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Nonminimal monopoles of the Dirac type as realization of the censorship conjecture

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We discuss a class of exact solutions of a three-parameter nonminimally extended Einstein-Maxwell model, which are attributed to nonminimal magnetic monopoles of the Dirac type. We focus on the investigation of the gravitational field of Dirac monopoles for those models, for which the singularity at the central point is hidden inside of an event horizon independently on the mass and charge of the object. We obtain the relationships between the nonminimal coupling constants, for which this requirement is satisfied. As explicit examples, we consider in detail two one-parameter models: first, the nonminimally extended Reissner-Nordström model (for the magnetically charged monopole) and, second, the Drummond-Hathrell model.

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I. INTRODUCTION

In 1969, Penrose formulated the so-called cosmic censorship conjecture [1], which assumes, in particular, that singularities have to be hidden inside of an event horizon and invisible to distant observers [2,3]. In the minimal Einstein theory, there exist a number of exact solutions, which can be considered as counterexamples to this censorship conjecture. For instance, the static spherically symmetric solutions to the Einstein equations with massless scalar field [4] *always* describe a naked singularity [5,6]. Naked singularities also appear, when we deal with the Reissner-Nordström metric, if $M^2 < Q^2_{(e)} + Q^2_{(m)}$ (M, $Q_{(e)}$, and $Q_{(m)}$ are the mass, electric and magnetic charges, respectively), or with the Kerr metric, if $M < |J|$ (J is an angular momentum). The solution for an individual electron with $M \ll |Q_{(e)}|$ (in the geometrical units) gives the simplest example of the naked singularity, because the gravitational attraction is negligible compared to the Coulomb repulsion, and the corresponding metric has no horizons.

We assume that a *nonminimal* interaction between electromagnetic and gravitational fields can eliminate this contradiction; i.e., the nonminimality results in the appearance of a new horizon, which hides the singular central point. Indeed, curvature coupling constants, which are involved in the nonminimal three-parameter Einstein-Maxwell model, can be naturally associated with characteristic lengths of the nonminimal interaction and, thus, at least one extra parameter r_q appears (see, e.g., [7,8]) in addition to the standard Schwarzschild radius r_g and Reissner-Nordström radius r_Q. This nonminimal extension sophisticates essentially the causal structure of space-time around the charged objects, and the appearance of an additional horizon, related to the censorship conjecture, becomes possible.

In order to illustrate this idea, we consider now exact solutions of the nonminimal Einstein-Maxwell model describing the magnetic monopoles of the Dirac type. In the minimal theory, the solution of this type demonstrates a naked singularity in the center; nevertheless, the curvature coupling is shown to lead to the hiding of this singularity inside of the nonminimal horizon. The exact three-parameter nonminimal solutions of the Dirac type can be represented in an explicit analytic form, which simplifies the discussion. These solutions can be considered as a direct reduction of the solutions, obtained for the nonminimal $SU(2)$ symmetric quasi-Abelian Wu-Yang monopole [9], to the model with $U(1)$ symmetry.

The paper is organized as follows. In Sec. II we discuss shortly the fundamentals of the model and represent a three-parameter family of exact solutions describing a nonminimal Dirac monopole. In Sec. III we consider relationships among three coupling constants, for which the space-time metric possesses a singularity "clothed" in a horizon for *arbitrary* mass and charge of the object. In Sec. IV A we consider nonminimal horizons for the exactly integrable model of the Reissner-Nordström type. In Sec. IV B we discuss in detail the one-parameter Drummond-Hathrell model, the horizon radius being obtained and estimated explicitly. In the last section we summarize the results.

II. THREE-PARAMETER FAMILY OF EXACT SOLUTIONS FOR NONMINIMAL MONOPOLES OF THE DIRAC TYPE

A. Nonminimally extended Einstein-Maxwell theory

The three-parameter nonminimal Einstein-Maxwell theory can be formulated in terms of the action functional

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$$S_{\rm NMEM} = \int d^4x\sqrt{-g}\left[\frac{R}{8\pi} + \frac{1}{2}F_{ik}F^{ik} + \frac{1}{2}\mathcal{R}^{ikmn}F_{ik}F_{mn}\right]. \tag{1}$$

