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Electroweak chiral Lagrangian from a natural topcolor-assisted technicolor model

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Based on previous studies on computing coefficients of the electroweak chiral Lagrangian from C. T. Hill's schematic topcolor-assisted technicolor model, we generalize the calculation to K. Lane's prototype natural topcolor-assisted technicolor model. We find that typical features of the model are qualitatively similar to those of Hill's, but Lane's model prefers a smaller technicolor group and the Z' mass must be smaller than 400 GeV. Furthermore, the S parameter is around the order of $+1$, mainly due to the existence of three doublets of techniquarks. We obtain the values for all coefficients of the electroweak chiral Lagrangian up to the order p^4. Apart from large negative four-fermion coupling values, the extended technicolor impacts on the electroweak chiral Lagrangian coefficients are small, since the techniquark self energy, which determines these coefficients, in general receives almost no influence from the extended technicolor induced four-fermion interactions except for its large momentum tail.

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I. INTRODUCTION

The topcolor-assisted technicolor (TC2) model realizes the electroweak symmetry breaking (EWSB) by joining technicolor (TC) and topcolor together to remove the objections that topcolor is unnatural and that TC cannot generate a large top mass. In the first schematic model proposed by C. T. Hill [1], EWSB is driven mainly by TC interactions and light quark and lepton masses are expected to be generated by extended technicolor (ETC). The third generation $(t, b)_{L,R}$ is arranged to transform with the usual quantum numbers under the gauge group $SU(3)_1 \otimes U(1)_1$, while (u, d), and (c, s) transform under a separate group $SU(3)_2 \otimes U(1)_2$. At the scale of order 1 TeV, $SU(3)_1 \otimes SU(3)_2 \otimes U(1)_1 \otimes U(1)_2$ is dynamically broken to the diagonal subgroup $SU(3)_C \otimes U(1)_Y$, and $SU(3)_1 \otimes U(1)_1$ interactions are supercritical for t quark leading top condensation, but subcritical for b quark causing no bottom condensation which achieve a large mass difference between t and b quarks.

As a candidate of a new physics model, before any new particles such as Z' or colorons predicted in the TC2 model appear in upcoming collider experiments, the behavior of the model in the low energy region for those discovered particles can be tested and described by its effective electroweak chiral Lagrangian (EWCL) [2–4] which, as a model independent platform of investigating EWSB mechanism, parametrizes the model by a set of coefficients. Starting from this EWCL, besides phenomenological research on fixing the coefficients of EWCL from experiments data, theoretical studies concentrate on computing the values of the coefficients from the detailed underlying model. Considering that TC2 model involves strongly-coupled dynamics for which traditionally perturbative expansion fails in computing the coefficients of EWCL. As in the previous paper [5],and based on our earlier systematic studies [6–10] on deriving the chiral Lagrangian and evaluating corresponding low energy constants (LECs) for pseudoscalar mesons from the first principle of QCD, we built up a formulation computing bosonic part of EWCL coefficients up to the order p^4 for one-doublet TC model [11] and Hill's schematic model [1]. This formulation is of general purposes, and it can be applied to many other strongly-coupled models. Then EWCL becomes a universal platform on which we can compare different underlying models with experiment data and extract the true physical theory of our real world. To achieve the aim of this comparison, the left theoretical works are to compute EWCL coefficients model by model. Present work is the second paper starting from Ref. [5] for series computations for various strongly coupled new physics models. Here we focus on K. Lane's prototype natural TC2 model [12].

In Hill's original model, effects of ETC interactions are only qualitatively estimated. Effective four-fermion interactions induced by ETC (EFFIIETC) are even not explicitly written in Ref. [1]. Accordingly, our previous computations [5] also do not involve possible ETC's contributions. By examining ETC effects, Chivukula, Dobrescu, and Terning (CDT) [13] argued that the TC2 proposal cannot be both natural and consistent with experimental measurements of the parameter $\rho = M_W^2/M_Z^2\cos^2\theta_W$. In the extreme case, even for degenerate up and down-type, the technifermions of third generation are likely to have custodial-isospin violating couplings to

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the strong $U(1)_1$ since part of m_t must arise from ETC, and this leads to large contributions to ρ parameter which contradicts with experiment data.[1] To overcome this difficulty, instead of conventional just one-doublet third generation technifermions, K. Lane and E. Eichten propose their model [12] by introducing two sets of technifermion doublets for third generation techniquarks with different $U(1)_1$ charges but up and down-type technifermions in the same doublet possessing the same $U(1)_1$ charges: $T^t_{L,R} = (U^t, D^t)_{L,R}$ giving the top quark its ETC mass; $T^b_{L,R} = (U^b, D^b)_{L,R}$ giving the bottom quark its ETC mass, these cut the intimate relation between custodial-isospin violation from techniquarks and $t - b$ mass difference. Because of this important role of ETC interactions in Lane's model, its effects in EWCL are worthy of examination. This paper is not only for computing EWCL coefficients of Lane's model, but also for investigating ETC effects on these coefficients.

In the next section, we apply our formulation developed in Ref. [5] to Lane's model [12]. We perform dynamical calculations through several steps: first we integrate in the Goldstone field U, then integrate out technigluons and techniquarks by solving the Schwinger-Dyson equation (SDE) for techniquarks and compute the effective action, then we further integrate out Z' and finally obtain the EWCL coefficients. Section III contains the discussion. In the Appendix, we list some requisite formulas.

II. DERIVATION OF EWCL FROM LANE'S MODEL

Considering a prototype natural TC2 model proposed by K. Lane and E. Eichten [12], the TC group is not specified in Ref. [12], but chosen to be $SU(N)$ in later Lane's improved model[14]. For definiteness, we take $G_{\rm TC} = SU(N)$. The gauge charge assignments of techniquarks in $G_{\rm TC}\otimes SU(3)_1\otimes SU(3)_2\otimes SU(2)_L\otimes U(1)_{Y_1}\otimes U(1)_{Y_2}$ are shown as Table I, for which we choose the case B solution[2] to the anomaly free conditions of Ref. [12].

The action of the symmetry breaking sector then is

$$S_{\rm SBS}[G^\alpha_\mu, A^A_{1\mu}, A^A_{2\mu}, W^a_\mu, B_{1\mu}, B_{2\mu}, \bar{T}^l, T^l, \bar{T}^t, T^t, \bar{T}^b, T^b] = \int d^4x(\mathcal{L}_{\rm gauge} + \mathcal{L}_{\rm techniquark} + \mathcal{L}_{\rm breaking} + \mathcal{L}_{4T}), \quad (1)$$

[1]In fact, the detailed up and down-type technifermions of third generation are formally arranged not to participate $U(1)_1$ interaction by vanishing their $U(1)_1$ charges in original Hill's model and then do not cause large contribution to ρ. This result is compatible with that obtained in Ref. [5]. But this naive arrangement is not realistic in the sense, as mentioned by CDT [13], that to give top and bottom (which must have different $U(1)_1$ charges to allow for their different masses) ETC masses, the different right-handed technifermions to which top and bottom quarks couple must have different $U(1)_1$ charges.

[2]Case A solution, as mentioned by K. Lane in Ref. [12], would not be possible to generate proper ETC masses for the t and b quarks and therefore not considered in this work.

TABLE I. Gauge charge assignments of techniquarks for prototype natural TC2 model given in Ref. [12]. These techniquarks are $SU(3)_1\otimes SU(3)_2$ singlets.

field	$SU(N)$	$SU(2)_L$	$U(1)_{Y_1}$	$U(1)_{Y_2}$
T^l_L	N	2	0	0
U^l_R	N	1	0	$\frac{1}{2}$
D^l_R	N	1	0	$-\frac{1}{2}$
T^t_L	N	2	-1	1
U^t_R	N	1	$-\frac{1}{2}$	1
D^t_R	N	1	$-\frac{1}{2}$	0
T^b_L	N	2	1	-1
U^b_R	N	1	$\frac{1}{2}$	0
D^b_R	N	1	$\frac{1}{2}$	-1

with different parts of Lagrangian given by

$$\mathcal{L}_{\rm gauge} = -\frac{1}{4}F^\alpha_{\mu\nu}F^{\alpha,\mu\nu} - \frac{1}{4}A^A_{1\mu\nu}A^{A\mu\nu}_1 - \frac{1}{4}A^A_{2\mu\nu}A^{A\mu\nu}_2 - \frac{1}{4}W^a_{\mu\nu}W^{a,\mu\nu} - \frac{1}{4}B_{1\mu\nu}B^{\mu\nu}_1 - \frac{1}{4}B_{2\mu\nu}B^{\mu\nu}_2, \quad (2)$$

$$\begin{aligned}\mathcal{L}_{\rm techniquark} &= \bar{T}^l\Big(i\not{\partial} - g_{\rm TC}t^\alpha\not{G}^\alpha - g_2\frac{\tau^a}{2}\not{W}^aP_L - \frac{1}{2}q_2\not{B}_2\tau^3P_R\Big)T^l \\ &+ \bar{T}^t\Big(i\not{\partial} - g_{\rm TC}t^\alpha\not{G}^\alpha - g_2\frac{\tau^a}{2}\not{W}^aP_L + q_1\not{B}_1P_L \\ &- q_2\not{B}_2P_L + \frac{1}{2}q_1\not{B}_1P_R - \Big(\frac{1}{2}+\frac{\tau^3}{2}\Big)q_2\not{B}_2P_R\Big)T^t \\ &+ \bar{T}^b\Big(i\not{\partial} - g_{\rm TC}t^\alpha\not{G}^\alpha - g_2\frac{\tau^a}{2}\not{W}^aP_L - q_1\not{B}_1P_L \\ &+ q_2\not{B}_2P_L - \frac{1}{2}q_1\not{B}_1P_R + \Big(\frac{1}{2}-\frac{\tau^3}{2}\Big)q_2\not{B}_2P_R\Big)T^b, \end{aligned}\quad (3)$$

$$\mathcal{L}_{4T} = \mathcal{H}_{\rm diag}, \quad (4)$$

$$\mathcal{H}_{\rm diag} = \frac{g^2_{\rm ETC}}{M^2_{\rm ETC}}\bar{T}^i_L\gamma^\mu T^i_L(b_U\bar{U}^j_R\gamma_\mu U^j_R + b_D\bar{D}^j_R\gamma_\mu D^j_R), \quad (5)$$

where $g_{\rm TC}$, g_2, q_1 and q_2 are the coupling constants of, respectively, $SU(N)$, $SU(2)_L$, $U(1)_{Y_1}$, and $U(1)_{Y_2}$ (since techniquarks are $SU(3)_1\otimes SU(3)_2$ singlets, corresponding coupling constants do not show up here); and the corresponding gauge fields (field strength tensors) are denoted by G^α_μ, W^a_μ, $B_{1\mu}$ and $B_{2\mu}$ ($F^\alpha_{\mu\nu}$, $W^a_{\mu\nu}$, $B_{1\mu\nu}$ and $B_{2\mu\nu}$) with the superscript α runs from 1 to N^2-1 and a from 1 to 3 ($SU(3)_1\otimes SU(3)_2$ gauge fields and field strength tensors are denoted by $A^A_{1\mu}$, $A^A_{2\mu}$ and $A^A_{1\mu\nu}$, $A^A_{2\mu\nu}$ with the superscript A runs from 1 to 8); $t^\alpha = \lambda^\alpha/2$ ($\alpha = 1,\ldots,N^2-1$) and τ^a ($a = 1, 2, 3$) are, respectively, Gell-Mann and Pauli matrices. $P_{\substack{R\\L}} = (1 \pm \gamma_5)/2$. Ordinary quarks are neglected, since we only discuss the bosonic part of EWCL.[3] For ETC induced four-fermion interactions $\mathcal{L}_{4T}$, although in original Ref. [12], except $\mathcal{H}_{\rm diag}$, there are other different kinds of interactions, such as $\mathcal{H}_{\bar{l}t\bar{t}b}$ and $\mathcal{H}_{\bar{l}b\bar{b}t}$, consider that these nondiagonal interactions will induce nondiagonal condensations which violate the preferred requirement $\langle \bar{U}^i_L U^j_R\rangle = \langle \bar{D}^i_L D^j_R\rangle \propto \delta_{ij}$ for $i,\ j = l,\ t,\ b$ given in Ref. [12], we drop them in our calculation.

