tbW vertex in the littlest Higgs model with T parity

F. Peñuñuri[1] and F. Larios[2]

[1]*Facultad de Ingenieria, Universidad Autónoma de Yucatán, A.P. 150, Cordemex, Mérida, Yucatan, México*
[2]*Departamento de Física Aplicada, CINVESTAV-Mérida, A.P. 73, 97310 Mérida, Yucatán, México*
(Received 3 November 2008; published 23 January 2009)

A study of the effective tbW vertex is done in the littlest Higgs model with T parity that includes the one-loop induced weak dipole coefficient f_{2R}. The top's width and the W-boson helicity in the $t \to bW^+$ decay as well as the t-channel and the s-channel modes of single top quark production at the LHC are then obtained for the tbW coupling. Our calculation is done in the Feynman-'t Hooft gauge, and we provide details of the analysis, like exact formulas (to all orders of the expansion variable v/f) of masses and mixing angles of all of the particles involved. Also, a complete and exact diagonalization (and normalization) of the scalar sector of the model is made.

DOI: 10.1103/PhysRevD.79.015013 PACS numbers: 12.60.Cn, 12.60.Fr

I. INTRODUCTION

The top quark plays a major role in the research program of the LHC. The top is the only quark that decays weakly before hadronization; therefore, we have an opportunity to study *bare* quark properties like spin, mass, and couplings [1,2]. Recent measurements of the single top quark production as well as the W helicity in the $t \to bW^+$ decay have been made by the D0 and CDF groups at the Tevatron, and these have (for the first time) set direct constraints on the effective tbW vertex [3]. On the other hand, the high production of top quarks at the LHC will make it possible to probe directly this vertex down to a few percent deviation level for the left-handed coefficient f_{1L} and to set limits of order 10^{-2} for f_{2R} and of order 10^{-1} for the right-handed f_{1R} and f_{1L} [4]. From the theoretical standpoint, observables that depend directly on the tbW coupling like single top production, the top's width, and the W helicity in the top's decay have been studied in models beyond the standard model (SM) like the minimal supersymmetric standard model [5] and the topcolor-assisted technicolor (TC2) model [6].

In the SM the Higgs boson receives large quadratic divergent corrections from the heavy gauge bosons and from this fermion. Models beyond the SM are studied that alleviate this problem; two important examples are supersymmetry and technicolor (and TC2) [7]. Another possible solution is provided by the recently proposed little Higgs models [8,9] (for a review, see Ref. [10]). In these models the quadratic divergent Higgs mass corrections get canceled at the one-loop level via the contribution from certain (very heavy) partners of the gauge bosons and the top quark (i.e. the $W_H^\pm$ boson, the Z_H boson, and the T quark). One explicit model has become well known, and it is called the "littlest Higgs model" (LH) [9]. The LH model is based on a nonlinear sigma model of an $SU(5)/SO(5)$ global symmetry breaking. It consists of two $SU(2) \times U(1)$ gauge symmetries that break down to the SM gauge symmetry at a certain scale f. The phenomenology of the model deals with heavy partners of the SM gauge bosons, like $W_H^\pm$, Z_H, and a heavy *photon* A_H, as well as a heavy partner of the top quark T [11]. These heavy partners mix with the lighter SM gauge bosons, and this gives rise to tree-level contributions to precision electroweak observables. Therefore, strong constraints have greatly limited the parameter range of the model (for instance: $f \geq 4$ TeV) [12]. A way out of this obstacle is given by implementing a new symmetry called T parity, where T-parity even and T-parity odd particles do not mix [13]. There is one model that is often studied in the literature; it is based on the previous LH model and is known as the littlest Higgs model with T parity (LHT) [14]. Electroweak precision constraints for the LHT model allow the scale f to be as low as $\sim$500 GeV [15]. This model has therefore received more attention recently, with many phenomenological studies on production and decays of the new heavy particles [16] as well as theoretical studies such as T-parity violation [17], top quark induced vacuum alignment [18], and two vacuum expectation value (VEV) scales f in LH models [19].

In this paper we study the tbW vertex in the context of the LHT. We will often refer to and will use the notation of Ref. [20]. A detailed explanation of the model can be found in Refs. [20,21]. In this work we focus on the interactions that are relevant to the study of the effective tbW vertex.

In the literature an expansion in powers of $\epsilon = v/f$ is usually made for the masses and mixing angles derived from the Lagrangian of the model. Here we have obtained the exact (all powers in $\epsilon \equiv v/f$) formulas for masses and mixings. Similar expressions have already appeared in Ref. [22], and we have found agreement. Moreover, we provide in detail the diagonalization procedure of the scalar sector, including the Goldstone bosons that are eaten by the gauge bosons and that participate in the one-loop calculation as is done in the Feynman-'t Hooft gauge. We provide Feynman rules that are not found in previous studies of the model.

The next section has the brief presentation of the LHT Lagrangians (kinetic and Yukawa) and the definition of mass eigenstate fields in terms of the original interaction

 © 2009 The American Physical Society

eigenstates. Then, in the following section, we will discuss the effective tbW vertex obtained from tree- and one-loop-level contributions. From this effective vertex we compute some of the observables associated to the top quark, like the top's decay width, the W-boson helicity in the $t \to bW^+$ decay, and the single top production process in the two most important modes: the t channel and the s channel.

II. THE LITTLEST HIGGS MODEL WITH T PARITY

The LHT model is based on a nonlinear sigma model for an $SU(5)/SO(5)$ symmetry breaking. The nonlinear Σ field is given as [15]

$$\Sigma = e^{2i\Pi/f}\Sigma_0, \quad \text{with} \quad \Sigma_0 = \begin{pmatrix} \mathbf{0}_{2\times 2} & \mathbf{0}_{2\times 1} & \mathbf{1}_{2\times 2} \\ \mathbf{0}_{1\times 2} & 1 & \mathbf{0}_{1\times 2} \\ \mathbf{1}_{2\times 2} & \mathbf{0}_{2\times 1} & \mathbf{0}_{2\times 2} \end{pmatrix}, \tag{1}$$

where $f \sim \mathcal{O}(1)$ TeV is the symmetry breaking scale known as the "pion decay constant." The "pion matrix" contains a total of 14 pion fields [15]:

$$\Pi = \begin{pmatrix} -\omega^0/2 - \eta/\sqrt{20} & -\omega^+/\sqrt{2} & -i\pi^+/\sqrt{2} & -i\phi^{++} & -i\phi^+/\sqrt{2} \\ -\omega^-/\sqrt{2} & \omega^0/2 - \eta/\sqrt{20} & (v+h+i\pi^0)/2 & -i\phi^+/\sqrt{2} & (-i\phi^0 + \phi_P^0)/\sqrt{2} \\ i\pi^-/\sqrt{2} & (v+h-i\pi^0)/2 & \sqrt{4/5}\eta & -i\pi^+/\sqrt{2} & (v+h+i\pi^0)/2 \\ i\phi^{--} & i\phi^-/\sqrt{2} & i\pi^-/\sqrt{2} & -\omega^0/2 - \eta/\sqrt{20} & -\omega^-/\sqrt{2} \\ i\phi^-/\sqrt{2} & (i\phi^0 + \phi_P^0)/\sqrt{2} & (v+h-i\pi^0)/2 & -\omega^+/\sqrt{2} & \omega^0/2 - \eta/\sqrt{20} \end{pmatrix}. \tag{2}$$

Seven of these fields get eaten by the gauge bosons of the model. The other seven become physical; in particular, the h field becomes the (little) Higgs field whose mass is protected from quadratic divergencies by the *collective symmetry breaking* mechanism of the little Higgs model [9]

An $[SU(2) \times U(1)]^2$ subgroup of the global $SU(5)$ symmetry is gauged. The gauged generators have the form

$$\begin{aligned} Q_1^a &= \begin{pmatrix} \sigma^a/2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ Y_1 &= \mathrm{diag}(3, 3, -2, -2, -2)/10, \\ Q_2^a &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\sigma^{a*}/2 \end{pmatrix}, \\ Y_2 &= \mathrm{diag}(2, 2, 2, -3, -3)/10. \end{aligned} \tag{3}$$

The kinetic term for the Σ field can be written as

$$\mathcal{L}_{\mathrm{kin}} = \frac{f^2}{8} \mathrm{Tr} D_\mu \Sigma (D^\mu \Sigma)^\dagger, \tag{4}$$

where

$$\begin{aligned} D_\mu \Sigma = \partial_\mu \Sigma - i \sum_j [g_j W_j^a (Q_j^a \Sigma + \Sigma Q_j^{aT}) \\ + g'_j B_j (Y_j \Sigma + \Sigma Y_j)], \end{aligned} \tag{5}$$

with $j = 1, 2$. Here B_j and W_j^a are the $U(1)_j$ and $SU(2)_j$ gauge fields, respectively, and g'_j and g_j are the corresponding coupling constants. The VEV Σ_0 breaks the extended gauge group $[SU(2) \times U(1)]^2$ down to the diagonal subgroup, which is identified with the standard model electroweak group $SU(2)_L \times U(1)_Y$.

The field H has the appropriate quantum numbers to be identified with the SM Higgs; after electroweak symmetry breaking (EWSB), it can be decomposed as $H = (-i\pi^+, [(v + h + i\pi^0)/\sqrt{2}])^T$, where $v = 246$ GeV is the EWSB scale.

The Lagrangian in Eq. (4) is invariant under T parity provided that $g_1 = g_2 (\equiv \sqrt{2} g)$ and $g'_1 = g'_2 (\equiv \sqrt{2} g')$. The T-parity gauge boson eigenstates (before EWSB) have the simple form $W_\pm = (W_1 \pm W_2)/\sqrt{2}$, $B_\pm = (B_1 \pm B_2)/\sqrt{2}$, where W_+ and B_+ are the standard model gauge bosons and are T-even, whereas W_- and B_- are the additional, heavy, T-odd states. From now on we will denote W_- and B_- as W_H and B_H, whereas W_+ and B_+ will be written simply as W and B. After EWSB, the T-even neutral states W^3 and B mix to produce the SM Z boson and the photon. Since they do not mix with the heavy T-odd states, the Weinberg angle is given by the usual SM relation $\tan\theta_w = g'/g$, and no corrections to precision electroweak observables occur at tree level. The mixing for the neutral gauge boson mass eigenstates is written as

$$\begin{aligned} \begin{pmatrix} Z \\ A \end{pmatrix} &= \begin{pmatrix} c_w & -s_w \\ s_w & c_w \end{pmatrix} \begin{pmatrix} W^3 \\ B \end{pmatrix}, \\ \begin{pmatrix} Z_H \\ A_H \end{pmatrix} &= \begin{pmatrix} c_H & s_H \\ -s_H & c_H \end{pmatrix} \begin{pmatrix} W_H^3 \\ B_H \end{pmatrix}, \end{aligned} \tag{6}$$

where $s_w = \sin\theta_w \simeq \sqrt{0.223}$ refers to the SM mixing angle and $s_H = \sin\theta_H$ refers to the heavy boson mixing angle. $\tan\theta_H$ ($\equiv t_H$) must satisfy the equation

$$t_H^2 + 2a_H t_H - 1 = 0, \quad \text{with}$$
$$a_H = 4\left(\frac{1}{t_w} - \frac{t_w}{5}\right) \Big/ s_v^2 - \left(\frac{1}{t_w} - t_w\right) \Big/ 2, \tag{7}$$

where $t_w = s_w/c_w \simeq 0.536$ and

$$s_v \equiv \sin\sqrt{2}\epsilon, \qquad c_v \equiv \cos\sqrt{2}\epsilon, \quad \text{with} \quad \epsilon \equiv \frac{v}{f}. \tag{8}$$

With this value of t_w we obtain

$$\tan\theta_H = \sqrt{a_H^2 + 1} - a_H \simeq 0.142\epsilon^2 - 0.068\epsilon^4. \tag{9}$$

The gauge boson mass terms obtained from Eq. (4) are as follows:

$$\begin{aligned} M_W^2 &= \left(\frac{fg}{2}\right)^2(1 - c_v), \qquad M_Z^2 = \left(\frac{fg}{2c_w}\right)^2(1 - c_v),\\ M_{W_H}^2 &= \left(\frac{fg}{2}\right)^2(3 + c_v), \qquad M_{Z_H}^2 = (fg)^2\left(1 - \frac{s_v^2}{8} + \delta_H\right)\\ M_{A_H}^2 &= (fg)^2\left(t_w^2\left(\frac{1}{5} - \frac{s_v^2}{8}\right) - \delta_H\right),\\ \delta_H &\equiv \frac{t_H c_H^2}{8}\left(2s_v^2 t_w + t_H\left[-8 + t_w^2\left(\frac{8}{5} - s_v^2\right) + s_v^2\right]\right). \end{aligned} \tag{10}$$

Similar expressions are given in Ref. [22]; our formulas are presented differently but agree with theirs (our mixing angle θ_H differs in sign).