Here $g = \det(g_{ik})$ is the determinant of the metric tensor g_{ik}, and R is the Ricci scalar. The Latin indices without parentheses run from 0 to 3. The Maxwell tensor F_{ik} is expressed, as usual, in terms of a potential four-vector A_k:

$$F_{ik} = \nabla_i A_k - \nabla_k A_i, \tag{2}$$

where the symbol ∇_i denotes the covariant derivative. The tensor $\mathcal{R}^{ikmn}$ is defined as follows (see [7]):

$$\begin{aligned}\mathcal{R}^{ikmn} \equiv{}& \frac{q_1}{2}R(g^{im}g^{kn} - g^{in}g^{km}) \\ &+ \frac{q_2}{2}(R^{im}g^{kn} - R^{in}g^{km} + R^{kn}g^{im} - R^{km}g^{in}) \\ &+ q_3R^{ikmn},\end{aligned} \tag{3}$$

where R^{ik} and R^{ikmn} are the Ricci and Riemann tensors, respectively, and q_1, q_2, and q_3 are the phenomenological parameters describing the nonminimal coupling of electromagnetic and gravitational fields. The variation of the action functional with respect to potential A_i yields

$$\nabla_k(F^{ik} + \mathcal{R}^{ikmn}F_{mn}) = 0. \tag{4}$$

In a similar manner, the variation of the action with respect to the metric yields

$$R_{ik} - \tfrac{1}{2}Rg_{ik} = 8\pi T^{\rm (eff)}_{ik}. \tag{5}$$

The effective stress-energy tensor $T^{\rm (eff)}_{ik}$ can be divided into four parts:

$$T^{\rm (eff)}_{ik} = T^{(M)}_{ik} + q_1T^{\rm (I)}_{ik} + q_2T^{\rm (II)}_{ik} + q_3T^{\rm (III)}_{ik}. \tag{6}$$

The first term

$$T^{(M)}_{ik} \equiv \tfrac{1}{4}g_{ik}F_{mn}F^{mn} - F_{in}F_k^{\ n} \tag{7}$$

is a stress-energy tensor of the pure electromagnetic field. The definitions of other three tensors are related to the corresponding coupling constants q_1, q_2, and q_3:

$$\begin{aligned}T^{\rm (I)}_{ik} ={}& RT^{(M)}_{ik} - \tfrac{1}{2}R_{ik}F_{mn}F^{mn} + \tfrac{1}{2}[\nabla_i\nabla_k - g_{ik}\nabla^l\nabla_l] \\ &\times[F_{mn}F^{mn}],\end{aligned} \tag{8}$$

$$\begin{aligned}T^{\rm (II)}_{ik} ={}& -\tfrac{1}{2}g_{ik}[\nabla_m\nabla_l(F^{mn}F^l_{\ n}) - R_{lm}F^{mn}F^l_{\ n}] \\ &- F^{ln}(R_{il}F_{kn} + R_{kl}F_{in}) - R^{mn}F_{im}F_{kn} \\ &- \tfrac{1}{2}\nabla^m\nabla_m(F_{in}F_k^{\ n}) \\ &+ \tfrac{1}{2}\nabla_l[\nabla_i(F_{kn}F^{ln}) + \nabla_k(F_{in}F^{ln})],\end{aligned} \tag{9}$$

$$\begin{aligned}T^{\rm (III)}_{ik} ={}& \tfrac{1}{4}g_{ik}R^{mnls}F_{mn}F_{ls} - \tfrac{3}{4}F^{ls}(F_i^{\ n}R_{knls} + F_k^{\ n}R_{inls}) \\ &- \tfrac{1}{2}\nabla_m\nabla_n[F_i^{\ n}F_k^{\ m} + F_k^{\ n}F_i^{\ m}].\end{aligned} \tag{10}$$

One may check directly that the tensor $T^{\rm (eff)}_{ik}$ satisfies the equation $\nabla^kT^{\rm (eff)}_{ik} = 0$.

Below, we consider nonminimally extended Einstein-Maxwell equations (4)–(10) for the case of the static spherically symmetric space-time metric

$$ds^2 = \sigma^2Ndt^2 - \frac{dr^2}{N} - r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{11}$$

where N and σ are functions of the radial variable r only.

