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Dilatonic dark matter and its experimental detection

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Assuming that the dilaton is the dark matter of the Universe, we propose an experiment to detect the relic dilaton using the electromagnetic resonant cavity, based on the dilaton-photon conversion in strong electromagnetic background. We calculate the density of the relic dilaton, and estimate the dilaton mass for which the dilaton becomes the dark matter of the Universe. With this we calculate the dilaton detection power in the resonant cavity, and compare it with the axion detection power in a similar resonant cavity experiment.

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I. INTRODUCTION

One of the important issues in cosmology is the search for the dark matter. Popular candidates of dark matter include axion, dilaton, and other weakly interacting massive particles known as WIMPs [1–3]. Although they differ completely in their origins, they have a common feature. All have very weak couplings to matter fields, which makes them excellent candidates of dark matter. In fact axion and dilaton could be viewed as different types of WIMPs, whose couplings to matter fields are made weak for different reasons. Nevertheless, the axion and dilaton have a remarkable similarity in that their couplings to the electromagnetic field and the fermionic matter fields are almost identical [1,2].

The dilaton is a universal scalar field which appears in all higher-dimensional unified theories (including the Kaluza-Klein theory and the superstring theory) which plays the role of the scalar graviton, and thus couples directly to all matter fields [4–6]. On the other hand, the axion is a pseudoscalar Goldstone boson generated by spontaneous breakdown of the Peccei-Quinn (PQ) $U_{\rm PQ}(1)$ symmetry which was introduced to solve the so-called "strong CP problem" in strong interaction [7,8]. But they have almost identical electromagnetic coupling, except that the dilation (being a scalar) couples to $F_{\mu\nu}^2$ while the axion (being a pseudoscalar) couples to $F_{\mu\nu}\tilde{F}_{\mu\nu}$. In this sense the dilaton and axion may be viewed as the scalar-pseudoscalar partners of each other. This is particularly true for the gravitational axion, the pseudoscalar graviton which has been proposed by Ni independent of the strong CP problem [9].

The axion has been believed to be one of the strong candidates of dark matter by many physicists, and experiments to detect it have been actively performed [1,10]. In comparison, the detection of the dilaton has not so actively been performed up to now, in spite of its theoretical importance. It is well-known that the dilaton generates the fifth force which can affect the Einstein's gravity in a fundamental way [11–13]. Moreover, as a massive scalar graviton it can naturally be viewed as a gravitationally interacting massive particle (GIMP), a WIMP whose interaction to matter fields is made very weak by the gravitational coupling. This makes the dilaton an excellent candidate of the dark matter in cosmology [2,14]. In this paper we study the dilaton as a candidate of dark matter in detail, and propose a dilaton detection experiment using an electromagnetic resonant cavity. In particular, we refine the existing estimate of the dilaton mass, calculate the dilaton detection power in the resonant cavity, and compare this with the axion detection power in similar experiments.

The paper is organized as follows. In Sec. II we briefly review the dilaton physics and the dilatonic fifth force, viewing the dilaton as a natural candidate of GIMPs. In Sec. III we discuss the role of dilaton in cosmology, and estimate the number density of the relic dilaton in the present Universe based on the dilaton decay to two photons and fermion-antifermion pairs. In Sec. IV we discuss the condition for the dilaton to be a candidate of dark matter, and refine the acceptable mass range of dilaton. In Sec. V we propose the experiment to detect the dilaton using an electromagnetic resonant cavity. We calculate the dilaton detection power in the resonant cavity, and compare it with the axion detection power in similar experiments. Finally in Sec. VI we discuss the physical implications of our analysis.

II. DILATONIC FIFTH FORCE

All known interactions are mediated by spin-one or spin-two fields. However, the unification of all interactions inevitably requires the existence of a fundamental spin-zero field. In fact, all modern unified theories (Kaluza-Klein theory, supergravity, and superstring) contain a fundamental scalar field called the dilaton, in particular, the Kaluza-Klein dilaton [4,11] and the string dilaton [6]. What makes this scalar field unique is that unlike others scalar fields such as the Higgs field, it couples directly to the (trace of the) energy-momentum tensor of the matter

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fields. As such it plays the role of the scalar graviton, and could generate the dilatonic fifth force which modifies Einstein's gravity in a fundamental way.

The dilaton has been proposed by many authors for different reasons and called by various names. But we emphasize that the dilaton as the scalar graviton has a long history. Dirac was the first who proposed such a scalar field, suggesting that the Newton's constant should actually be considered as a time-dependent parameter (and thus be treated as a scalar field) [15]. Soon after, Jordan and independently Brans and Dicke introduced the first dilaton known as the Brans-Dicke dilaton [16]. Unfortunately the Brans-Dicke dilaton was proposed as a massless scalar graviton, so that it must create a long range fifth force which is comparable to Newton's gravitational force. This contradicts the experiments which tell that such long range fifth force does not exist in nature [17,18]. This rules out the Brans-Dicke dilaton as unphysical. Subsequently the Kaluza-Klein dilaton [4] and the string dilaton [6] have been introduced, and later the dilaton was reinvented by many authors in so-called "the nonminimally coupled scalar models" and named graviscalar and/or graviexciton [19]. Among these the Kaluza-Klein dilaton has remarkable virtue. It appears as the internal graviton which represents the volume of the extra space, and acquires mass when the extra space has a nonvanishing curvature [2,11]. Thus it can naturally become a GIMP, gravitationally interacting massive particle, which can generate a short range fifth force which does not contradict with the known fifth-force experiments [4,11,14]. Moreover, as a GIMP which interacts more weakly than the popular WIMPs it can play the role of the dark matter in cosmology. Furthermore, the Kaluza-Klein dilaton (renamed the radion) has been argued to play a crucial role in resolving the hierarchy problem [20]. This is because it represents the volume of the extra space which modifies the higher-dimensional gravitational constant [2,11].

Similar scalar fields, for example, quintessence and chameleon, have been proposed to resolve various problems in cosmology [21,22]. But what makes the Kaluza-Klein dilaton unique is that it appears in all modern unified theories including the superstring, which are based on higher-dimensional physics. This justifies the theoretical *raison d'etre* of the dilaton. As importantly, it has well-defined features which can be tested by experiments. This is why the experimental verification of dilaton is very important.

As we have remarked an immediate consequence of the dilaton is the presence of dilatonic fifth force which modifies Einstein's gravitation [12,13]. To see how the dilaton affects the gravitation we have to know the mass of the dilation and its coupling strength to matter fields. In Kaluza-Klein theory the dilaton naturally acquires a mass when the extra space has nonvanishing curvature [2]. As for the dilatonic coupling to matter fields, the coupling may depend on the types of matter field it couples to [11]. But in practice only one type of coupling, the dilatonic coupling to the baryonic matter, is important because this is what we measure in experiments. So, only two parameters, the baryonic coupling constant and the mass of the dilaton, become important to describe the dilatonic fifth force. Let V_g and V_5 be the gravitational and fifth-force potentials and F_g and F_5 their forces between the two baryonic point particles separated by a distance r. From the dimensional argument, one may express the total potential and total force in the Newtonian limit as

$$\begin{aligned} V &= V_g + V_5 \simeq -\frac{\alpha_g}{r} - \frac{\alpha_5}{r}\exp(-\mu r), \\ F &= F_g + F_5 \simeq \frac{\alpha_g}{r^2} + \frac{\alpha_5}{r^2}(1+\mu r)\exp(-\mu r) \\ &= \frac{\alpha_g}{r^2}\{1 + \beta(1+\mu r)\exp(-\mu r)\}, \end{aligned} \tag{1}$$

where α_g, α_5 are the fine-structure constants of the gravitation and fifth force, and $\beta = \alpha_5/\alpha_g$ is the ratio between them. In terms of Feynman diagrams the first term represents one graviton exchange but the second term represents one dilaton exchange in the zero momentum transfer limit. In the Kaluza-Klein unification we have $\beta = n/(n+2)$ [2,11], but in general one may assume $\beta \simeq 1$ if one wants to identify the dilaton as a GIMP. On the other hand, it is important to keep in mind that in principle there is no *a priori* reason why the dilaton interaction has to be gravitational. So in principle it is good to leave α_5 and thus β as an arbitrary coupling constant, especially in the analysis of experiments.

