Unparticle and Higgs boson as composites

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We propose a generic framework in which the Higgs and the unparticle are both composite. The underlying theories are four-dimensional, asymptotically-free, nonsupersymmetric gauge theories with fermionic matter. We sketch a possible unification of these two sectors at a much higher scale resembling extended technicolor models. By construction our extensions are natural, meaning that there is no hierarchy problem. The coupling of the unparticle sector to the composite Higgs emerges as a four-Fermi operator. The bilinear unparticle operator near the electroweak scale has scaling dimension in the range $1 < d_{\mathcal{U}} < 3$. We investigate, in various ways, the breaking of scale invariance induced by the electroweak scale resulting in an unparticle condensate. The latter acts as a natural infrared cutoff or hadronic scale. We give the low-energy effective theory valid near the electroweak scale. The unparticle-Higgs mixing is found to be suppressed within our framework.

It is an exciting possibility that new strong dynamics could be discovered at the LHC. The hope is fueled by the fact that some of the best motivated extensions of the standard model (SM) break the electroweak symmetry dynamically [1,2]. The new models, passing the precision electroweak tests, are summarized in [3]. It is then interesting to explore the possibility to accommodate the unparticle scenario [4] into a natural setting featuring four-dimensional strongly interacting dynamics.

Georgi's original idea is that at high energy there is an ultraviolet (UV) sector coupled to the SM through the exchange of messenger fields with a large mass scale $M_{\mathcal{U}}$. Below that scale two things happen consecutively. First, the messenger sector decouples, resulting in contact interactions between the SM and the unparticle sector. Second, the latter flows into a nonperturbative infrared (IR) fixed point at a scale $\Lambda_{\mathcal{U}} \ll M_{\mathcal{U}}$ hence exhibiting scale invariance;

$$\mathcal{L} \sim \mathcal{O}_{\mathrm{UV}}\mathcal{O}_{\mathrm{SM}} \rightarrow \mathcal{O}_{\mathcal{U}}\mathcal{O}_{\mathrm{SM}}. \qquad (1)$$

The UV unparticle operator is denoted by $\mathcal{O}_{\mathrm{UV}}$ and it posses integer dimension d_{UV}. When the IR fixed point is reached the operator $\mathcal{O}_{\mathrm{IR}} \equiv \mathcal{O}_{\mathcal{U}}$ acquires a noninteger scaling dimension $d_{\mathcal{U}}$ through dimensional transmutation

$$|\langle 0|\mathcal{O}_{\mathcal{U}}|P\rangle| \sim (\sqrt{P^2})^{d_{\mathcal{U}}-1}. \qquad (2)$$

This defines the matrix element up to a normalization factor. In the regime of exact scale invariance the spectrum of the operator $\mathcal{O}_{\mathcal{U}}$ is continuous, does not contain isolated particle excitations and might be regarded as one of the reasons for the name "unparticle." The unparticle propagator carries a *CP*-even phase[1] [6,7] for spacelike momentum. Effects were found to be most unconventional for noninteger scaling dimension $d_{\mathcal{U}}$, e.g. [4,6,8].

The coupling of the unparticle sector to the SM (1) breaks the scale invariance of the unparticle sector at a certain energy. Such a possibility was first investigated with naive dimensional analysis (NDA) in Ref. [9] via the Higgs-unparticle coupling of the form

$$\mathcal{L}^{\,\mathrm{eff}} \sim \mathcal{O}_{\mathcal{U}}|H|^2. \qquad (3)$$

The dynamical interplay of the unparticle and Higgs sector in connection with the interaction (3) has been studied in [10]. It was found, for instance, that the Higgs VEV induces an unparticle VEV, which turned out to be infrared (IR) divergent for their assumed range of scaling dimension and forced the authors to introduce various IR regulators [10,11].

In this work we elevate the unparticle scenario to a natural extension of the SM by proposing a generic framework in which the Higgs and the unparticle sectors are both *composites* of elementary fermions. We use four-dimensional, nonsupersymmetric asymptotically-free gauge theories with fermionic matter. This framework allows us to address, in principle, the dynamics beyond the use of scale invariance *per se*.

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[1]The resulting *CP* violation was found to be consistent with the *CPT* theorem [5].

The Higgs sector is replaced by a walking technicolor model (TC), whereas the unparticle one corresponds to a gauge theory developing a nonperturbative[2] IR fixed point (conformal phase).[3,4] By virtue of TC there is no hierarchy problem. We sketch a possible unification of the two sectors, embedding the two gauge theories in a higher gauge group. The model resembles the ones of extended technicolor and leads to a simple explanation of the inter-action between the Higgs and the unparticle sectors.

The paper is organized as follows. In Sec. I A we de-scribe the basic scenario. Thereafter we address the for-mation of the unparticle VEV in Sec. I B and identify the VEV as the natural IR cutoff in connection with the dy-namical (constituent) fermion mass. The comparison with the IR cutoff suggested by NDA is presented in Appendix B. In Sec. II we give some more details about the unified framework. The low-energy effective Lagrangian, which could also be taken as a starting point, is given in Sec. II A. The regularized unparticle propagator with IR and UV cutoff is discussed in Sec. II B. The normalization of the unparticle operator to our specific model is discussed in Appendix C. In a further section II C we discuss the mixing of the Higgs with the unparticle based upon the previously given effective Lagrangian. In Appendix D we comment on the proposed unparticle limit of the model presented in [28]. The paper ends with an outlook in Sec. III, where possible future directions of research in collider physics, lattice, and dark matter are discussed. For example, we put forward the idea of the *unbaryon* as a possible dark matter candidate.

I. THE HIGGS AND UNPARTICLE AS COMPOSITES

A. Scenario

Our building block is an extended $G_{T \times U} \equiv SU(N_T) \times SU(N_U)$ technicolor (TC) gauge theory. The matter content constitutes of techniquarks Q_f^a charged under the represen-tation R_T of the TC group $SU(N_T)$ and Dirac techni-unparticle fermions Ψ_s^A charged under the representation R_U of the unparticle group $SU(N_U)$, where $a/A = 1 \ldots \dim[R_{T/U}]$ and $f/s = 1 \ldots F/S$ denote gauge and

FIG. 1. Schematic scenario. The ordering of the energy scales $\Lambda_{\mathcal{U}}$ and $\Lambda_{\mathcal{T}}$ is not of any importance.

flavor indices, respectively. We will first describe the (walking) TC and (techni)unparticle sectors separately before addressing their common dynamical origin. A graphical illustration of the scenario is depicted in Fig. 1 as a guidance for the reader throughout this section.

In the TC sector the number of techniflavors, the matter representation and the number of colors are arranged in such a way that the dynamics is controlled by a near conformal (NC) IR fixed point.[5] In this case the gauge coupling reaches almost a fixed point around the scale $\Lambda_{\mathcal{T}} \gg M_W$, with M_W the mass of the electroweak gauge boson. The TC gauge coupling, at most, gently rises from this energy scale down to the electroweak one. The cou-pling is said to *walk*.[6] Around the electroweak scale the TC dynamics triggers the spontaneous breaking of the electro-weak symmetry through the formation of the technifer-mion condensate, which therefore has the quantum numbers of the SM Higgs boson. The associated Goldstone bosons (technipions) then become the longitu-dinal degrees of freedom of the electroweak bosons in

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[2]We note that the Banks-Zaks [12] type IR points, used to illustrate the unparticle sector in [4], are accessible in perturba-tion theory. This yields anomalous dimensions of the gauge singlet operators which are close to the perturbative ones, resulting in very small unparticle type effects.

[3]Strictly speaking conformal invariance is a larger symmetry than scale invariance but we shall use these terms interchange-ably throughout this paper. We refer the reader to Ref. [13] for an investigation of the differences.

[4]Only very recently has it been possible to *directly* investigate, via lattice simulations, the dynamics of a number of gauge theories [14–24] expected to develop or to be very close an IR fixed point [25–27]. The bulk of the lattice results support the theoretical expectations [25,27].
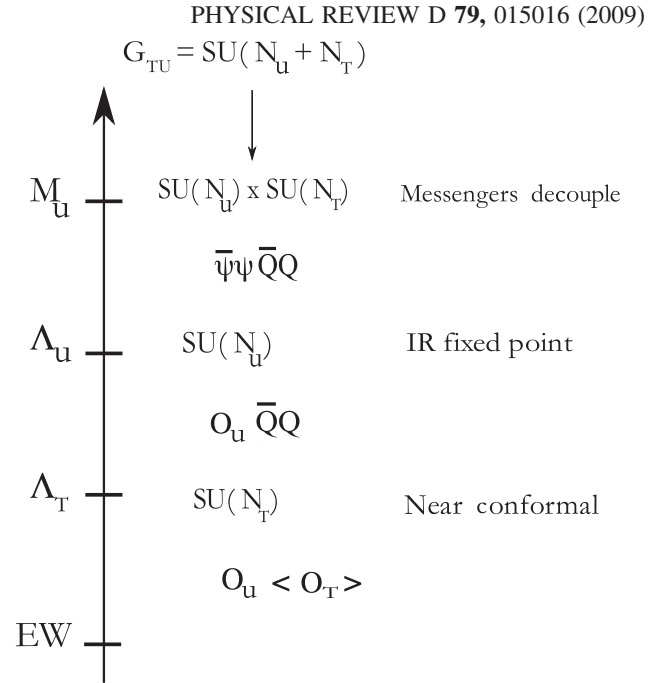
[5]There are a number of ways of achieving (near) conformal dynamics as summarized in [29]. The state-of-the-art phase diagram [26,27] and new tools [25] to construct viable NC nonsupersymmetric gauge theory are reported in [3].