B. Minimal solution with naked singularity as a starting point

In the minimal Einstein-Maxwell theory, the exact static spherically symmetric solution of the Reissner-Nordström type is the following:

$$\sigma(r) = 1, \qquad N(r) = 1 - \frac{2M}{r} + \frac{Q^2_{(e)} + Q^2_{(m)}}{r^2}. \tag{12}$$

When $M < \sqrt{Q^2_{(e)} + Q^2_{(m)}}$, there are no horizons, and the central point $r = 0$ is classified as the naked singularity. When $Q_{(e)} = 0$ and $M < |Q_{(m)}|$, one deals with a magnetic naked singularity.

The nonminimal Einstein-Maxwell model for the static spherically symmetric space-time and central electric and magnetic charges was studied for two special sets of the coupling constants: The first one satisfies the equalities $q_1 + q_2 + q_3 = 0$ and $2q_1 + q_2 = 0$ (see, e.g., [8,10–12]), and the second one relates to $q_1 + q_2 = 0$ and $q_3 = 0$ [8].

C. Nonminimal Dirac monopoles

Here we assume that electric charge is absent: $Q_{(e)} = 0$. One can check directly that Eqs. (2) and (4) are satisfied identically, when the potential of the electromagnetic field A_i and the field strength tensor F_{ik} outside a pointlike magnetic charge $Q_{(m)}$ have the form

$$A_k = \frac{Q_{(m)}}{\sqrt{4\pi}}(1 - \cos\theta)\delta^\varphi_k, \tag{13}$$

$$F_{ik} = \frac{Q_{(m)}}{\sqrt{4\pi}}\sin\theta(\delta^\theta_i\delta^\varphi_k - \delta^\theta_k\delta^\varphi_i). \tag{14}$$

Surprisingly, these quantities depend neither on the radial variable r nor on the coupling parameters q_1, q_2, and q_3. Thus, the well-known solution with a monopole-type magnetic field satisfies the nonminimally extended Maxwell equations. As a next step, we solve the Einstein equations, which can be reduced for the given ansatz to the following pair of key equations:

$$\frac{\sigma'}{\sigma}\left(1 - \frac{\kappa q_1}{r^4}\right) = \frac{\kappa}{r^5}(10q_1 + 4q_2 + q_3), \tag{15}$$

$$rN'\left(1-\frac{\kappa q_1}{r^4}\right)+N\left[1+\frac{\kappa}{r^4}(13q_1+4q_2+q_3)\right]$$
$$=1-\frac{\kappa}{2r^2}+\frac{\kappa}{r^4}(q_1+q_2+q_3). \tag{16}$$

When $q_1 \neq 0$, these key equations give the following three-parameter family of solutions:

$$\sigma=\left(1-\frac{\kappa q_1}{r^4}\right)^\beta, \qquad \beta\equiv\frac{10q_1+4q_2+q_3}{4q_1}, \tag{17}$$

$$N=1-\frac{2M}{r}\left(1-\frac{\kappa q_1}{r^4}\right)^{-(\beta+1)}+\frac{\kappa}{2r}\int_r^\infty\frac{dx}{x^2}\left[1+\frac{6}{x^2}(4q_1+q_2)\right]\left(1-\frac{\kappa q_1}{x^4}\right)^\beta\times\left(1-\frac{\kappa q_1}{r^4}\right)^{-(\beta+1)}. \tag{18}$$

In the special case, when $q_1 = 0$, the two-parameter family of solutions takes the form

$$\sigma=\exp\left[-\frac{\kappa(4q_2+q_3)}{4r^4}\right], \tag{19}$$

$$N=1-\frac{2M}{r}\exp\left[\frac{\kappa(4q_2+q_3)}{4r^4}\right]+\frac{\kappa}{2r}\int_r^\infty\frac{dx}{x^2}\left(1+\frac{6q_2}{x^2}\right)\times\exp\left[\frac{\kappa(4q_2+q_3)}{4}\left(\frac{1}{r^4}-\frac{1}{x^4}\right)\right]. \tag{20}$$

Here κ is a convenient positive constant with the dimensionality of area $\kappa=2Q^2_{(m)}$, and M is a constant of integration describing the asymptotic mass of the monopole. These solutions are direct $U(1)$ analogs of the nonminimal Wu-Yang monopole solutions obtained in Ref. [9], and they may be indicated as the nonminimal Dirac monopoles. Clearly, when $q_1=q_2=q_3=0$, the obtained solutions reduce to the minimal one (12) with $Q_{(e)}=0$.