A. The Yukawa couplings for quarks

The little Higgs model introduces a heavy partner of the top quark (the heavy top quark) with the purpose of canceling out the one-loop quadratic divergent radiative corrections to the Higgs mass [9]. When T parity is implemented in the fermion sector of the model, we require the existence of mirror partners for each of the original fermions. This means that for the third family we have, in addition to the usual bottom and top quarks, the mirror bottom and the mirror top, as well as the heavy top quark with its own mirror quark.

The T-parity invariant Yukawa Lagrangian of the LHT model is separated into four parts that generate masses for mirror quarks, down-type quarks, the first two generations of up-type quarks, and finally the top quark and its heavy partner. It is the latter that is defined in such a way that the top quark quadratic radiative corrections to the Higgs mass are canceled. We are interested only in the Yukawa Lagrangian of the third family. A presentation that includes the first two families and the corresponding Cabibbo-Kobayashi-Maskawa mixing can be found in Ref. [23]. For the purpose of our work we consider only the mirror and top quark Yukawa Lagrangians [20]:

$$\begin{aligned} \mathcal{L}_\kappa &= -\kappa f(\bar\Psi_2\xi\Psi_c + \bar\Psi_1\Sigma_0\Omega\xi^\dagger\Omega\Psi_c) + \text{H.c.},\\ \mathcal{L}_{\text{down}} &= \frac{i\lambda_d}{2\sqrt{2}} f\epsilon_{ij}\epsilon_{xyz}[\bar\Psi_2'\Sigma_{iy}\Sigma_{jz}X\\ &\quad - (\bar\Psi_1'\Sigma_0)_x\tilde\Sigma_{iy}\tilde\Sigma_{jz}\tilde X]d_R^+,\\ \mathcal{L}_t &= -\frac{\lambda_1 f}{2\sqrt{2}}\epsilon_{ijk}\epsilon_{xy}[(\bar Q_1)_i\Sigma_{jx}\Sigma_{ky} - (\bar Q_2\Sigma_0)_i\tilde\Sigma_{jx}\tilde\Sigma_{ky}]u_R^+\\ &\quad - \lambda_2 f(\bar U_{L_1}U_{R_1} + \bar U_{L_2}U_{R_2}) + \text{H.c.}, \end{aligned} \tag{11}$$

where $\xi \equiv e^{i\Pi/f}$, $\Omega \equiv \text{diag}\{1, 1, -1, 1, 1\}$, and $\tilde\Sigma \equiv \Sigma_0\Omega\Sigma^\dagger\Omega\Sigma_0$. ϵ_{ijk} and ϵ_{xy} are antisymmetric tensors where $ijk = 1, 2, 3$ and $xy = 4, 5$. The Lagrangian $\mathcal{L}_{\text{down}}$ that gives mass to the bottom quark will be not be used in our calculation as we take $\lambda_d \equiv 0$. The details of this Lagrangian can be found in Ref. [20]. Nevertheless, we provide Feynman rules for $\lambda_d \neq 0$.

In order to obtain the (exact) expressions for masses and mixings, we will use the VEV of the field Σ as given in Eq. (A2), as well as the VEV of the ξ field:

$$\langle\xi\rangle_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0\\ 0 & \frac{1+c_v'}{2} & \frac{i}{\sqrt{2}}s_v' & 0 & \frac{c_v'-1}{2}\\ 0 & \frac{i}{\sqrt{2}}s_v' & c_v' & 0 & \frac{i}{\sqrt{2}}s_v'\\ 0 & 0 & 0 & 1 & 0\\ 0 & \frac{c_v'-1}{2} & \frac{i}{\sqrt{2}}s_v' & 0 & \frac{1+c_v'}{2} \end{pmatrix}, \tag{12}$$

where $c_v' \equiv \cos(\epsilon/\sqrt{2})$ and $s_v' \equiv \sin(\epsilon/\sqrt{2})$. The u_R^+, U_{R_1}, and U_{R_2} quark fields are right-handed $SU(2)$ singlets. The upper plus sign in u_R^+ denotes that it is a T-even (T-parity eigenstate) fermion. With the other two (U_{R_1} and U_{R_2}), we can define T-even and T-odd linear combinations: $U_R^\pm \equiv (U_{R_1} \mp U_{R_2})/\sqrt{2}$ [20].

On the other hand, the Ψ and Q fields are left-handed $SU(5)$ multiplets defined as

$$\begin{aligned} \Psi_1 &= \begin{pmatrix} id_1\\ -iu_1\\ 0\\ 0\\ 0 \end{pmatrix}, \qquad \Psi_2 = \begin{pmatrix} 0\\ 0\\ 0\\ id_2\\ -iu_2 \end{pmatrix},\\ Q_1 &= \begin{pmatrix} id_1\\ -iu_1\\ U_{L_1}\\ 0\\ 0 \end{pmatrix}, \qquad Q_2 = \begin{pmatrix} 0\\ 0\\ U_{L_2}\\ id_2\\ -iu_2 \end{pmatrix}. \end{aligned} \tag{13}$$

From these we define the T-parity eigenstates $u_L^\pm \equiv (u_1 \mp u_2)/\sqrt{2}$, $d_L^\pm \equiv (d_1 \mp d_2)/\sqrt{2}$, and $U_L^\pm \equiv (U_{L_1} \mp U_{L_2})/\sqrt{2}$.

The Ψ_c multiplet is composed of 5 right-handed T-odd quark fields:

$$\Psi_c \equiv (ib^-, -ia^-, \chi^-, in^-, -ip^-)^T. \tag{14}$$

It turns out that two linear combinations of these become the right-handed mass eigenstates of the mirror top and mirror bottom quarks. The other three linear combinations are extra T-odd fermions that are assumed to have very large Dirac masses, so that they decouple from the main theory [14,21].

Below, we write down the mass eigenstates that arise from the Lagrangian in Eq. (11) for the top and its heavy partner:

$$\begin{pmatrix} t^+_{L(R)} \\ T^+_{L(R)} \end{pmatrix} = \begin{pmatrix} c_{L(R)} & -s_{L(R)} \\ c_{L(R)} & s_{L(R)} \end{pmatrix} \begin{pmatrix} u^+_{L(R)} \\ U^+_{L(R)} \end{pmatrix}, \tag{15}$$

where $c_{L(R)} = \cos\theta_{L(R)}$, $s_{L(R)} = \sin\theta_{L(R)}$, and $\theta_{L(R)}$ is the mixing angle of the left (right) top and heavy top quarks.

The mixing angles must satisfy two equations, which we write in terms of $\tan\theta_{L(R)}$:

$$r(1 + c_v) + \sqrt{2} r s_v \tan\theta_L - 2\tan\theta_R = 0,$$
$$\sqrt{2} r s_v \tan\theta_R - 2\tan\theta_L - r(1 + c_v)\tan\theta_L \tan\theta_R = 0, \tag{16}$$

with $r \equiv \lambda_1/\lambda_2$.

The solutions of these equations are

$$\tan\theta_L = \sqrt{a_m^2 + 1} - a_m,$$
$$\tan\theta_R = \frac{r}{2}(1 + c_v + \sqrt{2} s_v \tan\theta_L),$$
$$a_m \equiv \frac{1 + c_v - 3s_v^2/2 + 2/r^2}{\sqrt{2} s_v (1 + c_v)}.$$

The masses of the top and its heavy partner are

$$m_{t^+} = \tfrac{1}{2} f\lambda_2 c_L c_R [2\tan\theta_L \tan\theta_R + r(\sqrt{2} s_v - (1 + c_v)\tan\theta_L)],$$
$$m_{T^+} = \tfrac{1}{2} f\lambda_2 c_L c_R [2 + r\tan\theta_R (1 + c_v + \sqrt{2} s_v \tan\theta_L)].$$

Expanding in powers of ϵ,

$$\tan\theta_L \simeq \frac{r^2}{1 + r^2}\epsilon + \frac{r^4 + 2r^2 - 5}{6(1 + r^2)^3} r^2 \epsilon^3,$$
$$\tan\theta_R \simeq r + \frac{r^2 - 1}{1 + r^2}\frac{r}{2}\epsilon^2,$$
$$m_{t^+} \simeq \frac{f\lambda_1}{\sqrt{1 + r^2}}\left(\epsilon - \frac{2 + r^2 + 2r^4}{6(1 + r^2)^2}\epsilon^3\right), \tag{17}$$
$$m_{T^+} \simeq f\lambda_2\sqrt{1 + r^2}\left(1 - \frac{r^2}{2(1 + r^2)^2}\epsilon^2\right),$$
$$\cos\theta_L \equiv c_L \simeq 1 - \frac{r^4}{2(1 + r^2)^2}\epsilon^2.$$

Our formulas for the mixing angles and masses are in agreement with those of Ref. [22], where $\alpha \equiv \theta_R$ and $\beta \equiv \theta_L$.

The T-odd top and heavy top quarks are defined as

$$t^-_L = u^-_L, \qquad T^-_L = U^-_L, \qquad t^-_R = p'^-_R, \qquad T^-_R = U^-_R, \tag{18}$$

where the p'^- field comes from the redefinition of the right-handed T-odd fields of Eq. (14):

$$\begin{pmatrix} p'^-_R \\ a'^-_R \\ \chi'^-_R \end{pmatrix} = \begin{pmatrix} \frac{1+c'_v}{2} & \frac{c'_v-1}{2} & \frac{-1}{\sqrt{2}} s'_v \\ \frac{c'_v-1}{2} & \frac{1+c'_v}{2} & \frac{-1}{\sqrt{2}} s'_v \\ \frac{1}{\sqrt{2}} s'_v & \frac{1}{\sqrt{2}} s'_v & c'_v \end{pmatrix} \begin{pmatrix} p^- \\ a^- \\ \chi^- \end{pmatrix}. \tag{19}$$

Notice that the mass of t^- comes from the mirror fermion Lagrangian, whereas the mass of T^- comes from the top quark Lagrangian [see Eq. (11)]:

$$m_{t^-} = \sqrt{2} f\kappa, \qquad m_{T^-} = f\lambda_2.$$

For the calculations in this work, we will set $\kappa = 1$ so that the masses of mirror fermions are just $\sqrt{2} f$. The presence of the LHT mirror fermions is vital for the good high energy behavior of the model; in particular, they play an essential role in the scattering process $u\bar{u} \to W_H^+ W_H^-$ [20]. Our choice of $\kappa = 1$ and the corresponding values of the T-odd fermion mass respects the unitarity bounds of this process, as well as the limits coming from the contributions to the four fermion contact interaction $e^+e^- \to q\bar{q}$ [21].