[6]Such models are known as walking TC [30]. They are preferred over QCD-like TC models by the electroweak preci-sion data. In particular, the S-parameter receives a negative contribution for NC models [31]. A large class of phenomeno-logically viable models have been identified [3,27,29,32] of which minimal walking technicolor (MWT) and partially gauged technicolor (PGT) constitute two relevant examples.

exact formal analogy to the SM. In the simplest TC models the technipion decay constant F_T is related to the weak scale as $2M_W = gF_T$ (g is the weak coupling constant) and therefore $F_T \simeq 250$ GeV. The TC scale, analogous to Λ_{QCD} for the strong force, is roughly $\Lambda_{\mathrm{TC}} \sim 4\pi F_T$.

Now we turn our attention to the unparticle sector. Here the total number of massless techni-unparticle flavors S is balanced against the total number of colors N_U in such a way that the theory, *per se*, is asymptotically free and admits a nonperturbative IR fixed point. The energy scale around which the IR fixed point starts to set in is indicated with $\Lambda_{\mathcal{U}} \gg M_W$.

It might be regarded as natural to assume that the unparticle and the TC sectors have a common dynamical origin, e.g. are part of a larger gauge group at energies above $\Lambda_{\mathcal{T}}$ and $\Lambda_{\mathcal{U}}$. We would like to point out that the relative ordering between $\Lambda_{\mathcal{T}}$ and $\Lambda_{\mathcal{U}}$ is of no particular relevance for our scenario. The low-energy relics of such a unified-type model are four-Fermi operators allowing the two sectors to communicate with each other at low energy. The unparticle sector will then be driven away from the fixed point due to the appearance of the electroweak scale in the TC sector.

The model, of which further details are presented in Sec. II resembles models of extended technicolor (ETC) [3], where the techni-unparticles play the role of the SM fermions. We refer to these type of models as extended techni-unparticle (ETU) models.[7] At very high energies $E \gg M_{\mathcal{U}}$ the gauge group $G_{T \times U}$ is thought to be embedded in a simple group $G_{TU} \supset G_{T \times U}$. At around the scale $M_{\mathcal{U}}$ the ETU group is broken to $G_{TU} \rightarrow G_{T \times U}$ and the heavy gauge fields receive masses of the order of $M_{\mathcal{U}}$ and play the role of the messenger sector. Below the scale $M_{\mathcal{U}}$ the massive gauge fields decouple and four-Fermi operators emerge, which corresponds to the first step of the scenario, e.g. Eq. (1) and Fig. 1. Without committing to the specific ETU dynamics the interactions can be parametrized as:

$$\mathcal{L}^{\,\mathrm{eff}}_{<M_{\mathcal{U}}} = \alpha \frac{\bar{Q}Q\bar{\Psi}\Psi}{M_{\mathcal{U}}^2} + \beta \frac{\bar{Q}Q\bar{Q}Q}{M_{\mathcal{U}}^2} + \gamma \frac{\bar{\Psi}\Psi\bar{\Psi}\Psi}{M_{\mathcal{U}}^2}. \quad (4)$$

The coefficients α, β, and γ (the latter should not be confused with an anomalous dimension) are of order one, which can be calculated if the gauge coupling g_{TU} is perturbative. The Lagrangian (4) is the relic of the ETU (ETC) interaction and gives rise to two sources of dynamical chiral symmetry breaking in addition to the *intrinsic* dynamics of the groups $G_{T/U}$. These are contact interactions of the type emphasized in [34]. Firstly, when one fermion pair acquires a VEV then the α-term turns into a tadpole and induces a VEV for the other fermion pair. This is what happens to the unparticle sector when the TC

sector, or the SM Higgs [10], breaks the electroweak symmetry. Second, the γ term corresponds to a Nambu–Jona-Lasino type interaction which may lead to the formation of a VEV, for sufficiently large γ. This mechanism leads to breaking of scale invariance even in the absence of any other low-energy scale. Let us parenthetically note that this mechanism is operative in models of top condensation, cf. the TC report [35] for an overview. However, based on our analysis in Appendix A we shall neglect this mechanism in the sequel of this paper. We shall refer to these two mechanism as α/γ-induced condensates.

At the scale $\Lambda_{\mathcal{U}} \gg M_W$ the unparticle gauge sector flows into an IR fixed point and the UV operator $\mathcal{O}_{\mathrm{UV}} = \bar{\Psi}\Psi$ becomes the *composite unparticle* operator $\mathcal{O}_{\mathrm{IR}} \equiv \mathcal{O}_{\mathcal{U}}$ with scaling dimension $d_{\mathcal{U}} \equiv 3 - \gamma_{\mathcal{U}}$,[8]

$$(\bar{\Psi}\Psi)_{\mathrm{UV}} \sim \Lambda_{\mathcal{U}}^{\gamma_{\mathcal{U}}} \mathcal{O}_{\mathcal{U}}; \qquad M_W \ll \Lambda_{\mathcal{U}} \ll M_{\mathcal{U}}. \quad (5)$$

Note, the anomalous dimension $\gamma_{\mathcal{U}}$ of the operator has to satisfy $\gamma_{\mathcal{U}} \leq 2$ due to unitarity bounds of the representations of the conformal group [36]. The Lagrangian then simply becomes

$$\mathcal{L}^{\,\mathrm{eff}}_{\Lambda_{\mathcal{U}}} = \alpha' \frac{\bar{Q}Q\Lambda_{\mathcal{U}}^{\gamma_{\mathcal{U}}}\mathcal{O}_{\mathcal{U}}}{M_{\mathcal{U}}^2} + \beta' \frac{\bar{Q}Q\bar{Q}Q}{M_{\mathcal{U}}^2} + \gamma' \frac{\Lambda_{\mathcal{U}}^{2\gamma_{\mathcal{U}}}\mathcal{O}_{\mathcal{U}}\mathcal{O}_{\mathcal{U}}}{M_{\mathcal{U}}^2}. \quad (6)$$

This realizes the second step in the scenario, cf. Fig. 1 and Eq. (1). The matching coefficients α', β', γ' (6) are related to α, β, γ (4) by order one coefficients. The α-term in Eq. (6) is similar to the unparticle-Higgs interaction in Eq. (3).

The composite operator $\bar{Q}Q$ can be treated in analogy to $\bar{\Psi}\Psi$ in (5),

$$(\bar{Q}Q)_{\mathrm{UV}} \sim \Lambda_{\mathcal{T}}^{\gamma_{\mathcal{T}}} \mathcal{O}_{\mathcal{T}}; \qquad \Lambda_{\mathrm{TC}} \ll \Lambda_{\mathcal{T}} \ll M_{\mathcal{U}}, \quad (7)$$

up to logarithmic corrections which are negligible. Contrary to the unparticle sector the TC gauge dynamics break scale invariance through the formation of an *intrinsic condensate*

$$\langle \mathcal{O}_{\mathcal{T}} \rangle_{\Lambda_{\mathcal{T}}} \simeq w_T^{\gamma_{\mathcal{T}}} \Lambda_{\mathrm{TC}}^{d_{\mathcal{T}}} \equiv w_T^{\gamma_{\mathcal{T}}} \Lambda_{\mathrm{TC}}^{3-\gamma_{\mathcal{T}}}, \qquad w_T \equiv \left(\frac{\Lambda_{\mathcal{T}}}{\Lambda_{\mathrm{TC}}}\right). \quad (8)$$

The estimate of the VEV is based on scaling from QCD and renormalization group evolution.

The relevant terms contained in the low-energy effective theory around the electroweak scale are:[9]

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[7] The work by Georgi and Kats [33] on a two dimensional example of unparticles triggered this work.

[8] The parametrization $d_{\mathcal{U}} \equiv 3 - \gamma_{\mathcal{U}}$ will be standard throughout the entire paper and in the text the scaling dimension $d_{\mathcal{U}}$ and the anomalous dimension $\gamma_{\mathcal{U}}$ will be used interchangeably.

[9] Note that in QCD-like TC models (the gauge coupling displays a running behavior rather than a walking one) one would set $\gamma_{\mathcal{T}} \simeq 0$ in Eqs. (8) and (9).

$$\mathcal{L}^{\mathrm{eff}}_{\Lambda_{\mathcal{T}}} = \alpha' w_T^{\gamma_{\mathcal{T}}} \frac{\Lambda_{\mathrm{TC}}^3 \Lambda_{\mathcal{U}}^{\gamma_{\mathcal{U}}} \mathcal{O}_{\mathcal{U}}}{M_{\mathcal{U}}^2} + \gamma' \frac{\Lambda_{\mathcal{U}}^{2\gamma_{\mathcal{U}}} \mathcal{O}_{\mathcal{U}} \mathcal{O}_{\mathcal{U}}}{M_{\mathcal{U}}^2} + \dots \tag{9}$$

This step involves another matching procedure but we shall not introduce further notation here and denote the matching coefficients by simple primes only. As stated previously the TC condensate drives the TC gauge sector away from the fixed point and the coupling increases towards the IR. The sector is then replaced by a low-energy effective chiral Lagrangian featuring the relevant composite degrees of freedom [35,37]. The lightest isosinglet composite scalar, with $J^{\mathrm{PC}} = 0^{++}$ quantum numbers, is the *composite Higgs*. For further details cf. Appendix F of Ref. [3] as well as [38]. The linear realization of the chiral Lagrangian of the TC model will be discussed in Sec. II.