III. CONDITIONS FOR THE ABSENCE OF NAKED SINGULARITY

In Refs. [9,13], we attracted special attention to the solution (17)–(20) with a regular metric. In particular, it was shown that, when $q_1=-q$, $q_2=4q$, $q_3=-6q$, and q is positive, there are no horizons if the mass of the monopole is less than some critical mass $M_{(\mathrm{crit})}$. Now we focus on the analysis of the metrics, which have at least one horizon for *arbitrary* mass and magnetic charge $Q_{(m)}$, and we search for the relevant relationships among the coupling constants q_1, q_2, and q_3. It is convenient to divide our analysis into three parts for the cases $q_1<0$, $q_1=0$, and $q_1>0$, respectively.

A. First case: $q_1 < 0$

The main problem we are going to solve is the following: for what values of q_1, q_2, and q_3 the equation

$$N(r)=0 \tag{21}$$

has at least one positive solution, when the parameters $M\geq 0$ and $\kappa>0$ are arbitrary. For the derivation of basic inequalities, we use the following method. First, taking into account (18), we rewrite Eq. (21) in the form

$$2M=r\left(1+\frac{\kappa|q_1|}{r^4}\right)^{\beta+1}+\frac{\kappa}{2}\int_r^\infty\frac{dx}{x^2}\left[1+\frac{6}{x^2}(4q_1+q_2)\right]\times\left(1+\frac{\kappa|q_1|}{x^4}\right)^\beta. \tag{22}$$

Second, we make the replacement $z=r(\kappa|q_1|)^{-1/4}$ in this equation, thus introducing a new dimensionless variable z. Third, we rewrite the obtained equation as follows:

$$\frac{1}{2}\sqrt{\frac{\kappa}{|q_1|}}=S(z), \tag{23}$$

$$S(z)\equiv\frac{1}{\int_z^\infty d\tau\tau^{-2}(1+\tau^{-4})^\beta}\times\left[\left(12-\frac{3q_2}{|q_1|}\right)\int_z^\infty d\tau\tau^{-4}(1+\tau^{-4})^\beta+\frac{2M}{(\kappa|q_1|)^{1/4}}-z(1+z^{-4})^{\beta+1}\right]. \tag{24}$$

Since for negative q_1 the expression $(1+\tau^{-4})$, obtained by replacement, does not take on a zero value, the function $S(z)$ is continuous in the interval $z\in(0;+\infty)$. At the limiting case $z\to+\infty$, this function takes on the negative infinite value $\lim_{z\to+\infty}S(z)=-\infty$. We assume that the equality (23) should be fulfilled for arbitrary magnetic charge, i.e., for an arbitrary non-negative value of the parameter $\sqrt{\kappa/|q_1|}$. Thus, the function $S(z)$ should reach an infinite value at least in one of the points of the interval $z\in(0,+\infty)$. Being continuous at $z>0$, the function $S(z)$ can reach infinity only at $z=0$. Consequently, one should estimate the behavior of $S(z)$ in the vicinity of this point. The simple analysis shows that in this limit $S(z)$ tends to infinity, when $\beta\geq-3/4$. In addition, the infinite value is positive, i.e., $S(0)=+\infty$, when $12-\frac{3q_2}{|q_1|}>4\beta+3$ only. After the substitution of the expression for β from (17), we obtain the basic inequalities

$$13q_1+4q_2+q_3\leq 0, \qquad q_1+q_2+q_3>0. \tag{25}$$

B. Second case: $q_1 = 0$

When q_1 vanishes, we take Eq. (20) instead of (18) and exponential function $\exp\{\kappa(4q_2+q_3)/4r^4\}$ instead of $(1+\kappa|q_1|/r^4)^\beta$. The procedure for obtaining the basic inequalities is similar to the one used in the previous case, and it yields the same inequalities (25).

