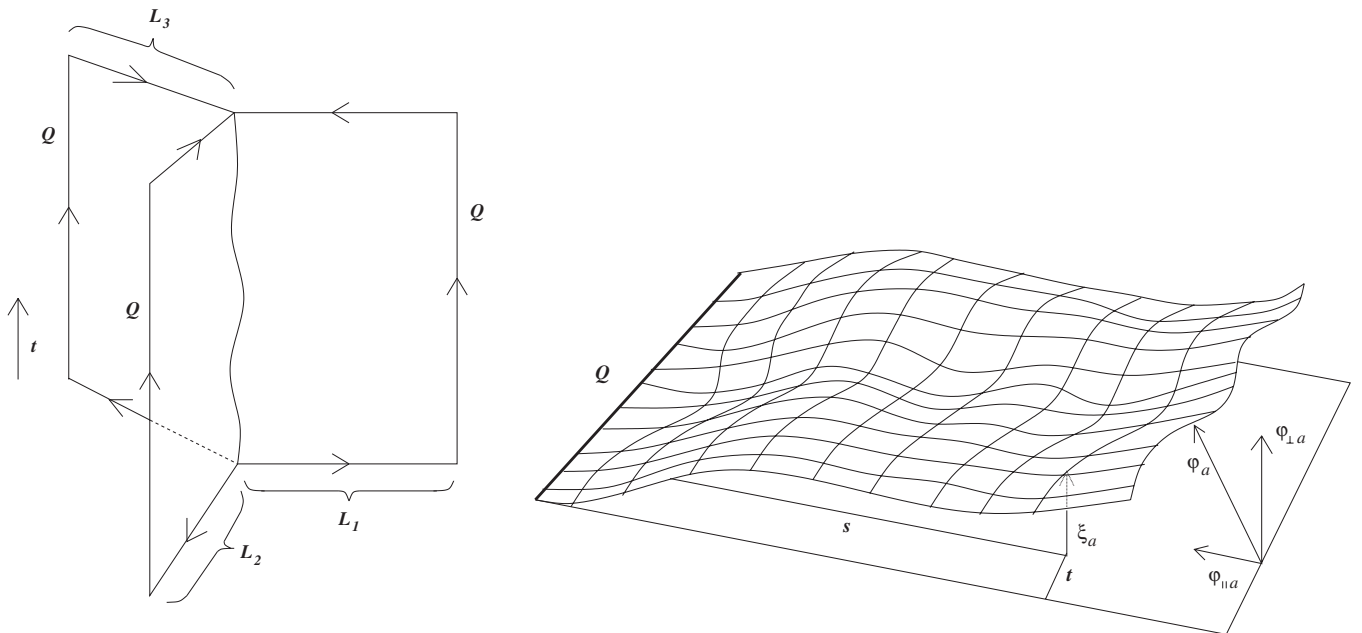

FIG. 2.   Left-hand side: World sheets spanned by the fluctuating strings during their time evolution. Right-hand side: Surface of one of the string world sheets.

this bosonic string model is nonrenormalizable (because it is nonpolynomial) and anomalous (except in 26 spacetime dimensions) [5,39,40], it can be considered a legitimate starting point in the construction of an effective theory, and it can be shown to agree with the Polchinski-Strominger effective model [19] up to and including the next-to-leading order [21]. Let $X^\mu(\boldsymbol{\zeta})$, with $\mu \in \{1, \ldots, D\}$, denote a map from the world sheet to the spacetime, embedding the world sheet; the induced metric of Eq. (2.1) is given by

$$g^{ij} = \frac{\partial X^\mu}{\partial \zeta_i} \frac{\partial X_\mu}{\partial \zeta_j}. \qquad (2.2)$$

To proceed with the quantum treatment, it is natural to fix the reparametrization and Weyl invariance to the "physical" gauge, allowing us to describe the transverse displacements (for our purposes, the Weyl anomaly can be neglected, because it vanishes in the limit of large distances). This means that only transverse fluctuations $\boldsymbol{\xi}_a(t, s)$ of the string world sheets $a \in \{1, \ldots, n\}$ around the classical configuration are considered as physical. The time t and parameter s label the position on string world sheet (blade) a. In particular, s denotes the spatial distance from the worldline of the quark a, i.e. the classical position of the junction is given by $s = L_a$. The junction worldline $\boldsymbol{\varphi}(t)$ fluctuates within the hyperplane spanned by the quarks (changing the minimal area of the blades) as well as in the $D - n$ remaining transverse spatial directions. From continuity, we obtain the boundary conditions for the transverse fluctuations $\boldsymbol{\xi}_a(t, s)$:

$$\boldsymbol{\xi}_a(t, L_a + \boldsymbol{e}_a \cdot \boldsymbol{\varphi}(t)) = \boldsymbol{\varphi}_{\perp a}(t), \qquad (2.3)$$

where $\boldsymbol{\varphi}_{\perp a} \equiv \boldsymbol{\varphi} - \boldsymbol{e}_a(\boldsymbol{e}_a \cdot \boldsymbol{\varphi})$. The transverse fluctuations $\boldsymbol{\xi}_a(t, s)$ vanish at the location of the quarks ($s = 0$), and are periodic in the time t, with period T.

For technical reasons we assume the junction itself to have a finite mass m. This results in a static energy and in a kinetic term. The parameter value m should not affect the large-distance results, $L_a \sigma \gg m$, that we present below, and indeed it cancels in the calculation. Expanding the NG action around the equilibrium configuration yields

$$S = S_\| + \frac{\sigma}{2} \sum_{a,i} \int_{\Gamma_a} d^2\zeta \frac{\partial \boldsymbol{\xi}_a}{\partial \zeta_i} \cdot \frac{\partial \boldsymbol{\xi}_a}{\partial \zeta_i} + m\left(T + \frac{1}{2}\int_0^T dt|\dot{\boldsymbol{\varphi}}|^2\right), \qquad (2.4)$$

where again ζ_1, ζ_2 are world sheet parameters and

$$S_\| = \sigma \sum_a \left(L_a T + \int dt \boldsymbol{e}_a \cdot \boldsymbol{\varphi}(t)\right) = \sigma L_Y T. \qquad (2.5)$$

$L_Y = \sum_a L_a$ above denotes the total string length. (Note that $\sum_a \boldsymbol{e}_a = \boldsymbol{0}$.) In the $T \to \infty$ limit the string thickness can be extracted from the partition function, which to leading nontrivial order is given by

$$Z = e^{-(\sigma L_Y + m)T} \int \mathcal{D}\boldsymbol{\varphi} \exp\left(-\frac{m}{2}\int dt|\dot{\boldsymbol{\varphi}}|^2\right) \prod_{a=1}^3 Z_a(\boldsymbol{\varphi}), \qquad (2.6)$$

where $Z_a(\boldsymbol{\varphi})$ denotes the partition function for the fluctuations of a given blade that is bounded by the junction worldline $\boldsymbol{\varphi}(t)$:

$$Z_a(\boldsymbol{\varphi}) = \int \mathcal{D}\boldsymbol{\xi}_a \exp\left(-\frac{\sigma}{2}\int|\partial\boldsymbol{\xi}_a|^2\right). \qquad (2.7)$$

The string partition functions $Z_a(\boldsymbol{\varphi})$ are Gaussian functional integrals and can be calculated as follows:

$$Z_a(\boldsymbol{\varphi}) = e^{-(\sigma/2)\int|\partial\boldsymbol{\xi}_{\min,a}|^2}|\det(-\Delta_{\Gamma_a})|^{-(D-2)/2}, \qquad (2.8)$$

where $\boldsymbol{\xi}_{\min,a}$ is the minimal-area solution for given $\boldsymbol{\varphi}(t)$. Δ_{Γ_a} denotes the Laplacian acting on the domain (blade) Γ_a. $\boldsymbol{\xi}_{\min,a}(t, s)$ is harmonic and satisfies the boundary conditions Eq. (2.3) [35]. Below we will evaluate this expression to the leading order in terms of the fluctuations $\boldsymbol{\varphi}$.

In contrast to the mesonic setup, the world sheets Γ_a are in general no rectangles. However, the determinant in Eq. (2.8) can still be calculated by decomposing the boundary $\boldsymbol{\varphi}(t)$ of Γ_a into a sum over Fourier modes and conformally mapping the resulting domains to rectangles, as shown in Appendix A. Carrying out this mapping and taking the limit $T \to \infty$, Jahn and de Forcrand [35] derived the subleading term of the ($n = 3$) baryonic potential V_{qqq}:

$$V_{qqq}(L_1, L_2, L_3) = \sigma \sum_a L_a + V^\| + (D - 3)V^\perp + \mathcal{O}(L_a^{-2}),$$

$$V^\| = -\frac{\pi}{24}\sum_a \frac{1}{L_a} + \int_0^\infty \frac{dw}{2\pi} \ln\left[\frac{1}{3}\sum_{a<b}\coth(wL_a)\coth(wL_b)\right], \qquad (2.9)$$

$$V^\perp = -\frac{\pi}{24}\sum_a \frac{1}{L_a} + \int_0^\infty \frac{dw}{2\pi} \ln\left[\frac{1}{3}\sum_a \coth(wL_a)\right].$$