With this in mind one may try to measure the coupling constant and the range of the fifth force experimentally. A recent torsion-balance fifth-force experiment puts the upper bound of the range of the fifth force to be around 56 μm with 95% confidence level (with $\beta \simeq 1$) [17,18]. This tells that the dilaton mass has to be larger than 10^{-2} eV. But in the following we will simply treat the dilaton mass an undetermined parameter, and find an independent estimate of the dilaton mass based on the assumption that the dilaton is the dark matter of the Universe.

III. RELIC DILATON IN COSMOLOGY

In the early Universe the dilaton starts with the thermal equilibrium at the beginning and decouples from other sources very early near the Planck time. Moreover, since its coupling to matter fields is very weak, it may easily survive in the present Universe and become the dark matter of the Universe [2,14]. In this section we discuss the relic dilaton.

From the dimensional argument one may assume the dilatonic coupling strength to matter fields to be gm/m_p, where g is the dimensionless coupling constant and m is the mass of the relevant matter (e.g., quarks and gluons). But at high temperature (at $T \gg m$), the coupling strength can be written as gT/m_p. With this one can easily estimate the dilaton creation (and annihilation) cross section as [14]

$$\sigma \simeq g^2\left(\frac{T}{m_p}\right)^2 \times \frac{1}{T^2}, \tag{2}$$

with the transition rates Γ

$$\Gamma \simeq N\sigma v \simeq g^2\left(\frac{T}{m_p}\right)^2 \times T, \tag{3}$$

where N and v are the density of the matter and the speed of the dilaton. Similarly the dilaton scattering cross section and the interaction rate are given by

$$\sigma \simeq g^4\left(\frac{T}{m_p}\right)^4 \times \frac{1}{T^2}, \qquad \Gamma \simeq N\sigma v \simeq g^4\left(\frac{T}{m_p}\right)^4 \times T. \tag{4}$$

On the other hand, the Hubble expansion rate in the early Universe is given by $H \simeq T^2/m_p$. So, letting $\Gamma \simeq H$ we find the dilaton decoupling temperature

$$T_D \simeq \frac{m_p}{g^{4/3}}. \tag{5}$$

This confirms that the dilaton is thermally produced at the beginning, and decouplesfrom the other matters around the Planck time when $g \simeq 1$.

The dilaton becomes unstable and decays into ordinary matter. A typical decay process is the two-photon process and the fermion-antifermion pair production process which may be described by the following interaction Lagrangian [14],

$$\mathcal{L}_{\rm int} \simeq \sqrt{16\pi G}\left\{-\frac{g_1}{4}\sigma F_{\mu\nu}F^{\mu\nu} - g_2 m\sigma\bar{\psi}\psi\right\}, \tag{6}$$

where g_1 and g_2 are dimensionless coupling constants, m is the mass of the fermion, and σ is the dilaton field. We could also include the following dilaton-fermion interaction in (6)

$$\sqrt{16\pi G}\{g_3\sigma\bar{\psi}\gamma^\mu\partial_\mu\psi + g_4\partial_\mu\sigma\bar{\psi}\gamma^\mu\psi\}. \tag{7}$$

But we will concentrate on (6) in the following, since we find that the inclusion of these interactions do not change our main conclusions in this paper. Notice that the dilaton interaction is (just like the gravity) governed by the overall dimensional coupling constant $\sqrt{16\pi G}$, which sets the strength of the dilaton interaction. So, with $g_1 \simeq g_2 \simeq 1$, the dilaton becomes a GIMP. But notice that we can always change (if we like) the strength of dilaton interaction by adjusting the dimensionless coupling constants g_1 and g_2. This is good, because in principle there is no *a priori* reason why the dilaton interaction has to be gravitational as we have remarked. So we will leave the dimensionless coupling constants arbitrary in the following.

The Lagrangian (6) should be compared to the following axion interaction Lagrangian given by [1,9]

$$\mathcal{L}_{\rm int} \simeq -\alpha_\gamma a F_{\mu\nu}\tilde{F}^{\mu\nu} - i\alpha_f a\bar{\psi}\gamma^5\psi, \tag{8}$$

where a is the axion field, α_γ and α_f are the axion coupling constants. This confirms that dilaton and axion behave as the scalar-pseudoscalar counterparts of each other.

Consider the interaction between dilaton and photon first. Let $\alpha_1 = g_1\sqrt{16\pi G}/4$ be the dimensional coupling constant and denote the dilaton mass by μ. The differential dilaton decay rate to two photons at tree level is given by

$$\begin{aligned} d\Gamma_{\sigma\to\gamma\gamma} &= \frac{1}{2p^0}\sum_{\lambda,\lambda'=\pm1}\frac{1}{2!}(2\pi)^4\delta^{(4)}(p^\mu - k^\mu - k'^\mu)|M|^2 \\ &\quad\times \frac{d^3\vec{k}}{(2\pi)^3 2k^0}\frac{d^3\vec{k}'}{(2\pi)^3 2k'^0}, \\ M &= -i\alpha_1(k_\mu\epsilon_\nu(k,\lambda) - k_\nu\epsilon_\mu(k,\lambda))(k'^\mu\epsilon'^\nu(k',\lambda') \\ &\quad - k'^\nu\epsilon'^\mu(k',\lambda')), \end{aligned} \tag{9}$$

where p^μ and k^μ, k'^μ are the 4-momenta of the incoming dilaton and the outgoing photons, M is the reduced Feynman matrix element, $\epsilon^\mu(k,\lambda)$ and $\epsilon'^\mu(k',\lambda')$ are the transverse polarization vectors of photons. It is simple to calculate the matrix element in the center of momentum (COM) frame where $\vec{k}' = -\vec{k}$,

$$\begin{aligned} &(k_\mu\epsilon_\nu(k,\lambda) - k_\nu\epsilon_\mu(k,\lambda))(k'^\mu\epsilon'^\nu(k',\lambda') - k'^\nu\epsilon'^\mu(k',\lambda')) \\ &\quad = 2(k_\mu k'^\mu)(\epsilon_\nu(k,\lambda)\epsilon'^\nu(k',\lambda')), \\ &k_\mu k'^\mu = -2|\vec{k}|^2, \qquad \sum_{\lambda,\lambda'}|\epsilon_\nu(k,\lambda)\epsilon'^\nu(k',\lambda')|^2 = 2, \end{aligned} \tag{10}$$

and we get the following decay rate:

$$\begin{aligned} \Gamma_{\sigma\to\gamma\gamma} &= \frac{\alpha_1^2}{2\pi^2\mu}\int d^3\vec{k}d^3\vec{k}'|\vec{k}|^2\delta^{(4)}(k^\mu + k'^\mu - p^\mu) \\ &= \frac{\alpha_1^2}{2\pi^2\mu}\int d^3\vec{k}d^3\vec{k}'|\vec{k}|^2\delta(k^0 + k'^0 - \mu)\delta^{(3)}(\vec{k} + \vec{k}') \\ &= \frac{\alpha_1^2}{2\pi^2\mu}\int d^3\vec{k}|\vec{k}|^2\delta(2k^0 - \mu)(\text{COM frame}) \\ &= \frac{\alpha_1^2}{\pi\mu}\int d|\vec{k}||\vec{k}|^4\delta(|\vec{k}| - \mu/2) = \frac{\alpha_1^2\mu^3}{16\pi}. \end{aligned} \tag{11}$$