B. Unparticle VEV $\langle \mathcal{O}_{\mathcal{U}} \rangle$

We shall now investigate how the α-term in (9) induces a VEV for the unparticle. In a theory with canonically normalized fields the mass term, if present, has to be included for the minimization of the potential. In the unparticle setup we do not have explicit mass terms but there is a continuous mass spectrum and it is not immediately clear how to proceed. Yet we can make use of the deconstructed version of unparticles, proposed by Stephanov [39], in order to imitate the situation of isolated masses [10]. The continuous spectrum is deconstructed into an infinite tower of massive particles,

$$\mathcal{O}_{\mathcal{U}}(x) = \sum_n f_n \varphi_n(x); \qquad f_n^2 = \Delta^2 \frac{B_{d_{\mathcal{U}}}}{2\pi} (M_n^2)^{d_{\mathcal{U}}-2} \tag{10}$$

$$M_n^2 = n\Delta^2,$$

with an adjusted residuum f_n and a finite spacing Δ in units of the mass. The spectral function $\rho_{d_{\mathcal{U}}}$ of the operator above is given by

$$\rho_{d_{\mathcal{U}}}(P^2)\theta(P_0) = \sum_n \delta(P^2 - P_n^2)|\langle 0|\mathcal{O}_{\mathcal{U}}|P_n\rangle|^2 \rightarrow \rho_{d_{\mathcal{U}}}(s)$$

$$\equiv \frac{B_{d_{\mathcal{U}}}}{2\pi} s^{d_{\mathcal{U}}-2} \theta(s). \tag{11}$$

The square root of $B_{d_{\mathcal{U}}}$ defines the strength with which a state couples to the unparticle operator. The sum above is a mnemonic for the sum over all the possible states in the Hilbert space. The propagator then follows from the spectral function from the Källén-Lehmann representation (C3). The factor $B_{d_{\mathcal{U}}}$ corresponds to $A_{d_{\mathcal{U}}}$ in [4], but we have chosen to denote it by a different letter since, as we shall see, our model demands a different form. The value $d_{\mathcal{U}} = 2$ in (11) is the dividing value between the IR and UV sensitive domains and will play a crucial role later on. Other regularizations than the one in Eq. (10) are possible [39]. Therefore, no physical interpretation should be attached to it. The Lagrangian (9) with added mass terms

becomes

$$\mathcal{L} = \bar{\alpha} \sum_n f_n \varphi_n + \bar{\gamma} \sum_{n,m} f_n f_m \varphi_n \varphi_m - \frac{1}{2} \sum_n M_n^2 \varphi_n^2, \tag{12}$$

with

$$\bar{\alpha} \equiv \alpha' w_T^{\gamma_{\mathcal{T}}} \frac{\Lambda_{\mathrm{TC}}^3 \Lambda_{\mathcal{U}}^{\gamma_{\mathcal{U}}}}{M_{\mathcal{U}}^2}, \qquad \bar{\gamma} \equiv \gamma' \frac{\Lambda_{\mathcal{U}}^{2\gamma_{\mathcal{U}}}}{M_{\mathcal{U}}^2}, \tag{13}$$

being the prefactors of $\mathcal{O}_{\mathcal{U}}$ and $\mathcal{O}_{\mathcal{U}}^2$ in (9), with mass dimensions $[\bar{\alpha}] = \gamma_{\mathcal{U}} + 1$ and $[\bar{\gamma}] = 2\gamma_{\mathcal{U}} - 2$. The equation of motion (e.o.m.) for the operator φ_n is

$$\bar{\alpha} f_n + 2\bar{\gamma} f_n \left(\sum_m f_m \varphi_m \right) - M_n^2 \varphi_n = 0. \tag{14}$$

A simple recursive relation follows from these relations,

$$\varphi_n = n^{(d_{\mathcal{U}}-4)/2} \varphi_1. \tag{15}$$

Inserting this result into the e.o.m. for φ_1 we obtain,

$$\langle \varphi_1 \rangle = \frac{\bar{\alpha} b_{d_{\mathcal{U}}} \Delta^{d_{\mathcal{U}}-3}}{1 - 2\bar{\gamma}(b_{d_{\mathcal{U}}} \Omega_\Delta)}, \qquad b_{d_{\mathcal{U}}} \equiv \sqrt{\frac{B_{d_{\mathcal{U}}}}{2\pi}}, \tag{16}$$

where Ω_Δ is the sum over the modes,

$$\Omega_\Delta \equiv \Delta^{d_{\mathcal{U}}-3} \sum_n \left(\frac{f_n}{b_{d_{\mathcal{U}}}} \right) \frac{\varphi_n}{\varphi_1} = \Delta^2 \sum_n (n\Delta^2)^{d_{\mathcal{U}}-3}.$$

The quantities $\langle \varphi_n \rangle$ are then obtained from the recursion relation (15) and the unparticle VEV is the sum of its deconstructed parts (10),

$$\langle \mathcal{O}_{\mathcal{U}} \rangle_\Delta = \sum_n f_n \langle \varphi_n \rangle = \frac{\bar{\alpha}}{1 - 2\bar{\gamma}(b_{d_{\mathcal{U}}} \Omega_\Delta)} (b_{d_{\mathcal{U}}} \Omega_\Delta). \tag{17}$$

Solving this equation with appropriate UV and IR regularizations is the main goal of the rest of this section. The unparticle condensate will be connected with the IR cutoff, which implies that Eq. (17) has to be solved in a self-consistent way. Removing the discrete regularization, the sum Ω_Δ is converted into an integral, which we shall regularize with an IR and UV regulator for later convenience,

$$\Omega(\Lambda_{\mathrm{IR}}, \Lambda_{\mathrm{UV}}) = \lim_{\Delta \to 0} \Omega_\Delta = \int_{\Lambda_{\mathrm{IR}}^2}^{\Lambda_{\mathrm{UV}}^2} ds\, s^{d_{\mathcal{U}}-3}$$

$$= \left(\frac{(\Lambda_{\mathrm{UV}}^2)^{d_{\mathcal{U}}-2} - (\Lambda_{\mathrm{IR}}^2)^{d_{\mathcal{U}}-2}}{d_{\mathcal{U}} - 2} \right). \tag{18}$$

We note that when the quadratic term is removed, i.e. $\gamma \to 0$ in (9), the problem reduces to a single unparticle operator coupled to an external source, of which the interaction (3) is a special case when the Higgs assumes a VEV. In this limit the result in Eq. (17) indeed reduces to the expression found in Ref. [10].

The integral $\Omega(\Lambda_{\mathrm{IR}}, \Lambda_{\mathrm{UV}})$ (18) is sensitive to the UV cutoff for $d_{\mathcal{U}} > 2$ and to the IR cutoff for $d_{\mathcal{U}} < 2$. The

effective theory for the unparticle operator is valid up to the scale $\Lambda_{\mathcal{U}}$ and is therefore a UV cutoff of the theory. Moreover at energies larger than $\Lambda_{\mathcal{T}}$ there is no $\bar{Q}Q$ condensate, which implies $\bar{\alpha} \to 0$ (13) and therefore the modes above $\Lambda_{\mathcal{T}}$ do not contribute to the VEV in Eq. (17). So effectively the UV cutoff is the lower of the two scales,

$$\Lambda_{\mathrm{UV}} \simeq \min(\Lambda_{\mathcal{U}}, \Lambda_{\mathcal{T}}). \tag{19}$$

The constituent fermion mass[10] m_{const} provides a natural IR cutoff:

$$\Lambda_{\mathrm{IR}} \simeq 2m_{\mathrm{const}} \simeq 2|\langle \mathcal{O}_{\mathcal{U}} \rangle|^{1/d_{\mathcal{U}}}. \tag{20}$$

For numerical estimates we have chosen the factor two in front of the condensate based on the crude idea of identi-

fying the IR cutoff with a possible lightest meson of mass roughly twice the constituent mass. However, this choice does not affect the qualitative nature of our results.

Assuming the UV and IR cutoffs as in Eqs. (19) and (20) respectively the unparticle condensate can then be extracted from Eq. (17) in terms of the scales $\{\Lambda_{\mathrm{TC}}, \Lambda_{\mathcal{U}}, \Lambda_{\mathcal{T}}, M_{\mathcal{U}}\}$. and the anomalous dimensions $\gamma_{\mathcal{U}}$ and $\gamma_{\mathcal{T}}$. The parameters obey the following hierarchies,

$$\Lambda_{\mathrm{TC}} \ll \Lambda_{\mathcal{U}}, \qquad \Lambda_{\mathcal{T}} \ll M_{\mathcal{U}}, \qquad \gamma_{\mathcal{U}} < \gamma_{\mathcal{T}}, \tag{21}$$

cf. Fig. 1 for the scales and Sec. II for an explanation concerning the relation of the anomalous dimensions. We investigate Eq. (17) analytically in the following three regimes

	valid near	sensitive

Approximate solution of Eq.(17) for $\langle \hat{\mathcal{O}}_{\mathcal{U}} \rangle$

$$\langle \hat{\mathcal{O}}_{\mathcal{U}} \rangle (|\langle \hat{\mathcal{O}}_{\mathcal{U}} \rangle|^{(2\gamma_{\mathcal{U}}-2)/(3-\gamma_{\mathcal{U}})} - 2C_1\gamma') - C_0\alpha' = 0 \qquad \begin{cases} d_{\mathcal{U}} \gtrsim 1 \\ \gamma_{\mathcal{U}} \lesssim 2 \end{cases} \qquad \mathrm{IR}$$

$$C_1 = \frac{b_{d_{\mathcal{U}}}}{\gamma_{\mathcal{U}}-1}\left(\frac{\Lambda_{\mathcal{U}}^2}{M_{\mathcal{U}}^2}\right)\left(\frac{(2\Lambda_{\mathrm{TC}})^2}{\Lambda_{\mathcal{U}}^2}\right)^{1-\gamma_{\mathcal{U}}} \qquad C_0 = C_1\left(\frac{\Lambda_{\mathrm{TC}}}{\Lambda_{\mathcal{U}}}\right)^{\gamma_{\mathcal{U}}} w_T^{\gamma_{\mathcal{T}}} \tag{22}$$

$$\langle \hat{\mathcal{O}}_{\mathcal{U}} \rangle \simeq -\alpha' b_2 \left(\frac{\Lambda_{\mathcal{U}}\Lambda_{\mathrm{TC}}}{M_{\mathcal{U}}^2}\right) w_T^{\gamma_{\mathcal{T}}} \log\left[\frac{\Lambda_{\mathrm{TC}}^2}{\Lambda_{\mathcal{U}}^2} 4|\langle \hat{\mathcal{O}}_{\mathcal{U}} \rangle|\right] \qquad \begin{cases} d_{\mathcal{U}} = 2 \\ \gamma_{\mathcal{U}} = 1 \end{cases} \qquad \mathrm{IR} \sim \mathrm{UV} \tag{23}$$

$$\langle \hat{\mathcal{O}}_{\mathcal{U}} \rangle \simeq +\alpha' \frac{b_{d_{\mathcal{U}}}}{1-\gamma_{\mathcal{U}}}\left(\frac{\Lambda_{\mathcal{U}}^2}{M_{\mathcal{U}}^2}\right)\left(\frac{\Lambda_{\mathrm{TC}}^2}{\Lambda_{\mathcal{U}}^2}\right)^{\gamma_{\mathcal{U}}} w_T^{\gamma_{\mathcal{T}}} \qquad \begin{cases} d_{\mathcal{U}} \lesssim 3 \\ \gamma_{\mathcal{U}} \gtrsim 0 \end{cases} \qquad \mathrm{UV}. \tag{24}$$