We confirm this result. For the equilateral case $L = L_1 = L_2 = L_3$ Eq. (2.9) simplifies to

$$V_{qqq,\Delta}(L) = 3\sigma L - \frac{D-3}{16}\frac{\pi}{L} + \mathcal{O}(L^{-2}). \qquad (2.10)$$

III. STRING THICKNESS AT THE JUNCTION

The bosonic string model yields a prediction for the thickness of the fluctuating strings. The width of the junction itself can be calculated by taking the expectation value:

$$\langle \varphi^2 \rangle = \frac{\int \mathcal{D}\varphi \varphi^2 e^{-S}}{\int \mathcal{D}\varphi e^{-S}}. \qquad (3.1)$$

The action S is defined in Eq. (2.4) and can also be read off from the partition function Eq. (2.6). We split the string width into contributions $\langle \varphi^{\perp 2} \rangle$, perpendicular to the hyperplane spanned by the n quarks, and $\langle \varphi^{\parallel 2} \rangle$ within the hyperplane of the quarks:

$$\langle \varphi^2 \rangle = \langle \varphi^{\perp 2} \rangle + \langle \varphi^{\parallel 2} \rangle. \qquad (3.2)$$

Note that $\varphi^\perp$ lives within a $(D-n)$-dimensional subspace and $\varphi^\parallel$ fluctuates in the remaining $n-1$ spatial directions. This differs from the definitions with respect to a given blade a of Sec. II above, $\varphi_{\perp a}$ [$(D-2)$-dimensional fluctuations] and $\varphi_{\parallel a}$ (one-dimensional fluctuations).

In the $T \to \infty$ limit, the perpendicular contribution reads

$$\langle \varphi^{\perp 2} \rangle = (D-n)\frac{1}{\pi}\int_0^\infty dw \frac{1}{mw^2 + \sigma w \sum_a \coth(wL_a)}. \qquad (3.3)$$

We present more details of the calculation in Appendix B. For n strings of identical length $L = L_1 = L_2 = \cdots = L_n$ we obtain

$$\langle \varphi^{\perp 2} \rangle = (D-n)\frac{1}{\pi}\int_0^\infty dw \frac{1}{mw^2 + n\sigma w \coth(wL)}. \qquad (3.4)$$

We split this integral at $w = C/L$, where C is an arbitrary L-independent constant. The first part of integration is subleading in L. We can choose C large enough, such that the approximation $\coth(C) \approx 1$ holds. Neglecting subleading terms in L, the second part of integration gives

$$\int_{C/L}^\infty dw \frac{1}{mw^2 + n\sigma w \coth(wL)} \simeq \frac{1}{n\sigma}\ln L. \qquad (3.5)$$

Therefore, to leading order, the result reads

$$\langle \varphi^{\perp 2} \rangle = \frac{D-n}{n}\frac{1}{\pi\sigma}\ln\frac{L}{L_0}, \qquad (3.6)$$

where we have absorbed an arbitrary constant into L_0; the width of the junction, orthogonal to the plane spanned by the quarks, grows logarithmically with the distance. L_0 will

depend on D, n, and the microscopic (ultraviolet) details of the gauge model.

We can perform a consistency check by comparing the above result to the mesonic case ($n = 2$). The logarithmic behavior of the width of a flux tube connecting two quarks in the string picture was predicted many years ago by Lüscher, Münster, and Weisz [4]. The result they found for $D = 4$ is

$$\delta^2 \sim \frac{1}{\pi M^2}\ln\frac{L'}{\lambda}, \qquad (3.7)$$

where M^2 denotes the string tension and λ represents a cutoff scale. The effective string width was studied again by Caselle *et al.* [26] and by Gliozzi [41], calculating the deviation of the transverse coordinates of the string from the respective Green function. The result they obtained for the mean squared width w_0^2 in $D = 3$, determined at the symmetry point of the string world sheet is

$$w_0^2 = \frac{1}{2\pi\sigma}\ln\frac{R}{R_c}, \qquad (3.8)$$

where σ denotes the string tension, R the interquark distance, and R_c is an ultraviolet scale.