With this we get the following lifetime of the dilaton

$$\tau_{\sigma\to\gamma\gamma} = \frac{1}{\Gamma_{\sigma\to\gamma\gamma}} = \frac{16m_p^2}{g_1^2\mu^3}. \tag{12}$$

Notice that when $\mu \simeq m_p$, the dilaton (with $g_1 \simeq 1$) has a very short lifetime.

Now consider the dilaton-fermion interaction, and let $\alpha_2 = g_2\sqrt{16\pi G}m$ be the dimensionless coupling constant. The differential decay rate of dilaton to fermion and anti-fermion pair at tree level is written as

$$d\Gamma_{\sigma\to\bar\psi\psi} = \frac{1}{2p^0}\sum_{s,s'=\pm(1/2)}(2\pi)^4\delta^{(4)}(p^\mu - k^\mu - k'^\mu) \times |M|^2\frac{d^3\vec k}{(2\pi)^3 2k^0}\frac{d^3\vec k'}{(2\pi)^3 2k'^0},$$
$$M = -i\alpha_2\bar u(k,s)v(k',s'), \tag{13}$$

where p^μ and k^μ, k'^μ are the 4-momenta of the incoming dilaton and the outgoing fermion-antifermion pair, and s, s' are the fermion spin indices. Using the well-known sum rule [23],

$$\sum_{s,s'=\pm(1/2)}|\bar u(k,s)v(k',s')|^2 = 4(-k_\mu k'^\mu - m^2), \tag{14}$$

we have the following decay rate:

$$\begin{aligned}\Gamma_{\sigma\to\bar\psi\psi} &= \frac{\alpha_2^2}{8\pi^2 p^0}\int\frac{d^3\vec k}{k^0}\frac{d^3\vec k'}{k'^0}(-k_\mu k'^\mu - m^2)\delta^{(4)}(k^\mu + k'^\mu - p^\mu)\\ &= \frac{\alpha_2^2}{8\pi^2 p^0}\times\frac{\mu^2-4m^2}{2}\int\frac{d^3\vec k}{k^0}\frac{d^3\vec k'}{k'^0}\delta^{(3)}(\vec k + \vec k' - \vec p)\delta(k^0 + k'^0 - p^0)\\ &= \frac{\alpha_2^2}{8\pi^2\mu}\times\frac{\mu^2-4m^2}{2}\int\frac{d^3\vec k}{(k^0)^2}\delta(2k^0-\mu)(\text{COM frame})\\ &= \frac{\alpha_2^2}{2\pi\mu}\times\frac{\mu^2-4m^2}{2}\int_{k^0=\mu/2}d|\vec k|\frac{|\vec k|^4}{(k^0)^2}\delta(2\sqrt{m^2+|\vec k|^2}-\mu) = \frac{\alpha_2^2\mu}{8\pi}\times\left[1-\left(\frac{2m}{\mu}\right)^2\right]^{3/2}.\end{aligned} \tag{15}$$

So we have the following lifetime of the dilaton:

$$\tau_{\sigma\to\bar\psi\psi} = \frac{1}{\Gamma_{\sigma\to\bar\psi\psi}} = \frac{m_p^2}{2g_2^2m^2\mu}\left[1-\left(\frac{2m}{\mu}\right)^2\right]^{-3/2}. \tag{16}$$

Notice that this becomes comparable to (12) only when $m \sim 0.32\times\mu$, so that the two-photon decay becomes the dominant decay of a dilaton in general.

The dilaton number density n after the decoupling is given by the well-known equation [3]

$$\frac{d(nR^3)}{dt} = -\frac{1}{\tau}(nR^3), \qquad \frac{dn}{dt} + 3Hn = -\frac{1}{\tau}n, \tag{17}$$

where τ is the total lifetime, R is the scale factor of the Friedmann-Robertson-Walker metric, and H is the Hubble parameter. From this we have the familiar expression

$$n(t) = n_D\left(\frac{R_D}{R}\right)^3\exp\left(\frac{-t}{\tau}\right), \tag{18}$$

where the subscript D denotes the decoupling time. Note that the factor $1/R^3$ represents the dilution of the dilaton due to Hubble expansion. To find the present dilaton number density notice that in the highly relativistic regime (i.e., when $T \gg \mu$), the particle number density is given by [3]

$$\begin{aligned} n_b &= \frac{\zeta(3)}{\pi^2}gT^3 \quad (\text{for a boson}),\\ n_f &= \frac{3}{4}\frac{\zeta(3)}{\pi^2}gT^3 \quad (\text{for a fermion}),\end{aligned} \tag{19}$$

where g is now the internal degrees of freedom (not the coupling constant) of the relevant particle and $\zeta(x)$ is the Riemann's zeta function. So, at the time of dilaton decoupling, the dilaton number density n_D is given by

$$n_D = \frac{\zeta(3)}{\pi^2}T_D^3 \simeq \frac{1.202}{\pi^2}T_D^3. \tag{20}$$

On the other hand, the total entropy density s of the Universe is given by [3]

$$s = \frac{2\pi^2}{45}g_*T^3, \qquad g_* = \sum_{\text{bosons}}g_i\left(\frac{T_i}{T}\right)^3 + \frac{7}{8}\sum_{\text{fermions}}g_i\left(\frac{T_i}{T}\right)^3, \tag{21}$$

where g_i and T_i are the internal degrees of freedom and the thermal equilibrium temperature of the i-th particle, and T is the thermal temperature of the photon. At present we have $g_{*0} \simeq 3.91$ (with a photon and three types of light neutrinos), but at the Planck time we have $g_* \simeq 106.75$ according to the standard model [3]. Now, the total entropy conservation of the Universe in the comoving volume tells that $g_{*D}T_D^3R_D^3 = g_{*0}T_0^3R_0^3$. From this we get (with $T_0 \simeq$ 2.73 K) the present dilaton number density $n(t_0)$,

$$\begin{aligned} n(t_0) &= n_D\left(\frac{R_D}{R_0}\right)^3\exp\left(\frac{-t_0}{\tau}\right) = \frac{\zeta(3)}{\pi^2}T_D^3\left(\frac{R_D}{R_0}\right)^3\exp\left(\frac{-t_0}{\tau}\right)\\ &= \frac{\zeta(3)}{\pi^2}\frac{g_{*0}}{g_{*D}}T_0^3\exp\left(\frac{-t_0}{\tau}\right) \simeq 7.5\exp\left(\frac{-t_0}{\tau}\right)\ \text{cm}^{-3}.\end{aligned} \tag{22}$$

Note that the coefficient 7.5 cm^{-3} would be the present dilaton number density if the dilaton had not been decaying at all, which is half the present number density of the massless graviton.

