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Strong decays of radially excited mesons in a chiral approach

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We study radial excitations of pseudoscalar and vector $q\bar{q}$ mesons within a chiral approach. We derive a general form for a chiral Lagrangian describing processes involving excited pseudoscalar and vector mesons. The parameters of the chiral Lagrangian are fitted using data and previous calculations in the framework of the $^3P_0$ model. Finite-width effects are examined and predictions for mesons previously not discussed are given. Available experimental data is analyzed whenever possible. Possible hints for exotic mesons and open interpretation issues are discussed.

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I. INTRODUCTION

The identification of non-$q\bar{q}$ mesons is a fascinating topic, being deeply connected to the nontrivial relationship between the QCD Lagrangian and the physical, observable meson states. Therefore, great experimental and theoretical efforts are being deployed in order to identify glueballs, hybrid mesons, and others (for a recent review, see e.g. [1,2]). However, exotic mesons may also possess nonexotic quantum numbers and, *a priori*, might not be distinguishable from ordinary $q\bar{q}$ states. The understanding of the decay pattern of excited pseudoscalar and vector mesons is therefore essential in order to identify exotic candidates, such as glueballs, hybrids, or meson molecules which carry ordinary quantum numbers. As no systematic theory exists in this nonperturbative regime, one has to rely on models in order to describe the decays.

The spectrum and the decay pattern of excited mesons have been under study for many years. Descriptions of $q\bar{q}$-meson masses and decay properties have been done starting in the early eighties. In [3], the radial excitations of light mesons have been considered in a phenomenological quark model, and the issue of the pseudoscalar isoscalars has already been adressed in [4]. The decay rates found will later be included in our discussion. In Ref. [5], mesons (from the pion to the upsilon) have been considered in a relativized quark model. Meson masses and couplings have been calculated in a unified framework—soft QCD. Decay amplitudes have been computed within the elementary emission model.

The properties of vector-meson excitations were recently studied, e.g. the ρ-like mesons in [6]. On the interpretation of states on the experimental side, there is still some ongoing debate. A recent analysis is given in [7,8]. The problem of excited scalar and tensor mesons has been under heavy investigation as well. The assignment of the structure of f_0 and f_2 (possibly with admixture of a glueball with corresponding quantum numbers) is an open issue, studied e.g. in [9].

In a series of papers [10–15], the masses of excited scalar, pseudoscalar, and vector-meson states (the first excitation) and their respective couplings are studied in a chiral Lagrangian with form factors derived within the Nambu-Jona-Lasinio (NJL) quark model. Chiral symmetry breaking is described by the gap equation. Also, experimental candidates for the theoretical states are identified and the main strong decay widths are calculated. These will be confronted with our results. Another approach to excited mesons has been performed in Ref. [16,17], using the Bethe-Salpeter equation, yielding masses and amplitudes of bound states in given J^P channels.

The $^3P_0$ model was used in several works to compute strong decays and serves today as the main reference for calculations of decay widths. Decay properties for a wide mass range of mesons have been analyzed in [18–20]. Previously this model has been applied to the vector radial excitations in [21].

The most convenient language for the treatment of light hadrons at small energies was elaborated in the context of the chiral perturbation theory (ChPT) [22–25], the effective low-energy theory of the strong interaction. The goal of the present paper is the tree-level study of the decays of excited pseudoscalar and vector mesons within a chiral approach motivated by ChPT. We derive a general form of the chiral Lagrangian describing processes involving excited pseudoscalar and vector mesons. Parameters of the Lagrangian are fitted using data and previous calculations in the framework of the $^3P_0$ model. This work is continuation of a series of papers [26–28], where we analyzed the decays of scalar (including mixing with the scalar glueball) and tensor mesons (including mixing with the tensor glueball) using a phenomenological chiral approach. Here, we consider the strong decays of the first and second radial excitations in the pseudoscalar channel and the first excitation in the vector channel. Note that we do not consider mixing or additional components such as glueballs, yet. Since the experimental data is still very poor in this regime, a fit to the data would not be very

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useful. Whenever values are available, comparison will be done and evaluated.