In Ref. [12], an operator effecting $SU(3)_1 \otimes SU(3)_2 \otimes U(1)_1 \otimes U(1)_2$ breaking to $SU(3)_C \otimes U(1)_Y$ is needed. We introduce a 3×3 matrix scalar field Φ to take the role of this operator to break $SU(3)_1 \otimes SU(3)_2 \otimes U(1)_{Y_1} \otimes U(1)_{Y_2}$ to $SU(3)_C \otimes U(1)_Y$ which leads to massive colorons and Z'. This scalar field transforms as $(\bar{3}, 3, \frac{5}{6}, -\frac{5}{6})$ under the group $SU(3)_1 \otimes SU(3)_2 \otimes U(1)_{Y_1} \otimes U(1)_{Y_2}$ which leads to the covariant derivative

$$D_\mu\Phi = \partial_u\Phi + i\Phi\left(h_1\frac{\lambda^{A*}}{2}A^A_{1\mu} - \frac{5}{6}q_1B_{1\mu}\right) - i\left(h_2\frac{\lambda^A}{2}A^A_{2\mu} - \frac{5}{6}q_2B_{2\mu}\right)\Phi,$$

with h_1 and h_2 being the coupling constants of $SU(3)_1 \otimes SU(3)_2$ and corresponding Lagrangian can be written as

$$\mathcal{L}_{\rm H} = \frac{1}{2}\,{\rm tr}[(D_\mu\Phi)^\dagger(D^\mu\Phi)] + V(\Phi), \qquad (6)$$

in which potential $V(\Phi)$ is assumed to cause vacuum condensate $\Phi_{ij} = \tilde{v}\delta_{ij}$ and the leading effects can be obtained by just replacing Φ with its vacuum expectation value in (6),

$$\mathcal{L}_H|_{\Phi=\tilde{v}} = \frac{1}{4}\frac{g_3^2\tilde{v}^2}{\sin^2\theta\cos^2\theta}B^A_\mu B^{A\mu} + \frac{25}{72}\frac{g_1^2\tilde{v}^2}{\sin^2\theta'\cos^2\theta'}Z'_\mu Z'^\mu, \qquad (7)$$

where the SM $U(1)_Y$ field B_μ with generator $Y = Y_1 + Y_2$ and the $U(1)'$ field Z'_μ (the gluon A^A_μ and coloron B^A_μ) are defined by orthogonal rotations with mixing angle θ' (θ):

$$\begin{pmatrix} B_{1\mu} & B_{2\mu}\end{pmatrix} = \begin{pmatrix} Z'_\mu & B_\mu\end{pmatrix}\begin{pmatrix}\cos\theta' & -\sin\theta' \\ \sin\theta' & \cos\theta'\end{pmatrix}, \qquad (8a)$$

$$\begin{pmatrix} A^A_{1\mu} & A^A_{2\mu}\end{pmatrix} = \begin{pmatrix} B^A_\mu & A^A_\mu\end{pmatrix}\begin{pmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{pmatrix}, \qquad (8b)$$

with

$$g_1 \equiv q_1\sin\theta' = q_2\cos\theta' \qquad g_3 \equiv h_1\sin\theta = h_2\cos\theta. \qquad (9)$$

The coloron field B^A_μ does not couple to other fields except to ordinary fermions at present order of approximation, so we can ignore their contributions to bosonic part of EWCL.[4] i.e. we can take

$$\mathcal{L}_{\rm breaking} = \frac{1}{2}M_0^2Z'_\mu Z'^\mu \qquad M_0^2 = \frac{25}{36}\frac{g_1^2\tilde{v}^2}{\sin^2\theta'\cos^2\theta'}. \qquad (10)$$

With the above preparations, the strategy to derive the EWCL from Lane's model can be formulated as

$$\exp(iS_{\rm EW}[W^a_\mu, B_\mu]) = \int\mathcal{D}\bar{T}^l\mathcal{D}T^l\mathcal{D}\bar{T}^t\mathcal{D}T^t\mathcal{D}\bar{T}^b\mathcal{D}T^b \times \mathcal{D}G^\alpha_\mu\mathcal{D}Z'_\mu\exp[iS_{\rm SBS}[G^\alpha_\mu, 0, 0, W^a_\mu, B_{1\mu}, B_{2\mu}, \bar{T}^l, T^l, \bar{T}^t, T^t, \bar{T}^b, T^b]] \qquad (11)$$

$$= \mathcal{N}[W^a_\mu, B_\mu]\int\mathcal{D}\mu(U)\exp(iS_{\rm eff}[U, W^a_\mu, B_\mu]), \qquad (12)$$

where A^A_μ related to $A^A_{1\mu}$ and $A^A_{2\mu}$ through (8b) is ordinary gluon field, $U(x)$ is a dimensionless unitary unimodular matrix field in EWCL, and $\mathcal{D}\mu(U)$ denotes the normalized functional integration measure on U. The normalization factor $\mathcal{N}[W^a_\mu, B_\mu]$ is determined through requirements that when the TC and ETC interactions are switched off, $S_{\rm eff}[U, W^a_\mu, B_\mu]$ must vanish. This leads to the following electroweak gauge fields W^a_μ, B_μ dependent $\mathcal{N}[W^a_\mu, B_\mu]$,

$$\mathcal{N}[W^a_\mu, B_\mu] = \int\mathcal{D}\bar{T}^l\mathcal{D}T^l\mathcal{D}\bar{T}^t\mathcal{D}T^t\mathcal{D}\bar{T}^b\mathcal{D}T^b\mathcal{D}G^\alpha_\mu \times \mathcal{D}Z'_\mu e^{iS_{\rm SBS}|_{\rm ignore\ \ TC,ETC,A^A_{1\mu}=A^A_{2\mu}=0}}. \qquad (13)$$

Since there are many steps in deriving EWCL, we discuss them separately in the following subsections.

A. Integrating in Goldstone field U

In terms of Z' and B fields given by (8a), we can rewrite techniquark interaction (3) as

$$\mathcal{L}_{\rm techniquark} = \bar{\psi}(i\partial\!\!\!/ - g_{\rm TC}t^\alpha G\!\!\!/^\alpha + V\!\!\!/ + A\!\!\!/\gamma^5)\psi, \qquad (14)$$

where all three doublets techniquarks are arranged in one by six matrix $\psi = (U^l, D^l, U^t, D^t, U^b, D^b)^T$ and

[3]For top quark, its effect should be considered due to its large mass comparable to symmetry breaking scale. There is an EFFIIETC $\mathcal{H}_{\bar{t}t} = \frac{g^2_{\rm ETC}}{M^2_{\rm ETC}}\bar{t}_L\gamma^\mu U^t_L\bar{U}^t_R\gamma_\mu t_R$ + H.c. responsible for top mass. This interaction should be included in our calculation in principle, and if top quark has nonzero condensation, this interaction will contribute to techniquark self energy. Since Ref. [12] treats this term as a perturbation, we can ignore it at leading order of our coefficients' computations.

[4]One can consider higher order corrections by including in (7) the quantum fluctuation effects of field Φ. Since these effects depend on details of symmetry breaking mechanism which are not specified in Ref. [12], in order not to deviate original Lane's model too much, we ignore them in the present paper.

$$V_\mu = \left(-\frac{1}{2}g_2\frac{\tau^a}{2}W^a_\mu - \frac{1}{2}g_1\frac{\tau^3}{2}B_\mu\right)\otimes \mathbf{I} + Z_{V\mu}, \quad (15)$$

$$A_\mu = \left(\frac{1}{2}g_2\frac{\tau^a}{2}W^a_\mu - \frac{1}{2}g_1\frac{\tau^3}{2}B_\mu\right)\otimes \mathbf{I} + Z_{A\mu}, \quad (16)$$

with $\mathbf{I} = \mathrm{diag}(1,1,1)$, $Z_{V\mu} = \mathrm{diag}(Z^l_{V\mu}, Z^t_{V\mu}, Z^b_{V\mu})$, $Z_{A\mu} = \mathrm{diag}(Z^l_{A\mu}, Z^t_{A\mu}, Z^b_{A\mu})$, and

$$\begin{aligned} Z^l_{V\mu} &= \frac{1}{4}g_1\tan\theta' Z'_\mu\tau^3, \\ Z^t_{V\mu} &= g_1 Z'_\mu\left[\frac{3}{4}\cot\theta' + \left(\frac{3}{4}+\frac{1}{4}\tau^3\right)\tan\theta'\right], \\ Z^b_{V\mu} &= g_1 Z'_\mu\left[-\frac{3}{4}\cot\theta' - \left(\frac{3}{4}-\frac{1}{4}\tau^3\right)\tan\theta'\right], \end{aligned} \quad (17)$$

$$\begin{aligned} Z^l_{A\mu} &= \frac{1}{4}g_1\tan\theta' Z'_\mu\tau^3, \\ Z^t_{A\mu} &= g_1 Z'_\mu\left[-\frac{1}{4}\cot\theta' + \left(-\frac{1}{4}+\frac{1}{4}\tau^3\right)\tan\theta'\right], \\ Z^b_{A\mu} &= g_1 Z'_\mu\left[\frac{1}{4}\cot\theta' + \left(\frac{1}{4}+\frac{1}{4}\tau^3\right)\tan\theta'\right]. \end{aligned} \quad (18)$$

The Lagrangian (1) is locally $SU(2)_L\times U(1)_Y$ invariant and approximately globally $SU(6)_L\times SU(6)_R$ invariant. We introduce a local 2×2 operator $O(x)$ as $O(x)\equiv \mathrm{tr}_{lc}[T^l_L(x)\bar{T}^l_R(x) + T^t_L(x)\bar{T}^t_R(x) + T^b_L(x)\bar{T}^b_R(x)]$ with tr_{lc} as the trace with respect to Lorentz and TC indices. The transformation of $O(x)$ under $SU(2)_L\times U(1)_Y$ is $O(x)\to V_L(x)O(x)V_R^\dagger(x)$ (with $V_L = e^{i(\tau^a/2)\theta^a}$ and $V_R = e^{i(\tau^3/2)\theta^0}$). Then we decompose $O(x)$ as $O(x) = \xi_L^\dagger(x)\sigma(x)\xi_R(x)$ with the $\sigma(x)$ represented by a Hermitian matrix describes the modular degree of freedom; while $\xi_L(x)$ and $\xi_R(x)$ are represented by unitary matrices describe the phase degree of freedom of $SU(2)_L$ and $U(1)_Y$ respectively. Now we define a new field $U(x)$ as $U(x)\equiv\xi_L^\dagger(x)\xi_R(x)$ which is the nonlinear realization of the Goldstone boson field in EWCL. Subtracting the $\sigma(x)$ field, we find that the present decomposition results in a constraint $\xi_L(x)O(x)\xi_R^\dagger(x) - \xi_R(x)O^\dagger(x)\xi_L^\dagger(x) = 0$, the functional expression of it is

$$\int \mathcal{D}\mu(U)\mathcal{F}[O]\delta(\xi_L O\xi_R^\dagger - \xi_R O^\dagger\xi_L^\dagger) = \text{const.}, \quad (19)$$

where $\mathcal{D}\mu(U)$ is an effective invariant integration measure; $\mathcal{F}[O]$ only depends on O. Substituting identity (19) into (11), we obtain

$$\int \mathcal{D}G^\alpha_\mu\mathcal{D}\bar\psi\mathcal{D}\psi\mathcal{D}Z'_\mu \exp(iS_{\mathrm{SBS}}|_{A^A_{1\mu}=A^A_{2\mu}=0}) = \int\mathcal{D}\mu(U)\mathcal{D}Z'_\mu\exp(iS_{Z'}[U,W^a_\mu,B_\mu,Z'_\mu]), \quad (20)$$

where $\mathcal{D}\bar\psi\mathcal{D}\psi$ is the shorthand notation for $\mathcal{D}\bar{T}^l\mathcal{D}T^l\mathcal{D}\bar{T}^t\mathcal{D}T^t\mathcal{D}\bar{T}^b\mathcal{D}T^b$ and

$$\begin{aligned} S_{Z'}[U,W^a_\mu,B_\mu,Z'_\mu] = &\int d^4x\left(-\frac{1}{4}W^a_{\mu\nu}W^{a,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}Z'_{\mu\nu}Z'^{\mu\nu} + \frac{1}{2}M_0^2 Z'_\mu Z'^\mu\right) \\ &- i\log\int\mathcal{D}G^\alpha_\mu\mathcal{D}\bar\psi\mathcal{D}\psi\mathcal{F}[O]\delta(\xi_L O\xi_R^\dagger - \xi_R O^\dagger\xi_L^\dagger) \\ &\times\exp\left\{i\int d^4x\left[-\frac{1}{4}F^\alpha_{\mu\nu}F^{\alpha,\mu\nu} + \bar\psi(i\partial\!\!\!/ - g_{\mathrm{TC}}t^\alpha G\!\!\!/^\alpha + V\!\!\!/ + A\!\!\!/\gamma^5)\psi + \mathcal{L}_{4T}\right]\right\}. \end{aligned} \quad (21)$$