We divide the string connecting quark and antiquark into two parts of equal length (up to small longitudinal fluctuations), connected in the middle by a junction. We can then apply Eq. (3.6) for $D = 3$ and $D = 4$. The above predictions indeed coincide with our results where we identify $\sigma = M^2$, $L = 2L' = 2R$, and $L_0 = 2\lambda = 2R_c$.

Now let us turn to the string width within the plane of the quarks. This is only well defined for $n \geq 3$ and the most interesting case is the $n = 3$ baryon. In contrast to the perpendicular width $\langle \varphi^{\perp 2} \rangle$, our calculation applies to $n = 3$ only since we assume the sources to lie in a two-dimensional plane. In this case the $n > 3$ minimal string configuration will usually contain more than one junction, unless junctions are fixed at the positions of quarks and do not fluctuate. It turns out that our calculation of $\langle \varphi^{\parallel 2} \rangle$ cannot easily be generalized to higher dimensional planes, i.e. to $n > 3$. In the baryonic equilateral case we obtain [Eq. (B23)]

$$\langle \varphi^{\parallel 2}_{qqq,\Delta} \rangle = \frac{4}{3\sigma\pi}\int_0^\infty \frac{dw}{w^2\tilde{m}/L + (w - w^3 a/L^2)\coth(w)}, \qquad (3.9)$$

where $\tilde{m} = 2m/(3\sigma)$ and $a = (D-2)/(12\pi\sigma)$. The result for nonequilateral configurations can be obtained from Eq. (B19) below.

We split the integral in analogy to the above discussion of the perpendicular fluctuations. However, one finds that, like in the calculation of the baryonic potential, a pole emerges. If one views the NG action as the first term within an effective string theory then this pole has to be canceled by counterterms arising from the inclusion of higher dimensional operators. In this sense it should not affect the

leading order result. Assuming this, the parallel contribution to the width of the junction turns out to be

$$\langle \boldsymbol{\varphi}_{qqq}^{\|2} \rangle = \frac{4}{3} \frac{1}{\pi\sigma} \ln\frac{L}{L_c}, \qquad (3.10)$$

where again subleading contributions are suppressed and L_c is an undetermined constant. Note that the coefficient above is by a factor $4/(D-3)$ larger than the one in front of the logarithm within the expression for $\langle \boldsymbol{\varphi}^{\perp2} \rangle$ of Eq. (3.6). One factor $2/(D-3)$ corresponds to the ratio of independently fluctuating parallel over perpendicular components while another factor of 2 is expected from the stronger restoring force for perpendicular displacements, relative to parallel ones.

IV. CONCLUSIONS

In this article, we studied the width of the junction of flux tubes in baryonlike systems composed of infinitely heavy, static color sources (quarks) at large distances L from the junction. Assuming the low-energy aspects to be governed by the dynamics of the bosonic Nambu-Goto string model, we have shown that the width of the junction grows logarithmically with the distance between the quarks. In particular the quadratic width orthogonal to the $(n-1)$-dimensional plane spanned by n equidistant quarks [the baryons of SU(n) gauge theories] in $D \geq n$ spacetime dimensions reads [Eq. (3.6)]

$$\langle \boldsymbol{\varphi}^{\perp2} \rangle = \frac{D-n}{n} \frac{1}{\pi\sigma} \ln\frac{L}{L_0}. \qquad (4.1)$$

This also applies to (and generalizes) the mesonic case ($n = 2$). The corresponding result for general geometries with a Steiner junction can be obtained from Eq. (3.3). The width within the plane of the sources also grows logarithmically as a function of the separation and we have calculated this for $n = 3$. The result for the equilateral case is displayed in Eq. (3.10) while the general result can be calculated from Eq. (B19). We also confirm the result of Ref. [35] for the baryonic potential, Eqs. (2.9) and (2.10).

The mesonic flux-tube width has already been investigated in lattice simulations of different gauge theories [15,24–30]. While most of these studies confirm the logarithmic broadening of the mesonic string, without much lattice spacing dependence, it should be noted that Ref. [25] found such broadening only at fixed lattice spacings but the string width to actually shrink, possibly to zero, if the continuum limit was taken. This is also incompatible with the result of Ref. [42] of a vanishing overlap between a thin string state and the ground state wave function. Further lattice studies are required to resolve this controversy.