The $^3P_0$ model has been used to predict many decay widths of mesons within a large mass range, from the GeV scale [18,19] up to charmonia [20]. Its phenomenological success has made it a popular tool for the estimation of decay widths. The model describes the strong open flavor decays of mesons as a $q\bar{q}$-pair production process. The two quarks of the produced pair separate to form the final mesons together with the two quarks of the initial meson. The $q\bar{q}$ pair is assumed to be produced in a $J^{PC} = 0^{++}$ state corresponding to vacuum quantum numbers. This is the main assumption of the model. Simple harmonic oscillator functions are used for the incoming and outgoing particles, the decay amplitudes then have analytical expressions. Predictions obtained within the model are of a rather qualitative type, but have been shown to agree roughly with experiment, although large deviations can occur.

For many of the meson masses assumed in [18–20] better experimental data has become available now or different candidates for the theoretical states are being currently discussed. Therefore, in addition to the fit of our parameters, we comment on the inclusion of finite-width effects and the impact of mass changes on the resulting decay widths. Different scenarios are analyzed in order to identify excited pseudoscalar and vector mesons.

In the present paper, we proceed as follows. In Sec. II, we present the chiral Lagrangian which will be used and the generic expressions for the decay widths. The fit to the $^3P_0$ model within the different channels will be performed in the following section and we will discuss the consequences of available experimental data on the decays. We summarize our results in Sec. IV where we also discuss further possible applications.

II. CHIRAL LAGRANGIAN FOR EXCITED PSEUDOSCALAR AND VECTOR MESONS

The lowest-order chiral Lagrangian describing decays $P^* \to VP(V)$ and $V^* \to PP(V)$ of excited vector V^*, pseudoscalar P^* (first radial excitation), and P^{**} (second radial excitation) mesons involving also their ground states (pseudoscalars P and vectors V) is motivated by ChPT [22–25], and reads as

$$\begin{aligned}\mathcal{L} &= \frac{F^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi_+\rangle + \mathcal{L}^P_{\rm mix} - \frac{1}{2}\sum_{\mathcal{V}=V,V^*}\Big\langle D^\rho \mathcal{V}_{\rho\nu} D_\mu \mathcal{V}^{\mu\nu} - \frac{1}{2}M^2_{\mathcal{V}}\mathcal{V}_{\mu\nu}\mathcal{V}^{\mu\nu}\Big\rangle \\ &+ \frac{1}{2}\sum_{\mathcal{P}=P^*,P^{**}}\langle D^\mu \mathcal{P} D_\mu \mathcal{P} - M^2_{\mathcal{P}}\mathcal{P}^2\rangle + c_{P^*PV}\langle \mathcal{V}_{\mu\nu}[u^\mu, D^\nu \mathcal{P}^*]\rangle + c_{P^*VV}\langle \epsilon^{\mu\nu\alpha\beta}\mathcal{P}^*\{\mathcal{V}_{\mu\nu}, \mathcal{V}_{\alpha\beta}\}\rangle \\ &+ c_{P^{**}PV}\langle \mathcal{V}_{\mu\nu}[u^\mu, D^\nu \mathcal{P}^{**}]\rangle + c_{P^{**}VV}\langle \epsilon^{\mu\nu\alpha\beta}\mathcal{P}^{**}\{\mathcal{V}_{\mu\nu}, \mathcal{V}_{\alpha\beta}\}\rangle + c_{V^*PP}\langle \mathcal{V}^*_{\mu\nu}[u^\mu, u^\nu]\rangle \\ &+ c_{V^*VP}\langle \epsilon^{\mu\nu\alpha\beta}U\{\mathcal{V}_{\mu\nu}, \mathcal{V}^*_{\alpha\beta}\}\rangle. \end{aligned} \tag{1}$$

Here we use the following notation: The symbols $\langle\cdots\rangle$, $[\cdots]$, and $\{\cdots\}$ occurring in Eq. (1) denote the trace over flavor matrices, commutator, and anticommutator, respectively. $U = u^2 = \exp(iP\sqrt{2}/F)$ is the chiral field collecting ground state pseudoscalar fields in the exponential parametrization with