From (12), S_{eff} relates to $S_{Z'}$ by

$$\mathcal{N}[W^a_\mu,B_\mu]e^{iS_{\mathrm{eff}}[U,W^a_\mu,B_\mu]} = \int\mathcal{D}Z'_\mu e^{iS_{Z'}[U,W^a_\mu,B_\mu,Z'_\mu]} \quad (22)$$

To match the correct normalization, we introduce the logarithm function as the normalization factor $\int\mathcal{D}\bar\psi\mathcal{D}\psi e^{i\int d^4x\bar\psi(i\partial\!\!\!/+V\!\!\!/+A\!\!\!/\gamma^5)\psi} = \exp\mathrm{Tr}\log(i\partial\!\!\!/ + V\!\!\!/ + A\!\!\!/\gamma^5)$ and then take a special $SU(2)_L\times U(1)_Y$ rotation, as $V_L(x) = \xi_L(x)$ and $V_R(x)=\xi_R(x)$, on both numerator and denominator,

$$\begin{aligned} S_{Z'}[U,W^a_\mu,B_\mu,Z'_\mu] = &\int d^4x\left(-\frac{1}{4}W^a_{\mu\nu}W^{a,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}Z'_{\mu\nu}Z'^{\mu\nu} + \frac{1}{2}M_0^2 Z'_\mu Z'^\mu\right) - i\,\mathrm{Tr}\log(i\partial\!\!\!/ + V\!\!\!/ + A\!\!\!/\gamma^5) \\ &- i\log\frac{\int\mathcal{D}G^\alpha_\mu\mathcal{D}\bar\psi_\xi\mathcal{D}\psi_\xi\mathcal{F}[O_\xi]\delta(O_\xi - O_\xi^\dagger)e^{iS'}}{\int\mathcal{D}\bar\psi_\xi\mathcal{D}\psi_\xi e^{iS'|_{\text{ignore TC,ETC}}}} \end{aligned} \quad (23)$$

$$S' = \int d^4x\left[-\frac{1}{4}F^\alpha_{\mu\nu}F^{\alpha,\mu\nu} + \bar\psi_\xi(i\partial\!\!\!/ - g_{\mathrm{TC}}t^\alpha G\!\!\!/^\alpha + V\!\!\!/_\xi + A\!\!\!/_\xi\gamma^5)\psi_\xi + \mathcal{L}_{\xi 4T}\right], \quad (24)$$

where the rotated fields are denoted by subscript ξ and they are defined as follows

$$T^i_\xi = P_L\xi_L(x)T^i_L(x) + P_R\xi_R(x)T^i_R(x), \qquad i=l,t,b \qquad O_\xi(x)\equiv\xi_L(x)O(x)\xi_R^\dagger(x) \qquad Z'_{\xi,\mu}(x)\equiv Z'_\mu(x), \quad (25)$$

$$g_2\frac{\tau^a}{2}W^a_{\xi,\mu}(x)\equiv\xi_L(x)\bigg[g_2\frac{\tau^a}{2}W^a_\mu(x)-i\partial_\mu\bigg]\xi^\dagger_L(x),\quad (26)$$

$$g_1\frac{\tau^3}{2}B_{\xi,\mu}(x)\equiv\xi_R(x)\bigg[g_1\frac{\tau^3}{2}B_\mu(x)-i\partial_\mu\bigg]\xi^\dagger_R(x).\quad (27)$$

and $\mathcal{L}_{\xi4T}$ is $\mathcal{L}_{4T}$ with TC fields replaced by rotated ones. It can be shown that

$$\mathcal{L}_{\xi4T}=\mathcal{L}_{4T}.\quad (28)$$

Action (23) can be further decomposed as

$$\begin{aligned}S_{Z'}[U,W^a_\mu,B_\mu,Z'_\mu]=\int d^4x\bigg(&-\frac{1}{4}W^a_{\mu\nu}W^{a,\mu\nu}-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}\\&-\frac{1}{4}Z'_{\mu\nu}Z'^{\mu\nu}+\frac{1}{2}M_0^2Z'_\mu Z'^\mu\bigg)\\&+S_{\text{norm}}[U,W^a_\mu,B_\mu,Z'_\mu]\\&+S_{\text{anom}}[U,W^a_\mu,B_\mu,Z'_\mu],\end{aligned}\quad (29)$$

where

$$S_{\text{norm}}[U,W^a_\mu,B_\mu]=-i\log\int\mathcal{D}G^\alpha_\mu\mathcal{D}\bar\psi_\xi\mathcal{D}\psi_\xi\mathcal{F}[O_\xi]\times\delta(O_\xi-O^\dagger_\xi)e^{iS'},\quad (30)$$

and

$$\begin{aligned}iS_{\text{anom}}[U,W^a_\mu,B_\mu,Z'_\mu]=&\,\mathrm{Tr}\log(i\partial\!\!\!/+V\!\!\!/+A\!\!\!/\gamma^5)\\&-\mathrm{Tr}\log(i\partial\!\!\!/+V\!\!\!/_\xi+A\!\!\!/_\xi\gamma^5).\end{aligned}\quad (31)$$

The transformations of the rotated fields under $SU(2)_L\times U(1)_Y$ are $\psi_\xi(x)\to h(x)\psi_\xi(x)$, $O_\xi(x)\to h(x)O_\xi(x)h^\dagger(x)$ with $h(x)$ describes a hidden local $U(1)$ symmetry. Thus, the chiral symmetry $SU(2)_L\otimes U(1)_Y$ covariance of the unrotated fields has been transferred totally to the hidden symmetry $U(1)$ covariance of the rotated fields.

B. Integrating out techinigluons and techniquarks

With the technique developed in Ref. [5], the technigulon fields in Eq. (3) can be formally integrated out with the help of full n-point Green's function of the G^α_μ-field $G^{\alpha_1\ldots\alpha_n}_{\mu_1\ldots\mu_n}$,

$$\begin{aligned}e^{iS_{\text{norm}}[U,W^a_\mu,B_\mu,Z'_\mu]}=&\int\mathcal{D}\bar\psi_\xi\mathcal{D}\psi_\xi\mathcal{F}[O_\xi]\delta(O_\xi-O^\dagger_\xi)\\&\times\exp\bigg\{i\int d^4x[\bar\psi_\xi(i\partial\!\!\!/+V\!\!\!/_\xi+A\!\!\!/_\xi\gamma^5)\psi_\xi\\&+\mathcal{L}_{\xi4T}]+\sum_{n=2}^\infty\int d^4x_1\ldots d^4x_n\frac{(-ig_{\text{TC}})^n}{n!}\\&\times G^{\alpha_1\ldots\alpha_n}_{\mu_1\ldots\mu_n}(x_1,\ldots,x_n)J^{\mu_1}_{\xi,\alpha_1}(x_1)\ldots J^{\mu_n}_{\xi,\alpha_n}\\&\times(x_n)\bigg\},\end{aligned}\quad (32)$$

where effective source $J^{\alpha\mu}_\xi(x)$ is identified as $J^{\alpha\mu}_\xi(x)\equiv\bar\psi_\xi(x)t^\alpha\gamma^\mu\psi_\xi(x)$.

1. Schwinger-Dyson equation for techniquark propagator

To show that the TC interaction indeed induces the condensation $\langle\bar\psi\psi\rangle\neq0$ which triggers EWSB, we explicitly calculate the behavior of techniquark propagator $S^{\sigma\rho}(x,x')\equiv\langle\psi^\sigma_\xi(x)\bar\psi^\rho_\xi(x')\rangle$ in the following. *Neglecting the factor* $\mathcal{F}[O_\xi]\delta(O_\xi-O^\dagger_\xi)$, the total functional derivative of the integrand with respect to $\bar\psi^\sigma_\xi(x)$ is zero,

$$\begin{aligned}0=\int\mathcal{D}\bar\psi_\xi\mathcal{D}\psi_\xi\frac{\delta}{\delta\bar\psi^\sigma_\xi(x)}\exp\bigg[&\int d^4x(\bar\psi_\xi I+\bar I\psi_\xi)\\&+i\int d^4x[\bar\psi_\xi(i\partial\!\!\!/+V\!\!\!/_\xi+A\!\!\!/_\xi\gamma^5)\psi_\xi+\mathcal{L}_{\xi4T}]\\&+\sum_{n=2}^\infty\int d^4x_1\ldots d^4x_n\frac{(-ig_{\text{TC}})^n}{n!}G^{\alpha_1\ldots\alpha_n}_{\mu_1\ldots\mu_n}(x_1,\ldots,x_n)\\&\times J^{\mu_1}_{\xi,\alpha_1}(x_1)\ldots J^{\mu_n}_{\xi,\alpha_n}(x_n)\bigg],\end{aligned}\quad (33)$$

where $I(x)$ and $\bar I(x)$ are the external sources for techniquark fields, respectively, $\bar\psi_\xi(x)$ and $\psi_\xi(x)$; and which leads to SDE for techniquark propagators,

$$S_X(x,y)=\langle X(x)\bar X(y)\rangle\qquad X=U^l_\xi,D^l_\xi,U^t_\xi,D^t_\xi,U^b_\xi,D^b_\xi.\quad (34)$$

The detail derivation procedure is similar to that in Ref. [5]. The only difference is that now we have EFFIIETC in the theory. The final obtained SDE[5] is

$$\begin{aligned}i\Sigma_X(x,y)=&\,C_2(N)g^2_{\text{TC}}G_{\mu\nu}(x,y)[\gamma^\mu S_X(x,y)\gamma^\nu]\\&-iC_X\gamma_\mu[P_LS_X(x,x)P_L\\&+P_RS_X(x,x)P_R]\gamma^\mu\delta(x-y),\end{aligned}\quad (35)$$

with techniquark self energy defined as

$$\begin{aligned}i\Sigma_X(x,y)\equiv&\,S_X^{-1}(x,y)+i[i\partial\!\!\!/_x+V\!\!\!/_\xi(x)+A\!\!\!/_\xi(x)\gamma_5]\\&\times\delta(x-y),\end{aligned}\quad (36)$$

[5]Notice that in original studies of QCD chiral Lagrangian, SDE was derived as stationary equation of effective action [6,7]. While in generalizing the formulation to EWCL in Ref. [5], to simplify the procedure, we change the original complex derivation of SDE from effective action to explicit path integral evaluation in which we have taken approximations of neglecting higher-point Green's functions and factorizing four point Green's function. The effects of these approximations are equivalent to take the lowest order in dynamical perturbation theory and the improved ladder approximation in original stationary equation approach. The judgment for the validity of these approximations relies on the results for QCD chiral Lagrangian, for which the obtained results with this approach more or less agree with the experimentally determined LEC's. In present work, we just follow the process proposed in Ref. [5].

and technigluon propagator $G^{\alpha\beta}_{\mu\nu}(x, y) = \delta^{\alpha\beta} G_{\mu\nu}(x, y)$. $C_2(N) = (N^2 - 1)/(2N)$ is Casimir operator from $(t^\alpha t^\alpha)_{ab} = C_2(N)\delta_{ab}$ for the fundamental representation of TC group $SU(N)$. Further C_X is effective ETC induced four fermion coupling which is

$$C_{U^l_\xi} = C_{U^t_\xi} = C_{U^b_\xi} = \frac{g^2_{\rm ETC}}{M^2_{\rm ETC}} b_U$$
$$C_{D^l_\xi} = C_{D^t_\xi} = C_{D^b_\xi} = \frac{g^2_{\rm ETC}}{M^2_{\rm ETC}} b_D. \quad (37)$$

In the following, we first consider the case of $V_{\xi,\mu} = A_{\xi,\mu} = 0$. In this situation, the technigluon propagator in Landau gauge is $G^{\alpha\beta}_{\mu\nu}(x, y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} G_{\mu\nu}(p^2)$ with $G_{\mu\nu}(p^2) = \frac{i}{-p^2[1+\Pi(-p^2)]}(g_{\mu\nu} - p_\mu p_\nu/p^2)$. And the techniquark self energy and propagator are respectively

$$\begin{pmatrix} \Sigma_X(x, y) \\ S_X(x, y) \end{pmatrix} = \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} \times \begin{pmatrix} \Sigma_X(-p^2) \\ S_X(-p^2) \end{pmatrix}, \quad (38)$$

with $S_X(p) = i/[\not p - \Sigma_X(-p^2)]$. Substitute the above results into the SDE and parametrize the technigluon propagator as $\alpha_{\rm TC}[(p_E - q_E)^2] \equiv g^2_{\rm TC}/(4\pi[1 + \Pi(p^2_E)])$ for the Euclidean momentum p_E, q_E, and we obtain the following integration equation which with angular approximation $\alpha_{\rm TC}[(p_E - q_E)^2] = \alpha_{\rm TC}(p^2_E)\theta(p^2_E - q^2_E) + \alpha_{\rm TC}(q^2_E)\theta(q^2_E - p^2_E)$. It can be further reduced to differential equation,

$$i\Sigma_X(-p^2) = 4\int \frac{d^4q}{(2\pi)^4}\left\{\frac{3\pi C_2(N)\alpha_{\rm TC}[-(p-q)^2]}{(p-q)^2} + C_X\right\} \times \left[\frac{\Sigma_X(-q^2)}{q^2 - \Sigma^2_X(-q^2)}\right]. \quad (39)$$

Once the above equation presents a nonzero solution, we obtain the nontrivial techniquark condensate

$$\langle \bar X(x) X'(x)\rangle = -4N\delta_{XX'}\int \frac{d^4p_E}{(2\pi)^4}\frac{\Sigma_X(p^2_E)}{p^2_E + \Sigma^2_X(p^2_E)}, \quad (40)$$

which breaks $SU(2)_L \otimes U(1)_1 \otimes U(1)_2$ to the subgroup $U(1)_{\rm em}$.