The question if and at what distances our string predictions become valid can be addressed by lattice simulations of baryonic configurations in SU(3) gauge theory at large L in $D = 3$ and $D = 4$ spacetime dimensions. While this is numerically quite challenging, at least the simplified case of $D = 3$ Z_3 gauge theory can be mapped to a two-dimensional Potts model, allowing for precise numerical simulations [34,43]. These show consistency with the potential of Eq. (2.9).

APPENDIX A: CONFORMAL MAPPING OF A WORLD SHEET TO A RECTANGLE

Here we provide the ingredients for the calculation of the fluctuations at the junction. We follow Ref. [35], conformally mapping the blade (see Fig. 2) to a rectangle. The minimal-area solution for a fixed position of the junction, $\boldsymbol{\xi}_{\min,a}(t, s)$, is harmonic and satisfies the boundary conditions Eq. (2.3):

$$\Delta\boldsymbol{\xi}_{\min,a} = 0, \qquad \boldsymbol{\xi}_{\min,a}(t, L_a + \boldsymbol{e}_a \cdot \boldsymbol{\varphi}(t)) = \boldsymbol{\varphi}_{\perp a}(t). \qquad (A1)$$

The determinant in Eq. (2.8) is computed with Dirichlet boundary conditions on the domain $\Gamma_a = \{(t, s)|0 \leq s \leq L_a + \boldsymbol{e}_a \cdot \boldsymbol{\varphi}(t)\}$. In terms of the Fourier components $\boldsymbol{\varphi}_w$ of $\boldsymbol{\varphi}(t)$, $\boldsymbol{\xi}_{\min,a}$ is given by

$$\boldsymbol{\xi}_{\min,a} = \frac{1}{\sqrt{T}} \sum_w \boldsymbol{\varphi}_{w,\perp a} \frac{\sinh(ws)}{\sinh(wL_a)} e^{iwt} + \mathcal{O}(\varphi^2), \qquad (A2)$$

where $w = 2\pi n/T$. The integral in Eq. (2.8), which represents the change in the minimal area due to the transverse fluctuations $\boldsymbol{\varphi}_{\perp a}$, can now be calculated:

$$\int_{\Gamma_a} d^2\zeta \sum_i \frac{\partial\boldsymbol{\xi}_{\min,a}}{\partial\zeta_i} \cdot \frac{\partial\boldsymbol{\xi}_{\min,a}}{\partial\zeta_i}$$
$$= \sum_w w \coth(wL_a)|\boldsymbol{\varphi}_{w,\perp a}|^2 + \mathcal{O}(\varphi^3). \qquad (A3)$$

The determinant in Eq. (2.8) is obtained by mapping the domain Γ_a conformally to a rectangle $L_a' \times T$. Note that the conformal map $f_a(z) = z + \sum_w c_{wa} e^{wz}$ has to be complex differentiable. Its coefficients c_{wa} are fixed by the constraints:

$$f_a(i\mathbb{R}) = i\mathbb{R},$$
$$f_a(L_a' + it) = L_a + \boldsymbol{e}_a \cdot \boldsymbol{\varphi}(t) + it + \mathcal{O}(\varphi^2). \qquad (A4)$$

One easily sees that $L_a' = L_a + \frac{1}{\sqrt{T}} \boldsymbol{e}_a \cdot \boldsymbol{\varphi}_0$. To leading order in $\boldsymbol{\varphi}$ the conformal map is then given by

$$f_a(z) = z + \frac{1}{\sqrt{T}} \sum_{w \neq 0} \frac{e_a \cdot \varphi_w}{\sinh(wL_a)} e^{wz} + \mathcal{O}(\varphi^2). \quad \text{(A5)}$$

This conformal mapping changes the Laplacian by a scalar factor:

$$\Delta_{\Gamma_a} = e^{2\rho_a(z)} \Delta_{L_a' \times T}, \qquad \rho_a(z) = -\frac{1}{2} \ln|\partial_z f_a|^2. \quad \text{(A6)}$$