We have chosen to normalize the unparticle VEV,

$$\langle \hat{\mathcal{O}}_{\mathcal{U}} \rangle \equiv \frac{\langle \mathcal{O}_{\mathcal{U}} \rangle}{\Lambda_{\mathrm{TC}}^{d_{\mathcal{U}}}},$$

to the chiral symmetry breaking scale of the TC sector. Equations (22) and (24) are sensitive to the IR and UV domain of (18). The solutions are valid in a small neighborhood of $d_{\mathcal{U}} \gtrsim 1$ and $d_{\mathcal{U}} \lesssim 3$, respectively. Note, the UV sensitive domain $d_{\mathcal{U}} \lesssim 3$ corresponds to a perturbative Banks-Zaks type fixed point [12]. Equation (23) represents the domain which is equally sensitive to the IR and UV. We have set $d_{\mathcal{U}} = 2$ strictly for presentational convenience only. The γ-term is solely important for (22) or more precisely is of the same order as the α-term for typical values of the model parameters. For $\alpha', \gamma' \geq 0$ all solutions are positive. In Fig. 2 (left) we have plotted the IR

cutoff (20) as a function of $\gamma_{\mathcal{U}}$ for different $\gamma_{\mathcal{T}}$ up to the bound $\gamma_{\mathcal{U}} \leq \gamma_{\mathcal{T}}$ (21). The input values, which are thought to be typical, are indicated in the caption.

The breaking of scale invariance, due to the coupling to the Higgs sector, was investigated in an earlier reference by the use of naïve dimensional analysis (NDA) [9]. In Appendix B we compare their results with ours. At the parametric level we find,

$$\Lambda_{\mathrm{IR}} \gtrsim \Lambda_{\mathrm{IR}}^{\mathrm{NDA}}, \tag{25}$$

cf. Fig. 2 (right). The difference being caused by the fact that in the NDA analysis it is implicitly assumed that the unparticle sector scales with the IR cutoff whereas in our model the unparticle condensate can also be sensitive to the UV domain. Parametric equality is reached in the region of IR sensitivity, e.g. (22). Needless to say that with NDA factors of 4π can go unnoticed. In connection with the latter, a similar criticism could apply to our prescription in (20). Nevertheless it appears to us that it is physically motivated and to some extent is backed up from our empirical knowledge of QCD.

So far we have not specified the normalization factor $B_{d_{\mathcal{U}}}$ introduced in Eqs. (10) and (11). In Appendix C we motivate the following formula

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[10]In QCD the condensate induces a dynamical mass, the so-called constituent quark mass. An estimate can be obtained by extending the definition of the perturbative pole mass to include additional terms from the operator product expansion [40]. Adapting the situation to the case of a nontrivial fixed point leads to $(m_{\mathrm{const}})^{d_{\mathcal{U}}} \simeq -g_{\mathcal{U}}^2 C_{d_{\mathcal{U}}} \langle \mathcal{O}_{\mathcal{U}} \rangle$, where $C_{d_{\mathcal{U}}}$ an order one coefficient which is not calculable due to virulent strong interaction effects. The lowest order QCD result is recovered by setting $d_{\mathcal{U}} \to 3$ and $C_{d_{\mathcal{U}}} \to 1$.
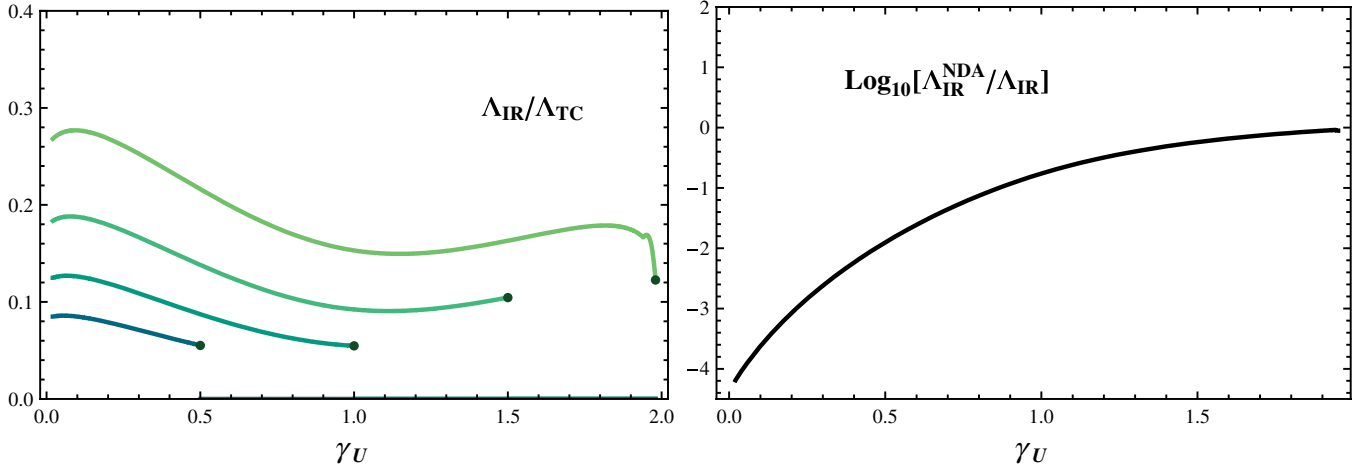
FIG. 2 (color online). (left) $\Lambda_{\mathrm{IR}}/\Lambda_{\mathrm{TC}}$ as a function of $\gamma_\mathcal{U}$ up to the constraint $\gamma_\mathcal{U} \leq \gamma_\mathcal{T}$. The actual value of $\gamma_\mathcal{T}$ can therefore be read-off from the endpoint of the curve. (right) Logarithm of the ratio of IR cutoffs against $\gamma_\mathcal{U}$ for $\gamma_\mathcal{T} = 1$. The influence of the γ-term is completely negligible for the chosen input values. The dependence on $\gamma_\mathcal{T}$ is very mild and we have chosen somewhat arbitrarily $\gamma_\mathcal{T} = 3/2$. Trivial factors, like $b_{d_\mathcal{U}}$, are fixed such that equality of Λ_{IR} and $\Lambda_{\mathrm{IR}}^{\mathrm{NDA}}$ is reached for $\gamma = 0$ in the domain $\gamma_\mathcal{U} = 2$. In both figures we have chosen $N_U = 4$ in the interpolation formula (26). Furthermore, the hierarchies of scales (21) are set to $\Lambda_{\mathrm{TC}}:(\Lambda_\mathcal{U} = \Lambda_\mathcal{T}):M_\mathcal{U} = 1:10^1:10^3$ and the coefficients α' and γ' to a value of unity.

$$(B_{d_\mathcal{U}})_{\mathrm{interpol}} = 2\pi(d_\mathcal{U} - 1) + \left(\frac{N_U}{16\pi} - \pi\right)(d_\mathcal{U} - 1)^2,$$

(26)

as the interpolation formula between the value of B_3, which is determined by the free-fermion loop in our model, and the behavior around B_1, which is model independent [36]. As previously stated it differs from the normalization factor $A_{d_\mathcal{U}}$ in Ref. [4].

In the next section we will discuss an ETU model in some more detail.

II. A SCHEMATIC ETU MODEL

We imagine that at an energy much higher than the electroweak scale, the theory is described by a gauge theory

$$\mathcal{L}^{\mathrm{UV}} = -\frac{1}{2}\,\mathrm{Tr}[\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}] + \sum_{\mathcal{F}=1}^{F} \bar{\xi}_\mathcal{F}(i\slashed{\partial} + g_{\mathrm{TU}}\slashed{\mathcal{A}})\xi_\mathcal{F}$$

$$+ \ldots$$

(27)

where $\mathcal{A}$ is the gauge field of the $SU(N_T + N_U)$ group and gauge indices are suppressed. $(\xi_\mathcal{F}^\mathcal{A})^T = (Q^1 \ldots Q^{N_T}, \Psi^1 \ldots \Psi^{N_U})_\mathcal{F}$ is the fermion field unifying the technifermion and TC matter content. The dots in (27) stand for the $SU(3) \times SU(2)_L \times U(1)_Y$ gauge fields and their interactions to the SM fermions and technifermions. There is no elementary Higgs field in this formulation. Unification of the TC and techni-unparticle dynamics, as outlined in Sec. I, constrains the flavor symmetry of the two sectors to be identical at high energies. The matter content and the number of technifermions (TC + techni-unparticles) is

chosen, within the phase diagram in [3], such that the theory is asymptotically free at high energies. The non-Abelian global flavor symmetry is $SU_L(F) \times SU_R(F)$.

At an intermediate scale $M_\mathcal{U}$, much higher than the scale where the unparticle and TC subgroup become strongly coupled, the dynamics is such that $SU(N_T + N_U)$ breaks to $SU(N_T) \times SU(N_U)$. Only two flavors (i.e. one electroweak doublet) are gauged under the electroweak group. The global symmetry group breaks explicitly to $G_F = SU_L(2) \times SU_R(2) \times SU_L(F - 2) \times SU_R(F - 2)$. At this energy scale the weak interactions are, however, negligible and we can safely ignore it.