C. Third case: $q_1 > 0$

When q_1 is positive, the situation differs essentially from that of the two previous cases. First of all, the metric (11), (17), and (18) is ill-defined for a fractional β, when $r < \sqrt[4]{\kappa q_1}$. If β is an integer, the metric has a singularity at $r < \sqrt[4]{\kappa q_1}$. Therefore, we have to restrict our consideration by the interval $r < \sqrt[4]{\kappa q_1}$ only. Let us show now that no horizon for arbitrary mass and magnetic charge exists for this interval. The procedure of finding of basic inequalities is similar to that of the first case, but now we obtain a modified auxiliary function $\tilde{S}(z)$ instead of $S(z)$ [see (24)]:

$$\tilde{S}(z) \equiv -\frac{1}{\int_z^\infty d\tau \tau^{-2}(1-\tau^{-4})^\beta} \times \left[\left(12+\frac{3q_2}{q_1}\right)\int_z^\infty d\tau\tau^{-4}(1-\tau^{-4})^\beta - \frac{2M}{(\kappa q_1)^{1/4}} + z(1-z^{-4})^{\beta+1}\right]. \tag{26}$$

The function $\tilde{S}(z)$ is continuous in the interval $z \in (1; +\infty)$ and $\lim_{z\to+\infty}\tilde{S}(z) = -\infty$. In order to resolve the equation

$$\frac{1}{2}\sqrt{\frac{\kappa}{q_1}} = \tilde{S}(z), \tag{27}$$

for arbitrary magnetic charge, we should require that $\tilde{S}(z)$ tends to positive infinity at $z \to 1$, i.e., $\lim_{z\to 1}\tilde{S}(z) = +\infty$. However, $\tilde{S}(1)$ is finite, and thus it is impossible.

D. Basic inequalities

Summing up the results of the three previous subsections, we can presume that in the nonminimal model under consideration the metric (11) and (17)–(20) has at least one event horizon for arbitrary values of the mass $M \geq 0$ and magnetic charge $Q_{(m)}$, when the three following inequalities are valid:

$$q_1 \leq 0, \qquad 13q_1 + 4q_2 + q_3 \leq 0, \qquad q_1 + q_2 + q_3 > 0. \tag{28}$$

Since the first and second inequalities are unstrict, there are three interesting particular cases.

1. $13q_1 + 4q_2 + q_3 \neq 0$

If the second inequality is strict, i.e., $\beta \neq -3/4$, the value of the function $N(r)$ at the center is finite and negative:

$$N(0) = \frac{(q_1+q_2+q_3)}{(13q_1+4q_2+q_3)} < 0. \tag{29}$$

Since $N(\infty) = 1 > 0$, and $N(r)$ is a continuous function, there is at least one point at $r > 0$, say, r^*, in which $N(r^*) = 0$. This fact demonstrates explicitly that the singular point of origin $r = 0$ is hidden inside of an event horizon.

2. $13q_1 + 4q_2 + q_3 = 0$ *and* $q_1 \neq 0$

When $\beta = -3/4$ and $q_1 \neq 0$, the function $N(r)$ behaves in the vicinity of $r = 0$ as

$$N(r) \sim A\ln r, \qquad A = \frac{3(4q_1+q_2)}{q_1} > 0. \tag{30}$$

Thus, at the point of origin $N(0) = -\infty$, and one has at least one solution of the equation $N(r) = 0$, as in the previous case.

3. $13q_1 + 4q_2 + q_3 = 0$ *and* $q_1 = 0$

When $\beta = -3/4$ and $q_1 = 0$, one obtains that q_2 is negative, and at $r \to 0$ the function $N(r)$ behaves as

$$N(r) \sim -\frac{\kappa|q_2|}{r^4}. \tag{31}$$

Thus, the values $N(0)$ are now infinite, but also negative, confirming our conclusion that there exists at least one point with $N(r^*) = 0$.

The inequalities (28) can be rewritten in the simple form using the following reparametrization:

$$q_1 = -Q_1, \qquad q_2 = 4Q_1 - Q_2 - Q_3, \qquad q_3 = -3Q_1 + Q_2 + 4Q_3. \tag{32}$$

In these new terms, the basic inequalities read

$$Q_1 \geq 0, \qquad Q_2 \geq 0, \qquad Q_3 > 0, \tag{33}$$

separating the first octant with two boundary planes in the auxiliary three-dimensional space of parameters Q_1, Q_2, and Q_3. A true number of horizons for each set of q_1, q_2, and q_3, satisfying (28) depends on relations among the mass, charge, and coupling constants. Below, we consider a number of exact solutions illustrating our conclusions.