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}. \tag{2}$$

D_μ denotes the chiral and gauge-invariant derivative, $u_\mu = iu^\dagger D_\mu U u^\dagger$ is the chiral field, $\chi_\pm = u^\dagger \chi u^\dagger \pm u\chi^\dagger u$, $\chi = 2B(s + ip)$, $s = \mathcal{M} + \cdots$, and $\mathcal{M} = \mathrm{diag}\{\hat{m}, \hat{m}, m_s\}$ is the current quark mass (we restrict to the isospin symmetry limit with $m_u = m_d = \hat{m}$); B is the quark vacuum condensate parameter and F the pion decay constant. The matrices $\mathcal{V}$ and $\mathcal{V}^*$ represent the nonets of ground state $\{\rho^\pm, \rho^0, K^{*\pm}, K^{*0}, \bar{K}^{*0}, \phi\}$ and first radially excited $\{\rho^\pm(1450), \rho^0(1450), \omega(1420), K^{*\pm}(1680), K^{*0}(1680), \bar{K}^{*0}(1680), \phi(1680)\}$ vector mesons in tensorial representation [23,25]. $\mathcal{P}^*$ and $\mathcal{P}^{**}$ denote the nonets of the first $\{\pi^\pm(1300), \pi^0(1300), K^\pm(1460), K^0(1460), \bar{K}^0(1460), \eta(1295), \eta(1475)\}$ and second $\{\pi^\pm(1800), \pi^0(1800), K^\pm(1830), K^0(1830), \bar{K}^0(1830), \eta(1760), \eta(2225)\}$ radially excited pseudoscalar meson fields (see explicit form of $\mathcal{V}$, $\mathcal{V}^*$, $\mathcal{P}^*$, and $\mathcal{P}^{**}$ in Appendix A). The constants c_{P^*PV}, c_{P^*VV}, $c_{P^{**}PV}$, $c_{P^{**}VV}$, c_{V^*PP}, c_{V^*VP} define the couplings between the corresponding types of mesons, respectively. Following [25] we encode in $\mathcal{L}^P_{\rm mix}$ an additional contribution to the mass of the η_0 (due to the axial anomaly) and the η_0-η_8 mixing term:

$$\mathcal{L}^P_{\rm mix} = -\frac{1}{2}\gamma_P\eta_0^2 - z_P\eta_0\eta_8, \tag{3}$$

(the parameters γ_P and z_P are in turn related to the parameters M_{η_1} and $\tilde{d}_m$ of [25]). The physical, diagonal states η and η' are given by

$$\eta_0 = \eta'\cos\theta_P - \eta\sin\theta_P, \qquad \eta_8 = \eta'\sin\theta_P + \eta\cos\theta_P, \tag{4}$$

where θ_P is the pseudoscalar mixing angle. We follow the standard procedure [25,28–31] and diagonalize the corresponding η_0-η_8 mass matrix to obtain the masses of η and η'. By using $M_\pi = 139.57$ MeV, $M_K = 493.677$ MeV (the physical charged pion and kaon masses), $M_\eta = 547.75$ MeV and $M_{\eta'} = 957.78$ MeV, the mixing angle is determined as $\theta_P = -9.95°$, which corresponds to the tree-level result (see details in Ref. [30]). Correspondingly, one finds $M_{\eta_0} = 948.10$ MeV and $z_P = -0.105$ GeV^2. Higher order corrections in ChPT cause a doubling of the absolute value of the pseudoscalar mixing angle [30]; we restrict our work to the tree-level evaluation, we therefore consistently use the corresponding tree-level result of $\theta_P = -9.95°$. In the present approach we do not include the neutral pion when considering mixing in the pseudoscalar sector, because we work in the isospin limit. For all pseudoscalar mesons we use the unified leptonic decay constant F, which is identified with the pion decay constant $F = F_\pi = 92.4$ MeV. A more accurate analysis including higher orders should involve the individual couplings of the pseudoscalar mesons (for a detailed discussion see Refs. [24]). For the radial excitations and the ground state vector mesons, we assume ideal mixing, so that the flavor content of the excited ϕ and η_{ss} is completely given by $s\bar{s}$, the content of ω and η_{nn} by $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$.