For completeness, let us write down the masses of the T-even and T-odd bottom quarks. The mass of b^+ is given by the down-type Yukawa Lagrangian given in Eq. (11) (see Ref. [20]):

$$m_{b^+} = \frac{f}{\sqrt{2}}\lambda_d s_v c_v^{-1/4}, \qquad m_{b^-} = \sqrt{2} f\kappa.$$

Notice that the formulas we have obtained are exact (to all orders in the ϵ expansion); in particular, the mirror fermion masses are equal for t^- and b^- quarks. We remind the reader that in our calculation we take the mass of b^+ as zero ($\lambda_d \equiv 0$). Feynman rules with the mass eigenstates can be found in Appendix B.

III. THE $\bar{t}bW^+$ COUPLING IN THE LHT MODEL

Let us define the effective $\bar{t}bW^+$ coupling as follows:

$$\mathcal{L}_{tbW} = \frac{g}{\sqrt{2}} W^-_\mu \bar{b}\gamma^\mu (f_{1L} P_L + f_{1R} P_R) t - \frac{g}{\sqrt{2} M_W}\partial_\nu W^-_\mu \bar{b}\sigma^{\mu\nu}(f_{2L} P_L + f_{2R} P_R) t + \text{H.c.}, \tag{20}$$

where we have used the mass scale m_W that is also used in the literature [24–26].

In the SM the values of the form factors at tree level are $f_{1L} = V_{tb} \simeq 1$ and $f_{1R} = f_{2L} = f_{2R} = 0$. Radiative corrections to the factors f_{1R} and f_{2L} must be zero if we neglect the mass of the bottom quark. We take $m_b \equiv 0$ in this work, so we set $f_{1R} = f_{2L} \equiv 0$ for this study. These couplings can be probed by studying the top decay $t \to bW^+$ and the single top production processes [24,27]. The dimension-5 coupling f_{2R} is different from zero at one loop: we obtain $f_{2R}^{\rm SM} = 0.002\,01$ $(0.002\,14)$ for $m_H = 120\,(150)$ GeV. This value seems to be too small to be probed at the LHC [4]. In fact, the dominant radiative corrections for the top width or single top production comes from QCD [27]. We would like to know if the coefficient f_{2R} predicted by the LHT model could be large enough to be measured at the LHC.

In the LHT model, the coefficient f_{1L} is modified at tree level by the tT mixing angle θ_L ($f_{1L} = c_L$). The tree-level tbW vertex is reduced by the factor c_L, and this translates into a lower production at the LHC [28].

We have performed the one-loop contribution to f_{2R} in the LHT model. We have worked in the Feynman-'t Hooft gauge, where there are a total of 47 diagrams to compute if we take the bottom quark mass as zero. Some of the diagrams are shown in Fig. 1. (We provide the Feynman rules necessary for such a calculation in Appendix B.) Notice that the Goldstone bosons in the original Lagrangian have to be diagonalized and normalized. We have done all of this exactly (at all orders in powers of ϵ) in Appendix A. The exponential expansion in the Lagrangians of Eqs. (4) and (11) generate vertices of dimension 4 and higher that contribute at one loop to f_{2R}. As it turns out, the contribution from the dimension-4 terms to the tbW vertex are finite, whereas the contribution from the higher dimension terms is divergent. This is no surprise because the LHT model is a nonrenormalizable effective low energy model with a cutoff scale ($\Lambda \sim 4\pi f$). In principle, all of the operators that are consistent with the symmetries of the LHT model should be considered [15]. In our study, we disregard effects from higher dimension terms and keep only the contribution from the dimension-4 couplings that render a finite result [29].

Concerning the specific numerical values used for the parameters of the model, we have chosen values from the allowed region of f vs λ_2 that is shown in Fig. 1 of Ref. [22]. The mass of the top quark is taken $m_t = 173$ GeV, and this sets the value of $\lambda_1 \simeq 1$ (with a very small dependence on the value of f). The masses of the t^- and b^- mirror quarks is taken as $\sqrt{2}f$. The masses of the physical T-odd scalars $\phi'^{\pm}$, ϕ'^0, and $\phi_P'^0$ are taken as $\sqrt{2}m_H f/v$ ($= 0.69f$ for $m_H = 120$ GeV) as it is done in the literature [11,21]. As is well known, in the Feynman-'t Hooft gauge the masses of Goldstone bosons ($\pi'^{\pm}$, π'^0, $\omega'^{\pm}$, ω'^0, and η') are equal to the masses of their corresponding gauge bosons which are given in Eq. (10). We have chosen a range of the scale f between 550 and 1550 GeV. Smaller values of f are prohibited by the low energy data [15]. Higher values are allowed but not interesting as the value of f_{2R} remains essentially constant above the 1.5 TeV scale.

The variation of f_{2R} from $f_{2R}^{\rm SM}$ as predicted by the LHT model turns out to be of the order expected by a one-loop correction. In fact, $\Delta f_{2R}/f_{2R}^{\rm SM}$ is under 20% for the allowed values of the scale f and the Yukawa coefficient λ_2. We show the variation of f_{2R} as a function of the scale f in Fig. 2. We also show in Fig. 3 how the variation in f_{2R} diminishes with higher values of the Higgs mass. In contrast to the SM the LHT model allows for higher values of m_H and is still compatible with the electroweak precision data [15]; however, we have found that for bigger m_H the deviation in f_{2R} gets smaller and thus less interesting. (Observe in Fig. 1 that the Higgs field also appears in the contributions from the LHT heavy states.) From now on we will assume a fixed value $m_H = 120$ GeV.

It is possible to obtain the variation in the top width, the W helicity in the $t \to bW^+$ decay, as well as the s and t channels of the single top production processes once we have the effective tbW coupling. A general analysis of this coupling and the observables mentioned has been done in Ref. [24]. Let us apply this approach to the effective tbW coupling as predicted by the LHT model. The total $t \to bW^+$ decay width of the top quark can be written as a sum of the contributions from each of the three polarizations of the W^+ boson:

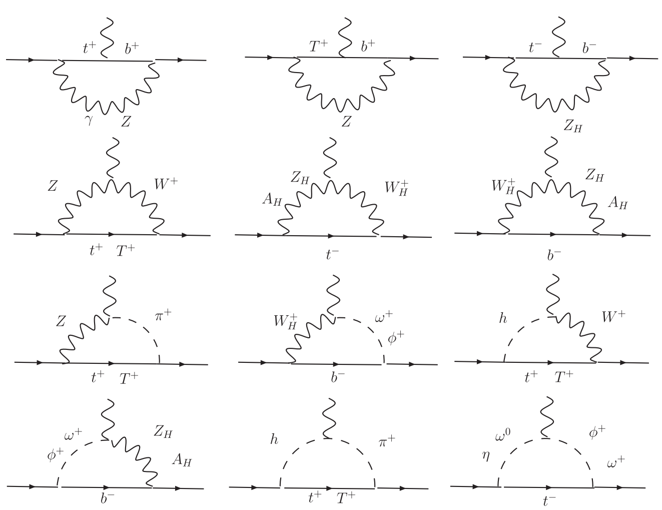

FIG. 1. Some Feynman diagrams that give rise to the f_{2R} coupling of the tbW vertex in the LHT model.

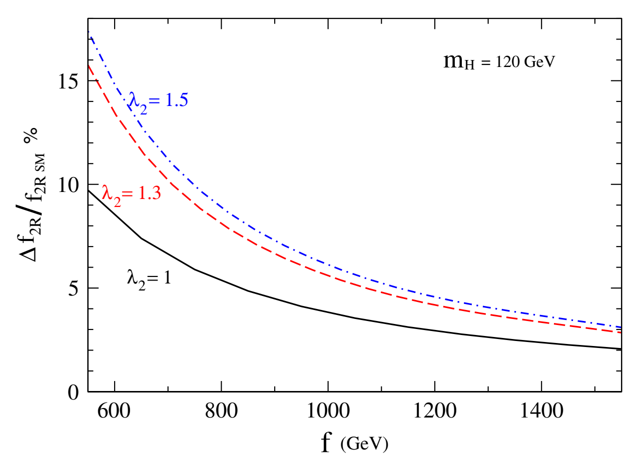

FIG. 2 (color online). The f_{2R} variation for $m_H = 120$ GeV.

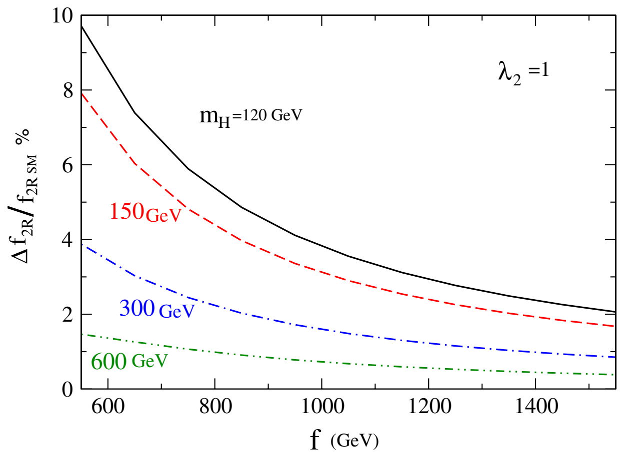

FIG. 3 (color online). The f_{2R} variation for several values of m_H.

$$\begin{aligned} \Gamma_t &= \Gamma_0 + \Gamma_- + \Gamma_+ \\ &= \frac{g^2 m_t}{64\pi}\frac{(a_t^2-1)^2}{a_t^4}(a_t^2 L_0^2 + 2T_m^2 + 2T_p^2), \\ L_0^2 &= (f_{1L} + f_{2R}/a_t)^2 + (f_{1R} + f_{2L}/a_t)^2, \\ T_m^2 &= (f_{1L} + a_t f_{2R})^2, \qquad T_p^2 = (f_{1R} + a_t f_{2L})^2, \\ a_t &= \frac{m_t}{m_W}. \end{aligned} \tag{21}$$

From this expression we define the W-helicity ratios

$$f_0 \equiv \frac{\Gamma_0}{\Gamma_t}, \qquad f_- \equiv \frac{\Gamma_-}{\Gamma_t}, \quad \text{and} \quad f_+ \equiv \frac{\Gamma_+}{\Gamma_t}.$$

Notice that the T_p coefficient is zero for $m_b = 0$. However, we are including it here for the sake of completeness. For $f_+ = 0$ we have that $f_0 + f_-$ must be equal to one. Therefore, it is only necessary to study one of them. In this work we show the deviation in f_- predicted by the LHT model.

It is convenient to define the following effective terms:

$$\begin{aligned} x_0 &= L_0^2 - 1, \qquad x_5 = a_t^2(f_{2R} + f_{2L})^2, \\ x_m &= T_m^2 - 1, \qquad x_p = T_p^2. \end{aligned} \tag{22}$$

Then, the W-helicity ratios and the single top production cross section for the s and t channels are given by

$$\begin{aligned} f_0 &= \frac{a_t^2(1+x_0)}{a_t^2(1+x_0) + 2(1+x_m+x_p)}, \\ f_+ &= \frac{2x_p}{a_t^2(1+x_0) + 2(1+x_m+x_p)}, \\ f_- &= \frac{2(1+x_m)}{a_t^2(1+x_0) + 2(1+x_m+x_p)}, \\ \sigma_t &= \sigma_t^{\rm SM} + a_0 x_0 + a_m x_m + a_p x_p + a_5 x_5, \\ \sigma_s &= \sigma_s^{\rm SM} + b_0 x_0 + b_m x_m + b_p x_p + b_5 x_5. \end{aligned} \tag{23}$$

TABLE I. The single top production cross section coefficients of Eq. (23) for $m_t = 173$ GeV. The last column is the Born level production cross section in the SM. All in units of picobarns.

t channel:	a_0	a_m	a_p	a_5	$\sigma_t^{\rm SM}$
Tevatron	0.995	−0.089	−0.181	0.336	0.906
LHC (t)	174.2	−22.2	−38.1	78.2	151.99
LHC ($\bar{t}$	111.9	−23.4	−14.6	48.7	88.46
s channel:	b_0	b_m	b_p	b_5	$\sigma_s^{\rm SM}$
Tevatron	−0.094	0.040	0.040	0.263	0.306
LHC (t)	−1.58	6.30	6.30	7.02	4.716
LHC ($\bar{t}$)	−0.944	3.83	3.83	3.76	2.884

The numerical values of the a_j and b_j coefficients are given in Ref. [24] for a mass of the top quark $m_t = 178$ GeV. In Table I we show their values for $m_t = 173$ GeV. We have used the CTEQ6L1 parton distribution function when integrating over the parton luminosities [30].