To obtain the numerical solution of Eq. (39), we take the running constant $\alpha_{\rm TC}(p^2)$, the same as that used in Eq. (49) of Ref. [5], for which there are three input parameters: N, N_f, and $\Lambda_{\rm TC}$. N as the TC number is a free parameter, and we take four different values $N = 3, 4, 5, 6$ to estimate its effects. $N_f = 6$ is due to three doublets of techniquarks. The scale of TC interaction $\Lambda_{\rm TC}$ will be fixed from $f = 250$ GeV determined later in (71). There is another parameter due to the presence of EFFIIETC. We denote its dimensionless value by $b \equiv C\Lambda^2_{\rm TC}$ with C, introduced in (37) as coefficients of EFFIIETC. Consider that C is proportional to $1/M^2_{\rm ETC}$, the dimensionless parameter $b \propto \Lambda^2_{\rm TC}/M^2_{\rm ETC}$ should be very small. We take the physical cutoff of the equation to be the scale of ETC and $\Lambda = \Lambda_{\rm ETC} = 100\Lambda_{\rm TC}$. The results of $\Sigma(p^2_E)$ are depicted in Fig. 1 in which dashed lines are for a positive b and different Ns; while solid lines are for different negative bs and $N = 3$. From which, we find

(1) For $N = 3$ and positive b, EFFIIETC influences $\Sigma(p^2_E)$ very little except for its large momentum tail. We have changed coupling b by enlarging its magnitude 100 times, the general form of $\Sigma(p^2_E)$ almost does not change. For $N = 3$ and a negative b, above $b = -0.00300$ the change in $\Sigma(p^2_E)$ is small. Below $b = -0.00300$, we see the explicit change of $\Sigma(p^2_E)$ which at large momentum region exhibits typical slowly damping asymptotic behavior due to the existence of four-fermion coupling. To check the validity of the phenomena, we have changed the differential equation to the original integration equation for SDE with and without angular approximation $\alpha_{\rm TC}[(p_E - q_E)^2] = \alpha_{\rm TC}(p^2_E)\theta(p^2_E - q^2_E) + \alpha_{\rm TC}(q^2_E)\theta(q^2_E - p^2_E)$ and increased the cutoff of the theory, but all obtain the similar result. For $N = 4, 5, 6$, we can find similar phenomena as the case of $N = 3$ which are not written down here, since later we will show that present model prefers smaller N and then the final result of our calculation will be only limited in the case of $N = 3$.

(2) For large momentum tail of $\Sigma(p^2_E)$, we find that if the positive b is larger than some critical value, $\Sigma(p^2_E)$ will be negative as momentum becomes large which indicates the possible oscillation. These values are $b_{N=3} = 2.45 \times 10^{-4}$, $b_{N=4} = 2.26 \times 10^{-4}$, $b_{N=5} = 2.15 \times 10^{-4}$, and $b_{N=6} = 2.08 \times 10^{-4}$. Considering that $b_N \propto \Lambda^2_{\rm TC}/\Lambda^2_{\rm ETC}$ must be very small, we take $b = 2.08 \times 10^{-4}$ as a typical value of our computation. To exhibit the differences of

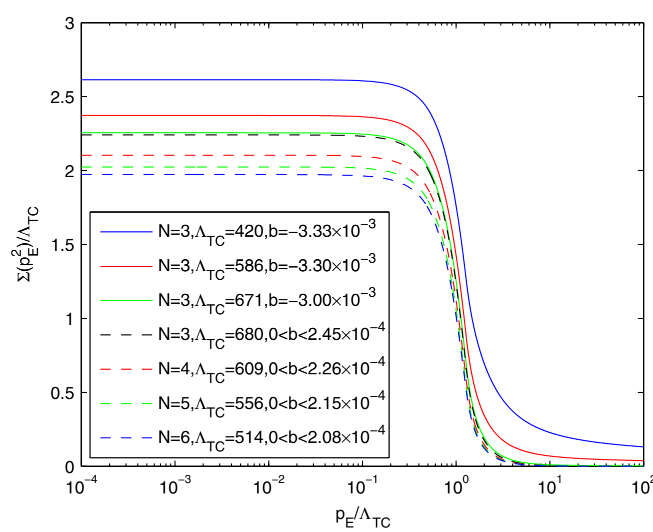

FIG. 1 (color online). Techniquark self energy $\Sigma(p^2_E)$. $\Lambda_{\rm TC}$ is in unit of GeV and is fixed by $f = 250$ GeV.

tails for different bs, we draw diagrams of $\Sigma(p_E^2)$ with $b = 2.08 \times 10^{-4}$ and $b = 0$ together in Fig. 2. We find that the differences show up only in the tail of self energy at momentum beyond $50\Lambda_{\rm TC}$ and bellow that limit, there is almost no difference. We further find that for fixed $f = 250$ GeV, from the later result of (71), both $b = 0$ and $b = 2.08 \times 10^{-4}$ cases lead to almost the same $\Lambda_{\rm TC}$.

If we further take $b_U = b_D = b$, Σ_X equals for each techniflavor and we can neglect subscript X. Then with the technique developed in Ref. [5], we can show that if the function $\Sigma(\partial_x^2)\delta(x-y)$ is the solution of the SDE in the case $V_{\xi,\mu} = A_{\xi,\mu} = 0$, we can replace its argument ∂_x by the minimal-coupling covariant derivative $\bar{\nabla}_x \equiv \partial_x - iV_\xi(x)$ and use it, *i.e.*, $\Sigma(\bar{\nabla}_x^2)\delta(x-y)$, as an approximate solution of the SDE in the case $V_{\xi,\mu} \neq 0$ and $A_{\xi,\mu} \neq 0$.

2. Effective action

Starting from (32), the exponential multifermion terms on the right-hand side of the equation can be written explicitly as

$$\sum_{n=2}^{\infty}\int d^4x_1\ldots d^4x_n \frac{(-ig_{\rm TC})^n}{n!} G^{\alpha_1\ldots\alpha_n}_{\mu_1\ldots\mu_n}(x_1,\ldots,x_n) J^{\mu_1}_{\xi,\alpha_1}(x_1)\ldots \times J^{\mu_n}_{\xi,\alpha_n}(x_n) \approx \int d^4x d^4x' \bar{\psi}^\sigma_\xi(x)\Pi_{\sigma\rho}(x,x')\psi^\rho_\xi(x'), \tag{41}$$

$$\Pi_{\sigma\rho}(x,x') = \sum_{n=2}^{\infty}\Pi^{(n)}_{\sigma\rho}(x,x') \approx \Pi^2_{\sigma\rho}(x,x'), \tag{42}$$

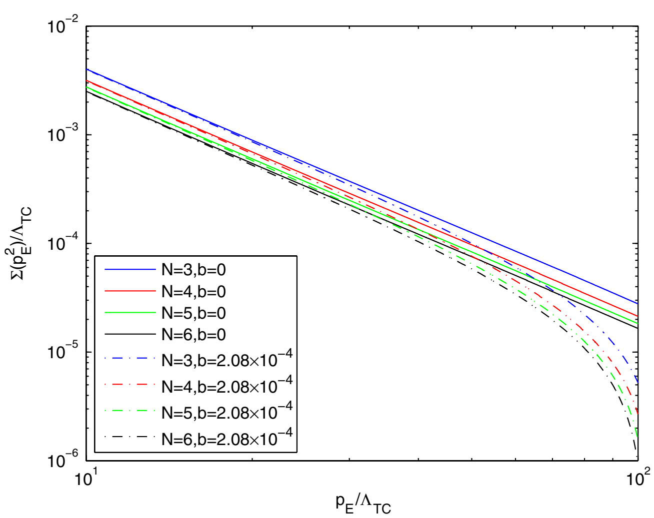

FIG. 2 (color online). The tail of techniquark self energy $\Sigma(p_E^2)$ exhibits ETC effects.

$$\Pi^{(2)}_{\sigma\rho}(x,x') = -g^2_{\rm TC} G^{\alpha_1\alpha_2}_{\mu_1\mu_2}(x,x')[t_{\alpha_1}\gamma^{\mu_1}S(x,x')t_{\alpha_2}\gamma^{\mu_2}]_{\sigma\rho}, \tag{43}$$

where we have taken the approximation that (a). *replacing the summation over 2n-fermion interactions with parts of them by their vacuum expectation values (VEVs)*; (b). *only keeping the leading four fermion interactions*. The approximation (a) can be seen as some kinds of average field approximation and the approximation (b) is a low energy approximation in which we ignore the high dimension operators. The effects of these two approximations are that they lead finally to Tr log(...) terms given in (44) which is originally obtained by taking large N_c limit and lowest order in dynamical perturbation theory in Ref. [7,8]. We base validity of these approximations on the previous experience of the agreement between our theoretical computation for QCD chiral Lagrangian and experiment data. For the $\mathcal{L}_{\xi 4T}$ term in (32), we use the same average field approximation given above. Combining with the result (35) and neglecting the factor $\mathcal{F}[O_\xi]\delta(O_\xi - O_\xi^\dagger)$ in Eq. (32),[6] we obtain

$$\begin{aligned} S_{\rm norm}[U, W^a_\mu, B_\mu] &\approx -i\log\int\mathcal{D}\bar{\psi}_\xi\mathcal{D}\psi_\xi\exp\Big[i\int d^4x\bar{\psi}_\xi \\ &\quad\times(i\not\partial + \not{V}_\xi + \not{A}_\xi\gamma^5)\psi_\xi - i\int d^4x d^4x' \\ &\quad\times\bar{\psi}^\sigma_\xi(x)\Sigma_{\sigma\rho}(x,x')\psi^\rho_\xi(x')\Big] \\ &\approx -i\,{\rm Tr}\log[i\not\partial + \not{V}_\xi + \not{A}_\xi\gamma^5 - \Sigma(\bar{\nabla}^2)], \end{aligned} \tag{44}$$

where $\Sigma(\bar{\nabla}^2)$ in techniflavor space is block diagonal. Notice that the arguments of Tr log are block diagonal which enable us to compute them block by block,

$$\begin{aligned} S_{\rm norm}[U, W^a_\mu, B_\mu] &= \sum_{\eta=1}^{3} -i{\rm Tr}\log[i\not\partial + \not{v}^n + \not{a}^\eta\gamma^5 - \Sigma(\bar{\nabla}^{\eta,2})] \\ &= \sum_{\eta=1}^{3}\int d^4x{\rm tr}_f[(F_0^{1D})^2 a^{\eta 2} - \mathcal{K}_1^{1D}(d_\mu a^{\eta\mu})^2 \\ &\quad - \mathcal{K}_2^{1D}(d_\mu a^\eta_\nu - d_\nu a^\eta_\mu)^2 + \mathcal{K}_3^{1D}(a^{\eta 2})^2 \\ &\quad + \mathcal{K}_4^{1D}(a^\eta_\mu a^\eta_\nu)^2 - \mathcal{K}_{13}^{1D}V^\eta_{\mu\nu}V^{\eta\mu\nu} \\ &\quad + i\mathcal{K}_{14}^{1D}a^\eta_\mu a^\eta_\nu V^{\eta\mu\nu}] + \mathcal{O}(p^6), \end{aligned} \tag{45}$$

for which $\bar{\nabla}^\eta_\mu \equiv \partial_\mu - iv^\eta_\xi$ and from (15) to (18) and (25) to (27),