IV. DILATON AS A DARK MATTER CANDIDATE

The above analysis implies that the dilaton with a proper mass can easily survive to present time, and could become the dark matter of the Universe. Assuming this is the case, we can estimate the mass of the dilaton. It has been argued that there are two mass ranges of the relic dilaton, $\mu_1 \simeq$ 500 eV and $\mu_2 \simeq 270$ MeV, in which the relic dilaton could be the dominant matter of the Universe [14]. This is because the dilaton with mass larger than μ_2 does not survive long enough to become the dominant matter of the Universe, and the dilaton with mass smaller than μ_1 survives but fails to be dominant due to its low mass. The dilaton with mass in between cannot be seriously considered because it would overclose the Universe. In this section we refine the above result.

According to recent cosmological observations, the dark matter occupies about 23% of the critical density $\rho_c = 3H_0^2/(8\pi G) \simeq 10.5h^2$ keV cm^{-3}, where h is the dimensionless Hubble parameter in units of 100 km sec^{-1} Mpc^{-1}. On the other hand, the "dark energy" characterized by the cosmological constant is believed to occupy about 70% of the total energy of the Universe [24]. So for the dilaton to be the dark matter of the Universe we must have the following requirement [14]:

$$\rho(\mu) = \mu \times 7.5 \exp\left(\frac{-t_0}{\tau(\mu)}\right) \text{cm}^{-3} = 0.23 \times \frac{3H_0^2}{8\pi G} \simeq 0.23 \times 10.5h^2 \ (\text{keV cm})^{-3}, \tag{23}$$

where $\rho(\mu)$ is the dilaton mass density. At the same time, the energy density $\tilde{\rho}(\mu)$ of the daughter particles (photons and light fermions) coming from the dilaton decay should be negligible compared to the critical density. This gives the second requirement

$$\tilde{\rho}(\mu) \ll \rho_c. \tag{24}$$

To find the dilaton mass which satisfies these constraints, we have to know the coupling constants g_1 and g_2. In Kaluza-Klein unification they are given by [11]

$$g_1 = \sqrt{\frac{n+2}{n}}, \qquad g_2 = \sqrt{\frac{n}{n+2}}. \tag{25}$$

But in the following we will leave them as free parameters, although our favorite values are $g_1 \simeq g_2 \simeq 1$. Now, with $t_0 = 1.37 \times 10^{10}$ yr $= 4.33 \times 10^{17}$ sec and $h \simeq 0.7$, we obtain the numerical solutions of the first constraint (23) shown in Table I. As we see in the table, it has two solutions for the dilaton mass and lifetime for given coupling constants. We denote the smaller one by μ_1 and τ_1 and the larger one by μ_2 and τ_2 in the table. In our numerical calculations, the decay channels we considered are $\gamma\gamma$, $\nu\bar{\nu}$, e^+e^-, $\mu^+\mu^-$ processes. So when $g_1 \simeq g_2 \gtrsim 5 \times 10^{-2}$, our calculations are exact. But when $g_1 \simeq g_2 \lesssim 10^{-2}$, the dilaton has larger mass and can decay into other heavier particles like $\tau^+\tau^-$. But even in the latter case, the two-photon decay probability is far greater than the fermion-antifermion decay probability except when $m \simeq 0.32 \times \mu$ (in which case we have $\Gamma_{\sigma\to\bar{\psi}\psi} \simeq 1.49 \times \Gamma_{\sigma\to\gamma\gamma}$) as we have remarked, and the error in evaluating the dilaton mass in the latter case is at most 20% or so.

TABLE I. The coupling constants versus dilaton mass and lifetime, where we have assumed $g_1 \simeq g_2$. Here the smaller mass is denoted by μ_1 and larger mass is denoted by μ_2, and τ_1 and τ_2 are the lifetime of μ_1 and μ_2.

$g_1 \simeq g_2$	μ_1	τ_1 (sec)	μ_2	τ_2 (sec)
10	160 eV	3.84×10^{33}	75.6 MeV	3.62×10^{16}
5	160 eV	1.53×10^{34}	121 MeV	3.49×10^{16}
1	160 eV	3.84×10^{35}	276 MeV	3.29×10^{16}
0.5	160 eV	1.53×10^{36}	445 MeV	3.19×10^{16}
0.1	160 eV	3.84×10^{37}	1.68 GeV	2.94×10^{16}
0.05	160 eV	1.53×10^{38}	2.76 GeV	2.85×10^{16}
10^{-2}	160 eV	3.84×10^{39}	8.37 GeV	2.66×10^{16}
10^{-3}	160 eV	3.84×10^{41}	40.0 GeV	2.45×10^{16}
10^{-4}	160 eV	3.84×10^{43}	191 GeV	2.25×10^{16}

Note that the smaller mass μ_1 is insensitive to the values of the coupling constants, while the larger mass μ_2 increases as the coupling constants decrease. On the other hand, the lifetime τ_1 is sensitive to the values of the coupling constants, while the lifetime τ_2 remains of the same order for all values of the coupling constants.

With $g_1 \simeq g_2 \simeq 1$ we can plot the dilaton density $\rho(\mu)$ against its mass μ, which is shown in Fig. 1. Note that $\rho(\mu)$ (denoted by ρ_d in the figure) starts from zero and approaches to the maximum value of about $1.08 \times 10^5 \rho_c$ at $\mu \simeq 103$ MeV, and again decreases to zero when μ goes to infinity. More importantly, $\rho(\mu)$ exceeds the dark matter density in the range 160 eV $< \mu <$ 276 MeV. This means that when $\mu <$ 160 eV or $\mu >$ 276 MeV, the dilaton undercloses the Universe, but when 160 eV $< \mu <$ 276 MeV it overcloses the Universe. This immediately rules out the dilaton with mass range 160 eV $< \mu <$ 276 MeV. Moreover, we have two possible mass ranges which are of particular interest, $\mu_1 \simeq$ 160 eV with lifetime

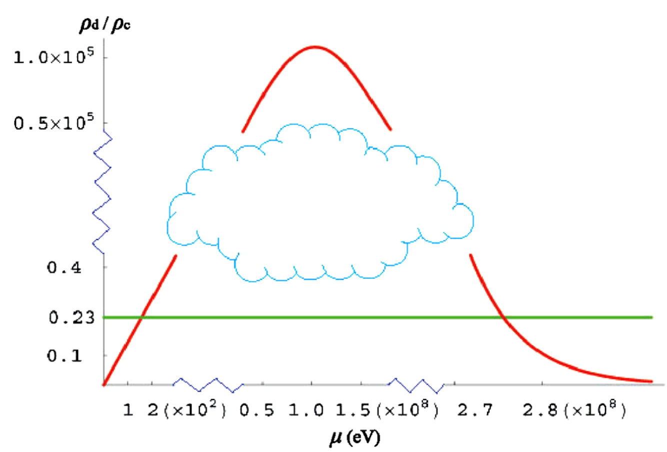

FIG. 1 (color online). The dilaton mass density $\rho(\mu)$ denoted by ρ_d (with respect to the critical density ρ_c) versus the dilaton mass μ, obtained with $g_1 \simeq g_2 \simeq 1$.