IV. EXPLICIT EXAMPLES OF EXACT SOLUTIONS WITH NONMINIMAL HORIZONS

A. Nonminimal solution of the Reissner-Nordström type with $q_1 = 0$, $4q_2 + q_3 = 0$

The given set of parameters relates to the third (special) case, considered in the previous subsection. When q_1 vanishes and $q_3 = -4q_2$, the formulas (19) and (20) yield

$$\sigma(r) = 1, \qquad N(r) = 1 - \frac{2M}{r} + \frac{\kappa}{2r^2} + \frac{\kappa q_2}{r^4}. \tag{34}$$

We deal with the one-parameter nonminimal generalization of the Reissner-Nordström solution. This exact solution is characterized by the infinite central value $N(0)$, this value being negative if $q_2 < 0$. Thus, starting from $N(\infty) = 1 > 0$, the continuous function $N(r)$ tends to $N(0) = -\infty$ and crosses the line $N = 0$ at least once *for arbitrary mass and charge*. In other words, the equation $N(r) = 0$ leads to the quartic equation

$$r^4 - 2Mr^3 + \frac{\kappa}{2}r^2 + \kappa q_2 = 0, \tag{35}$$

which has at least one positive real root and, thus, guarantees that the space-time possesses at least one horizon for arbitrary mass and charge. For this case, the inequalities (28) yield that $-3q_2 > 0$, in agreement with our conclusion.

Generic requirements for M, κ, and q_2, which classify the number of nonminimal horizons, can be found using the well-known Ferrari method (see, e.g., [14]); nevertheless, we restrict ourselves to two explicit examples only, demonstrating the cases with one and three horizons.

1. $M = 0$: One horizon

In the minimal model, the condition $M = 0$ leads to the Reissner-Nordström solution with a naked singularity. In the nonminimal model, the quartic equation (35) reduces to the biquadratic one, and, clearly, the only positive real root is

$$r = r_{(\mathrm{H})} = \frac{1}{2}\sqrt{\kappa}\sqrt{\sqrt{1 + \frac{16|q_2|}{\kappa}} - 1}. \tag{36}$$

In the minimal limit $q_2 \to 0$, the radius of the horizon $r_{(\mathrm{H})}$ tends to zero. When $|q_2| \ll \kappa$, $r_{(\mathrm{H})} \to \sqrt{2|q_2|}$; when $|q_2| \gg \kappa$, $r_{(\mathrm{H})} \to (\kappa|q_2|)^{1/4}$.

2. $\kappa = 2M^2$: Three horizons

In the minimal model, the condition $\kappa = 2M^2$ (or, equivalently, $M^2 = Q^2_{(m)}$) introduces the so-called extreme Reissner-Nordström black hole, for which two horizons coincide. For the nonminimal model, Eq. (35) can be presented as a product of two quadratic equations. Clearly, for arbitrary mass there exists the positive real root

$$r_{(\mathrm{H}1)} = \frac{M}{2}\left(1 + \sqrt{1 + \frac{4\sqrt{2|q_2|}}{M}}\right). \tag{37}$$

In addition, when $M > 4\sqrt{2|q_2|}$, there are two roots else

$$r_{(\mathrm{H}2,3)} = \frac{M}{2}\left(1 \pm \sqrt{1 - \frac{4\sqrt{2|q_2|}}{M}}\right). \tag{38}$$

When $q_2 \to 0$, one obtains from (37) and (38)

$$r_{(\mathrm{H}1)} \simeq M + \sqrt{2|q_2|}, \qquad r_{(\mathrm{H}2)} \simeq M - \sqrt{2|q_2|}, \qquad r_{(\mathrm{H}3)} \simeq \sqrt{2|q_2|}. \tag{39}$$

This means that nonminimal coupling removes the degeneration, which appears if the mass coincides with the charge, and splits the double horizon of the extreme Reissner-Nordström magnetic black hole into two space-apart horizons with the radii $r_{(\mathrm{H}1)}$ and $r_{(\mathrm{H}2)}$, respectively. The radius of the third nonminimal horizon $r_{(\mathrm{H}3)}$ tends to zero at vanishing coupling parameter q_2.

B. Nonminimal model of the Drummond-Hathrell type

The one-parameter Drummond-Hathrell model arises from the calculation of the one-loop QED corrections to the Einstein-Maxwell Lagrangian in curved space-time [15]. For this model $q_1 = -5q$, $q_2 = 13q$, and $q_3 = -2q$, where $q = \frac{\alpha\lambda^2}{180\pi}$ ($\alpha \approx 1/137$ is the fine structure constant, and $\lambda \approx 4 \times 10^{-13}$ m is the Compton wavelength of the electron). Clearly,

$$q_1 \leq 0, \qquad 13q_1 + 4q_2 + q_3 = -15q < 0, \qquad q_1 + q_2 + q_3 = 6q > 0; \tag{40}$$

i.e., this set of the coupling constants satisfies basic inequalities (28).