[6] This approximation is also used in Eq. (33) and was first introduced in Ref. [7], where $\mathcal{F}[O_\xi]$ is dropped out, because it belongs to the $1/N_c$ order. $\delta(O_\xi - O_\xi^\dagger)$ is exponentialized there by introducing auxiliary field Θ and it is shown in Eq. (17) in Ref. [7] that in large N_c limit, there is no contribution of corresponding term due to Eq. (15) there.

$$v^\eta_\mu = -\frac{1}{2}g_2\frac{\tau^a}{2}W^a_{\xi,\mu} - \frac{1}{2}g_1\frac{\tau^3}{2}B_{\xi,\mu} + Z^\eta_{V\mu},$$
$$a^\eta_\mu = \frac{1}{2}g_2\frac{\tau^a}{2}W^a_{\xi,\mu} - \frac{1}{2}g_1\frac{\tau^3}{2}B_{\xi,\mu} + Z^\eta_{A\mu} \qquad \eta = l, t, b, \tag{46}$$

where $d_\eta a^\eta_\nu \equiv \partial_\mu a^\eta_\nu - i[v^\eta_\mu, a^\eta_\nu]$, $V^\eta_{\mu\nu} \equiv i[\partial_\mu - iv^\eta_\mu, \partial_\nu - iv^\eta_\nu]$. F_0^{1D} and $\mathcal{K}_i^{1D}$ coefficients with superscript $1D$ to denote that they are from one doublet TC model discussed in Ref. [5] which are functions of techniquark self energy $\Sigma(p^2)$ and detailed expressions of them are already given in Eq. (36) of Ref. [7] with the replacement of $N_c \to N$.

For anomaly part, the U field dependent part can be produced by the normal part with vanishing techniquark self energy Σ, i.e.

$$iS_{\rm anom}[U, W^a_\mu, B_\mu] = {\rm Tr}\log(i\not\partial + \not V + \not A\gamma^5) - iS_{\rm norm}[U, W^a_\mu, B_\mu]|_{\Sigma=0}. \tag{47}$$

Notice that the pure gauge field part independent of U field is irrelevant to EWCL. Combined with (45), above relation implies

$$iS_{\rm anom}[U, W^a_\mu, B_\mu] = {\rm Tr}\log(i\not\partial + \not V + \not A\gamma^5) + i\sum_{\eta=1}^{3}\int d^4x\,{\rm tr}_f[-\mathcal{K}_1^{1D,({\rm anom})}(d_\mu a^{\eta\mu})^2 - \mathcal{K}_2^{1D,({\rm anom})}(d_\mu a^\eta_\nu - d_\nu a^\eta_\mu)^2 + \mathcal{K}_3^{1D,({\rm anom})}(a^{\eta 2})^2 + \mathcal{K}_4^{1D,({\rm anom})}(a^\eta_\mu a^\eta_\nu)^2 - \mathcal{K}_{13}^{1D,({\rm anom})}V^\eta_{\mu\nu}V^{\eta\mu\nu} + i\mathcal{K}_{14}^{1D,({\rm anom})}a^\eta_\mu a^\eta_\nu V^{\eta\mu\nu}] + \mathcal{O}(p^6), \tag{48}$$

with

$$\mathcal{K}_i^{1D,({\rm anom})} = -\mathcal{K}_i^{1D}|_{\Sigma=0} \qquad i = 1, 2, 3, 4, 13, 14 \tag{49}$$

where we have used the result that $F_0^{1D}|_{\Sigma=0} = 0$. Combining normal and anomaly part contributions together, with the help of (29), we finally find

$$S_{Z'}[U, W^a_\mu, B_\mu, Z'_\mu] = \int d^4x\left(-\frac{1}{4}W^a_{\mu\nu}W^{a,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}Z'_{\mu\nu}Z'^{\mu\nu} + \frac{1}{2}M_0^2 Z'_\mu Z'^\mu\right) - i\,{\rm Tr}\log(i\not\partial + \not V + \not A\gamma^5) + \sum_{\eta=1}^{3}\int d^4x\,{\rm tr}_f[(F_0^{1D})^2 a^{\eta 2} - \mathcal{K}_1^{1D,\Sigma\neq 0}(d_\mu a^{\eta\mu})^2 - \mathcal{K}_2^{1D,\Sigma\neq 0}(d_\mu a^\eta_\nu - d_\nu a^\eta_\mu)^2 + \mathcal{K}_3^{1D,\Sigma\neq 0}(a^{\eta 2})^2 + \mathcal{K}_4^{1D,\Sigma\neq 0}(a^\eta_\mu a^\eta_\nu)^2 - \mathcal{K}_{13}^{1D,\Sigma\neq 0}V^\eta_{\mu\nu}V^{\eta\mu\nu} + i\mathcal{K}_{14}^{1D,\Sigma\neq 0}a^\eta_\mu a^\eta_\nu V^{\eta\mu\nu}] + \mathcal{O}(p^6). \tag{50}$$

With the help of (46) and (25) to (27), the above result can be further simplified to the form (A4) in which the explicitly U field dependence is displayed.

C. Integrating out Z'

We can further decompose (A4) into

$$S_{Z'}[U, W^a_\mu, B_\mu, Z'_\mu] = \tilde{S}_{Z'}[U, W^a_\mu, B_\mu, Z'_\mu] + S_{Z'}[U, W^a_\mu, B_\mu, 0], \tag{51}$$

where $\tilde{S}_{Z'}[U, W^a_\mu, B_\mu, Z'_\mu]$ is the Z' dependent part of $S_{\rm eff}[U, W^a_\mu, B_\mu, Z'_\mu]$. We find the Z' independent part $S_{Z'}[U, W^a_\mu, B_\mu, 0]$ is just the same as that of one-doublet TC model given in Ref. [5], the only difference is that now there is an extra overall factor 3 multiplied in front of all terms. The source of this factor comes from the fact that in the present model, instead of one doublet, we have three techniquark doublets. So switching off the effects from the Z' particle, contributions of the present TC2 model to bosonic part of EWCL are equivalent to those of the three-doublets TC model. In $\tilde{S}_{Z'}[U, W^a_\mu, B_\mu, Z'_\mu]$, in order to normalize the Z' field correctly, we introduce normalized field $Z'_{R,\mu}$ as

$$Z'_\mu = \frac{1}{c_{Z'}}Z'_{R,\mu}$$
$$c^2_{Z'} = 1 + g_1^2\Big[3\mathcal{K}\tan^2\theta' + 10\mathcal{K}(\tan\theta' + \cot\theta')^2 + \mathcal{K}_2^{1D,\Sigma\neq 0}(\tan\theta' + \cot\theta')^2 + \frac{3}{2}\mathcal{K}_2^{1D,\Sigma\neq 0}\tan^2\theta' + \frac{9}{2}\mathcal{K}_{13}^{1D,\Sigma\neq 0}(\tan\theta' + \cot\theta')^2 + \frac{3}{2}\mathcal{K}_{13}^{1D,\Sigma\neq 0}\tan^2\theta'\Big], \tag{52}$$

in terms of normalized field $Z'_{R,\mu}$, $\tilde{S}_{Z'}[U, W^a_\mu, B_\mu, Z'_\mu]$ become

$$\tilde{S}_{Z'}[U, W^a_\mu, B_\mu, Z'_\mu] = \int d^4x\Big[\frac{1}{2}Z'_{R,\mu}D_Z^{-1,\mu\nu}Z'_{R,\nu} + Z_R'^{,\mu}J_{Z,\mu} + Z_R^2 Z'_{R,\mu}J^\mu_{3Z} + g_{4Z}\frac{g_1^4}{c^4_{Z'}}Z_R'^{,4}\Big] \tag{53}$$

with

$$D_Z^{-1,\mu\nu} = g^{\mu\nu}(\partial^2 + M_{Z'}^2) - (1 + \lambda_Z)\partial^\mu\partial^\nu + \Delta_Z^{\mu\nu}(X), \tag{54}$$

$$M_{Z'}^2 = \frac{1}{c_{Z'}^2}\bigg\{M_0^2 + \frac{1}{2}(F_0^{1D})^2 g_1^2(\cot\theta' + \tan\theta')^2 + \frac{3}{4}(F_0^{1D})^2 g_1^2 \tan^2\theta'\bigg\}, \tag{55}$$

$$\lambda_Z = \frac{g_1^2}{c_{Z'}^2}\bigg[-\frac{1}{2}(\tan\theta' + \cot\theta')^2 - \frac{3}{4}\tan^2\theta'\bigg]\mathcal{K}_1^{1D,\Sigma\neq 0}, \tag{56}$$

$$\begin{aligned}\Delta_Z^{\mu\nu}(X) = \frac{1}{c_{Z'}^2}\bigg\{&\bigg(-\frac{3}{4}\mathcal{K}_1^{1D,\Sigma\neq 0} - \frac{3}{16}\mathcal{K}_3^{1D,\Sigma\neq 0} + \frac{3}{8}\mathcal{K}_{13}^{1D,\Sigma\neq 0} - \frac{3}{16}\mathcal{K}_{14}^{1D,\Sigma\neq 0}\bigg)g_1^2\tan^2\theta'\,\mathrm{tr}[X^\mu\tau^3]\mathrm{tr}[X^\nu\tau^3]\\ &+ \bigg[\frac{3}{2}\mathcal{K}_1^{1D,\Sigma\neq 0}\tan^2\theta' - \frac{1}{4}(\cot\theta' + \tan\theta')^2\mathcal{K}_3^{1D,\Sigma\neq 0} - \frac{1}{4}(\cot\theta' + \tan\theta')^2\mathcal{K}_4^{1D,\Sigma\neq 0} - \frac{3}{8}\mathcal{K}_4^{1D,\Sigma\neq 0}\tan^2\theta'\\ &- \frac{3}{4}\mathcal{K}_{13}^{1D,\Sigma\neq 0}\tan^2\theta' + \frac{3}{8}\mathcal{K}_{14}^{1D,\Sigma\neq 0}\tan^2\theta'\bigg]g_1^2\,\mathrm{tr}[X^\mu X^\nu] + g^{\mu\nu}\bigg[\bigg(-\frac{1}{8}(\cot\theta' + \tan\theta')^2 - \frac{3}{16}\tan^2\theta'\bigg)\mathcal{K}_3^{1D,\Sigma\neq 0}\\ &+ \frac{3}{16}\tan^2\theta'\mathcal{K}_4^{1D,\Sigma\neq 0} - \frac{1}{8}(\cot\theta' + \tan\theta')^2\mathcal{K}_4^{1D,\Sigma\neq 0} + \frac{3}{4}\tan^2\theta'\mathcal{K}_{13}^{1D,\Sigma\neq 0} - \frac{3}{8}\tan^2\theta'\mathcal{K}_{14}^{1D,\Sigma\neq 0}\bigg]g_1^2\,\mathrm{tr}[X^k X_k]\\ &+ g^{\mu\nu}\bigg[-\frac{3}{16}\mathcal{K}_4^{1D,\Sigma\neq 0} - \frac{3}{8}\mathcal{K}_{13}^{1D,\Sigma\neq 0} + \frac{3}{16}\mathcal{K}_{14}^{1D,\Sigma\neq 0})\bigg] \times g_1^2\tan^2\theta'\,\mathrm{tr}[X_k\tau^3]\mathrm{tr}[X^k\tau^3]\bigg\},\end{aligned} \tag{57}$$

$$J_Z^\mu = J_{Z0}^\mu + \frac{g_1^2\gamma}{c_{Z'}}\partial^\nu B_{\mu\nu} + \tilde{J}_Z^\mu, \tag{58}$$

$$J_{Z0\mu} = -\frac{3}{4c_{Z'}}i(F_0^{1D})^2 g_1\tan\theta'\,\mathrm{tr}[X_\mu\tau^3], \tag{59}$$

$$\gamma = 3\mathcal{K}\tan\theta' + \bigg(\frac{3}{2}\mathcal{K}_2^{1D,\Sigma\neq 0} + \frac{3}{2}\mathcal{K}_{13}^{1D,\Sigma\neq 0}\bigg)\tan\theta', \tag{60}$$