As mentioned above, the LHT model predicts a tree-level reduction of the tbW coupling. Therefore, an important feature of this model is that at tree level all single top production modes as well as the total decay width show the same proportional deviation from the SM prediction [28]. In our study, we want to consider the additional effect from the dimension-5 f_{2R} coupling that arises at the one-loop level in LHT.

The SM born level prediction of the $t \to bW^+$ width of the top quark is $\Gamma(t \to bW^+) = 1.5$ GeV for $m_t = 173$ GeV. There is a 10% decrease when QCD and electroweak corrections as well as nonzero m_b and finite W-boson width effects are considered [27,31]. In Fig. 4, we show the deviation in the total width of the top quark coming from the LHT model. The solid lines in Fig. 4 give the reduction in Γ_t as a function of the scale f and three different values of λ_2. For these lines both the effective f_{1L} and f_{2R} couplings are considered. Nonetheless, we also

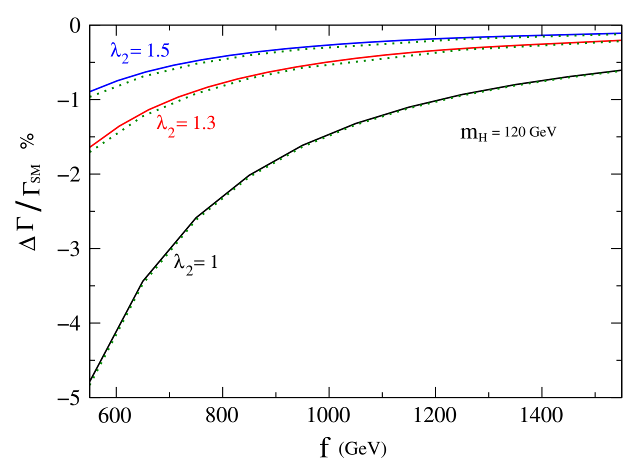

FIG. 4 (color online). The variation in the top width. Solid lines contain the contribution from both the f_{1L} and the f_{2R} couplings. Dotted lines are for $f_{2R} = 0$.

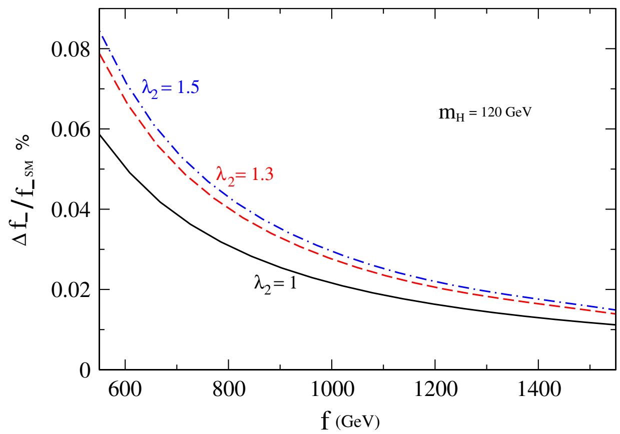

FIG. 5 (color online). The variation in the W-boson helicity ratio f_-. It would require a deviation of f_{2R} much bigger than 20% in order to have a significant change in f_-.

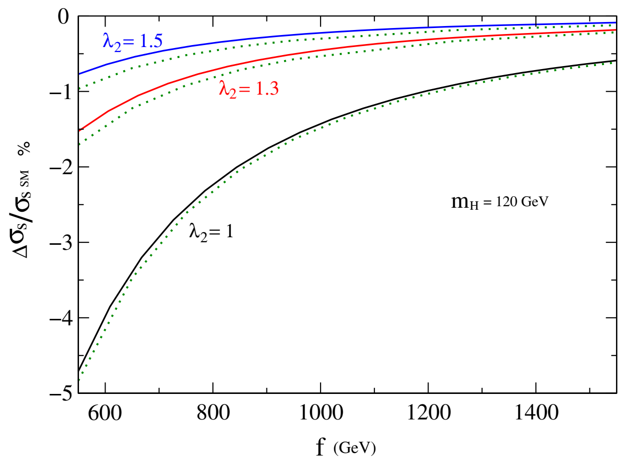

FIG. 7 (color online). The variation in the σ_s cross section. Solid lines contain the contribution from both the f_{1L} and the f_{2R} couplings. Dotted lines are for $f_{2R} = 0$.

show in dotted lines the same curves obtained when only the f_{1L} coupling is considered. Dotted and solid lines almost overlap: as expected, the change in Γ_t is driven mainly by the tree-level mixing with the heavy top. We conclude that the small changes in f_{2R} cannot be seen by measuring Γ_t. Also, notice that the (%) reduction in Fig. 4 is entirely due to the cosine of the tT mixing angle c_L, which according to Eq. (17) tends to 1 when either $\epsilon \rightarrow 0$ or $r = \lambda_1/\lambda_2 \rightarrow 0$.

As for the W-boson helicity ratios f_0 and f_-, in principle, these observables are more sensitive to the f_{2R} coupling. Notice that L_0 and T_m in Eq. (21) get exactly the same correction if only the f_{1L} is modified (we have set $f_{1R} = f_{2L} = 0$). This means that the ratios f_0 and f_- do not change at all from their SM values when we consider only the tree-level tbW coupling of the LHT model. However, when we consider the change in the f_{2R} coupling, we do observe a deviation that (unfortunately) turns

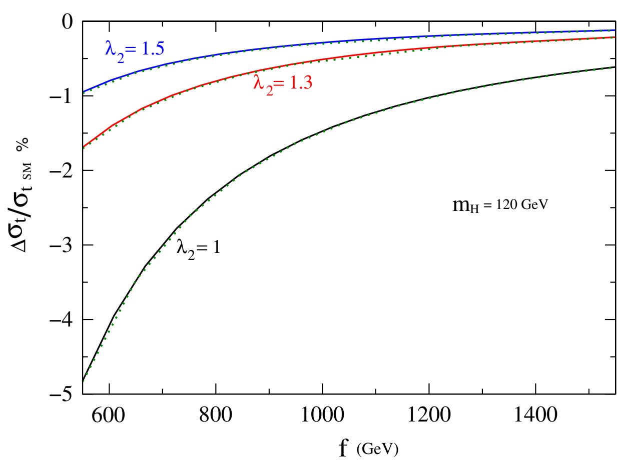

FIG. 6 (color online). The variation in the σ_t cross section. Solid lines contain the contribution from both the f_{1L} and the f_{2R} couplings. Dotted lines are for $f_{2R} = 0$.

out to be very small (of order less than 0.1%) as it is shown in Fig. 5. We conclude that the W-boson helicity ratios f_0 and f_- require a substantial deviation in the dimension-5 coupling f_{2R} in order to show significant changes from their SM values.

As mentioned above, the effective couplings f_{1L} and f_{2R} can also be probed with single top production. In comparison with the top decay width Γ_t and the W-helicity ratio f_-, the cross section could be more sensitive to the f_{2R} coupling. We show the deviation in the t-channel cross section in Fig. 6. Notice that the change when we go from considering the deviation in f_{1L} only (dotted lines) to considering both the deviations in f_{1L} and f_{2R} (solid lines) is hardly visible. This change is slightly more pronounced for the s-channel cross section as shown in Fig. 7. We can observe from the values of the effective coefficients in Table I that for this channel the f_{2R} coupling has a somewhat bigger effect through the x_5 and x_0 terms defined in Eq. (22). For instance, for a scale $f = 550$ GeV and $\lambda_2 = 1.5$ the tree level f_{1L} reduction expected in the LHT model brings about a 1.0% reduction in σ_s (dotted line), whereas the combined f_{1L} and f_{2R} deviations bring a smaller 0.8% reduction in σ_s (solid line). The reason for this can be seen in Fig. 2. The LHT contribution, on the one hand, decreases the value of f_{1L} and, on the other hand, increases the (positive) value of f_{2R} with respect to the SM. We thus have a small compensation in the value of the x terms in Eq. (22).

IV. CONCLUSIONS

Besides the SM electroweak parameters, the LHT model adds three more free parameters: κ, λ_2, and the scale f (which is associated with an estimated cutoff $\Lambda \sim 4\pi f$). We have chosen a value of the mirror fermion Yukawa $\kappa = 1$ that for our range of f gives T-odd fermion masses that are consistent with bounds from four fermion contact interactions $e^+e^- q\bar{q}$ and from unitarity in $u\bar{u} \to W_H^+ W_H^-$ scattering processes. Our study concentrates on the two other parameters: λ_2 (which drives m_{T^+}) and the scale f. As for the values of λ_2 and f we have chosen $\lambda_2 = 1, 1.3$, and 1.5 and $550 \leq f \leq 1550$ GeV as suggested by Ref. [22].

Because of the mixing between the top quark and its heavy partner T^+, the LHT model predicts a tree-level reduction of the dim 4 f_{1L} coupling that implies an expected proportional reduction in the top width and single top production. Changes in f_{1L} by themselves do not vary the predicted W helicity fractions in the $t \to bW$ decay. However, the contribution from the f_{2R} coupling could modify these fractions. The dim 5 weak dipole coupling f_{2R} arises at the one-loop level in the SM and in the LHT model. It is somewhat increased in size by the LHT model. However, the contribution to f_m (and f_0) is negligible. The increase in f_{2R} predicted by the LHT tends to counteract the effects of the reduction in f_{1L}. In any case, the effects from f_{2R} are very small. We have found that the tree-level analysis of the top's width and the single top cross section remains valid for the LHT model. Top quark observables like total width and single top production are sensitive to the mixing with the heavy T^+ partner. In particular, the LHC may probe deviations of the f_{1L} tbW coupling down to a few percent [4], and this would imply an indirect probe of the scale f (and the Yukawa λ_2) of the LHT model [see Eq. (17)]. Of course, there are direct tests of the new heavy states at the LHC that will give more precise determination of these parameters. A recent study (see Ref. [22]) has shown that signal events from T^+ and T^- production can be distinguished from SM backgrounds so that the mass and mixings of the top partners can be obtained with relatively good accuracies. Furthermore, other studies have shown that, since the mass of the T-odd fermions cannot be too heavy to be consistent with low energy data, they can be produced at high enough rates at the LHC [20].

ACKNOWLEDGMENTS

We thank C.-P. Yuan, Chuan-Ren Chen, M. A. Pérez, and R. Martinez for useful discussions. We thank Conacyt for support.