1. Number of horizons

In the Drummond-Hathrell model $\beta = 0$, and the metric functions $\sigma(r)$ and $N(r)$ take the following explicit form [16]:

$$\sigma(r) = 1, \qquad N(r) = \frac{r^4 - 2Mr^3 + \kappa r^2/2 - 2\kappa q}{r^4 + 5\kappa q}. \tag{41}$$

At the point of origin, $N(0) = -2/5 < 0$ in agreement with (29), as well as $N(\infty) = 1$; thus, at least one horizon exists for arbitrary mass and charge. Let us mention that $N(0) \neq 1$; consequently, the metric (41) possesses the so-called "mild" or "conic" singularity. This means that the metric functions themselves $\sigma(r)$ and $N(r)$ are finite at $r = 0$, whereas the Ricci scalar is infinite because of the term $[1 - N(r)]/r^2$. The same situation is described in Ref. [8] for the Fibonacci model.

In order to find the number of horizons for the metric (41), let us consider in more detail the roots of the numerator of $N(r)$, i.e., analyze the quartic equation

$$r^4 - 2Mr^3 + \frac{\kappa r^2}{2} - 2\kappa q = 0. \tag{42}$$

We divide the analysis into two cases: $\kappa > 96q$ and $\kappa \leq 96q$. When $\kappa > 96q$, it is convenient to introduce the following auxiliary quantities:

$$M_{1,2} = \frac{2r_{1,2}}{3} + \frac{\kappa}{6r_{1,2}}, \qquad r_{1,2} = \frac{\sqrt{\kappa}}{2}\left(1 \pm \sqrt{1 - \frac{96q}{\kappa}}\right)^{1/2}. \tag{43}$$

There are three different possibilities:

(i) $M_1 < M < M_2$: Equation (42) has three real positive solutions.

(ii) $M = M_1$ or $M = M_2$: There are two different solutions, since a couple of solutions coincide.

(iii) $M < M_1$ or $M > M_2$: Equation (42) has only one real positive solution.

When $\kappa \leq 96q$, Eq. (42) has only one positive real root for arbitrary mass M. In other words, for arbitrary magnetic charge (i.e., for any κ), one can find at least one horizon attributed to any mass M, and the naked singularity does not exist in the nonminimal Drummond-Hathrell model.

As a simple explicit illustration let us assume that the monopole mass M is vanishing. Then the single positive solution to (42) can be written in the explicit form

$$r_{h0} = \frac{\sqrt{\kappa}}{2}\left(\sqrt{1+\frac{32q}{\kappa}}-1\right)^{1/2}. \tag{44}$$

If $q = 0$, this horizon turns into the point of origin. When $q \ll \kappa$, r_{h0} tends to $2\sqrt{q}$; when $q \gg \kappa$, $r_{h0} \approx \sqrt[4]{2\kappa q}$. Thus, this horizon is essentially nonminimal.

2. Numerical estimation of the radius of the nonminimal horizon

The nonminimal Drummond-Hathrell model is especially attractive, since all of the parameters of the model can be directly estimated. Indeed, the value of q can be readily estimated as $q = \frac{\alpha\lambda^2}{180\pi} \approx 2 \times 10^{-30}$ m^2. The quantity $\sqrt{\kappa}$ is proportional to the magnetic charge $Q_{(m)}$, and for a magnetic monopole with unit charge it can be estimated as $\sqrt{\kappa} \simeq 10^{-34}$ m [17]. Thus, we deal with the case $q \gg \kappa$, the inequality $\kappa \leq 96q$ is valid, and there is only one horizon according to our previous analysis. The radius of nonminimal horizon can be found now from the formula

$$r_h \approx (2\kappa q)^{1/4} \sim 10^{-25} \text{ m}. \tag{45}$$

The choice of this formula can be motivated as follows. The mass of the monopole is unknown, but we assume that it is less than the Planck mass, which guarantees that $M \ll \sqrt{\kappa}$. Then, using Eq. (44) for vanishing mass, and taking into account that $q \gg \kappa$, we obtain (45). Thus, our conclusion is that the nonminimal horizon in the Drummond-Hathrell model has the radius of the order 10^{-25} m. This value is much greater than the Planck length $L_{\rm pl} \sim 10^{-35}$ m but is much smaller than the Compton wavelength of the electron $\lambda \approx 4 \times 10^{-13}$ m.