$$\begin{aligned}\tilde{J}_Z^\mu = \frac{1}{c_{Z'}}\bigg\{&\frac{3}{4}ig_1\tan\theta'\mathcal{K}_1^{1D,\Sigma\neq 0}\{\mathrm{tr}[U^\dagger(D^\nu D_\nu U)U^\dagger D^\mu U\tau^3] - \tan\theta'\,\mathrm{tr}[U^\dagger(D^\nu D_\nu U)\tau^3 U^\dagger D^\mu U + \partial^\mu(U^\dagger D^\nu D_\nu U\tau^3)]\}\\ &+ \frac{3}{2}(-\mathcal{K}_2^{1D,\Sigma\neq 0} + \mathcal{K}_{13}^{1D,\Sigma\neq 0})g_1\tan\theta'\partial_\nu\,\mathrm{tr}[\bar{W}^{\mu\nu}\tau^3] + \frac{3i}{4}\bigg(\frac{1}{4}\mathcal{K}_3^{1D,\Sigma\neq 0} - \frac{1}{4}\mathcal{K}_4^{1D,\Sigma\neq 0} - \mathcal{K}_{13}^{1D,\Sigma\neq 0} + \frac{1}{2}\mathcal{K}_{14}^{1D,\Sigma\neq 0}\bigg)\\ &\times g_1\tan\theta'\,\mathrm{tr}[X^\nu X_\nu]\mathrm{tr}[X^\mu\tau^3] + \frac{3i}{4}\bigg(\frac{1}{2}\mathcal{K}_4^{1D,\Sigma\neq 0} + \mathcal{K}_{13}^{1D,\Sigma\neq 0} - \frac{1}{2}\mathcal{K}_{14}^{1D,\Sigma\neq 0}\bigg)g_1\tan\theta'\,\mathrm{tr}[X^\mu X_\nu]\mathrm{tr}[X^\nu\tau^3]\\ &- \frac{3}{4}(\mathcal{K}_{13}^{1D,\Sigma\neq 0} - \frac{1}{4}\mathcal{K}_{14}^{1D,\Sigma\neq 0})g_1\tan\theta'\,\mathrm{tr}[\bar{W}^{\mu\nu}(X_\nu\tau^3 - \tau^3 X_\nu)] + \frac{3}{2}i(\mathcal{K}_{13}^{1D,\Sigma\neq 0} - \frac{1}{4}\mathcal{K}_{14}^{1D,\Sigma\neq 0})g_1\tan\theta'\partial_\nu\,\mathrm{tr}[X^\mu X^\nu\tau^3]\bigg\},\end{aligned} \tag{61}$$

$$g_{4Z} = (\mathcal{K}_3^{1D,\Sigma\neq 0} + \mathcal{K}_4^{1D,\Sigma\neq 0})\bigg[\frac{3}{128}\tan^4\theta' + \frac{3}{32}\tan^2\theta'(\cot\theta' + \tan\theta')^2 + \frac{1}{64}(\cot\theta' + \tan\theta')^4\bigg], \tag{62}$$

$$J_{3Z}^\mu = \frac{-i}{c_{Z'}^3}(\mathcal{K}_3^{1D,\Sigma\neq 0} + \mathcal{K}_4^{1D,\Sigma\neq 0})g_1^3\bigg[\frac{3}{32}\tan^3\theta' + \frac{3}{16}(\cot\theta' + \tan\theta')^2\tan\theta'\bigg]\mathrm{tr}[X^\mu\tau_3]. \tag{63}$$

Perform the loop expansion to (22), the result of Z' field integration is

$$S_{\rm eff}[U, W^a_\mu, B_\mu] - i\log\mathcal{N}[W^a_\mu, B_\mu] = \tilde{S}_{Z'}[Z'_c, U, W^a, B] + \text{loop terms}, \tag{64}$$

with classical field Z'_c satisfies

$$\frac{\partial}{\partial Z'_{c,\mu}(x)}[\tilde{S}_{Z'}[Z'_c, U, W^a, B] + \text{loop terms}] = 0, \tag{65}$$

and

$$-i\log\mathcal{N}[W^a_\mu, B_\mu] = [\tilde{S}_{Z'}[Z'_c, U, W^a, B] + \text{loop terms}]_{\Sigma=0}, \tag{66}$$

which is obtained from (13) and the fact that when we switch off TC and ETC interactions, techniquark self energy vanishes. With (53), the solution is

$$Z'^{\mu}_c(x) = -D^{\mu\nu}_Z J_{Z,\nu}(x) + O(p^3) + \text{loop terms}. \tag{67}$$

Then

$$S_{\rm eff}[U, W^a_\mu, B_\mu] - i\log\mathcal{N}[W^a_\mu, B_\mu] = \int d^4x\bigg[-\frac{1}{2}J_{Z,\mu}D^{\mu\nu}_Z J_{Z,\nu} - J_{3Z,\mu'}(D^{\mu'\nu'}_Z J_{Z,\nu'}) \times (D^{\mu\nu}_Z J_{Z,\nu})^2 + g_{4Z}\frac{g_1^4}{c_{Z'}}(D^{\mu\nu}_Z J_{Z,\nu})^4\bigg] + \text{loop terms}, \tag{68}$$

where $D^{-1,\mu\nu}_Z D_{Z,\nu\lambda} = D^{\mu\nu}_Z D^{-1}_{Z,\nu\lambda} = g^\mu_\lambda$ and it is not difficult to show that if we are accurate up to order p^4, then order p of Z'_c solution is enough, and all contributions from order p^3 of Z'_c belong to order p^6. The loop terms in (68) should be small, at least they are expected not to change the qualitative picture of the model. Otherwise the physics related to Z' of the model will deviate from its designers since original discussions are mainly limited in tree order. With this expectation, we can ignore loop terms which will simplify the further computation. With the help of (54), (58), and (68),

$$S_{\rm eff}[U, W^a_\mu, B_\mu] - i\log\mathcal{N}[W^a_\mu, B_\mu] = \int d^4x\bigg[-\frac{1}{2}J_{Z0,\mu}D^{\mu\nu}_Z J_{Z0,\nu} - \frac{1}{M^2_{Z'}}J_{Z0,\mu}\Big(\tilde{J}^\mu_Z + \frac{g_1^2\gamma}{c_{Z'}}\partial_\nu B^{\mu\nu}\Big) - \frac{1}{M^6_{Z'}}J_{3Z,\mu}J^\mu_{Z0}J^2_{Z0} + \frac{g_{4Z}g_1^4}{c^4_{Z'}M^8_{Z'}}J^4_{Z0}\bigg]. \tag{69}$$

Ignoring terms higher than order p^4, we find $S_{\rm eff}[U, W^a_\mu, B_\mu]$ has the exact form of standard EWCL up to order p^4. We can then read out the corresponding coefficients. The results will be given in the next subsection. The normalization factor now is

$$\begin{aligned}-i\log\mathcal{N}[W^a_\mu, B_\mu] = \int d^4x\bigg[&-\Big(\frac{1}{4}+\frac{3}{4}\mathcal{K}g_2^2+\frac{3}{8}\mathcal{K}_2^{1D,\Sigma\neq0}g_2^2 + \frac{3}{8}\mathcal{K}_{13}^{1D,\Sigma\neq0}g_2^2\Big)W^a_{\mu\nu}W^{a,\mu\nu}\\ &-\Big(\frac{1}{4}+\frac{3}{4}\mathcal{K}g_1^2+\frac{3}{8}\mathcal{K}_2^{1D,\Sigma\neq0}g_1^2 + \frac{3}{8}\mathcal{K}_{13}^{1D,\Sigma\neq0}g_1^2 + \frac{3(F_0^{1D})^2}{8M^2_{Z'}}\beta_1 g_1^2 + \beta_1 g_1^2\cot\theta'\gamma\Big)B_{\mu\nu}B^{\mu\nu}\bigg].\end{aligned} \tag{70}$$

D. Coefficients of EWCL

From $S_{\rm eff}[U, W^a_\mu, B_\mu]$ obtained in the last subsection, we can read out coefficients of EWCL. The p^2 order coefficients are

$$f^2 = 3(F_0^{1D})^2 \qquad \beta_1 = \frac{3(F_0^{1D})^2 g_1^2\tan^2\theta'}{8c^2_{Z'}M^2_{Z'}}. \tag{71}$$

Combining with (10) and (55) and T parameter $\alpha T = 2\beta_1$ given in Ref. [2], we further obtain

$$\beta_1 = \frac{1}{2}\alpha T = \frac{12}{(\frac{200\tilde{v}^2}{3f^2} + 16)(1+\cot^2\theta')^2 + 24}, \tag{72}$$

then T is positive and uniquely determined by θ' and $\tilde{v}/f$. It is bounded above and the upper limit is $3/(5 + 25\tilde{v}^2/3f^2)\alpha \leq 9/(40\alpha)$, since we know $\tilde{v} \geq f$. In the following numerical computations, for simplicity, we all take $\tilde{v} = f$. The p^4 order coefficients are

$$\alpha_1 = 3(1-2\beta_1)L_{10}^{1D} + \frac{3(F_0^{1D})^2}{2M_{Z'}^2}\beta_1 - 2\gamma\beta_1\cot\theta', \qquad \alpha_2 = -\frac{3}{2}(1-2\beta_1)L_9^{1D} + \frac{3(F_0^{1D})^2}{2M_{Z'}^2}\beta_1 - 2\gamma\beta_1\cot\theta',$$
$$\alpha_3 = -\frac{3}{2}(1-2\beta_1)L_9^{1D}, \qquad \alpha_4 = 3L_2^{1D} + 6\beta_1 L_9^{1D} + \frac{3(F_0^{1D})^2}{2M_{Z'}^2}\beta_1, \qquad \alpha_5 = 3L_1^{1D} + \frac{3}{2}L_3^{1D} - \frac{3(F_0^{1D})^2}{2M_{Z'}^2}\beta_1 - 6\beta_1 L_9,$$
$$\alpha_6 = -\frac{3(F_0^{1D})^2}{2M_{Z'}^2}\beta_1 - 6\beta_1(4L_1^{1D} + L_9^{1D}) + \beta_1^2[(1+\cot^2\theta')^2(48L_1^{1D} + 8L_3^{1D}) + 24L_1^{1D}],$$
$$\alpha_7 = \frac{3(F_0^{1D})^2}{2M_{Z'}^2}\beta_1 - 2\beta_1(3L_3^{1D} + 6L_1^{1D} - 3L_9^{1D}) + \beta_1^2[(1+\cot^2\theta')^2(24L_1^{1D} + 4L_3^{1D}) + 6\tan\theta'(L_3^{1D} + 2L_1^{1D})],$$
$$\alpha_8 = -\frac{3(F_0^{1D})^2}{2M_{Z'}^2}\beta_1 + 12\beta_1 L_{10}^{1D}, \qquad \alpha_9 = -\frac{3(F_0^{1D})^2}{2M_{Z'}^2}\beta_1 + 6\beta_1(L_{10}^{1D} - L_9^{1D}),$$
$$\alpha_{10} = 4\beta_1^2(18L_1^{1D} + 3L_3^{1D}) + 32\beta_1^4 g_{4Z}\cot^4\theta' - \beta_1^3(144L_1^{1D} + 24L_3^{1D})[1 + 2(1+\cot^2\theta')^2],$$
$$\alpha_{11} = \alpha_{12} = \alpha_{13} = \alpha_{14} = 0, \tag{73}$$

where L_i relate to $\mathcal{K}_i^{1D,\Sigma\neq 0}$ coefficients through

$$\mathcal{K}_2^{1D,\Sigma\neq 0} = L_{10}^{1D} - 2H_1^{1D},$$
$$\mathcal{K}_3^{1D,\Sigma\neq 0} = 64L_1^{1D} + 16L_3^{1D} + 8L_9^{1D} + 2L_{10}^{1D} + 4H_1^{1D},$$
$$\mathcal{K}_4^{1D,\Sigma\neq 0} = 32L_1^{1D} - 8L_9^{1D} - 2L_{10}^{1D} - 4H_1^{1D},$$
$$\mathcal{K}_{13}^{1D,\Sigma\neq 0} = -L_{10}^{1D} - 2H_1^{1D},$$
$$\mathcal{K}_{14}^{1D,\Sigma\neq 0} = -4L_{10}^{1D} - 8L_9^{1D} - 8H_1^{1D}. \tag{74}$$

Several features of these results are:

(1) The contributions to p^4 order coefficients are divided into two parts: the three doublets TC model contribution (equals to 3 times of one doublet TC model discussed in Ref. [5]) and the Z' contribution.