$\tau_1 \simeq 3.84 \times 10^{35}$ sec and $\mu_2 \simeq 276$ MeV with lifetime $\tau_2 \simeq 3.29 \times 10^{16}$ sec, which makes the dilaton the dominant matter of the Universe.

So far we have assumed that the dilaton occupies all of the dark matter, about 23% of the critical density ρ_c. But even when we loosen this constraint, we get a similar result. Varying the dilaton mass density, we obtain the result shown in Table II with $g_1 \simeq g_2 \simeq 1$. The result shows that μ_1 and τ_1 are sensitive to the change of dilaton mass density, but μ_2 and τ_2 are not much affected by that. Moreover, the generic feature of the dilaton physics remains the same.

Now, we have to make sure that the dilaton mass should also satisfy the second constraint (24). To check this, notice that the 160 eV dilaton is almost stable because $\tau_1 \simeq 8.1 \times 10^{17} t_0$. So the energy density of the daughter particles must be negligible compared to the energy density of the dilaton. This means that this dilaton can easily satisfy the second constraint (24). On the other hand, most of the 276 MeV dilaton should have decayed by now, because $\tau_2 \simeq 6.9 \times 10^{-2} t_0$. Indeed only 0.5×10^{-6} of the heavy dilaton which survives now make up the present dark matter, so that the energy density of the daughter particles becomes much bigger than that of the surviving dilaton. This means that the daughter particles from the heavy dilaton overclose the Universe, and thus cannot satisfy the second constraint. This effectively rules out the heavy dilaton. So only the 160 eV dilaton can be accepted as the dark matter candidate.

The above discussion tells that there are two constraints on the dilaton mass, the experimental constraint from the fifth force and the theoretical constraint from cosmology. Clearly these constraints restrict the allowed range of the dilaton mass. Putting the two constraints together we obtain Fig. 2, which shows the allowed range of the dilaton mass versus the relative fine-structure constant $\beta = \alpha_5/\alpha_g$ of the fifth force. Notice that the cosmological constraint tells that the range of the fifth force can not be smaller than 10^{-9} m.

At this point one might object to our calculations, because our calculation neglected the time dilatation effect of the dilaton motion. Indeed the dilaton lifetimes (12) and (16) calculated in the COM frame do not accurately describe the lifetimes in the comoving frame. This is because

TABLE II. The dilaton mass and lifetime versus the ratio ρ_d/ρ_c. Here the coupling constants g_1 and g_2 are set to be 1.

ρ_d/ρ_c	μ_1	τ_1 (sec)	μ_2	τ_2 (sec)
100%	686 eV	4.85×10^{33}	270 MeV	3.65×10^{16}
23%	160 eV	3.84×10^{35}	276 MeV	3.29×10^{16}
10%	68.6 eV	4.85×10^{36}	280 MeV	3.12×10^{16}
4%	27.4 eV	7.58×10^{37}	284 MeV	2.93×10^{16}
1%	6.86 eV	4.85×10^{39}	291 MeV	2.68×10^{16}
0.5%	3.43 eV	3.88×10^{40}	294 MeV	2.59×10^{16}

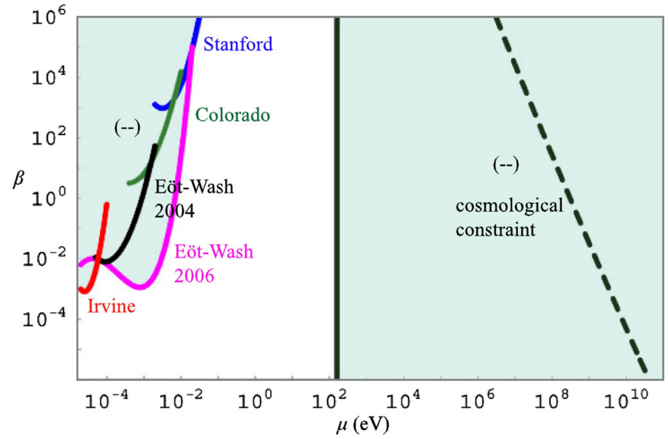

FIG. 2 (color online). The allowed mass μ of dilaton (unshaded region), where we leave $\beta = \alpha_5/\alpha_g$ arbitrary. The shaded region marked by (−) is the excluded region, and the dotted line represents the mass of the heavy dilaton whose daughter particles overclose the Universe.

after decoupling the dilatons in general are not at rest in the comoving frame, since they move with energy comparable to Planck mass when they decouple. So in the early Universe one must take into account the time dilatation effect of the dilaton motion to find the correct dilaton lifetimes (especially when the dilaton is light). Notice, however, as the Universe expands the dilatons will slow down quickly as their velocities redshift. This implies that the time dilatation effect quickly becomes negligible. To see this, notice that the time $t_{\rm NR}$ when the dilaton becomes nonrelativistic is given by [3]

$$t_{\rm NR} \simeq 1.2 \times 10^7 \left(\frac{\rm keV}{\mu}\right)^2 \left(\frac{T_{\rm NR}}{T_\gamma}\right)^2 \ {\rm sec}$$
$$= 1.2 \times 10^7 \left(\frac{\rm keV}{\mu}\right)^2 \left(\frac{g_{*\rm NR}}{g_{*D}}\right)^{2/3} \ {\rm sec}, \qquad (26)$$
$$\frac{t_{\rm EQ}}{t_{\rm NR}} = \left[\frac{\mu/{\rm eV}}{17(\Omega_0 h^2)(T_{NR}/T_\gamma)}\right]^2,$$

where $T_{\rm NR} \simeq \mu/3 \simeq 53.3$ eV is the temprature when the dilaton becomes nonrelativistic, T_γ and $g_{*\rm NR}$ are the temperature of the photon and the total relativistic degrees of freedom at $t_{\rm NR}$. So the dilaton in this case becomes nonrelativistic at $t_{\rm NR} \simeq 5.17 \times 10^7$ sec, well before the matter-radiation equilibrium era $t_{\rm EQ} \simeq 4.36 \times 10^{10} (\Omega_0 h^2)^{-2}$ sec $\simeq 1.82 \times 10^{11}$ sec. This justifies the above calculations which neglect the time dilatation effect.

The dark matter dilaton has the following characteristics. With mass $\mu \simeq 160$ eV, the possible decay channels of the dilaton are the $\gamma\gamma$ and three $\nu\bar{\nu}$ processes. But with lifetime $\tau \simeq 8.1 \times 10^{17} t_0$ this dilaton is almost stable. To see whether this can be hot or cold dark matter, we should estimate the free-streaming distance $\lambda_{\rm FS}$ of the dilaton, which is given by [3,14],

$$\lambda_{\rm FS} \simeq 0.2\ {\rm Mpc}\left(\frac{\rm keV}{\mu}\right)\left(\frac{T_{\rm NR}}{T_\gamma}\right)\left[\ln\left(\frac{t_{\rm EQ}}{t_{\rm NR}}\right)+2\right]$$
$$= 0.2\ {\rm Mpc}\left(\frac{\rm keV}{\mu}\right)\left(\frac{g_{*\rm NR}}{g_{*D}}\right)^{1/3}\left[\ln\left(\frac{t_{\rm EQ}}{t_{\rm NR}}\right)+2\right]. \quad (27)$$

Now, with $g_{*D} \simeq 106.75$ and $g_{*\rm NR} \simeq 3.91$ we get $\lambda_{\rm FS} \simeq 4.2$ Mpc. So the free-streaming distance of the dilaton becomes less than the typical structure formation scale $\lambda_{\rm EQ} \simeq 13(\Omega_0 h^2)^{-1} \simeq 18.6$ Mpc. This qualifies the 160 keV dilaton as a cold dark matter.