V. DISCUSSION

The logic of the development of the nonminimal Einstein-Maxwell theory prompts that the phenomenologically introduced coupling constants q_1, q_2, and q_3, which have the dimensionality of area, either have to be associated with some known constants of Nature or some new nonminimal radii should be introduced and properly motivated. One attempt to realize this idea was made in Ref. [7], where the approach based on the symmetry of the susceptibility tensor $\mathcal{R}^{ikmn}$ (3) is proposed. In Refs. [8,9,13,18], special sets of coupling parameters were found, for which the metric functions of nonminimally coupled systems happened to be regular, and the absence of singularity became one of the arguments for the nonminimal extension of the Einstein-Maxwell theory.

Here we analyzed a new possibility to fix the coupling constants, which is related to the censorship conjecture. We discussed the three-parameter family of exact solutions of the nonminimal Einstein-Maxwell model, which can be associated with magnetic monopoles of the Dirac type. We have shown explicitly that the singular point $r = 0$ appears to be hidden by some nonminimal horizon independently on the mass and magnetic charge, when the basic inequalities (28) are satisfied. In terms of new appropriate parameters Q_1, Q_2, and Q_3 [see (32)] such a kind of nonminimal clothing is possible, when these new parameters belong to the first octant of the auxiliary three-dimensional **Q** space (including two of three separating planes). As it was shown by the example of the nonminimal Drummond-Hathrell model (see Sec. IV B), the radius of the nonminimal event horizon can be estimated as $r_h \sim 10^{-25}$ m; i.e., it can be greater by 10 orders than the Planck length $L_{\rm pl}$. In forthcoming papers we intend to analyze nonminimal models with electric charge and the dyonic model in order to find analogous necessary conditions prescribed by the censorship conjecture. We believe that the combination of requirements obtained for nonminimal magnetic monopoles, electrically charged objects, and dyons could fix the choice of coupling constants and define unambiguously the radius of the event horizon $r_{(P)}$ associated with the censorship conjecture, proposed by Penrose.

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[1] R. Penrose, Riv. Nuovo Cimento Soc. Ital. Fis. **1**, 252 (1969).
[2] P. K. Townsend, arXiv:gr-qc/9707012.
[3] R. M. Wald, *General Relativity* (University of Chicago Press, Chicago, 1984).
[4] M. Wyman, Phys. Rev. D **24**, 839 (1981).
[5] M. D. Roberts, Gen. Relativ. Gravit. **21**, 907 (1989).
[6] A. Hardell and H. Dehnen, Gen. Relativ. Gravit. **25**, 1165 (1993).
[7] A. B. Balakin and J. P. S. Lemos, Classical Quantum Gravity **22**, 1867 (2005).
[8] A. B. Balakin, V. V. Bochkarev, and J. P. S. Lemos, Phys.

Rev. D **77**, 084013 (2008).
[9] A. B. Balakin and A. E. Zayats, Phys. Lett. B **644**, 294 (2007).
[10] G. W. Horndeski, Phys. Rev. D **17**, 391 (1978).
[11] G. W. Horndeski, J. Math. Phys. (N.Y.) **19**, 668 (1978).
[12] F. Müller-Hoissen and R. Sippel, Classical Quantum Gravity **5**, 1473 (1988).
[13] A. B. Balakin and A. E. Zayats, Gravitation Cosmol. **14**, 86 (2008).
[14] G. A. Korn and T. M. Korn, *Mathematical Handbook for Scientists and Engineers* (Dover, New York, 2000), p. 25.
[15] I. T. Drummond and S. J. Hathrell, Phys. Rev. D **22**, 343 (1980).
[16] A. E. Zayats, Ph.D. thesis, Kazan State University, 2008 (in Russian).
[17] A. B. Balakin, S. V. Sushkov, and A. E. Zayats, Phys. Rev. D **75**, 084042 (2007).
[18] A. B. Balakin and W. Zimdahl, Phys. Rev. D **71**, 124014 (2005).