At the scale $M_\mathcal{U}$ there are the Q_c^i fermions—with $i = 1, \ldots, F$ and $c = 1, \ldots, N_T$—as well as the Ψ_u^i ones—with $i = 1, \ldots, F$ and $u = 1, \ldots, N_U$. Assigning the indices $i = 1, 2$ to the fermions gauged under the electroweak group we observe that not only the TC fermions are gauged under the electroweak but also the techni-unparticles. To ensure that the unparticle sector is experimentally not too visible we have to assume a mechanism that provides a large mass to the charged techni-unparticle fermions. In reality this is quite a difficult task, since we do not want to break the SM weak symmetry explicitly.[11] Our treatment below, however, is sufficiently general to be straightforwardly adapted to various model constructions.

As already stated in the first section, the number of flavors and colors for the TC and unparticle gauge groups

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[11]One could for instance unify the flavor symmetry of the unparticles with the technicolor gauge group into an ETC group. This would also produce a Lagrangian of the type (4). The TC fermions would be charged under the electroweak group separately.

$SU(N_T)$ and $SU(N_U)$ have to be arranged such that the former is NC and the latter is conformal. This enforces the conditions:

$$F \leq F^*_{N_T}, \qquad F^*_{N_U} \leq F - 2. \qquad (28)$$

F^*_N denotes the critical number of flavors, for a given number of colors N, above which the theory develops an IR fixed point. Recall that two unparticle flavors are decoupled and hence $F \to F - 2$ in the second inequality in (28).

According to the conjectured all order beta function [25] F^*_N is

$$F^*_N = \frac{11N}{\gamma^* + 2}, \qquad (29)$$

for an $SU(N)$ gauge theory with matter in the fundamental representation. This restricts F^*_N inasmuch as the critical anomalous dimension has to satisfy the unitarity bound $\gamma^* \leq 2$ [36]. The ladder approximation, for instance, yields $\gamma^* \approx 1$ [41]. Combining Eqs. (28) and (29) we arrive at the following allowed window for the number of flavors:

$$\frac{11N_U}{\gamma^* + 2} + 2 \leq F \leq \frac{11N_T}{\gamma^* + 2}. \qquad (30)$$

The anomalous dimension of the mass operator for the unparticle and TC fermions at the fixed point are

$$\gamma_U = \frac{11N_U - 2F + 4}{F - 2}, \qquad \gamma_T = \frac{11N_T - 2F}{F}. \qquad (31)$$

They follow from the conjectured all order beta function [25]. For walking TC γ_T is, in fact, very near γ^* and F is very close to the upper bound of Eq. (30). Conformality of the unparticle sector requires γ_U to be smaller than γ^*. Summarizing:

$$\gamma_U < \gamma^* \lesssim \gamma_T. \qquad (32)$$

A. Low-energy description

Below the scale M_U all four-Fermi interactions have to respect the flavor symmetry G_F. The most general four-Fermi operators have been classified in [42] and the coefficient of the various operators depend on the specific model used to break the unified gauge theory. Upon Fierz rearrangement, the operators of greatest phenomenological relevance are,

$$\mathcal{L}^{\mathrm{eff}} = \left(\frac{G}{2} \bar{\Psi}_L \Sigma \Psi_R + \mathrm{H.c.}\right) + \frac{G'}{2M_U^2} (\bar{\Psi}_L \Psi_R)(\bar{\Psi}_R \Psi_L)$$
$$+ \ldots, \qquad (33)$$

the scalar-scalar interactions of Eq. (4). Here Σ is the quark bilinear,

$$\Sigma_i^j \sim (Q_{Li} \bar{Q}_R^j)_{\mathrm{UV}}, \qquad i = 1, \ldots, F. \qquad (34)$$

The flavor indices are contracted and the sum starts from the index value 3; the first two indices correspond to the Ψ's charged under the electroweak force, which are decoupled at low energy. The fermion bilinear becomes the unparticle operator (7),

$$(\mathcal{O}_U)_i^j = \frac{\Psi_{Li} \bar{\Psi}_R^j}{\Lambda_U^{\gamma_U}}. \qquad (35)$$

The matrix Σ at energies near the electroweak symmetry breaking scale is identified with the interpolating field for the mesonic composite operators.

To investigate the coupling to the composite Higgs we write down the low-energy effective theory using linear realizations. We parametrize the complex $F \times F$ matrix Σ by

$$\Sigma = \frac{\sigma + i\Theta}{\sqrt{F}} + \sqrt{2}(i\Pi^a + \tilde{\Pi}^a)T^a, \qquad (36)$$

where $(\sigma, \tilde{\Pi})$ and (Θ, Π) have 0^{++} and 0^{-+} quantum numbers, respectively. The Lagrangian is given by

$$\mathcal{L}^{\mathrm{eff}} = \frac{1}{2} \mathrm{Tr}[(D\Sigma)^\dagger D\Sigma] - k_1(\hat{\mathrm{Tr}}[\Sigma^\dagger \mathcal{O}_U] + \mathrm{H.c.})$$
$$- k_2 \hat{\mathrm{Tr}}[\mathcal{O}_U \mathcal{O}_U^\dagger] - m_{\mathrm{ETC}}^2 \sum_{a=4}^{F^2-1} \frac{\Pi^a \Pi^a}{2}$$
$$- V(\Sigma, \Sigma^\dagger), \qquad (37)$$

where

$$D\Sigma = \partial \Sigma - igW\Sigma + ig'\Sigma B T_R^3, \quad \text{and} \quad W = W^a T_L^a, \qquad (38)$$

and $\mathrm{Tr}[T_{L/R}^a T_{L/R}^b] = \delta^{ab}/2$. The coefficients k_1 and k_2 are directly proportional to the α' and γ' coefficients in (6). The *hat* on some of the traces indicates that the summation is only on the flavor indices from 3 to F. Three of the Goldstone bosons play the role of the longitudinal gauge bosons and the remaining ones receive a mass m_{ETC}^2 from an ETC mechanism. We refer the reader to Ref. [35] for discussion of different ETC models with mechanisms for sufficiently large mass generation. The first term in the Lagrangian is responsible for the mass of the weak gauge bosons and the kinetic term for the remaining Goldstone bosons. The VEV's for the flavor-diagonal part of the unparticle operator, reduces to the computation performed in the previous section. The potential term preserves the global flavor symmetry G_F. Up to dimension four, including the determinant responsible for the η' mass in QCD, the terms respecting the global symmetries of the TC theory are:

$$V(\Sigma, \Sigma^\dagger) = -\frac{m^2}{2} \mathrm{Tr}[\Sigma^\dagger \Sigma] + \frac{\lambda_1}{F} \mathrm{Tr}[\Sigma^\dagger \Sigma]^2$$
$$+ \lambda_2 \mathrm{Tr}[(\Sigma^\dagger \Sigma)^2] - \lambda_3(\det\Sigma + \det\Sigma^\dagger). \qquad (39)$$

The coefficient m^2 is positive to ensure chiral symmetry breaking in the TC sector. The Higgs VEV enters as follows,

$$\sigma = v + h, \quad \text{with } F_T = \sqrt{\frac{2}{F}}v \simeq 250 \text{ GeV}. \qquad (40)$$

F here is the number of flavors and h the composite field with the same quantum numbers as the SM Higgs. The particles σ, Θ, $\tilde{\Pi}$ all have masses of the order of v. The Higgs mass, the Higgs VEV, and the Θ mass, for instance, are

$$v^2 = \frac{m^2}{(\lambda_1 + \lambda_2 - \lambda_3)}, \qquad m_h^2 = 2m^2, \qquad m_\Theta^2 = 4v^2\lambda_3^2, \qquad (41)$$

up to corrections of the order of $O(\Lambda_{\text{TC}}^2/M_{\mathcal{U}}^2)$ due to contributions from α-terms.

The lightest pseudoscalars of the unparticle sector are the pseudo Goldstone bosons emerging from the explicit breaking of the global flavor symmetry in the unparticle sector. Their mass can be read off from the linear term in $O_{\mathcal{U}}$ of the effective Largrangian (37)

$$m_{\tilde{\Pi}_{\mathcal{U}}}^2 \simeq \Lambda_{\text{TC}}^2 \left(\frac{\Lambda_{\text{TC}}}{M_{\mathcal{U}}^2}\right)^2 \left(\frac{\Lambda_{\mathcal{U}}}{\Lambda_{\text{TC}}}\right)^{\gamma_{\mathcal{U}}} \left(\frac{\Lambda_{\mathcal{I}}}{\Lambda_{\text{TC}}}\right)^{\gamma_{\mathcal{I}}}. \qquad (42)$$

B. Regularized unparticle propagator

In our model the unparticle propagator to be used for phenomenology, defined from the Källén-Lehmann representation (C3), is

$$\Delta_{\mathcal{U}}(q^2, \Lambda_{\text{UV}}^2, \Lambda_{\text{IR}}^2) = -\frac{B_{d_{\mathcal{U}}}}{2\pi} \int_{\Lambda_{\text{IR}}^2}^{\Lambda_{\text{UV}}^2} \frac{ds\, s^{d_{\mathcal{U}}-2}}{s - q^2 - i0} + \text{s.t.} \qquad (43)$$