(2) All corrections from Z' particle are at least proportional to β_1 which vanish if the mixing disappears by $\theta' = 0$.

(3) Since $L_{10}^{1D} < 0$, combining with positive β_1, (73) then tells us α_8 is negative. Then $U = -16\pi\alpha_8$ coefficient given in Ref. [2] is always positive in the present model.

(4) α_1 and α_2 depend on γ which from (60) further rely on an extra parameter $\mathcal{K}$. We can combine (52) and (71) together to fix $\mathcal{K}$,

$$\frac{(F_0^{1D})^2 g_1^2\tan^2\theta'}{8\beta_1 M_{Z'}^2} = \frac{1}{3} + g_1^2\bigg[\mathcal{K}\tan^2\theta' + \frac{10}{3}\mathcal{K}(\tan\theta' + \cot\theta')^2 + \frac{1}{3}\mathcal{K}_2^{1D,\Sigma\neq 0}(\tan\theta' + \cot\theta')^2 + \frac{1}{2}\mathcal{K}_2^{1D,\Sigma\neq 0}\tan^2\theta' + \frac{3}{2}\mathcal{K}_{13}^{1D,\Sigma\neq 0}(\tan\theta' + \cot\theta')^2 + \frac{1}{2}\mathcal{K}_{13}^{1D,\Sigma\neq 0}\tan^2\theta'\bigg]. \tag{75}$$

Once $\mathcal{K}$ is fixed, with the help of (A5), we can determine the ratio of infrared cutoff κ and ultraviolet cutoff Λ. In Fig. 3, we draw the κ/Λ as function of T and $M_{Z'}$, we find natural criteria $\Lambda > \kappa$ offers stringent constraints on the allowed region for T and $M_{Z'}$ that present theory prefer small Z' mass (< 0.4 TeV) and small TC group. For example, $T < 0.035$ for $M_{Z'} = 0.3$ TeV and $N = 3$, $T < 0.25$ for $M_{Z'} = 0.2$ TeV and $N = 6$, $T < 0.74$ for $M_{Z'} = 0.2$ TeV and $N = 3$. In Fig. 4, we draw

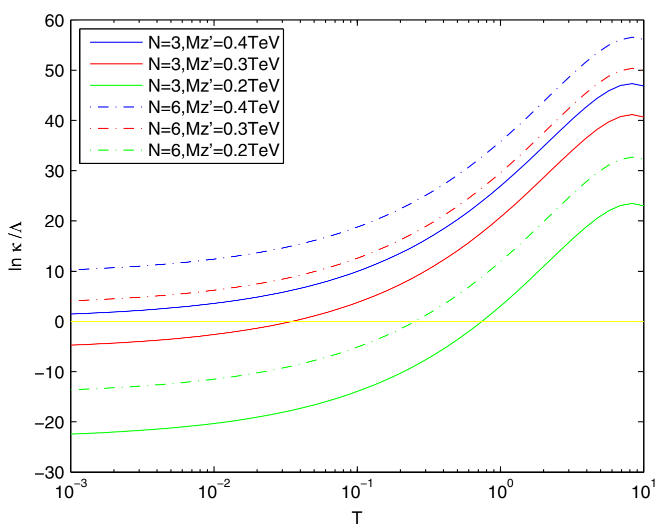

FIG. 3 (color online). The ratio of infrared cutoff and ultraviolet cutoff κ/Λ as function of T parameter and Z' mass in unit of TeV.

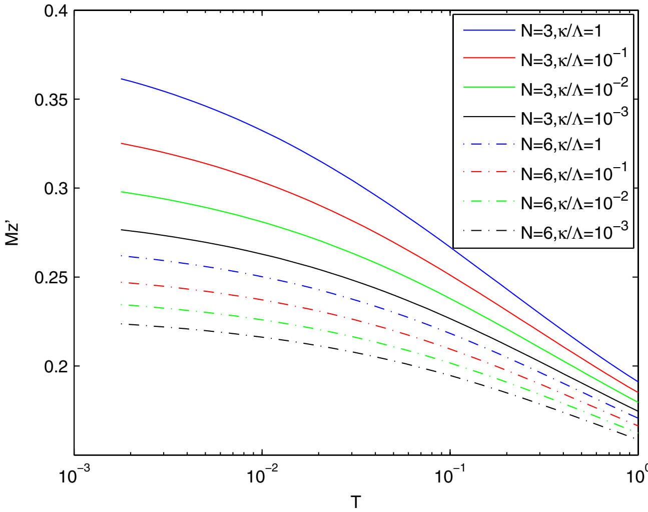

FIG. 4 (color online). Z' mass in unit of TeV as function of T parameter and κ/Λ.

Z' mass as function of T parameter and κ/Λ. The line of $\kappa/\Lambda = 1$ gives the upper bound of Z' mass $M_{Z'} < 0.4$ TeV, which is already beyond the experiment limit given by Ref. [15,16]. To check whether this bound is reliable, we have changed coupling of EFFIIETC by either enlarging its magnitude 100 times or reversing its sign, the results all almost do not change. The special case of $b = -3.33 \times 10^{-3}$ also has no effect here. To examine the reason why the present model causes smaller $M_{Z'}$ than that from Hill's model, we consider the situation of very tiny θ' and κ/Λ, then the leading term in the right-hand side of (75) is $\frac{10}{3} g_1^2 \mathcal{K} \cot^2\theta'$. Combining with Eq.

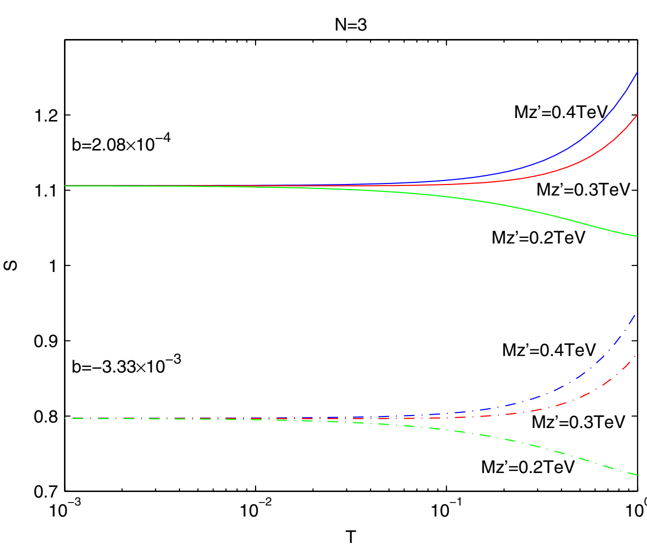

FIG. 5 (color online). S parameter for Lane's natural TC2 model.

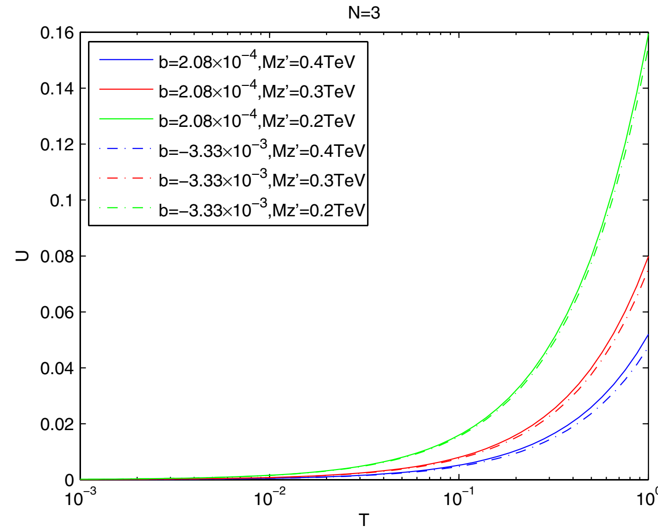

FIG. 6 (color online). U parameter for Lane's natural TC2 model.

(72), we find that (75) in this extreme case gives $M_{Z'} = F_0\sqrt{\frac{31}{120}\mathcal{K}} \simeq f\sqrt{\mathcal{K}}/(2\sqrt{3})$. For Hill's model, we obtain the result that $M_{Z'} = F_0\sqrt{\mathcal{K}}/2 = f\sqrt{\mathcal{K}}/2$. So Z' mass is smaller than that in Hill's model by a factor $1/\sqrt{3}$ due to the identification of F_0 with $f/\sqrt{3}$ now in (71) but with f in Hill's model. Considering the smaller TC group will allow relative larger Z' mass. In following discussions, we only limit us in the case of $N = 3$.

(5) For a typical case with $b = -3.33 \times 10^{-3}$, except for coefficients F_0^{1D} and $\mathcal{K}_1^{1D}$ which receive relative large corrections from ETC interaction, all other $\mathcal{K}_i^{1D}$ coefficients only feel small ETC effects.

With $f = 250$ GeV, all EWCL coefficients depend on two physical parameters β_1 and $M_{Z'}$. Combined with $\alpha T = 2\beta_1$, we can use the present experimental result for the T parameter to fix β_1. In Fig. 5 and 6, we draw graphs for the $S = -16\pi\alpha_1$ and $U = -16\pi\alpha_8$ in terms of the T parameter, respectively. We take three typical Z' masses $M_{Z'} = 0.2, 0.3, 0.4$ TeV for references. For S parameter, we find that all values of it are at order of 1. This can be understood as that at region of small T parameter, the main contribution to S parameter comes from the three doublets TC model which results in positive S. This result roughly equals to $-3L_{10}^{1D}$ which is 3 times larger than the corresponding value in Hill's model due to the existence of three doublets techniquarks. We also find that large negative b will reduce the value of S, but considering that the value $b = -0.00333$ corresponding to $g_{\mathrm{ETC}}^2 b_U = -0.00333\Lambda_{\mathrm{ETC}}^2/\Lambda_{\mathrm{TC}}^2$ is already large enough, we do not think any larger negative bs will have physical meaning. For U parameter, we find it is positive and below 0.2.

Considering that the facts of small $M_{Z'}$ and relative large S are all not favored by present precision measurements of the SM, we just leave the analytic formulas for other α_i coefficients there and will not draw diagrams for them any further.

III. DISCUSSION

In this paper, we generalize the calculation in Ref. [5] for C. T. Hill's schematic TC2 model to K. Lane's prototype natural TC2 model. We find that, similar to Hill's model, coefficients of EWCL for the Lane's model are divided into a direct TC and ETC interaction part, a TC and topcolor induced effective Z' particle contribution part, and an ordinary quarks contribution part. The first two parts are computed in this paper. We show that the direct TC and ETC interaction part is 3 times larger than the corresponding part of Hill's model due to the existence of three techniquark doublets, while effective Z' contributions are different from Hill's model due to changes of $U(1)_1 \otimes U(1)_2$ group representation arrangements and are at least proportional to the p^2 order parameter β_1 in EWCL. Typical features of the model are that it only allows positive S, T, and U parameters. S is around 1 which is roughly 3 times larger than that in original Hill's model due to the existence of three doublets of techniquarks, and T parameter varies in the range $0 \sim 9/(40\alpha)$. Analytical expression (73) for five p^4 order coefficients α_3, α_4, α_5, α_8, and α_9 exactly equal to 3 times of those obtained from Hill's model in Ref. [5]. These coefficients include all three custodial symmetry conserve ones. The Z' mass is bounded from 0.4 TeV and larger $M_{Z'}$ prefers smaller N. Compared to the results obtained in Ref. [5] for C. T. Hill's TC2 model, the results from Lane's first natural TC2 model deviate more from the experiment data. This calls for improvement of the model.

In fact, the present model is only a prototype natural TC2 model. Many details of the model are not even specified in the original paper [12], prohibiting us from performing the computation more accurately and leaving us more space to improve the dynamics. One typical nonspecified effect is the walking dynamics. As mentioned by K. Lane, the TC of the model is expected to be a walking gauge theory. This is a new feature different from the conventional gauge theory, and this walking is not explicitly realized in the present prototype model, since techniquarks are in fundamental representation of the TC group and the number of techniquarks is not large enough. Another unspecified detail is the $SU(3)_1 \otimes SU(3)_2$ symmetry-breaking mechanism. It is now simulated without detailed dynamics content by introducing an effective scalar field Φ which transforms as $(\bar{3}, 3, \frac{5}{6}, -\frac{5}{6})$ under the group $SU(3)_1 \otimes SU(3)_2 \otimes U(1)_{Y_1} \otimes U(1)_{Y_2}$ and corresponding interaction potential $V(\Phi)$. Introducing scalar fields, which is only how the theory is effective, deviates the basic idea of TC models. All these shortcomings are overcome in the improved model [14]. Considering that this new model is much more complex and different than the present one, it involves different dynamics and therefore requires more analysis and computation techniques. For example, the condensations of the techniquarks are block diagonal in three doublets flavor space now but not for the additional two doublets techniquarks newly added in the improved model (which is more like the case A solution of the paper [12], while in the present paper we only discuss the case B solution as mentioned in footnote [2]). In order to make our discussion less complex and to specially exhibit the result of Lane's first natural TC2 model, in this paper we limit ourselves to the primary prototype model and focus our attention on figuring out the analytical expressions for the coefficients of EWCL, estimating possible constrains to the model and identifying the effects of ETC interactions. We leave the discussion of the new improved model for a future paper.