The variation of the determinant of the Laplacian with respect to a holomorphic mapping of Γ onto some other region $\tilde{\Gamma}$ via the function $f(z)$ can be calculated by means of the Alvarez-Polyakov formula (see, e.g. Ref. [2]):

$$\ln\frac{\det(-\Delta_\Gamma)}{\det(-\Delta_{\tilde{\Gamma}})} = \frac{1}{12\pi} \int_{\partial\Gamma} d\tau \frac{\epsilon_{ij} z'^i z''^j}{z'^2} \ln|\partial_z f|^2$$
$$+ \frac{1}{12\pi} \int_\Gamma d^2 z \partial_z \ln|\partial_z f|^2 \partial_{\bar{z}} \ln|\partial_z f|^2. \quad \text{(A7)}$$

Here $z(\tau)$ is an arbitrary parametrization of $\partial\Gamma$ and $z' = dz/d\tau$. In our case the first integral above vanishes, and thus, from the conformal map Eq. (A5), we obtain to leading order

$$\int_{L_a' \times T} d^2 z \partial_z \ln|\partial_z f_a|^2 \partial_{\bar{z}} \ln|\partial_z f_a|^2$$
$$= \sum_w w^3 |e_a \cdot \varphi_w|^2 \coth(wL_a) + \mathcal{O}(\varphi^3), \quad \text{(A8)}$$

where we used the fact that the Fourier coefficients satisfy $\varphi_{-w} = \varphi_w^*$. In Ref. [2] the rectangle with periodic boundary conditions in time is further conformally mapped onto a circle resulting in

$$\det(-\Delta_{L_a' \times T}) = \eta^2\left(\frac{iT}{2L_a'}\right), \quad \text{(A9)}$$

where $\eta(\tau)$ denotes the Dedekind η function. Collecting the above results, we obtain for the determinant of the Laplacian with respect to the blade a:

$$\det(-\Delta_{\Gamma_a}) = \eta^2\left(\frac{iT}{2L_a'}\right)$$
$$\times \exp\left(-\frac{1}{12\pi} \sum_w w^3 \coth(wL_a)|e_a \cdot \varphi_w|^2\right). \quad \text{(A10)}$$

APPENDIX B: CALCULATION OF THE WIDTH OF THE JUNCTION $\langle\varphi^2\rangle$

In this appendix we provide more steps for the calculation of Eqs. (3.3) and (3.9). The calculation is carried out for n quarks located in a plane. This configuration with only one junction might not be stable for more than three quarks [38]. However, the result for the orthogonal contribution can easily be generalized to configurations of n quarks, distributed in a $(n-1)$-dimensional hyperplane.

The thickness of the string at the junction can be calculated taking the expectation value of φ^2 [see Eq. (2.4)]:

$$\langle\varphi^2\rangle = \frac{\int \mathcal{D}\varphi \varphi^2 e^{-S}}{\int \mathcal{D}\varphi e^{-S}}. \quad \text{(B1)}$$

To do this, we have to consider integrals

$$\int \mathcal{D}\varphi \exp\left[-\frac{m}{2} \int_0^T dt |\dot{\varphi}|^2\right.$$
$$+ \sum_{a=1}^n \left(-\frac{\sigma}{2} \int d^2\zeta \sum_i \frac{\partial\xi_{\min,a}}{\partial\zeta_i} \cdot \frac{\partial\xi_{\min,a}}{\partial\zeta_i}\right.$$
$$+ \left.\left.\frac{D-2}{24\pi} \sum_w w^3 \coth(wL_a)|e_a \cdot \varphi_w|^2\right)\right]. \quad \text{(B2)}$$

We can replace the integral in the first term above by a sum over Fourier components:

$$\int_0^T dt |\dot{\varphi}|^2 = \sum_w w^2 |\varphi_w|^2. \quad \text{(B3)}$$

We denote the plane that is spanned by the spatial unit vectors of the n strings $e_a = (e_{a,x}, e_{a,y}, 0, \ldots)$ as the x-y plane. These n unit vectors obey the relation $\sum_a e_a = \mathbf{0}$. The x-y components of φ (or any other vectors) carry the "$\|$" superscript. We obtain

$$\sum_a \coth(wL_a)|e_a \cdot \varphi_w^\||^2$$
$$= |\varphi_{w,x}^\||^2 \sum_a e_{a,x}^2 \coth(wL_a) + |\varphi_{w,y}^\||^2 \sum_a e_{a,y}^2 \coth(wL_a)$$
$$+ 2(\text{Re}(\varphi_{w,x}^\|)\text{Re}(\varphi_{w,y}^\|) + \text{Im}(\varphi_{w,x}^\|)\text{Im}(\varphi_{w,y}^\|))$$
$$\times \sum_a e_{a,x} e_{a,y} \coth(wL_a). \quad \text{(B4)}$$