For $d_{\mathcal{U}} > 2$ the integral is sensitive to the UV completion, of which the subtraction terms (s.t.) are a mnemonic. More precisely, the part which is sensitive to Λ_{UV} is ambiguous due to the presence of, in principle computable, counterterms, which are expected to be of order one.[12] This will limit, in practice, the predictivity of the theory. Modeling the UV and IR transition regions by hard cutoffs is of course a crude model. Yet this should not be relevant as long as q^2 is sufficiently far away from these cutoffs. Whereas, for q^2 close to the cutoff, the integral has an endpoint singularity which is, to a great extent, a model artefact. The situation could be ameliorated for instance by smearing the momentum with a smooth probability density. Because of the breaking of the scale invariance there will be single and multiparticle states appearing in the spectrum, which will affect the $q^2 \sim \Lambda_{\text{IR}}^2$ behavior. Having made these statements, we now turn to the evaluation of the integral in (43). It can be expressed as the difference of an IR and UV part,

$$\Delta_{\mathcal{U}}(q^2, \Lambda_{\text{UV}}^2, \Lambda_{\text{IR}}^2) = f_{d_{\mathcal{U}}}(\Lambda_{\text{IR}}^2, q^2 + i0) - f_{d_{\mathcal{U}}}(\Lambda_{\text{UV}}^2, q^2 + i0), \qquad (44)$$

given by,

$$f_{d_{\mathcal{U}}}(\Lambda^2, q^2) \equiv \underbrace{\left[\frac{B_{d_{\mathcal{U}}}}{2\pi} \frac{(\Lambda^2)^{d_{\mathcal{U}}-2}}{d_{\mathcal{U}}-2}\right] {}_2F_1\left(1, 2 - d_{\mathcal{U}}, 3 - d_{\mathcal{U}}, \frac{q^2}{\Lambda^2}\right)}_{\equiv \bar{f}_{d_{\mathcal{U}}}(q^2/\Lambda^2)}. \qquad (45)$$

For later convenience we give the behavior of the function $\bar{f}_{d_{\mathcal{U}}}(x)$ for small and large arguments appropriate for the respective domains:

$$\bar{f}_{d_{\mathcal{U}}}(x + i0) = \begin{cases} a_0 + a_1 x + O(x^2) & x \ll 1 \quad \text{UV} \\ a_{d_{\mathcal{U}}-2}(-x - i0)^{d_{\mathcal{U}}-2} + a_{-1}\frac{1}{x} + O(\frac{1}{x^2}) & x \gg 1 \quad \text{IR} \end{cases}, \qquad (46)$$

where the leading coefficients are given by

$$a_0 = 1, \qquad a_{-1} = -\frac{\Gamma(1 - d_{\mathcal{U}})\Gamma(3 - d_{\mathcal{U}})}{\Gamma(2 - d_{\mathcal{U}})^2}$$

$$a_1 = \frac{d_{\mathcal{U}} - 2}{d_{\mathcal{U}} - 3}, \qquad a_{d_{\mathcal{U}}-2} = \Gamma(3 - d_{\mathcal{U}})\Gamma(d_{\mathcal{U}-1}). \qquad (47)$$

1. IR region: $1 < d_{\mathcal{U}} < 2$

In the domain $1 < d_{\mathcal{U}} < 2$ the regularized propagator is close to the propagator without IR and UV regularization presented in [6]. From the expansion (46) one immediately obtains,

$$\lim_{\Lambda_{\text{IR}} \to 0 \Lambda_{\text{UV}} \to \infty} \Delta_{\mathcal{U}}(q^2, \Lambda_{\text{UV}}^2, \Lambda_{\text{IR}}^2)$$

$$= \frac{B_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)}(-q^2 - i0)^{d_{\mathcal{U}}-2} \qquad (48)$$

$$1 < d_{\mathcal{U}} < 2,$$

using $\Gamma(z)\Gamma(1 - z)\sin(\pi z) = \pi$. Note that for finite cut-

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[12]The counterterms are expected to be of order one in a theory which is not fine-tuned. This is also known under the term: "naturalness." In our model the UV completion is known and the counterterms could in principle be determined, but in practice this is outside the scope of our possibilities.
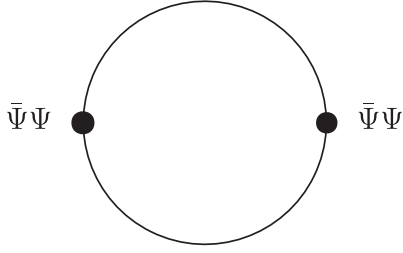
FIG. 3.   Fermion bubble with scalar vertices, corresponding to the unparticle propagator in the limit $d_\mathcal{U} \to 3$.

offs the UV part of the propagator is suppressed by $(\Lambda_{IR}/\Lambda_{UV})^{2(2-d_\mathcal{U})}$ and is therefore of minor importance for $d_\mathcal{U}$ close to 1.

2. UV region: $2 \leq d_\mathcal{U} < 3$

As previously stated, for $d_\mathcal{U} > 2$ the UV part becomes increasingly dominant and manifests itself in the appearance of counterterms. In fact in the strict limit $d_\mathcal{U} \to 3$, for example, the UV contribution is formally the same as the fermion loop contribution to the Higgs mass in the SM, e.g. Fig. 3, which is quadratically divergent. The effective theory is valid for $q^2 \ll \Lambda_{UV}^2$ and therefore the coefficient a_0 (46) is relevant for the UV part of the propagator (44). In practice this means that only a single counterterm, the one associated with a_0 is relevant. As stated earlier, by naturalness the counterterms are expected to be of order one.

Note that the limit $d_\mathcal{U} \to 2$ leads to a particularly simple expression

$$\lim_{d_\mathcal{U} \to 2} \Delta_\mathcal{U}(q^2, \Lambda_{UV}^2, \Lambda_{IR}^2) = \frac{B_{d_\mathcal{U}}}{2\pi} \log\left(\frac{-q^2 - i0}{\Lambda_{UV}^2}\right) + \text{s.t.},$$
(49)

where (s.t.) stands for subtraction terms (counterterms).

C. Unparticle-Higgs mixing

We shall now turn to the question of the mixing of the unparticle and the Higgs. Our findings resemble results from extra dimensional models. E.g. the model called HEIDI [43], where the continuous spectrum is mimicked by an infinite tower of narrowly spaced Kaluza-Klein modes. The difference is that our model is inherently four dimensional and that the parameters, such as the IR cutoff and the strength of the unparticle-Higgs coupling, are related to each other. Our model is also different from the one in Ref. [10] since, although both are in four dimensions, the Higgs and unparticle coupling emerges dynamically within a UV complete theory.

The interaction term from Eq. (37)

$$\mathcal{L}^{\text{mix}} = -g_{h\mathcal{O}_\mathcal{U}} h \mathcal{O}_\mathcal{U}, \qquad g_{h\mathcal{O}_\mathcal{U}} = \frac{k_1(F-2)}{\sqrt{F}},$$
(50)

introduces a mixing between the Higgs and the unparticle.

The constant k_1 has mass dimension $\gamma_\mathcal{U}$. Its size, on which we will comment below, is crucial for the qualitative nature of the physics. The Higgs propagator is obtained from inverting the combined Higgs-unparticle system

$$\Delta_{hh}(q^2) = \frac{1}{q^2 - m_h^2 - g_{h\mathcal{O}_\mathcal{U}}^2 \Delta_\mathcal{U}(q^2, \Lambda_{UV}^2, \Lambda_{IR}^2)}.$$
(51)

This, of course, results in unparticle corrections controlled by $g_{h\mathcal{O}_\mathcal{U}}$. The propagator can be rewritten in terms of a dispersion representation

$$\Delta_{hh}(q^2) = -\int \frac{ds \rho_{hh}(s)}{s - q^2 - i0},$$
(52)

where the density,

$$\int ds \rho_{hh}(s) = 1,$$
(53)

is automatically normalized to unity. The nonzero value of the coupling $g_{h\mathcal{O}_\mathcal{U}}$ results solely in a change of basis (or poles and cuts) of the intermediate particles but does not change the overall density of states. A direct way to derive (53) is to equate the representations (51) and (52), multiply them by q^2 and take the limit $q^2 \to \infty$ resulting in (53). Please note, this only works in the case where $d_\mathcal{U} < 3$, for which the interaction (50) is power counting renormalizable. If this condition is not fulfilled, one could resort to a subtracted dispersion relation.

The dispersion representation can be split into resonance[13] and continuum contributions,

$$\rho_{hh}(s) = \sum_i r_i \delta(s - \bar{m}_i^2) + \sigma(s).$$
(54)

The resonance contribution, if present, can then be obtained from the pole equation

$$\Delta_{hh}^{-1}(\bar{m}_i^2) = 0, \qquad r_i = \left| \frac{d\Delta_{hh}^{-1}(s)}{ds} \right|^{-1}_{s=\bar{m}_i^2} \leq 1.$$
(55)

The residues r_i are smaller (or equal) to one as a consequence of the normalization condition Eq. (53). The continuum is simply given by the cut,

$$\sigma(s) = \Theta(s - \Lambda_{IR}^2)\text{Im}[\Delta_{hh}^{-1}(s)],$$
(56)

which corresponds to the imaginary part; most familiar from the optical theorem.

To a large extent the spectral function is characterized by the zeros of the pole equation and the onset of the continuum relative to the poles. This will depend on the strength of the mixing and the anomalous dimension. Somewhat exotic effects can be obtained when the mixing term is

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[13]The pole description is only adequate in the narrow width approximation. The Higgs width is of course rather sizable in a theory of strong interactions. The presentation below is meant to be for illustrative purposes only.

made very large [28,44].[14] In our model the mixing is determined by k_1 (50). Its parametric value is given by

$$k_1 \sim \alpha' \Lambda_{\mathrm{TC}}^{\gamma_\mathcal{U}} \left(\frac{\Lambda_{\mathrm{TC}}}{M_\mathcal{U}}\right)^2 \left(\frac{\Lambda_\mathcal{U}}{\Lambda_{\mathrm{TC}}}\right)^{\gamma_\mathcal{U}} w_T^{\gamma_\mathcal{T}}, \qquad w_T = \left(\frac{\Lambda_\mathcal{T}}{\Lambda_{\mathrm{TC}}}\right)$$

(57)

which we have normalized to the TC scale

The value of k_1 is, of course, suppressed by the large scale $M_\mathcal{U}$ *per se*, but receives enhancements from the powers of the anomalous dimensions. For the maximal allowed anomalous dimensions $\gamma_\mathcal{U} \simeq \gamma_\mathcal{T} \simeq 2$ and a hierarchy of scales as indicated in the caption of Fig. 2 one finds $k_1 \Lambda_{\mathrm{TC}}^{-\gamma_u} \simeq \alpha' \cdot O(10^{-2})$. We therefore expect the coupling $g_{h\mathcal{O}_\mathcal{U}} \Lambda_{\mathrm{TC}}^{-\gamma_u}$ (50) to be considerably smaller than 1.