One could easily find that the dilaton with mass $\mu \simeq 276$ MeV has a free-streaming distance $\lambda_{\rm FS} \simeq 1.55 \times 10^{-5}$ Mpc, which is much shorter than that of the light dilaton [14]. This would have made the heavy dilaton an excellent cold dark matter. But of course, this dilaton is not acceptable as a dark matter because the daughter particles overclose the Universe.

V. DILATON DETECTION EXPERIMENT

So far, we have tried to estimate the dilaton mass based on the conjecture that the dilaton is the dark matter of the Universe. Now an important question is how to detect the relic dilaton and confirm such conjecture. Clearly one could try to establish the existence of the dilaton measuring the dilatonic fifth force [12,17]. But the above analysis implies that, if indeed the dilaton is the dark matter of the Universe, its detection by the conventional fifth-force experiments would be difficult because such dilaton generates an extremely short ranged fifth force. Fortunately new types of fifth-force experiments known as the Casimir regime experiments are under way at present [25]. It would be very interesting to see whether these experiments can detect such short range dilatonic fifth force.

In this section we propose a totally different type of experiment based on two-photon decay of the relic dilaton. Of course, one might try to detect the two-photon decay of the relic dilaton directly, searching for the monoenergetic x-ray signals from the sky [14]. Here we propose another type of experiment, a Sikivie-type experiment which detects the dilaton conversion to one photon in strong electromagnetic background. In this type of experiment the dilaton conversion rate can be greatly enhanced by two factors, first by the strong electromagnetic background and secondly by the large dilaton density of halo. It is clear that the conversion rate is enhanced by the strong background, because the conversion amplitude is proportional to the background field strength. Moreover, just as in the axion detection experiment, we can assume that our Galaxy halo is made of the relic dilaton if the dark matter is the dilaton. In this case the conversion rate will be enhanced by a factor 10^5, because the average energy density of the relic dilaton $0.23 \times 10.5h^2\ {\rm keV\,cm^{-3}} \simeq 1.18\ {\rm keV\,cm^{-3}}$ in the present Universe can be replaced by the Galaxy halo density $\rho_{\rm halo} \simeq 0.3\ {\rm GeV\,cm^{-3}}$ [10]. In the following we estimate the power of dilaton conversion to one photon in strong magnetic background, assuming that our Galaxy halo is made of dilaton.

Consider a rectangular cavity with three edges L_x, L_y, L_z and volume $V = L_x L_y L_z$ made of a perfect conductor, which has a strong magnetic background $\vec{B}_{\rm ext}(\vec{x}) = B_{\rm ext}(\vec{x})\hat{z}$ in the z direction inside, and consider the halo dilaton conversion in the cavity described by the interaction

$$\mathcal{L}_{\sigma\gamma\gamma} = -\alpha_1 \hat{\sigma} F_{\mu\nu}^2 = 2\alpha_1 \hat{\sigma}(\vec{E}^2 - \vec{B}^2),$$
$$\alpha_1 = \frac{1}{4} g_1 \sqrt{16\pi G}. \quad (28)$$

In this case the induced photon is described by TE mode (the magnetic wave) $\vec{B}(\vec{x}) = B(\vec{x})\hat{z}$, and the differential cross section $d\sigma$ of the dilaton conversion in the cavity is given by

$$d\sigma_{\vec{k},\lambda} = 2\pi\delta(k^0 - p^0)\frac{1}{2p^0 v}\frac{d^3\vec{k}}{(2\pi)^3 2k^0}|M|^2,$$
$$M = -i4\alpha_1 \vec{B}(\vec{x}) \cdot \vec{B}_{\rm ext}(\vec{q})$$
$$= -i4\alpha_1 k^0(\hat{\epsilon}(\vec{k},\lambda) \times \hat{k}) \cdot \vec{B}_{\rm ext}(\vec{q}), \quad (29)$$

where p^μ and k^μ are the 4-momenta of the dilaton and the induced photon, M is the Feynman reduced matrix element, $\hat{\epsilon}(\vec{k}, \lambda = \pm 1)$ and $\hat{k}$ are the 3-dimensional photon polarization vector and the unit vector in the direction of the photon momentum $\vec{k}$, $\vec{q} = \vec{k} - \vec{p}$ is the spatial momentum transfer, and $\vec{B}_{\rm ext}(\vec{q})$ is the Fourier transform of $\vec{B}_{\rm ext}(\vec{x})$. Note that in the classical background only energy is conserved, and the $\delta(k^0 - p^0)$ term represents this fact. Then the total cross section σ in the continuum limit is given as follows:

$$\sigma = \sum_{\lambda=\pm 1} 2\pi \int d^3\vec{k}\delta(k^0 - p^0)\frac{1}{2p^0 v}\frac{1}{(2\pi)^3 2k^0}|M|^2$$
$$= \frac{\alpha_1^2}{\pi^2 v}\sum_{\lambda=\pm 1}\int d^3\vec{k}\delta(k^0 - p^0)|\hat{B}(\vec{k},\lambda)\cdot\vec{B}_{\rm ext}(\vec{q})|^2, \quad (30)$$

where $\hat{B}(k,\lambda) = \hat{\epsilon}(\vec{k},\lambda) \times \hat{k}$ is the unit vector in the direction of the induced magnetic field $\vec{B}$.

Let the wave vector of the photon be $\vec{k} = (n_x\pi/L_x, n_y\pi/L_y, n_z\pi/L_z)$, where (n_x, n_y, n_z) are arbitrary integers. For TE modes, the boundary condition

$$B(z = 0, L_z) = 0, \qquad \frac{\partial B}{\partial x}(x = 0, L_x) = 0,$$
$$\frac{\partial B}{\partial y}(y = 0, L_y) = 0 \quad (31)$$

requires the induced magnetic field to assume the form

$$B = A\cos\left(\frac{n_x \pi x}{L_x}\right)\cos\left(\frac{n_y \pi y}{L_y}\right)\sin\left(\frac{n_z \pi z}{L_z}\right), \quad (32)$$

where A is a normalization constant. Notice that n_x and n_y cannot be zero simultaneously, and n_z must be a nonzero integer [26].