APPENDIX A: THE GOLDSTONE BOSON SECTOR IN THE 'T HOOFT-FEYNMAN GAUGE

In the LHT model, the charged fields $\omega^\pm$ and $\phi^\pm$ as well as the neutral fields ω^0, η, and ϕ_P^0, mix at order $(\frac{v}{f})^2$. It is a linear combination of these that is eaten by the heavy gauge bosons when the extended gauge group is broken down to $SU(2)_L \times U(1)_Y$. On the other hand, the π fields are T-parity even and do not mix with the other scalars. They are absorbed by the standard model W/Z bosons as usual. The fields h, ϕ^0, ϕ_P^0, and $\phi^\pm$ remain in the spectrum (after diagonalization). The basis of the kinetic Lagrangian of Eq. (4) is an exponential matrix that is usually computed up to the first few leading terms. However, it is possible to obtain the exact expressions for the kinetic ($\partial^\mu \phi \partial_\mu \phi$) scalar, the scalar-boson mixing ($W_\mu^+ \partial^\mu \pi^-$), and boson mass terms. [It is from the latter that the boson masses of Eq. (10) were obtained.]

In obtaining the following formulas, it is convenient to notice that the VEV value of the field matrix Π Eq. (2) is proportional to a matrix M_0:

$$M_0 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}, \tag{A1}$$

for which is easy to prove that $M_0^{2n+1} = 2^n M_0$ and $M_0^{2(n+1)} = 2^n M_0^2$, with $n = 0, 1, 2, 3, \ldots$. With these identities it can be shown that the VEV of Σ is [15]

$$\langle \Sigma \rangle_0 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & -\frac{1-c_v}{2} & \frac{i}{\sqrt{2}} s_v & 0 & \frac{1+c_v}{2} \\ 0 & \frac{i}{\sqrt{2}} s_v & c_v & 0 & \frac{i}{\sqrt{2}} s_v \\ 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1+c_v}{2} & \frac{i}{\sqrt{2}} s_v & 0 & -\frac{1-c_v}{2} \end{pmatrix}, \tag{A2}$$

where $s_v = \sin\sqrt{2}\epsilon$ and $\epsilon \equiv v/f$ as defined in Eq. (8). This expression can be used in the kinetic Lagrangian Eq. (4) to obtain the (exact) mixing and mass terms of the LHT model. The diagonalization and normalization of Goldstone boson fields has been discussed at order ϵ^2 for the charged sector in Ref. [15] and for the neutral sector in Ref. [32]. Below, we will make the same analysis for charged and neutral bosons exactly (at all orders in ϵ).

1. The charged $W^\pm$ bosons

Let us write down the part of the kinetic Lagrangian Eq. (4) that involves the charged bosons of the LHT model. It is convenient to put it in a matricial form:

$$\begin{aligned} \mathcal{L}_{\text{kin}} = {} & \partial^\mu \phi^{++} \partial_\mu \phi^{--} + 2\tilde{\kappa} \partial^\mu \pi^+ \partial_\mu \pi^- \\ & + fg \frac{1 - c_v}{\sqrt{2} y} (W_\mu^+ \partial^\mu \pi^- + W_\mu^- \partial^\mu \pi^+) \\ & + (\partial_\mu \omega^- \partial_\mu \phi^-) B \begin{pmatrix} \partial^\mu \omega^+ \\ \partial^\mu \phi^+ \end{pmatrix} \\ & + fg W_{H\mu}^+ \left(\frac{1+\kappa_0}{2} \; i \frac{1-\kappa_0}{2} \right) \begin{pmatrix} \partial^\mu \omega^- \\ \partial^\mu \phi^- \end{pmatrix} \\ & + fg W_{H\mu}^- \left(\frac{1+\kappa_0}{2} \; - i \frac{1-\kappa_0}{2} \right) \begin{pmatrix} \partial^\mu \omega^+ \\ \partial^\mu \phi^+ \end{pmatrix}, \end{aligned} \tag{A3}$$

with

$$B = \begin{pmatrix} \frac{1}{2} + \tilde{\kappa} & -i(\frac{1}{2} - \tilde{\kappa}) \\ i(\frac{1}{2} - \tilde{\kappa}) & \frac{1}{2} + \tilde{\kappa} \end{pmatrix},$$

$$\kappa_0 = \frac{s_v}{\sqrt{2}\epsilon} = 1 - \frac{2\epsilon^2}{3!} + \frac{4\epsilon^4}{5!} - \cdots, \tag{A4}$$

$$\tilde{\kappa} = \frac{1 - c_v}{2\epsilon^2} = \frac{1}{2!} - \frac{2\epsilon^2}{4!} + \frac{4\epsilon^4}{6!} - \cdots.$$

We then redefine the T-even charged scalar $\pi^\pm$ as well as the T-odd $\omega^\pm$ and $\phi^\pm$ to diagonalize the Lagrangian. The new T-even $\pi'^\pm$ field is given by

$$\pi^\pm \equiv \frac{\pm i}{\sqrt{2\tilde{\kappa}}} \pi'^\pm. \tag{A5}$$

An extra phase i multiplies the $\pi'^\pm$ field so that the $W^\pm \pi'^\pm$ mixing and the $SU(2)$ gauge-fixing terms become identical to the usual SM expressions [33]. The other two charged scalars are redefined as

$$\begin{pmatrix} \omega^+ \\ \phi^+ \end{pmatrix} \equiv T \begin{pmatrix} i\omega'^+ \\ \phi'^+ \end{pmatrix},$$
$$T = \frac{\hat{m}_{W_H}}{2\tilde{\kappa} + \kappa_0^2} \begin{pmatrix} 2\tilde{\kappa} + \kappa_0 & i(1 - \kappa_0)\sqrt{2\tilde{\kappa}} \\ i(2\tilde{\kappa} - \kappa_0) & (1 + \kappa_0)\sqrt{2\tilde{\kappa}} \end{pmatrix}, \tag{A6}$$

where $\hat{m}_{W_H} = \frac{M_{W_H}}{fg} = \sqrt{3 + c_v}/2$. Notice that the new $\omega'^\pm$ field has an extra phase factor $\pm i$ that is convenient to use so that the Feynman rules of this T-odd Goldstone boson resemble the rules of its T-even counterpart (the $\pi'^\pm$ boson). On the other hand, for the physical heavy T-odd $\phi'^\pm$ boson we choose not to insert the phase factor. The Feynman rules in Appendix B stand for the new $\pi'^\pm$, $\omega'^\pm$, $\phi'^\pm$, etc., fields, but we have dropped the $'$ symbol for simplicity.

In terms of the new (mass eigenstates) fields the Lagrangian becomes

$$\begin{aligned} \mathcal{L}_{\text{kin}} = {}& \partial^\mu \phi'^+ \partial_\mu \phi'^- + \partial^\mu \pi'^+ \partial_\mu \pi'^- \\ & + iM_W(W_\mu^- \partial^\mu \pi'^+ - W_\mu^+ \partial^\mu \pi'^-) + \partial^\mu \omega'^+ \partial_\mu \omega'^- \\ & + iM_{W_H}(W_{H\mu}^- \partial^\mu \omega'^+ - W_{H\mu}^+ \partial^\mu \omega'^-), \end{aligned} \tag{A7}$$

where the $W_{H\mu}^+ \partial^\mu \omega'^-$ mixing term is canceled (after integration by parts) when we add the usual gauge-fixing term:

$$\begin{aligned} \Delta\mathcal{L} = {}& \frac{-1}{\xi_L} |\partial^\mu W_\mu^\pm - iM_W \xi_L \pi'^\pm|^2 - \frac{1}{\xi_H} |\partial^\mu W_{H\mu}^\pm \\ & - iM_{W_H} \xi_H \omega'^\pm|^2. \end{aligned} \tag{A8}$$

To obtain Feynman rules it is convenient to use an expansion in powers of $\epsilon = v/f$:

$$T = \mathbf{1} + \begin{pmatrix} 1 & 4i \\ 2i & 1 \end{pmatrix} \frac{\epsilon^2}{24} + \begin{pmatrix} -\frac{61}{8} & 13i \\ \frac{19}{2}i & -1 \end{pmatrix} \frac{\epsilon^4}{720}. \tag{A9}$$

2. The neutral bosons

The neutral boson sector in the Lagrangian of Eq. (4) can also be written in the following matricial form (notice that $M_Z = \frac{vg}{2c_w}\sqrt{2\tilde{\kappa}}$):

$$\begin{aligned} \mathcal{L}_{\text{kin}} = {}& \frac{1}{2}(\partial^\mu h)^2 + \tilde{\kappa}(\partial^\mu \phi^0)^2 + \tilde{\kappa}(\partial^\mu \pi^0)^2 \\ & + \frac{vg}{c_w} \tilde{\kappa} Z^\mu \partial_\mu \pi^0 + \bar{x}^T A \bar{x} + A_H^\mu \bar{a}^T \bar{x} + Z_H^\mu \bar{b}^T \bar{x}, \end{aligned} \tag{A10}$$

with

$$\bar{x} = \begin{pmatrix} \partial_\mu \omega^0 \\ \partial_\mu \eta \\ \partial_\mu \phi_P^0 \end{pmatrix}, \qquad \bar{a} = \frac{fgc_H}{8} \begin{pmatrix} 8t_w - (4\kappa_1 + \kappa_2)x_\omega \\ (8t_w - 5\kappa_1 x_\omega)/\sqrt{5} \\ \sqrt{2}\kappa_1 x_\omega \end{pmatrix},$$

$$\bar{b} = \frac{fgc_H}{8} \begin{pmatrix} 8 - \kappa_1 x_\eta \\ (8 - \kappa_2 x_\eta)/\sqrt{5} \\ -\sqrt{2}\kappa_1 x_\eta \end{pmatrix},$$

$$A = \frac{1}{16} \begin{pmatrix} 7 + \kappa_0^2 & \sqrt{5}(1 - \kappa_0^2) & -\sqrt{2}(1 - \kappa_0^2) \\ \sqrt{5}(1 - \kappa_0^2) & 3 + 5\kappa_0^2 & \sqrt{10}(1 - \kappa_0^2) \\ -\sqrt{2}(1 - \kappa_0^2) & \sqrt{10}(1 - \kappa_0^2) & 6 + 2\kappa_0^2 \end{pmatrix},$$

and

$$\begin{aligned} &\kappa_1 = 1 - \kappa_0 c_v, && \kappa_2 = 3 + 5\kappa_0 c_v, && x_\omega = t_w + t_H, \\ &x_\eta = 1 - t_w t_H, && y_\omega = t_w - 5t_H, && y_\eta = 5 + t_w t_H, \end{aligned} \tag{A11}$$

where $t_w \equiv s_w/c_w$ and t_H are $\tan(\theta_w)$ and $\tan(\theta_H)$, respectively [see Eq. (7)]. To normalize the π^0 and ϕ^0 fields, we simply redefine $\pi'^0 \equiv \sqrt{2\tilde{\kappa}}\pi^0$ and $\phi'^0 \equiv \sqrt{2\tilde{\kappa}}\phi^0$. (The Higgs field h needs no redefinition.) We also redefine the other scalars to properly diagonalize the Lagrangian:

$$\begin{pmatrix} \omega^0 \\ \eta \\ \phi_P^0 \end{pmatrix} \equiv T \begin{pmatrix} \omega'^0 \\ \eta' \\ \phi_P'^0 \end{pmatrix}, \quad \text{with}$$
$$T = \frac{1}{\kappa_0\sqrt{2}\sqrt{1 + 3c_v^2}} D_1(t_{ij}) D_2, \tag{A12}$$

where T is conveniently written as a product of 3 matrices: D_1, (t_{ij}), and D_2. Two of them are diagonal matrices defined as $D_1 = \text{diag}(1, \sqrt{5}, \sqrt{2})$ and $D_2 = \text{diag}(1/d_\omega, 1/d_\eta, 1/2)$, with

$$d_\omega = \sqrt{8(t_w^2 + 5t_H^2) - 5s_v^2 x_\omega^2} \quad \text{and}$$
$$d_\eta = \sqrt{8(5 + t_w^2 t_H^2) - 5s_v^2 x_\eta^2}.$$

The matrix elements t_{ij} are as follows:

$$\begin{aligned}
t_{11} &= \kappa_0(5c_v^2 x_\omega + 2t_w) + c_v y_\omega, \\
t_{12} &= \kappa_0(5c_v^2 x_\eta - 2t_w t_H) - c_v y_\eta, \\
t_{13} &= \kappa_1, \\
t_{21} &= \kappa_0(c_v^2 x_\omega + 2t_H) - c_v y_\omega, \\
t_{22} &= \kappa_0(c_v^2 x_\eta + 2) + c_v y_\eta, \\
t_{23} &= -\kappa_1, \\
t_{31} &= -(\kappa_0 - c_v)y_\omega, \\
t_{32} &= (\kappa_0 - c_v)y_\eta, \\
t_{33} &= 1 + 3\kappa_0 c_v,
\end{aligned}$$

where the x's, y's, and κ's are defined in Eqs. (A4) and (A11).