We define A_x, A_y, and A_{Re} as

$$A_x = \left(\frac{\sigma}{2} w + \frac{(D-2)w^3}{24\pi}\right)\left[\sum_a e_{a,x}^2 \coth(wL_a)\right],$$
$$A_y = \left(\frac{\sigma}{2} w + \frac{(D-2)w^3}{24\pi}\right)\left[\sum_a e_{a,y}^2 \coth(wL_a)\right], \quad \text{(B5)}$$
$$A_{\text{Re}} = \left(\frac{\sigma}{2} w + \frac{(D-2)w^3}{24\pi}\right)\left[\sum_a e_{a,x} e_{a,y} \coth(wL_a)\right].$$

Note that $A_x + A_y$ as well as the combination $A_x A_y - A_{\text{Re}}^2$ are invariant under rotations within the x-y plane. Since the angles between the n strings are equal, we can parametrize the unit vectors by

$$e_a = (\cos(2\pi a/n), \sin(2\pi a/n), 0, \ldots). \quad \text{(B6)}$$

Thus we obtain

$$A_x A_y - A_{\text{Re}}^2 = \left(\frac{\sigma}{2}w + \frac{(D-2)w^3}{24\pi}\right)^2 \sum_{a<b} \underbrace{\sin^2\left(2\pi\frac{(a-b)}{n}\right)}_{\alpha_{ab}} \coth(wL_a)\coth(wL_b). \tag{B7}$$

In the case of $n = 3$, the geometrical coefficients are $\alpha_{ab} = 3/4$. This results in the simplification: $A_x A_y - A_{\text{Re}}^2 = \frac{3}{4} \times (\frac{\sigma}{2}w + \frac{(D-2)w^3}{24\pi})^2 \sum_{a<b} \coth(wL_a)\coth(wL_b)$.

We can split the integral over φ in Eq. (B2) using $|\varphi_{w,\perp a}|^2 = |\varphi_w|^2 - |\varphi_w \cdot e_a|^2$ into parts that are parallel and perpendicular to the plane of the quarks:

$$\int \mathcal{D}\varphi \exp\left[-\frac{m}{2}\int dt |\dot{\varphi}|^2 + \sum_{a=1}^{3}\left(-\frac{\sigma}{2}\int |\partial \xi_{\min,a}|^2 + \frac{D-2}{24\pi}\sum_w w^3 \coth(wL_a)|e_a \cdot \varphi_w|^2\right)\right]$$

$$= \int \mathcal{D}\varphi^\perp \exp\left[-\frac{1}{2}\sum_w\left(mw^2 + \sigma w\sum_a \coth(wL_a)\right)|\varphi_w^\perp|^2\right]$$

$$\times \int \mathcal{D}\varphi^\parallel \exp\left\{\sum_w\left[-\frac{1}{2}\left(mw^2 + \sigma w\sum_a \coth(wL_a)\right)|\varphi_w^\parallel|^2 + |\varphi_{w,x}^\parallel|^2 A_x + |\varphi_{w,y}^\parallel|^2 A_y + 2(\text{Re}(\varphi_{w,x}^\parallel)\text{Re}(\varphi_{w,y}^\parallel)\right.\right.$$

$$\left.\left. + \text{Im}(\varphi_{w,x}^\parallel)\text{Im}(\varphi_{w,y}^\parallel))A_{\text{Re}}\right]\right\}. \tag{B8}$$

Here, $\varphi_w^\perp$ are the $D-3$ components of φ_w that are perpendicular to the plane spanned by the quarks. We abbreviate the first functional integral above as I_1 and the second as I_2. The solutions of these Gaussian integrals read

$$I_1 = \left(\prod_{w>0}\frac{\pi}{mw^2 + \sigma w\sum_a \coth(wL_a)}\right)^{D-3}, \tag{B9}$$

$$I_2 = \prod_{w>0}\frac{\pi^2}{A_1 A_2 - 4A_{\text{Re}}^2}, \tag{B10}$$

where we defined

$$C_w = mw^2 + \sigma w\sum_a \coth(wL_a),$$

$$A_1 = C_w - 2A_x, \tag{B11}$$

$$A_2 = C_w - 2A_y.$$

We are interested in the expectation value

$$\langle \varphi^2 \rangle = \langle \varphi^{\perp 2} \rangle + \langle \varphi^{\parallel 2} \rangle = \frac{I_\perp}{I_1} + \frac{I_\parallel}{I_2}, \tag{B12}$$