Now, we have

$$\sum_{\lambda=\pm 1} |\hat{B}(\vec{k},\lambda)\cdot \vec{B}_{\rm ext}(\vec{q})|^2 = |\hat{k}\times B_{\rm ext}(\vec{q})\hat{z}|^2 = \frac{k_x^2+k_y^2}{(k^0)^2}|B_{\rm ext}(\vec{q})|^2, \tag{33}$$

$$\vec{B}_{\rm ext}(\vec{q}) = \int_V \exp(i\vec{q}\cdot\vec{x})\vec{B}_{\rm ext}(\vec{x})d^3\vec{x} = \int_V \exp(i\vec{q}\cdot\vec{x})B_{\rm ext}(\vec{x})\hat{z}d^3\vec{x} = B_{\rm ext}(\vec{q})\hat{z},$$

so that, changing the integration into summation as follows,

$$d^3\vec{k} = dk_x dk_y dk_z = \frac{\pi}{L_x}\frac{\pi}{L_y}\frac{\pi}{L_z}dn_x dn_y dn_z = \frac{\pi^3}{V}, \tag{34}$$

we get the following cross section:

$$\sigma = \sum_{\vec{k}} \frac{\pi\alpha^2}{Vv}\frac{(k_x^2+k_y^2)}{(k^0)^2}\delta(k^0-p^0)|B_{\rm ext}(\vec{q})|^2. \tag{35}$$

To proceed, we let

$$\vec{B}_{\rm ext}(\vec{x}) = B_0\cos(Qx)\hat{z} \tag{36}$$

and approximate $\vec{q} = (\vec{k}-\vec{p}) \sim \vec{k}$ since the incoming halo dilaton is highly nonrelativistic (with $v \sim 10^{-3}c$) [10]. In this case we have

$$\begin{aligned}|B_{\rm ext}(\vec{q})|^2 &= |\int_V d^3\vec{x}\exp(i\vec{q}\cdot\vec{x})B_0\cos(Qx)|^2 \\ &= B_0^2L_x^2L_y^2L_z^2\frac{\sin^2(\frac{k_yL_y}{2})}{(\frac{k_yL_y}{2})^2}\frac{\sin^2(\frac{k_zL_z}{2})}{(\frac{k_zL_z}{2})^2}\frac{1}{4} \\ &\times\left(\left[\frac{\sin(k_x-Q)L_x}{(k_x-Q)L_x}+\frac{\sin(k_x+Q)L_x}{(k_x+Q)L_x}\right]^2 + \left[\frac{1-\cos(k_x-Q)L_x}{(k_x-Q)L_x}+\frac{1-\cos(k_x+Q)L_x}{(k_x+Q)L_x}\right]^2\right).\end{aligned} \tag{37}$$

As we can see, $|B_{\rm ext}(\vec{q})|^2$ has the maximum value

$$|B_{\rm ext}(\vec{q})|^2_{\rm max} = \frac{B_0^2L_x^2L_y^2L_z^2}{\pi^2}, \tag{38}$$

when

$$k_x = \pm Q, \qquad k_zL_z = \pm\pi, \qquad k_yL_y = 0. \tag{39}$$

Note that $|B_{\rm ext}(\vec{q})|^2_{\rm max}$ would be highly suppressed without the external sinusoidal background, which is why we choose the sinusoidal external magnetic field (36).

We are interested in the dilaton with the mass range $\mu \gtrsim 0.1$ keV whose Compton wavelength is of order smaller than 2×10^{-7} cm. Considering the typical detector length scale L_x, L_y, $L_z \simeq 1$ m, and $(k^0)^2 = k_x^2+k_y^2+k_z^2 \simeq \mu^2$, we have $k_x \simeq k^0 \gg \max(k_y,k_z)$ since μL_x, $\mu L_y \gg 1$ in the resonance case. Thus we can use the following approximation:

$$k^0dk^0 = k_xdk_x + k_ydk_y + k_zdk_z \simeq k_xdk_x \Rightarrow dk^0 = \frac{k_x}{k^0}dk_x \simeq dk_x. \tag{40}$$

On the other hand, the number of additional modes due to the differential spread dk^0 around $k^0 = p^0$ is

$$dn_x = \frac{L_x}{\pi}dk_x \simeq \frac{L_x}{\pi}dk^0, \qquad \delta(k^0-p^0)dn_x = \frac{L_x}{\pi}. \tag{41}$$

Combining these relations, we finally obtain

$$\begin{aligned}\sigma &= \sum_{\vec{k}}\frac{\pi\alpha_1^2}{Vv}\frac{(k_x^2+k_y^2)}{(k^0)^2}\delta(k^0-p^0)|B_{\rm ext}(\vec{q})|^2 \\ &\simeq \frac{4\pi\alpha_1^2}{Vv}\frac{(k^0)^2}{(k^0)^2}\delta(k^0-p^0)dn_x\times|B_{\rm ext}(\vec{q})|^2_{\rm max} \\ &= \frac{16\pi\alpha_1^2}{Vv}\frac{L_x}{\pi}\times|B_{\rm ext}(\vec{q})|^2_{\rm max} \\ &= \frac{4\alpha_1^2}{\pi^2Vv}B_0^2L_x^2L_y^2L_z^2L_x = \frac{4\alpha_1^2}{\pi^2 v}B_0^2VL_x,\end{aligned} \tag{42}$$

and the following detection power P:

$$P = \mu n_d v\sigma = \left(\frac{4\alpha_1^2}{\pi^2}\right)\rho_dB_0^2L_xV, \tag{43}$$

where n_d is the dilaton number density and ρ_d is the dilaton energy density. Notice that the detection power depends on the energy density, not the mass, of dilaton.

This agrees with that of the axion detection power except for the numerical factor of order unity which comes from the different axion-photon coupling constant. In the case of the axion, the axion-photon interaction Lagrangian and axion detection power are given as follows [1]:

$$\begin{aligned}\mathcal{L}_{a\gamma\gamma} &= -\alpha_\gamma aF_{\mu\nu}\tilde{F}^{\mu\nu} = 4\alpha_\gamma a\vec{E}\cdot\vec{B}, \\ P_a &= 2\alpha_\gamma^2\rho_aB_0^2L_xV.\end{aligned} \tag{44}$$

As we have mentioned there are two types of axion, the popular axion from strong interaction and the gravitational axion proposed as a pseudoscalar graviton [7,9]. The difference is that for the popular axion the coupling constant α_γ is given by $g_\gamma\alpha/4\pi f_a$, where g_γ is a model-dependent dimensionless coupling constant of order one, α is the electromagnetic fine-structure constant, and f_a is the $U_{\rm PQ}(1)$ symmetry breaking scale. But for the gravitational axion α_γ is similar to our α_1 because this axion is the pseudoscalar partner of the dilaton. Other than this they are virtually identical.

We can compare the axion detection power with the dilaton detection power. Consider the popular axion first.