In this case there is generally a unique solution to the pole equation. In the IR region $1 < d_\mathcal{U} < 2$ the analysis can be made quantitative whereas in the UV region $2 < d_\mathcal{U} < 3$ the uncertainty due to sizable counterterms makes a quantitative assessment difficult. As explained in Sec. II B 2 these counterterms are expected to be comparable in size only for the leading coefficient a_0 (46).

At the qualitative level it is an interesting question of whether the Higgs resonance is below or above the threshold [10,43]. For the values chosen in the caption of Fig. 2 the Higgs resonance is close to the IR cutoff. On the other hand it could very well be that the scale $M_\mathcal{U}$ is closer to the GUT scale which would decrease the IR cutoff definitely below the composite Higgs mass scale. In Appendix D we comment on the unparticle limit of the HEIDI models and compare our parameters with the fit of that model to the excess of the Higgs search at LEP.

III. OUTLOOK

We introduced a framework in which the Higgs and the unparticle are both composite. The underlying theories are four-dimensional, asymptotically-free, nonsupersymmetric gauge theories with fermionic matter. We sketched a possible unification of these two sectors at a scale much higher than the electroweak scale. The resulting model resembles extended technicolor models and we termed it extended technicolor unparticle (ETU). The coupling of the unparticle sector to the SM emerges in a simple way and assumes the form of four-Fermi interactions below $M_\mathcal{U}$.

In our model the unparticle sector is coupled to the composite Higgs. Another possibility is to assume that the Higgs sector itself is unparticlelike, with a continuous

mass distribution. This unHiggs [45,46] could find a natural setting within walking technicolor, which is part of our framework. Of course, it is also possible to think of an unparticle scenario that is not coupled to the electroweak sector, where scale invariance is broken at a (much) lower scale. This could result in interesting effects on low-energy physics as extensively studied in the literature.

With respect to our model in the future one can:

(i) Study the composite Higgs production in association with a SM gauge boson, both for proton-proton (LHC) and proton-antiproton (Tevatron) collisions via the low-energy effective theory (37). In Refs. [47,48] it has been demonstrated that such a process is enhanced with respect to the SM, due to the presence of a light composite (techni)axial resonance.[15] The mixing of the light composite Higgs with the unparticle sector modifies these processes in a way that can be explored at colliders. Concretely, the transverse missing energy spectrum can be used to disentangle the unparticle sector from the TC contribution *per se*.

(ii) Use first principle lattice simulations to gain insight on the nonperturbative (near) conformal dynamics. It is clear from our analysis that this knowledge is crucial for describing and understanding unparticle dynamics. As a model example we have considered in the main text partially gauge technicolor introduced in [50]. These gauge theories are being studied on the lattice [22–24]. Once the presence of a fixed point is established, for example, via lattice simulations [18–21], the anomalous dimension of the fermion mass can be determined from the conjectured all order beta function [3,25], as done in Sec. II. Moreover, on the lattice one should be able to directly investigate the two-point function, i.e. the unparticle propagator.

(iii) Investigate different models at the ETU level. For example, one could adapt some models, introduced to generate masses to the SM fermions, in [51–57] to improve on our ETU model.

(iv) Study possible cosmological consequences of our framework. The lightest baryon of the unparticle gauge theory, the *unbaryon*, is naturally stable [due to a protected $U(1)$ unbaryon number] and therefore a possible dark matter candidate. Because of the fact that we expect a closely spaced spectrum of unbaryons and unparticle vector mesons, it shares properties in common with secluded models of dark matter [58] or previously discussed unparticle dark matter models [59].

Within our framework unparticle physics emerges as a natural extension of dynamical models of electroweak

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[14] It should also be mentioned that for very large mixing the theory typically becomes unstable. The pole equation has tachyonic solutions and vertices grow in an uncontrolled manner, indicating the appearance of a new vacuum. It is possible though that interesting effects could arise in a somewhat intermediate regime.

[15] A similar analysis within an extra-dimensional setup has been performed in [49].

symmetry breaking. As seen above the link opens the doors to yet unexplored collider phenomenology and possible new avenues for dark matter, such as the use of the unbaryon.

APPENDIX A: γ-INDUCED CONDENSATE $\langle \mathcal{O}_\mathcal{U} \rangle$

In this appendix we intend to sketch how the γ-term, in addition to the α-term in Eq. (1), can induce an unparticle VEV. The treatment essentially follows the Nambu–Jona-Lasino model [60]; a simple and concise summary of the latter is given in the appendix of Ref. [35]. The γ-term

$$\delta \mathcal{L}^{\mathrm{eff}}_{\Lambda_\mathcal{U}} = \gamma' \frac{\Lambda_\mathcal{U}^{2\gamma_u}}{M_\mathcal{U}^2} \mathcal{O}_\mathcal{U}^2 \qquad (A1)$$

can be rewritten into the following form

$$\delta \mathcal{L}^{\mathrm{eff}}_{\Lambda_\mathcal{U}} = \left(\sqrt{\gamma'} \frac{\Lambda_\mathcal{U}}{M_\mathcal{U}} \Lambda_\mathcal{U}^{\gamma_u} \mathcal{O}_\mathcal{U} H + \mathrm{H.c} \right) - \Lambda_\mathcal{U}^2 |H|^2 \quad (A2)$$

by the purely formal manipulation of introducing an auxiliary field H. The crucial question is then whether the coupling of the γ-term is large enough to enforce a dynamical VEV. This will be decided solely by the sign of the $|H|^2$-term. One has to integrate out the fermions between the scales $\Lambda_\mathcal{U}$ and μ. This is straightforward in the unparticle scenario since the propagator is known, up to UV and IR cutoffs. The $|H|^2$-term is then simply given by contracting the unparticle propagator (43) between two $\mathcal{O}_\mathcal{U} H$ interaction points; this leads to

$$\delta \mathcal{L}^{\mathrm{eff}}_\mu = -\Lambda_\mathcal{U}^2 |H|^2 \left(1 + \gamma' \frac{\Lambda_\mathcal{U}^{2\gamma_u}}{M_\mathcal{U}^2} \Delta_\mathcal{U}(-\mu^2, \Lambda_{\mathrm{UV}}^2, \Lambda_{\mathrm{IR}}^2) \right)$$
$$\equiv -m_H^2 |H|^2. \qquad (A3)$$

For $m_H^2 < 0$ the $|H|$ field acquires a VEV and induces an unparticle VEV through the gap equation. We remind the reader that the value of γ' is expected to be of the order one. In the range $\gamma_\mathcal{U} \sim 0 - 1$ the sign of m_H^2 is negative for $\gamma' > M_\mathcal{U}^2 / \Lambda_\mathcal{U}^2$ which would demand an unnatural enhancement of the γ' coefficient. For $\gamma_\mathcal{U} > 1$ the γ-term becomes a relevant operator and one could expect a qualitatively change in the picture. Around $\gamma_\mathcal{U} \lesssim 2$ the inequality becomes $\gamma' > (\Lambda_\mathcal{T}/\Lambda_\mathcal{U})^{2/3} (M_\mathcal{U}/\Lambda_\mathcal{U})^{2/3} (\Lambda_{\mathrm{TC}}/\Lambda_\mathcal{U})^{4/3}$ and could indeed lead to VEV at a scale comparable to the

one from the α VEV (17). To determine the value of the VEV we would need to evaluate the coefficient of the $|H|^4$-term which is a difficult task *per se* and beyond the scope of this paper.

APPENDIX B: COMPARISON WITH NAÏVE DIMENSIONAL ANALYSIS

In Ref. [9] it was pointed out that the interaction of a SM operator to the unparticle sector would act as a source of breaking the scale invariance. In the absence of an underlying model, the authors resorted to NDA. We will see here that the physics of the condensate or the anomalous dimension is, of course, not captured by such a generic approach as in Ref. [9].

The schematic notation in (1) made more precise [4] reads,

$$\mathcal{L}^{\mathrm{eff}} = M_\mathcal{U}^4 \left(\frac{\Lambda_\mathcal{U}}{M_\mathcal{U}} \right)^{d_{\mathrm{UV}} - d_\mathcal{U}} \left(\frac{O_{\mathrm{SM}}}{M_\mathcal{U}^{d_{\mathrm{SM}}}} \right) \left(\frac{\mathcal{O}_\mathcal{U}}{M_\mathcal{U}^{d_\mathcal{U}}} \right), \qquad (B1)$$

where $d_{\mathrm{SM/UV}}$ are the scaling dimensions of the SM operator and the unparticle operator in the UV. Assuming that $O_{\mathrm{SM}} \rightarrow v^{d_{\mathrm{SM}}}$ acquires a VEV at the electroweak scale v, NDA then suggests that scale invariance is broken at a scale $\Lambda_{\mathrm{IR}}^{\mathrm{NDA}}$,

$$(\Lambda_{\mathrm{IR}}^{\mathrm{NDA}})^4 \simeq \mathcal{L}^{\mathrm{eff}}(\mathcal{O}_\mathcal{U} \rightarrow (\Lambda_{\mathrm{IR}}^{\mathrm{NDA}})^{d_\mathcal{U}}, O_{\mathrm{SM}} \rightarrow v^{d_{\mathrm{SM}}}), \quad (B2)$$

when the term in Eq. (B1) is of the same size as a generic four dimensional operator of the unparticle sector. This leads to

$$\Lambda_{\mathrm{IR}}^{\mathrm{NDA}} \sim v \left(\frac{\Lambda_\mathcal{U}}{M_\mathcal{U}} \right)^{(d_{\mathrm{UV}} - d_\mathcal{U})/(4 - d_\mathcal{U})} \left(\frac{v}{M_\mathcal{U}} \right)^{(d_{\mathrm{SM}}/(4 - d_\mathcal{U})) - 1}. \tag{B3}$$

The above equation reduces to (3.3) in Ref. [9] for $O_{\mathrm{SM}} = |H|^2$ with $d_{\mathrm{SM}} = 2$.