where

$$I_\perp = \int \mathcal{D}\varphi^\perp \varphi^{\perp 2} \exp\left\{-\frac{1}{2}\sum_w\left[mw^2 + \sigma w\sum_a \coth(wL_a)\right]\right.$$

$$\left.\times |\varphi_w^\perp|^2\right\}, \tag{B13}$$

$$I_\parallel = \int \mathcal{D}\varphi^\parallel \varphi^{\parallel 2} \exp\left\{\sum_w\left[-\frac{1}{2}\left(mw^2 + \sigma w\sum_a \coth(wL_a)\right)\right.\right.$$

$$\times |\varphi_w^\parallel|^2 + |\varphi_{w,x}^\parallel|^2 A_x + |\varphi_{w,y}^\parallel|^2 A_y$$

$$\left.\left. + 2(\text{Re}(\varphi_{w,x}^\parallel)\text{Re}(\varphi_{w,y}^\parallel) + \text{Im}(\varphi_{w,x}^\parallel)\text{Im}(\varphi_{w,y}^\parallel))A_{\text{Re}}\right]\right\}.$$

$$\tag{B14}$$

Let us recall that $\varphi^\perp$ is $(D-3)$-dimensional. Performing the first integral yields

$$I_\perp = (D-3)\frac{2}{T}\left(\prod_{w'>0}\frac{\pi}{C_{w'}}\right)^{D-3}\sum_{w>0}\frac{1}{C_w}, \tag{B15}$$

so that

$$\langle \varphi^{\perp 2} \rangle = \frac{I_\perp}{I_1} = \frac{2}{T}\sum_{w>0}\frac{(D-3)}{mw^2 + \sigma w\sum_a \coth(wL_a)}. \tag{B16}$$

With $w = 2\pi n/T$, in the limit of large T we obtain

$$\langle \varphi^{\perp 2} \rangle = \frac{1}{\pi}\int_0^\infty dw \frac{D-3}{mw^2 + \sigma w\sum_a \coth(wL_a)}. \tag{B17}$$

The parallel contribution to the width of the junction is calculated analogously. We perform the integral $I_\parallel$ [Eq. (B14)]. In terms of A_1 and A_2 defined in Eq. (B11), one is left with

$$I_\parallel = \frac{2}{T}\left(\prod_{w>0}\frac{\pi^2}{A_1 A_2 - 4A_{\text{Re}}^2}\right)\sum_{w>0}\frac{A_1 + A_2}{A_1 A_2 - 4A_{\text{Re}}^2}. \tag{B18}$$

Therefore, combining $I_\parallel$ with Eq. (B10) yields

$$\langle \boldsymbol{\varphi}^{\|2} \rangle = \frac{I_\|}{I_2} = \frac{2}{T} \sum_{w>0} \frac{A_1 + A_2}{A_1 A_2 - 4A_{\mathrm{Re}}^2} = \frac{4}{3} \frac{1}{\sigma\pi} \int_0^\infty dw \frac{1}{w} \frac{\tilde{m}w + (1 - aw^2)C_1}{\tilde{m}^2 w^2 + 2\tilde{m}w(1 - aw^2)C_1 - 4aw^2(C_1)^2 + \frac{4}{3}(1 + aw^2)^2 C_2},$$

$$(\text{B19})$$

where

$$\tilde{m} = \frac{2}{3}\frac{m}{\sigma}, \qquad a = \frac{D-2}{12\pi\sigma}, \qquad (\text{B20})$$

$$C_1 = \frac{1}{3} \sum_a \coth(wL_a), \qquad (\text{B21})$$

$$C_2 = \frac{1}{3} \sum_{a<b} \alpha_{ab} \coth(wL_a) \coth(wL_b). \qquad (\text{B22})$$

For an equilateral baryon we have $\sum_{a<b} \alpha_{ab} = \frac{9}{4}$ and $L = L_1 = L_2 = L_3$. Thus, for this special case, we obtain

$$\langle \boldsymbol{\varphi}_{qqq,\Delta}^{\|2} \rangle = \frac{4}{3\sigma\pi} \int_0^\infty \frac{dw}{w^2 \tilde{m}/L + (w - w^3 a/L^2)\coth(w)}. \qquad (\text{B23})$$

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