It is convenient to make an expansion in powers of $\epsilon = v/f$:

$$T = \mathbf{1} + T_2\frac{\epsilon^2}{12} + T_4\frac{\epsilon^4}{6} + \cdots,$$

$$T_2 = \begin{pmatrix} 1/2 & -2.183\sqrt{5} & 2\sqrt{2} \\ \frac{2.183}{2}\sqrt{5} & 5/2 & -2\sqrt{10} \\ -\sqrt{2} & \sqrt{10} & 1 \end{pmatrix}, \tag{A13}$$

$$T_4 = \begin{pmatrix} -0.5682 & 0.6200 & \frac{41}{30\sqrt{2}} \\ 0.1315 & -0.6093 & \frac{-41}{6\sqrt{10}} \\ -0.1288 & 1.6184 & \frac{-17}{240} \end{pmatrix},$$

where we have taken $t_w = 0.536$; thus $(10 + t_w^2)/(5 - t_w^2) = 2.183$.

In terms of the new (mass eigenstates) fields the neutral boson Lagrangian becomes

$$\begin{aligned}
\mathcal{L}_{\text{kin}} &= \tfrac{1}{2}(\partial^\mu h)^2 + \tfrac{1}{2}(\partial^\mu \pi'^0)^2 + M_Z Z^\mu \partial_\mu \pi'^0 + \tfrac{1}{2}(\partial^\mu \omega'^0)^2 \\
&\quad + \tfrac{1}{2}(\partial^\mu \eta')^2 + \tfrac{1}{2}(\partial^\mu \phi_P'^0)^2 + \tfrac{1}{2}(\partial^\mu \phi'^0)^2 \\
&\quad + M_{Z_H} Z_H^\mu \partial_\mu \omega'^0 + M_{A_H} A_H^\mu \partial_\mu \eta',
\end{aligned} \tag{A14}$$

where the mixing terms like $Z^\mu \partial_\mu \pi'^0$ are canceled (after integration by parts) when we add the usual gauge-fixing terms:

$$\begin{aligned}
\Delta\mathcal{L} &= \frac{-1}{2\xi_A}(\partial_\mu A^\mu)^2 - \frac{1}{2\xi_Z}(\partial_\mu Z^\mu - M_Z \xi_Z \pi'^0)^2 \\
&\quad - \frac{1}{2\xi_{A_H}}(\partial_\mu A_H^\mu - M_{A_H}\xi_{A_H}\eta')^2 \\
&\quad - \frac{1}{2\xi_{Z_H}}(\partial_\mu Z_H^\mu - M_{Z_H}\xi_{Z_H}\omega'^0)^2.
\end{aligned} \tag{A15}$$

APPENDIX B: FEYNMAN RULES

We want to show the Feynman rules that we used. The scalar fields are not the original interaction eigenstates but the mass eigenstates that are written as $\pi'^{\pm}$, π'^0, etc. We have dropped the $'$ symbol to simplify the notation. Table II shows bosonic vertices that involve one charged W^- SM boson. Tables III, IV, and V show vertices for fermions and charged, T-even neutral and T-odd neutral scalar bosons, respectively. For the fermion-gauge boson interactions, we refer the reader to Tables V and VI of Ref. [20]. We have carefully verified that our rules agree with the ones there. We have written in Table VI some others that do not appear in Ref. [20]. Other types of interactions, like four-boson vertices or dimension-5 $\bar{f}f\phi\phi$ (ϕ any scalar) vertices, can be found in Ref. [21].

Please notice the definitions ($\epsilon = v/f$, $r = \lambda_1/\lambda_2$):

$$\begin{aligned}
S_{\lambda\nu\mu} &\equiv (p_- - p_+)_\lambda g_{\mu\nu} + (p_+ - p_0)_\mu g_{\nu\lambda} \\
&\quad + (p_0 - p_-)_\nu g_{\lambda\mu}, \\
q_w &\equiv \frac{10 + t_w^2}{5 - t_w^2},
\end{aligned}$$

$$A_R \equiv i\sqrt{2}\lambda_d\left[-1 + \frac{r^4}{2(1+r^2)^2}\epsilon^2\right],$$

$$A_L \equiv i\frac{\sqrt{2}\lambda_2 r}{\sqrt{1+r^2}}\left[1 - \frac{1+3r^4}{4(1+r^2)^2}\epsilon^2\right],$$

$$B_R \equiv i\frac{\sqrt{2}\lambda_d r^2}{1+r^2}\left[-\epsilon + \frac{5 - 2r^2 + 2r^4}{6(1+r^2)^2}\epsilon^3\right],$$

$$B_L \equiv i\sqrt{2}\lambda_2\frac{r^2}{\sqrt{1+r^2}}\left[1 - \frac{\epsilon^2}{4}\frac{3+r^4}{(1+r^2)^2}\right],$$

$$D_R \equiv (i+4)\kappa\frac{\epsilon^2}{24},$$

$$\begin{aligned}
D_L &\equiv \frac{ir\lambda_2}{\sqrt{2}\sqrt{1+r^2}}\bigg[-\epsilon + \frac{\epsilon^3}{24} \\
&\quad \times \frac{7 - 4i + (2-8i)r^2 + (19-4i)r^4}{(1+r^2)^2}\bigg],
\end{aligned}$$

$$\begin{aligned}
E_L &\equiv \frac{ir^2\lambda_2}{\sqrt{2(1+r^2)}}\bigg[-\epsilon \\
&\quad + \frac{19 + 4i + (2+8i)r^2 + (7+4i)r^4}{24(1+r^2)^2}\epsilon^3\bigg],
\end{aligned}$$

$$a_R \equiv -\kappa\left[1 - \frac{r^4\epsilon^2}{2(1+r^2)^2}\right],$$

TABLE II. Feynman rules for three boson W^- vertices. Some of these rules also appear in Ref. [32]. Here M_W and M_Z stand for the SM mass of the $W^\pm$ and Z bosons, respectively.

Interaction	Feynman rule	Interaction	Feynman rule
$W^-_\mu W^+_\nu A_\lambda$	$igs_\omega S_{\lambda\nu\mu}$	$W^-_\mu W^+_{H\nu}\phi^0_P$	$\sim\epsilon^4$
$W^-_\mu W^+_\nu Z_\lambda$	$igc_\omega S_{\lambda\nu\mu}$	$W^-_\mu \omega^+ A_{H\nu}$	$ig^2 f t_\omega(\frac{5}{20-4t_W^2}-\frac{1}{4})\epsilon^2 g_{\mu\nu}$
$W^-_\mu \pi^+ A_\nu$	$igs_\omega M_W(1-\frac{1}{12}\epsilon^2)g_{\mu\nu}$	$W^-_\mu \omega^+ Z_{H\nu}$	$-ig^2 f(1-\frac{3}{8}\epsilon^2)g_{\mu\nu}$
$W^-_\mu \pi^+ Z_\nu$	$-igs^2_\omega M_Z(1-\frac{1}{12}\epsilon^2)g_{\mu\nu}$	$W^-_\mu \phi^+ A_{H\nu}$	$g^2\frac{t_\omega}{4}f\epsilon^2 g_{\mu\nu}$
$W^-_\mu W^+_\nu h$	$igM_W(1-\frac{1}{3}\epsilon^2)g_{\mu\nu}$	$W^-_\mu \phi^+ Z_{H\nu}$	$ig^2\frac{f}{4}(\frac{2}{3}+\frac{5}{3}i)\epsilon^2 g_{\mu\nu}$
$W^-_\mu W^+_\nu \pi^0$	0	$W^-_\mu \omega^+ \omega^0$	$g(1-\frac{1}{8}\epsilon^2)(p_{\omega^+\mu}-p_{0\mu})$
$W^-_\mu \pi^+ h$	$i\frac{g}{2}[p_{\pi\mu}(1-\frac{5}{12}\epsilon^2)-p_{h\mu}(1-\frac{1}{12}\epsilon^2)]$	$W^-_\mu \omega^+ \eta$	$g\frac{\sqrt{5}}{12}\epsilon^2[(q_\omega-\frac{3}{2})p_{\eta\mu}-(q_\omega+\frac{1}{2})p_{\omega^+\mu}]$
$W^-_\mu \pi^+ \pi^0$	$\frac{g}{2}(p_{\pi^+\mu}-p_{0\mu})$	$W^-_\mu \omega^+ \phi^0$	$-\frac{g}{12\sqrt{2}}\epsilon^2[(1+2i)p_{\phi\mu}-p_{\omega\mu}]$
$W^-_\mu W^+_{H\nu} A_{H\lambda}$	$-igs_h S_{\lambda\nu\mu}$	$W^-_\mu \omega^+ \phi^0_P$	$ig\sqrt{2}\frac{1}{12}\epsilon^2[\frac{1}{2}p_{\phi\mu}-(\frac{1}{2}+i)p_{\omega\mu}$
$W^-_\mu W^+_{H\nu} Z_{H\lambda}$	$igc_h S_{\lambda\nu\mu}$	$W^-_\mu \phi^+ \omega^0$	$-\frac{g}{6}(1+\frac{7}{4}i)(p_{\phi^+\mu}-p_{\omega_0\mu})\epsilon^2$
$W^-_\mu W^+_{H\nu}\eta$	$\frac{\sqrt{5}}{2}g^2 f(1+q_\omega)\epsilon^2 g_{\mu\nu}$	$W^-_\mu \phi^+ \eta$	$-ig\frac{\sqrt{5}}{12}(\frac{5}{2}p_{\eta\mu}-\frac{1}{2}p_{\phi^+\mu})\epsilon^2$
$W^-_\mu W^+_{H\nu}\phi^0$	0	$W^-_\mu \phi^+ \phi^0$	$\frac{g}{\sqrt{2}}[p_{0\mu}-p_{\phi^+\mu}-\frac{1}{24}\epsilon^2(5p_{0\mu}-p_{\phi^+\mu})]$
$W^-_\mu W^+_{H\nu}\omega^0$	$g^2 f(1-\frac{3}{8}\epsilon^2)g_{\mu\nu}$	$W^-_\mu \phi^+ \phi^0_P$	$-i\frac{g}{\sqrt{2}}[p_{\phi^0\mu}-p_{\phi^+\mu}-\frac{1}{24}(5p_{\phi^0\mu}-p_{\phi^+\mu})\epsilon^2]$

TABLE III. Feynman rules for fermion-charged scalar vertices.