Since f_a is related to the axion mass m_a by $m_a \simeq 6$ eV $\times 10^6$ GeV$/f_a$, and the educated guess of the axion mass is around 10^{-6} eV or so, we have $f_a \simeq 6 \times 10^{12}$ GeV [1,10]. So we have (with $g_1/g_\gamma \simeq 1$)

$$\frac{P}{P_a} \simeq 1.9 \times 10^6 \times \left(\frac{g_1}{g_\gamma}\right)^2 \left(\frac{f_a}{m_p}\right)^2 \simeq 4.7 \times 10^{-7}. \quad (45)$$

This is a small number. But the reason why this is so small is because the dilaton-photon coupling α_1 is much smaller than the dilaton-axion coupling α_γ, due to the fact that f_a is much bigger than the Planck mass. Indeed, with $\rho_d \simeq \rho_{\rm halo} \simeq 0.5 \times 10^{-24}$ g/cm$^3 \simeq 0.3$ GeV/cm^3 and $B_0 = 10$ T, $L_x = L_y = L_z \simeq 1$ m, we get the dilaton detection power $P \simeq 1.42 \times 10^{-31}$ W with $g_1 \simeq 1$. This is 10^{-5} times smaller than the axion detection power in current experiments [1,10]. But there are two things worth keeping in mind here. First, for the gravitation axion, we expect $\alpha_1 \simeq \alpha_\gamma$ so that P_a becomes as small as P. So in this case the axion detection power becomes smaller than the popular axion detection power, and becomes comparable to the dilaton detection power. Secondly, a strong dilaton coupling to matter fields (i.e., a large α_1 or equivalently a large g_1) will certainly enhance the dilaton detection power. And this possibility has not been ruled out by experiment yet.

Of course, a strong dilaton coupling in principle could change the dilaton life-time (and the density of dilaton at present Universe), and thus change the outcome of our results. But our numerical calculation shown in Fig. 2 suggests that changing the coupling (β up to 10^6 in the figure) does not change our results very much, unless of course β becomes extremely large. This assures that our results, in particular (43), remain valid for a large range of α_1.

Notice that, due to the pseudoscalar coupling, the axion produces TM modes (the electric wave) rather than TE modes. Another notable difference between the dilaton and the axion is that for the dilaton the photon polarization is perpendicular to the external magnetic field, whereas for the axion the photon polarization is parallel to the external magnetic field.

Now, a few remarks are in order. First, the above result holds when we have the resonance $Q \simeq \mu$. But it seems very difficult to make static magnetic field of wavelength of order $\mu^{-1} \lesssim 10^{-7}$ cm with the current technology. However we may be able to set up x-ray range electromagnetic waves with $\omega_{\rm ext} = Q \simeq \mu$. In that case, the only change needed is to replace $\delta(k^0 - p^0)$ by $\delta(k^0 - p^0 \pm p^0)$ in the above calculation, which will make the detection power P twice as big. Second, the dilaton detection power appears too small to be considered realistic at present. On the other hand, we notice that the relevant technologies are developing fast [10], so that it may be possible to detect the halo dilaton in the near future. Third, we have used the magnetic background in the above calculation. With an electric background the detection power would have been proportional to the electric field energy density. In terms of the field energy density, 1 T corresponds to 300 MV/m since $E = cB$ in the meter-kilogram-second unit system. But the strongest magnetic field and electric fields currently available are around 50 T and 40 MV/m [27], respectively. So at present a magnetic background can give us larger detection power. Moreover, in the air the electric breakdown happens when the electric field is about 3 MV/m. This is why we have used the magnetic background in our calculation. And this is why one hardly uses an electric background in particle creation or annihilation experiments in laboratories.

VI. DISCUSSION

An urgent issue in higher-dimensional unified theories is to have experimental verification of the theories. In the absence of any direct evidence of the extra space, it is very important to find a clear way to test the idea of the higher-dimensional unification. In this context the Kaluza-Klein dilaton plays a crucial role. All higher-dimensional unified theories, including the superstring, predict the existence of the dilaton. Moreover, as the scalar graviton it has unique and unmistakable features which can be tested experimentally. Most importantly, in the absence of any direct evidence of the extra dimension, it is perhaps the only way to verify the idea of the higher-dimensional unification [2,14]. This makes the experimental verifications of the dilaton very important.

A conceptually simple and straightforward experiment to detect the dilaton is the fifth-force experiment [12]. As the scalar graviton the dilaton couples to all matters, so that it creates the fifth force which modifies Einstein's gravity. So using the fifth-force experiments one may be able to verify the dilaton. The problem with this is that it is very difficult to detect the dilatonic fifth force, because the dilaton coupling to matter fields is very weak. On the other hand this weak coupling makes the dilaton lifetime very large, so that the dilaton can easily survive to the present Universe. This makes the dilaton an excellent candidate of dark matter as a GIMP [14]. Our analysis tells that there is practically only one mass range $\mu \simeq 160$ eV for which the dilaton can be the dark matter. This cosmological constraint of dilaton mass implies that detecting the dilaton by the conventional fifth-force experiments would be very difficult, because the dilatonic fifth force is too short ranged. This implies that we must consider a new type of experiments to detect the dilaton. As we have remarked, the experiments based on the Casimir effect could be useful to detect such short range fifth force [25].

In this paper we have discussed a totally different type of experiment to detect the dilaton based on the dilaton-photon conversion in strong magnetic background, and calculated the dilaton detection power. The downside of our result is that (if we treat the dilaton as a GIMP) the dilaton detection power is a little too small to be detected at present. The reason again is that the dilaton coupling to matter fields (in this case the electromagnetic field) is assumed to be very weak (i.e., gravitational). But the upside of this result is that it is very difficult to exclude the idea of the dilaton as the dark matter of the Universe by experiments. It can compete very well with other candidates of dark matter, axion and WIMPs. This makes the experimental detection of the dilaton important.

The small dilaton detection power in (43) should not be interpreted to discourage the cavity experiment we proposed here. On the contrary it should make such experiment more desirable. Of course when the dilaton is identified as a GIMP, the dilaton detection power does become very small. As we have pointed out, however, there is always the possibility that the dilaton has a stronger coupling to matter fields because there is no *a priori* reason why the dilaton coupling must be gravitational. Obviously a strong dilaton-matter coupling (i.e., a large g_1) would enhance the dilaton detection power in cavity experiments very much, and could easily make it larger than the axion detection power. And only experiments can tell how strong the dilaton couples to matter fields. This provides a strong motivation to do the above dilaton detection experiment. We hope that our result in this paper could help us to detect the dilaton.

It must be emphasized that our calculation of the dilaton mass and lifetime is an order estimate based on the linear and first-order approximation. First of all the dilaton in general has an exponential interaction to matter fields, so that (6) should be viewed only a linear approximation [11,14]. Secondly there are many scalar gravitons in higher-dimensional unified theory which can be identified as a GIMP, and the dilaton is one of many possible GIMPs. In general in $(4 + n)$-dimensional unified space there are $n(n + 1)/2$ scalar gravitons in the zero mode approximation, the dilaton and the unimodular metric of the n-dimensional extra space, which can be identified as GIMPs [2,11]. (And in string theory, one has similar moduli fields [6].) But here we have considered only the dilaton. Moreover, in higher-dimensional unified theory there are more particles (for example, the above mentioned GIMPs of the extra-dimensional metric) than what we have in the standard model, so that the numerical values of g_* in (22), (26), and (27) must be modified. For these reasons our estimates of the dilaton lifetime and mass should be viewed as an order estimate.

It has been argued that the dilaton (renamed the radion) can play a crucial role in resolving the hierarchy problem [20]. This is because the dilaton represents the volume of the extra space which determines the higher-dimensional gravitational constant in terms of the Newton's constant [2,11]. Now, we emphasize that our estimate of the dilaton mass could be used to determine the size of the extra space. The reason is because the dilaton mass is determined by the curvature of the extra space, which depends on the volume of the extra space [2,11]. This tells one that the dilaton as the dark matter can also play a crucial role in determining the size of the extra space. The details of this and related subjects will be discussed elsewhere [28].

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