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APPENDIX: NECESSARY FORMULAS FOR EWCL

In this appendix, we list the necessary formulas needed in the text. With definitions in which

$$D_\mu U = \partial_\mu U + ig_2 \frac{\tau^a}{2} W_\mu^a U - ig_1 U \frac{\tau^3}{2} B_\mu, \tag{A1}$$

$$D_\mu U^\dagger = \partial_\mu U^\dagger - ig_2 U^\dagger \frac{\tau^a}{2} W_\mu^a + ig_1 \frac{\tau^3}{2} B_\mu U^\dagger, \tag{A2}$$

$$X_\mu = U^\dagger (D_\mu U) \qquad \bar{W}_{\mu\nu} = U^\dagger g_2 \frac{\tau^a}{2} W_{\mu\nu}^a U, \tag{A3}$$

we have

$$
\begin{aligned}
S_{Z'}[U,W^a_\mu,B_\mu,Z'_\mu] = \int d^4x\Big\{&-\frac{1}{4}W^a_{\mu\nu}W^{a,\mu\nu}-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}-\frac{1}{4}Z'_{\mu\nu}Z'^{\mu\nu}+\frac{1}{2}M_0^2Z'_\mu Z'^\mu-\mathcal{K}\Big[\frac{3}{4}g_1^2B_{\mu\nu}B^{\mu\nu}-\frac{3}{2}g_1^2\tan\theta' B_{\mu\nu}Z'^{\mu\nu}\\
&+\frac{3}{4}g_1^2\tan^2\theta' Z'_{\mu\nu}Z'^{\mu\nu}+\frac{5}{2}g_1^2(\tan\theta'+\cot\theta')^2Z'_{\mu\nu}Z'^{\mu\nu}+\frac{3}{4}g_2^2W^a_{\mu\nu}W^{a\mu\nu}\Big]+(F_0^{1D})^2\Big\{-\frac{3}{4}\mathrm{tr}[X^\mu X_\mu]\\
&+\frac{1}{4}g_1^2(\cot\theta'+\tan\theta')^2Z'^2+\frac{3}{8}g_1^2\tan^2\theta' Z'^2-i\frac{3}{4}g_1\tan\theta' Z'^\mu\,\mathrm{tr}[X_\mu\tau^3]\Big\}-\mathcal{K}_1^{1D,\Sigma\neq0}\Big\{-\frac{3}{4}\mathrm{tr}[U^\dagger(D^\mu D_\mu U)\\
&\times U^\dagger(D^\nu D_\nu U)+2U^\dagger(D^\mu D_\mu U)(D^\nu U^\dagger)(D_\nu U)]-\frac{3}{4}ig_1\tan\theta' Z'^\nu\,\mathrm{tr}[U^\dagger(D^\mu D_\mu U)U^\dagger D_\nu U\tau^3]\\
&+\frac{3}{4}ig_1\tan\theta' Z'^\nu\,\mathrm{tr}[U^\dagger(D^\mu D_\mu U)\tau^3U^\dagger D_\nu U]-\frac{3}{4}ig_1\tan\theta'\partial_\nu Z'^\nu\,\mathrm{tr}[U^\dagger(D^\mu D_\mu U)\tau^3]\Big\}+\frac{3}{8}\Big(\mathcal{K}_1^{1D,\Sigma\neq0}\\
&+\frac{1}{4}\mathcal{K}_3^{1D,\Sigma\neq0}-\frac{1}{4}\mathcal{K}_4^{1D,\Sigma\neq0}-\mathcal{K}_{13}^{1D,\Sigma\neq0}+\frac{1}{2}\mathcal{K}_{14}^{1D,\Sigma\neq0}\Big)\times[\mathrm{tr}(X^\mu X_\mu)]^2-\frac{3}{8}\Big(\mathcal{K}_1^{1D,\Sigma\neq0}+\frac{1}{4}\mathcal{K}_3^{1D,\Sigma\neq0}\\
&-\frac{1}{2}\mathcal{K}_{13}^{1D,\Sigma\neq0}+\frac{1}{4}\mathcal{K}_{14}^{1D,\Sigma\neq0}\Big)g_1^2\tan^2\theta' Z'^\mu Z'_\nu\,\mathrm{tr}[X_\mu\tau^3]\mathrm{tr}[X^\nu\tau^3]+\frac{1}{8}\Big[6\mathcal{K}_1^{1D,\Sigma\neq0}\tan^2\theta'-(\cot\theta'+\tan\theta')^2\\
&\times\mathcal{K}_3^{1D,\Sigma\neq0}-(\cot\theta'+\tan\theta')^2\mathcal{K}_4^{1D,\Sigma\neq0}-\frac{3}{2}\mathcal{K}_4^{1D,\Sigma\neq0}\tan^2\theta'-3\mathcal{K}_{13}^{1D,\Sigma\neq0}\tan^2\theta'+\frac{3}{2}\mathcal{K}_{14}^{1D,\Sigma\neq0}\tan^2\theta'\Big]\\
&\times g_1^2Z'^\mu Z'_\nu\,\mathrm{tr}[X_\mu X^\nu]+\Big[-\frac{1}{4}(\tan\theta'+\cot\theta')^2-\frac{3}{8}\tan^2\theta'\Big]\mathcal{K}_1^{1D,\Sigma\neq0}g_1^2(\partial_\mu Z'^\mu)^2-\frac{3}{8}(\mathcal{K}_2^{1D,\Sigma\neq0}\\
&+\mathcal{K}_{13}^{1D,\Sigma\neq0})(g_2^2W^{\mu\nu a}W_{\mu\nu a}+g_1^2B^{\mu\nu}B_{\mu\nu})+\frac{3}{4}(\mathcal{K}_2^{1D,\Sigma\neq0}-\mathcal{K}_{13}^{1D,\Sigma\neq0})g_1\,\mathrm{tr}[\bar{W}^{\mu\nu}\tau^3](B_{\mu\nu}-\tan\theta' Z'_{\mu\nu})\\
&+\frac{3}{4}(\mathcal{K}_2^{1D,\Sigma\neq0}+\mathcal{K}_{13}^{1D,\Sigma\neq0})\tan\theta' g_1^2Z'_{\mu\nu}B^{\mu\nu}-\frac{1}{4}\Big[\mathcal{K}_2^{1D,\Sigma\neq0}(\tan\theta'+\cot\theta')^2+\frac{3}{2}\mathcal{K}_2^{1D,\Sigma\neq0}\tan^2\theta'\\
&+9\mathcal{K}_{13}^{1D,\Sigma\neq0}(\tan\theta'+\cot\theta')^2+\frac{3}{2}\mathcal{K}_{13}^{1D,\Sigma\neq0}\tan^2\theta'\Big]g_1^2Z'_{\mu\nu}Z'^{\mu\nu}+\frac{3i}{4}\Big(\frac{1}{4}\mathcal{K}_3^{1D,\Sigma\neq0}-\frac{1}{4}\mathcal{K}_4^{1D,\Sigma\neq0}-\mathcal{K}_{13}^{1D,\Sigma\neq0}\\
&+\frac{1}{2}\mathcal{K}_{14}^{1D,\Sigma\neq0}\Big)g_1\tan\theta' Z'_\nu\,\mathrm{tr}[X^\mu X_\mu]\mathrm{tr}[X^\nu\tau^3]+\frac{1}{8}\Big[-\frac{1}{2}(\cot\theta'+\tan\theta')^2\mathcal{K}_3^{1D,\Sigma\neq0}-\frac{3}{4}\tan^2\theta'\mathcal{K}_3^{1D,\Sigma\neq0}\\
&+\frac{3}{4}\tan^2\theta'\mathcal{K}_4^{1D,\Sigma\neq0}-\frac{1}{2}(\cot\theta'+\tan\theta')^2\mathcal{K}_4^{1D,\Sigma\neq0}+3\tan^2\theta'\mathcal{K}_{13}^{1D,\Sigma\neq0}-\frac{3}{2}\tan^2\theta'\mathcal{K}_{14}^{1D,\Sigma\neq0}\Big]\\
&\times g_1^2Z'^2\,\mathrm{tr}[X^\mu X_\mu]-\frac{3}{16}(\mathcal{K}_3^{1D,\Sigma\neq0}+\mathcal{K}_4^{1D,\Sigma\neq0})ig_1^3\Big[\frac{1}{2}\tan^3\theta'+(\cot\theta'+\tan\theta')^2\tan\theta'\Big]Z'_\mu Z'^2\,\mathrm{tr}[X^\mu\tau_3]\\
&+\frac{1}{64}(\mathcal{K}_3^{1D,\Sigma\neq0}+\mathcal{K}_4^{1D,\Sigma\neq0})g_1^4\Big[\frac{3}{2}\tan^4\theta'+6\tan^2\theta(\cot\theta'\tan\theta')^2+(\cot\theta'+\tan\theta')^4\Big]Z'^4+\frac{3}{8}\Big(\frac{1}{2}\mathcal{K}_4^{1D,\Sigma\neq0}\\
&+\mathcal{K}_{13}^{1D,\Sigma\neq0}-\frac{1}{2}\mathcal{K}_{14}^{1D,\Sigma\neq0}\Big)\mathrm{tr}[X^\mu X_\nu]\mathrm{tr}[X_\mu X^\nu]+\frac{3i}{4}\Big(\frac{1}{2}\mathcal{K}_4^{1D,\Sigma\neq0}+\mathcal{K}_{13}^{1D,\Sigma\neq0}-\frac{1}{2}\mathcal{K}_{14}^{1D,\Sigma\neq0}\Big)g_1\tan\theta' Z'^\nu\,\mathrm{tr}\\
&\times[X_\mu X_\nu]\mathrm{tr}[X^\mu\tau^3]+\frac{3}{16}\Big(-\frac{1}{2}\mathcal{K}_4^{1D,\Sigma\neq0}-\mathcal{K}_{13}^{1D,\Sigma\neq0}+\frac{1}{2}\mathcal{K}_{14}^{1D,\Sigma\neq0}\Big)g_1^2\tan^2\theta' Z'^2\,\mathrm{tr}[X_\mu\tau^3]\mathrm{tr}[X^\mu\tau^3]\\
&+\frac{3i}{4}\Big(-\mathcal{K}_{13}^{1D,\Sigma\neq0}+\frac{1}{4}\mathcal{K}_{14}^{1D,\Sigma\neq0}\Big)g_1B_{\mu\nu}\,\mathrm{tr}[\tau^3X^\mu X^\nu]+\frac{3i}{2}\Big(-\mathcal{K}_{13}^{1D,\Sigma\neq0}+\frac{1}{4}\mathcal{K}_{14}^{1D,\Sigma\neq0}\Big)\mathrm{tr}[X^\mu X^\nu\bar{W}_{\mu\nu}]\\
&+\frac{3}{4}\Big(-\mathcal{K}_{13}^{1D,\Sigma\neq0}+\frac{1}{4}\mathcal{K}_{14}^{1D,\Sigma\neq0}\Big)g_1\tan\theta' Z'_\mu\,\mathrm{tr}[\bar{W}^{\mu\nu}(X_\nu\tau^3-\tau^3X_\nu)]+\frac{3i}{4}\Big(\mathcal{K}_{13}^{1D,\Sigma\neq0}-\frac{1}{4}\mathcal{K}_{14}^{1D,\Sigma\neq0}\Big)\\
&\times g_1\tan\theta' Z'_{\mu\nu}\,\mathrm{tr}[X^\mu X^\nu\tau^3]\Big\},
\end{aligned}
\tag{A4}
$$

$$
\mathcal{K}=-\frac{1}{48\pi^2}\Big(\log\frac{\kappa^2}{\Lambda^2}+\gamma\Big)\qquad \Lambda,\kappa\text{: ultraviolet and infrared cutoffs.}\tag{A5}
$$

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