In our work $O_{\mathrm{SM}} \rightarrow O_{\mathrm{TC}} = \bar{Q}Q$ (6) with $d_{\mathrm{SM}} = 3$. The role of the electroweak scale is taken by $v \rightarrow \Lambda_{\mathrm{TC}}$. The knowledge of the UV completion settles the question on the UV dimension; $d_{\mathrm{UV}} = 3$. Furthermore, the anomalous dimension $\gamma_\mathcal{T}$ introduces an additional multiplicative factor $w_T^{\gamma_\mathcal{T}}$ (9) to the Lagrangian density (B1) as an artefact of walking technicolor. Altogether this yields

$$\Lambda_{\mathrm{IR}}^{\mathrm{NDA}} \rightarrow \Lambda_{\mathrm{TC}} (w_T)^{\gamma_\mathcal{T}/(1 + \gamma_u)} \left(\frac{\Lambda_\mathcal{U}}{M_\mathcal{U}} \right)^{\gamma_u/(1 + \gamma_u)}$$
$$\times \left(\frac{\Lambda_{\mathrm{TC}}}{M_\mathcal{U}} \right)^{(2 - \gamma_u)/(1 + \gamma_u)}. \tag{B4}$$

The crucial question is then how this compares with the IR cutoff in (20). We find that, for generic values of the parameters, the condensate IR cutoff is

$$\Lambda_{\mathrm{IR}} \gtrsim \Lambda_{\mathrm{IR}}^{\mathrm{NDA}}, \tag{B5}$$

larger than the IR cutoff suggested by NDA. The essential

point is that the VEV is sensitive to the UV cutoff for $d_\mathcal{U} \gtrsim 2$ through the spectral integral (18), whereas there is no such notion in the NDA. In fact in the NDA, cf. Eq. (B2), it is built in that the unparticle operator assumes the IR-scale Λ_{IR}^{NDA}.

This suggests that parametric equality (B5) is reached in the IR sensitive domain $d_\mathcal{U} \gtrsim 1$. Most reassuringly it is verified that in this domain both IR cutoffs scale as $\Lambda_{TC}(\Lambda_\mathcal{U}/M_\mathcal{U})^{2/3}$. In the UV domain $d_\mathcal{U} \lesssim 3$ the scaling differs, $\Lambda_{IR}^{NDA} \sim \Lambda_{TC}(\Lambda_{TC}/M_\mathcal{U})^2$ and $\Lambda_{IR} \sim \Lambda_{TC}(\Lambda_\mathcal{U}/M_\mathcal{U})^{2/3}$. In Fig. 2 (right) the logarithm of the ratio of the two IR cutoffs is plotted against $\gamma_\mathcal{U}$ for specified input values and provides an example of the qualitative statement made above.

APPENDIX C: NORMALIZATION FACTOR $B_{d_\mathcal{U}}$

In this appendix we shall discuss the normalization factor $b_{d_\mathcal{U}} \equiv \sqrt{B_{d_\mathcal{U}}/(2\pi)}$ (11) and (16). This is a necessary task in order to extract quantitative results from the unparticle VEV equation (17). Generally we do not know the behavior of $B_{d_\mathcal{U}}$ as a function of $d_\mathcal{U} \equiv 3 - \gamma_\mathcal{U}$, except around $d_\mathcal{U} \gtrsim 1$ and at $d_\mathcal{U} = 3$. Firstly, it is a fact that at $d_\mathcal{U} = 1$ the operator $\mathcal{O}_\mathcal{U}$ is equivalent to a free field [36]. This fixes the normalization factor,

$$B_{d_\mathcal{U}} = 2\pi(d_\mathcal{U} - 1) + \mathcal{O}((d_\mathcal{U} - 1)^\eta)$$
$$\text{with } \eta > 1, \quad \text{for } d_\mathcal{U} \gtrsim 1, \quad \text{(C1)}$$

around $d_\mathcal{U} \gtrsim 1$ in a model independent way. Since with (C1) the spectral function (11) precisely produces the free massless field limit,

$$\lim_{d_\mathcal{U} \to 1^+} \rho_{d_\mathcal{U}}(s) = \lim_{d_\mathcal{U} \to 1^+} (d_\mathcal{U} - 1)s^{d_\mathcal{U}-2}\theta(s) = \delta(s), \quad \text{(C2)}$$

with unit residue. This is equivalent to Georgi's [4] requirement that $A_{d_\mathcal{U}}$, in the notation of that paper, has to reproduce the 1-particle phase space in that limit. Second, in our model at $d_\mathcal{U} = 3$ the fermions are free fields and the unparticle propagator, which we write in a Källén-Lehmann form,

$$\Delta_\mathcal{U}(q^2) \equiv -i \int d^4 x e^{iq \cdot x} \langle 0|T\mathcal{O}_\mathcal{U}(x)\mathcal{O}_\mathcal{U}^\dagger(0)|0\rangle$$
$$= -\int \frac{ds\rho_{d_\mathcal{U}}(s)}{s - q^2 - i0} + \text{s.t.,} \quad \text{(C3)}$$

has to reduce to the free fermion loop depicted in Fig. 3. The letters s.t. denote possible subtraction terms which are relevant for $d_\mathcal{U} > 2$ to be discussed in Sec. II B. This fixes the spectral function or the normalization factor $B_{d_\mathcal{U}}$ (11) at $d_\mathcal{U} = 3$ to

$$\rho_3(s) = s\frac{N_U}{8\pi^2} \leftrightarrow B_3 = \frac{N_U}{4\pi}. \quad \text{(C4)}$$

This value is different from

$$A_3 = 1/(256\pi^3) \quad \text{(C5)}$$

obtained from the normalization,

$$A_{d_\mathcal{U}} = \frac{16\pi^{5/2}}{(2\pi)^{2d_\mathcal{U}}} \frac{\Gamma(d_\mathcal{U} + 1/2)}{\Gamma(d_\mathcal{U} - 1)\Gamma(2d_\mathcal{U})}$$
$$= \frac{1}{2}\frac{1}{(4\pi)^{2d_\mathcal{U}-3}}\frac{1}{\Gamma(d_\mathcal{U})\Gamma(d_\mathcal{U} - 1)} \quad \text{(C6)}$$

in Ref. [4]. This is not surprising since in this reference it was proposed to adapt $A_{d_\mathcal{U}}$ as the analytic continuation of the phase space of an integer number of $d_\mathcal{U}$ massless particles. The operator $\mathcal{O}_\mathcal{U} = \varphi_0^{d_\mathcal{U}}$, with φ_0 denoting a free massless scalar field, is of course a special realization of the unparticle scenario for integer scaling dimension $d_\mathcal{U}$. We would like to emphasize that in Ref. [4] it was clearly stated that the actual normalization might be rather different from the one in a concrete model.

In the case at hand $\mathcal{O}_\mathcal{U}|_{d_\mathcal{U}=3} = \bar{\Psi}_0\Psi_0$ corresponds to two free fermions, instead of three free boson, which explains the difference. One could in principle generalize this scenario to higher powers of pairs of free fermion fields and adapt it as the normalization conditions for $B_{d_\mathcal{U}}$ via analytic continuation. Unfortunately it appears that no closed formula can be written down for this case. In order to obtain some quantitative results we resort to model $B_{d_\mathcal{U}}$ by a quadratic interpolation function,

$$(B_{d_\mathcal{U}})_{\text{interpol}} = 2\pi(d_\mathcal{U} - 1) + \left(\frac{N_U}{16\pi} - \pi\right)(d_\mathcal{U} - 1)^2. \quad \text{(C7)}$$

Please not that these interpolation formula is positive as required by a positive spectral function (11). We would like to emphasize once more that the only firmly known parts are B_3 (model dependent) and the behavior around B_1 (model independent).

APPENDIX D: UNPARTICLE LIMIT OF HEIDI MODELS

In the HEIDI model [28] the Higgs-Higgs propagator assumes the

$$\Delta_{hh}(q^2) = \frac{1}{q^2 - M^2 - c^2(m_0^2 - q^2 - i0)^{(d-6)/2}}, \quad \text{(D1)}$$

same form as in (51). The letter c denotes a dimensionful constant proportional to the mixing parameter, m_0 is the mass of the lowest Kaluza-Klein excitations and M is the tree-level Higgs mass. Comparing with (51) and (D1) it is readily seen that identifying $(d - 6)/2 = d_\mathcal{U} - 2$ and m_0 with Λ_{IR} leads to a qualitatively similar propagator.

The authors in Ref. [28] attempted to reproduce the strictly scale invariant unparticle propagator in Eq. (48)

[6]. This can be achieved by making the mixing arbitrarily large $c \gg 1$, adopting $m_0 \to 0$ and keeping M fixed. The Higgs-Higgs propagator then becomes the inverse unparticle propagator, which in turn looks like an unparticle propagator with reversed scaling power. On this basis the identification $(d-6)/2 = -(d_{\mathcal{U}} - 2) \to d_{\mathcal{U}} = 5 - d/2$ was proposed in [28]. In our model the unparticle itself couples to the techniquark or technihadrons directly and such a limit does therefore not seem necessary for unparticlelike effects in phenomenology.

Finally, the excess of the LEP data in the Higgs strahlung search of 2.3σ and 1.7σ around 98 GeV and 115 GeV, respectively, were fitted to the HEIDI model [28]. The first peak is interpreted as the Higgs resonance and the second one as the onset of the continuum. Comparing the fitted parameters in [43] with k_1 in Eq. (57) and input values as given in the caption of Fig. 2 it is seen that they are of the same order of magnitude.

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