Interaction	Feynman rule	Interaction	Feynman rule
$\bar{t}^+ b^+ \pi^+$	$A_R P_R + A_L P_L$	$\bar{t}^- b^+ \omega^+$	$i\frac{\lambda_d}{\sqrt{2}}[-\epsilon+(1-2i)\frac{1}{24}\epsilon^3]P_R + ikP_L$
$\bar{T}^+ b^+ \pi^+$	$B_R P_R + B_L P_L$	$\bar{T}^- b^+ \omega^+$	0
$\bar{t}^- b^- \pi^+$	$-i\frac{k}{4}(\epsilon+\frac{1}{24}\epsilon^3)\gamma_5$	$\bar{t}^+ b^- \phi^+$	$D_R P_R + D_L P_L$
$\bar{T}^- b^- \pi^+$	0	$\bar{T}^+ b^- \phi^+$	$i\frac{r^2 k}{24(1+r^2)}[1-4i]\epsilon^3 P_R + E_L P_L$
$\bar{t}^+ b^- \omega^+$	$i[a_R P_R + a_L P_L]$	$\bar{t}^- b^+ \phi^+$	$i\frac{\lambda_d}{\sqrt{2}}[\epsilon-\frac{(1+4i)}{24}\epsilon^3]P_R - i(4i+1)\frac{k}{24}\epsilon^2 P_L$
$\bar{T}^+ b^- \omega^+$	$i[b_R P_R + b_L P_L]$	$\bar{T}^- b^+ \phi^+$	0

TABLE IV. Feynman rules for fermion-neutral T-even scalar vertices.

Interaction	Feynman rule	Interaction	Feynman rule
$\bar{t}^+ t^+ h$	$i\frac{r\lambda_2}{\sqrt{1+r^2}}(-1+\frac{2+r^2+2r^4}{2(1+r^2)^2}\epsilon^2)$	$\bar{t}^+ t^+ \pi^0$	$-\frac{r\lambda_2}{\sqrt{1+r^2}}[1-\frac{(1+5r^4)}{4(1+r^2)^2}\epsilon^2]\gamma_5$
$\bar{T}^+ T^+ h$	$i\lambda_2[\frac{r^2}{(r^2+1)^{3/2}}\epsilon+\frac{r^2(3r^6+r^4+8r^2-5)}{6(r^2+1)^{7/2}}\epsilon^3]$	$\bar{T}^+ T^+ \pi^0$	$\frac{r^4\lambda_2}{(1+r^2)^{3/2}}[-\epsilon+\frac{19-4r^2+7r^4}{12(1+r^2)^2}\epsilon^3]\gamma_5$
$\bar{T}^+ t^+ h$	$i\frac{r^2\lambda_2}{\sqrt{1+r^2}}(G_R P_R + G_L P_L)$	$\bar{T}^+ t^+ \pi^0$	$-\frac{r^2\lambda_2}{\sqrt{1+r^2}}(J_R P_R + J_L P_L)$
$\bar{b}^+ b^+ h$	$i\lambda_d(-1+\frac{3}{4}\epsilon^2)$	$\bar{b}^+ b^+ \pi^0$	$\lambda_d\gamma_5$
$\bar{t}^- t^- h$	0	$\bar{t}^- t^- \pi^0$	$-\frac{k}{2\sqrt{2}}(\epsilon+\frac{1}{24}\epsilon^3)\gamma_5$
$\bar{T}^- T^- h$	0	$\bar{T}^- T^- \pi^0$	0
$\bar{T}^- t^- h$	0	$\bar{T}^- t^- \pi^0$	0
$\bar{b}^- b^- h$	0	$\bar{b}^- b^- \pi^0$	0

TABLE V. Feynman rules for fermion-neutral T-odd scalar vertices.

Interaction	Feynman rule	Interaction	Feynman rule
$\bar{t}^+ t^- \omega^0$	$K_R P_R + K_L P_L$	$\bar{t}^+ t^- \phi^0$	$\frac{ir\lambda_2}{\sqrt{2(1+r^2)}}[\epsilon-\frac{1-4r^2+7r^4}{12(1+r^2)^2}\epsilon^3]P_L$
$\bar{T}^+ t^- \omega^0$	$M_R P_R + M_L P_L$	$\bar{T}^+ t^- \phi^0$	$i\frac{r^2\lambda_2}{\sqrt{2(1+r^2)}}[\epsilon+\frac{-7+4r^2-r^4}{12(1+r^2)^2}\epsilon^3]P_L$
$\bar{T}^- t^+ \omega^0$	$\frac{r\lambda_2}{12\sqrt{1+r^2}}(2-q_\omega)\epsilon^2 P_R$	$\bar{T}^- t^+ \phi^0$	0
$\bar{T}^+ T^- \omega^0$	$\frac{q\omega-2}{12\sqrt{1+r^2}}r^2\lambda_2\epsilon^2 P_L$	$\bar{T}^+ T^- \phi^0$	0
$\bar{b}^+ b^- \omega^0$	$\frac{\kappa}{\sqrt{2}}[1+\frac{1+q\omega}{24}\epsilon^2]P_R$	$\bar{b}^+ b^- \phi^0$	0
$\bar{t}^+ t^- \eta$	$N_R P_R + N_L P_L$	$\bar{t}^+ t^- \phi^0_P$	$-\frac{\kappa\epsilon^2}{4}P_R + V_L P_L$
$\bar{T}^+ t^- \eta$	$\Xi_R P_R + \Xi_L P_L$	$\bar{T}^+ t^- \phi^0_P$	$\beta_R P_R + \beta_L P_L$
$\bar{T}^- t^+ \eta$	$-\frac{2r\lambda_2}{\sqrt{5(1+r^2)}}[1+\frac{1+6r^2-3r^4}{8(1+r^2)^2}\epsilon^2]P_R$	$\bar{T}^- t^+ \phi^0_P$	$\sim\epsilon^4$
$\bar{T}^+ T^- \eta$	$\frac{2r^2\lambda_2}{\sqrt{5(1+r^2)}}[1+\frac{r^4 6r^2-3}{8(1+r^2)^2}\epsilon^2]P_L$	$\bar{T}^+ T^- \phi^0_P$	0
$\bar{b}^+ b^- \eta$	$\frac{k}{\sqrt{10}}[1-\frac{5(2q\omega-1)}{24}\epsilon^2]P_R$	$\bar{b}^+ b^- \phi^0_P$	$i\frac{\lambda_d}{\sqrt{2}}(\frac{1}{3}\epsilon^3-\epsilon)$

TABLE VI. Some Feynman rules for fermion-gauge boson vertices. All of the other rules appear in Ref. [20] and in Ref. [32].

Interaction	Feynman rule	Interaction	Feynman rule
$\bar{b}^- t^- W^-_\mu$	$i\frac{g}{\sqrt{2}}\gamma_\mu$	$\bar{t}^+ t^+ Z_\mu$	$i\frac{g}{2c_\omega}\gamma_\mu[C_t P_L - \frac{4}{3}s^2_\omega P_R]$
$\bar{b}^- T^- W^-_\mu$	0	$\bar{T}^+ T^+ Z_\mu$	$i\frac{g}{2c_\omega}\gamma_\mu[C_T P_L - \frac{4}{3}s^2_\omega P_R]$
$\bar{b}^+ T^- W^-_{H\mu}$	0	$\bar{b}^+ b^+ Z_\mu$	$i\frac{g}{2c_\omega}\gamma_\mu[\frac{2}{3}s^2_\omega P_R + (\frac{2}{3}s^2_\omega - 1)P_L]$
$\bar{t}^\pm t^\pm A_\mu$	$i\frac{2}{3}e\gamma_\mu$	$\bar{t}^- t^- Z_\mu$	$i\frac{g}{2c_\omega}\gamma_\mu[c^2_\omega - \frac{1}{3}s^2_\omega]$
$\bar{T}^\pm T^\pm A_\mu$	$i\frac{2}{3}e\gamma_\mu$	$\bar{T}^- T^- Z_\mu$	$-i\frac{2g}{3c_\omega}\gamma_\mu[s^2_\omega]$
$\bar{b}^\pm b^\pm A_\mu$	$-i\frac{1}{3}e\gamma_\mu$	$\bar{T}^- t^- Z_\mu$	0
$\bar{T}^+ t^+ Z_\mu$	$i\frac{g}{2c_\omega}c_L s_L\gamma_\mu P_L$	$\bar{b}^- b^- Z_\mu$	$i\frac{g}{2c_\omega}\gamma_\mu[\frac{2}{3}s^2_\omega - 1]$

$$a_L \equiv \frac{r\lambda_2}{\sqrt{2}\sqrt{1+r^2}}\left(\epsilon - \frac{i\epsilon^3[2-7i+(4-2i)r^2+(2-19i)r^4]}{24(1+r^2)^2}\right),$$

$$b_R \equiv \frac{r^2\kappa}{1+r^2}\left[-\epsilon + \frac{5-2r^2+2r^4}{6(1+r^2)^2}\epsilon^3\right],$$

$$b_L \equiv \frac{r^2\lambda_2}{\sqrt{2(1+r^2)}}\left[\epsilon - \frac{19+2i+(2+4i)r^2+(7+2i)r^4}{24(1+r^2)^2}\epsilon^3\right],$$

$$G_R \equiv \frac{1}{r(1+r^2)}\left[\epsilon + \frac{3r^6-2r^4+8r^2-2}{6(1+r^2)^2}\epsilon^3\right],$$

$$G_L \equiv -1 + \frac{3+r^2+r^4}{2(1+r^2)^2}\epsilon^2,$$

$$J_R \equiv \frac{r}{1+r^2}\left[\epsilon - \frac{\epsilon^3(13-4r^2+13r^4)}{12(1+r^2)^2}\right],$$

$$J_L \equiv \frac{3\epsilon^2(1+r^4)}{4(1+r^2)^2} - 1,$$

$$K_R \equiv -\frac{\kappa}{\sqrt{2}}\left[1 - \left(1 + q_w + \frac{12r^4}{(1+r^2)^2}\right)\frac{\epsilon^2}{24}\right],$$

$$K_L \equiv \frac{r\lambda_2}{2\sqrt{1+r^2}}\left[\epsilon - \left(q_w + \frac{3-6r^2+15r^4}{(1+r^2)^2}\right)\frac{\epsilon^3}{24}\right],$$

$$M_R \equiv \frac{r^2\kappa}{\sqrt{2}(1+r^2)}\left(-\epsilon + \left[q_w + \frac{21-6r^2+9r^4}{(1+r^2)^2}\right]\frac{1}{24}\epsilon^3\right),$$

$$M_L \equiv \frac{r^2\lambda_2}{2\sqrt{1+r^2}}\left(\epsilon - \left[q_w + \frac{15-6r^2+3r^4}{(1+r^2)^2}\right]\frac{1}{24}\epsilon^3\right),$$

$$N_R \equiv \frac{\kappa}{\sqrt{10}}\left[1 + \left(10q_w - 5 - \frac{12r^4}{(1+r^2)^2}\right)\frac{\epsilon^2}{24}\right],$$

$$N_L \equiv -\frac{r\lambda_2}{2\sqrt{5(1+r^2)}}\left[\epsilon + \left(10q_w + \frac{17+46r^2+5r^4}{(1+r^2)^2}\right) \times \frac{\epsilon^3}{24}\right],$$

$$V_L \equiv -\frac{r\lambda_2}{\sqrt{2(1+r^2)}}\left[\epsilon - \frac{\epsilon^3(7+8r^2+13r^4)}{12(1+r^2)^2}\right],$$

$$\Xi_R \equiv \frac{r^2\kappa}{\sqrt{10}(1+r^2)}\left[\epsilon + \left(10q_w - \frac{25+2r^2+13r^4}{(1+r^2)^2}\right)\frac{\epsilon^3}{24}\right],$$

$$\Xi_L \equiv -\frac{r^2\lambda_2}{2\sqrt{5(1+r^2)}}\left[\epsilon + \left(10q_w + \frac{5+46r^2+17r^4}{(1+r^2)^2}\right) \times \frac{\epsilon^3}{24}\right],$$

$$\beta_R \equiv -\frac{r^2\kappa}{4(1+r^2)^2}\epsilon^3,$$

$$\beta_L \equiv \frac{r^2\lambda_2}{\sqrt{2(1+r^2)}}\left[-\epsilon + \frac{13+8r^2+7r^4}{12(1+r^2)^2}\epsilon^3\right],$$

$$C_t \equiv c_L^2(c_w^2 - \tfrac{1}{3}s_w^2) - \tfrac{4}{3}s_L^2 s_w^2,$$

$$C_T \equiv s_L^2(c_w^2 - \tfrac{1}{3}s_w^2) - \tfrac{4}{3}c_L^2 s_w^2.$$

[1] T. Han, arXiv:0804.3178; T. M. P. Tait, Nucl. Phys. B, Proc. Suppl. **177–178**, 11 (2008); A. Quadt, Eur. Phys. J. C **48**, 835 (2006); D. Chakraborty, J. Konigsberg, and D. Rainwater, Annu. Rev. Nucl. Part. Sci. **53**, 301 (2003); M. Beneke *et al.*, arXiv:hep-ph/0003033; F. Larios, R. Martinez, and M. A. Perez, Int. J. Mod. Phys. A **21**, 3473 (2006); R. Martinez, M. A. Perez, and N. Poveda, Eur. Phys. J. C **53**, 221 (2008); J. L. Diaz-Cruz, M. A. Perez, and J. J. Toscano, Phys. Lett. B **398**, 347 (1997).

[2] T. M. P. Tait, Phys. Rev. D **61**, 034001 (1999); T. M. P. Tait and C. P. Yuan, Phys. Rev. D **63**, 014018 (2000); Q. H. Cao, J. Wudka, and C. P. Yuan, Phys. Lett. B **658**, 50 (2007); Q. H. Cao and J. Wudka, Phys. Rev. D **74**, 094015 (2006); F. Larios, M. A. Perez, and C. P. Yuan, Phys. Lett. B **457**, 334 (1999); U. Baur, A. Juste, L. H. Orr, and D. Rainwater, Nucl. Phys. B, Proc. Suppl. **160**, 17 (2006); B. Grzadkowski and M. Misiak, Phys. Rev. D **78**, 077501 (2008); B. Sahin and I. Sahin, Eur. Phys. J. C **54**, 435 (2008).

[3] R. Schwienhorst *et al.* (D0 Collaboration and CDF Collaboration), arXiv:0805.2175; V. M. Abazov *et al.* (D0 Collaboration), Phys. Rev. Lett. **101**, 221801 (2008); **100**, 062004 (2008); C. I. Ciobanu *et al.* (CDF Collaboration and D0 Collaboration), arXiv:0809.2173.

[4] J. A. Aguilar-Saavedra, J. Carvalho, N. Castro, A. Onofre, and F. Veloso, Eur. Phys. J. C **53**, 689 (2008); J. A. Aguilar-Saavedra, Nucl. Phys. **B804**, 160 (2008); F. Hubaut, E. Monnier, P. Pralavorio, K. Smolek, and V. Simak, Eur. Phys. J. C **44**, Suppl. 1, 13 (2005).

[5] J. Cao, R. J. Oakes, F. Wang, and J. M. Yang, Phys. Rev. D **68**, 054019 (2003); M. Beccaria, C. M. Carloni Calame, G. Macorini, E. Mirabella, F. Piccinini, F. M. Renard, and C. Verzegnassi, Phys. Rev. D **77**, 113018 (2008).

[6] X. Wang, Q. Zhang, and Q. Qiao, Phys. Rev. D **71**, 014035 (2005); X. Wang, Y. Xi, Y. Zhang, and H. Jin, Phys. Rev. D **77**, 115006 (2008).

[7] S. Chivukula, arXiv:hep-ph/9803219.

[8] N. Arkani-Hamed, A. G. Cohen, and H. Georgi, Phys. Lett. B **513**, 232 (2001); N. Arkani-Hamed, A. G. Cohen, T. Gregoire, and J. G. Wacker, J. High Energy Phys. 08 (2002) 020; N. Arkani-Hamed, A. G. Cohen, E. Katz, A. E. Nelson, T. Gregoire, and J. G. Wacker, J. High Energy Phys. 08 (2002) 021; W. Skiba and J. Terning, Phys. Rev. D **68**, 075001 (2003).

[9] N. Arkani-Hamed, A. G. Cohen, E. Katz, and A. E. Nelson, J. High Energy Phys. 07 (2002) 034.

[10] M. Perelstein, Prog. Part. Nucl. Phys. **58**, 247 (2007); M. Schmaltz and D. Tucker-Smith, Annu. Rev. Nucl. Part. Sci. **55**, 229 (2005).

[11] T. Han, H. E. Logan, B. McElrath, and L. T. Wang, Phys. Rev. D **67**, 095004 (2003); G. Burdman, M. Perelstein, and A. Pierce, Phys. Rev. Lett. **90**, 241802 (2003); **92**, 049903(E) (2004); T. Han, H. E. Logan, and L. T. Wang, J. High Energy Phys. 01 (2006) 099; M. Perelstein, M. E. Peskin, and A. Pierce, Phys. Rev. D **69**, 075002 (2004); D. E. Kaplan, M. Schmaltz, and W. Skiba, Phys. Rev. D **70**, 075009 (2004); G. Azuelos *et al.*, Eur. Phys. J. C **39**, Suppl. 2, 13 (2005); J. A. Conley, J. L. Hewett, and M. P. Le, Phys. Rev. D **72**, 115014 (2005); C. X. Yue, L. Zhou, and S. Yang, Eur. Phys. J. C **48**, 243 (2006); C. X. Yue, S. Yang, and L. H. Wang, Europhys. Lett. **76**, 381 (2006); Y. B. Liu, J. F. Shen, and X. L. Wang, arXiv:hep-ph/0610350; L. Wang, W. Wang, J. M. Yang, and H. Zhang, Phys. Rev. D **75**, 074006 (2007); S. K. Kang, C. S. Kim, and J. Park, Phys. Lett. B **666**, 38 (2008); J. Boersma, Phys. Rev. D **74**, 115008 (2006); G. A. Gonzalez-Sprinberg, R. Martinez, and J. A. Rodriguez, Phys. Rev. D **71**, 035003 (2005); W. Kilian, D. Rainwater, and J. Reuter, Phys. Rev. D **74**, 095003 (2006); **74**, 099905(E) (2006); K. Cheung, C. S. Kim, K. Y. Lee, and J. Song, Phys. Rev. D **74**, 115013 (2006); C. S. Chen, K. Cheung, and T. C. Yuan, Phys. Lett. B **644**, 158 (2007); K. Cheung and J. Song, Phys. Rev. D **76**, 035007 (2007); K. Cheung, J. Song, P. Tseng, and Q. S. Yan, Phys. Rev. D **78**, 055015 (2008).

[12] C. Csaki, J. Hubisz, G. D. Kribs, P. Meade, and J. Terning, Phys. Rev. D **67**, 115002 (2003); **68**, 035009 (2003); J. L. Hewett, F. J. Petriello, and T. G. Rizzo, J. High Energy Phys. 10 (2003) 062; W. Kilian and J. Reuter, Phys. Rev. D **70**, 015004 (2004); Z. Han and W. Skiba, Phys. Rev. D **72**, 035005 (2005); M. C. Chen and S. Dawson, Phys. Rev. D **70**, 015003 (2004); C. x. Yue and W. Wang, Nucl. Phys. **B683**, 48 (2004); M. C. Chen, Mod. Phys. Lett. A **21**, 621 (2006).

[13] H. C. Cheng and I. Low, J. High Energy Phys. 08 (2004) 061; 09 (2003) 051; H. C. Cheng, arXiv:0710.3407; H. C. Cheng, I. Low, and L. T. Wang, Phys. Rev. D **74**, 055001 (2006); M. Perelstein, Pramana **67**, 813 (2006); W. Kilian, D. Rainwater, and J. Reuter, Phys. Rev. D **71**, 015008 (2005).

[14] I. Low, J. High Energy Phys. 10 (2004) 067.

[15] J. Hubisz, P. Meade, A. Noble, and M. Perelstein, J. High Energy Phys. 01 (2006) 135.

[16] C. F. Berger, M. Perelstein, and F. Petriello, arXiv:hep-ph/0512053; M. S. Carena, J. Hubisz, M. Perelstein, and P. Verdier, Phys. Rev. D **75**, 091701 (2007); A. Freitas and D. Wyler, J. High Energy Phys. 11 (2006) 061; M. M. Nojiri and M. Takeuchi, J. High Energy Phys. 10 (2008) 025; Y. B. Liu, J. F. Shen, and X. L. Wang, arXiv:hep-ph/0610350; Q. H. Cao, C. R. Chen, F. Larios, and C. P. Yuan, arXiv:0801.2998 [Phys. Rev. D (to be published)]; C. R. Chen, K. Tobe, and C. P. Yuan, Phys. Lett. B **640**, 263 (2006); Q. H. Cao and C. R. Chen, Phys. Rev. D **76**, 075007 (2007); K. Hsieh and C. P. Yuan, Phys. Rev. D **78**, 053006 (2008); V. Barger, W. Y. Keung, and Y. Gao, Phys. Lett. B **655**, 228 (2007); A. Datta, P. Dey, S. K. Gupta, B. Mukhopadhyaya, and A. Nyffeler, Phys. Lett. B **659**, 308 (2008).

[17] C. T. Hill and R. J. Hill, Phys. Rev. D **76**, 115014 (2007); **75**, 115009 (2007).

[18] B. Grinstein and M. Trott, J. High Energy Phys. 11 (2008) 064.

[19] R. Barcelo, M. Masip, and M. Moreno-Torres, Nucl. Phys. **B782**, 159 (2007).

[20] A. Belyaev, Chuan-Ren Chen, K. Tobe, and C.-P. Yuan, Phys. Rev. D **74**, 115020 (2006).

[21] J. Hubisz and P. Meade, Phys. Rev. D **71**, 035016 (2005).

[22] S. Matsumoto, T. Moroi, and K. Tobe, Phys. Rev. D **78**, 055018 (2008).

[23] J. Hubisz, S. J. Lee, and G. Paz, J. High Energy Phys. 06 (2006) 041; M. Blanke, A. Buras, A. Poschienrieder, C. Tarantino, S. Uhlig, and A. Weiler. J. High Energy Phys.

12 (**2006**) 003.
[24] C. R. Chen, F. Larios, and C. P. Yuan, Phys. Lett. B **631**, 126 (2005).
[25] G. L. Kane, G. A. Ladinsky, and C.-P. Yuan. Phys. Rev. D **45**, 124 (1992).
[26] F. del Aguila and J. A. Aguilar-Saavedra, Phys. Rev. D **67**, 014009 (2003).
[27] W. Bernreuther, J. Phys. G **35**, 083001 (2008).
[28] Q. H. Cao, C. S. Li, and C. P. Yuan, Phys. Lett. B **668**, 24 (2008).
[29] We refer the reader to the discussion in Ref. [15] concerning the electroweak precision constraints on the LHT and the effects of higher dimension operators.
[30] J. Pumplin, D. R. Stump, J. Huston, H. L. Lai, P. Nadolsky, and W. K. Tung, J. High Energy Phys. 07 (2002) 012.
[31] H. S. Do, S. Groote, J. G. Korner, and M. C. Mauser, Phys. Rev. D **67**, 091501 (2003), and references therein; G. Calderon and G. Lopez Castro, Int. J. Mod. Phys. A **23**, 3525 (2008).
[32] M. Blanke, A. Buras, A. Poschienrieder, C. Tarantino, S. Recksiegel, S. Uhlig, and A. Weiler J. High Energy Phys. 01 (2007) 066.
[33] W. Hollik, Reports No. MPI-PH-93-21 and No. BI-TP-93-16, 1993; see also T.-P. Cheng and L.-F. Li, *Gauge Theory of Elementary Particle Physics* (Clarendon Press, Oxford, 1984).