The width for a generic two-particle decay $A \to BC$ is given by

$$\Gamma(A \to BC) = f\,\frac{\lambda^{3/2}(m_A^2, m_B^2, m_C^2)}{\pi m_A^3}|M|^2, \tag{5}$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ is the Källén function; M denotes the amplitude squared including the parameter of the decay strength; f denotes an additional factor which is $\frac{1}{2}$ for $P^* \to VV$, $\frac{1}{6}$ for $V^* \to PP$, $\frac{1}{64}$ for $P^* \to PV$ and $\frac{1}{192}$ for $V^* \to VP$. The average over polarization has already been included; a factor of $\frac{1}{2}$ will have to be added when considering decays with two identical particles in the final state. The decay amplitudes M in tree level are given in the tables in Appendix B.

In the following analysis of the decay widths, we will often encounter decay modes which are kinematically strongly suppressed or forbidden, when using the central mass values. Because of the finite width of the decaying or the produced particles, these decays may however be enhanced considerably. We include this effect in our calculations by approximately taking into account the mass distribution of a meson with a certain width in the following way

$$\Gamma_{\text{full}}(A \to BC) = \int \Gamma(A \to BC)_{m_A=m} f(m)dm. \tag{6}$$

The function $f(m)$ is the mass distribution of a resonance with central mass M and total width Γ given by

$$f(m) = \begin{cases} 0, & m < A_{\text{thr}} \\ \frac{1}{4A_0}\frac{\Gamma^2}{(m-M)^2+\frac{1}{4}\Gamma^2}\cdot\frac{m-A_{\text{thr}}}{M-\Gamma-A_{\text{thr}}}, & A_{\text{thr}} < m < M-\Gamma \\ \frac{1}{4A_0}\frac{\Gamma^2}{(m-M)^2+\frac{1}{4}\Gamma^2}, & M-\Gamma < m < M+\Gamma \\ \frac{1}{4A_0}\frac{\Gamma^2}{(m-M)^2+\frac{1}{4}\Gamma^2}\cdot\frac{M+3\Gamma-m}{2\Gamma}, & M+\Gamma < m < M+3\Gamma \\ 0, & M+3\Gamma < m, \end{cases} \tag{7}$$

where A_{thr} denotes the threshold and A_0 is a normalization constant such that

$$\int f(m)dm = 1. \tag{8}$$

Also for broad final states an analogous integration, as suggested in Eq. (6), will be performed. Note, the spectral function $f(m)$ is taken as a Breit-Wigner form where the low-mass tail is modified to introduce a proper threshold cutoff A_{thr}. For the high-mass tail an additional regularization is introduced to keep the distribution for a finite range of mass values. Such parametrizations of the spectral function were originally used and tested in the study of proton-antiproton annihilations into mesons [32], where finite size effects can also play a relevant role. We will see that in many cases the inclusion of finite-width effects is important, especially when the central masses of the final state mesons lie near threshold.

III. FIT OF THE DECAY STRENGTHS AND RESULTS

In the present approach, relative rates are parameter free predictions, when staying within given nonets both in the initial and final state. To also obtain absolute values for the decay widths we use predictions of the $^3P_0$ model [18,19] to set a scale for the corresponding coupling constants. In particular, for a given set of decay modes we perform a χ^2 fit to the predictions of [18,19] and also give an explicit comparison.

For experimentally measured mesons we use the notation of the Particle Data Group (PDG); for the theoretical states we use π^*, η^*, etc. The fits to the $^3P_0$ model are performed without inclusion of finite-width effects since the authors in [18,19] do not include these either.

In our model, we do not include the radial excitations in the final state channels since this would increase the number of parameters too much. Since the radial excitations are not identified unambiguously yet, this would be a further source of uncertainty. However, the decay into excited mesons becomes only relevant when looking at the second excitation of the vector mesons, therefore this possible fit is not presently discussed.

A. First radial pseudoscalar excitation

1. Experimental situation

The common interpretation (see e.g. Ref. [33]) of the first radial excitation of the pseudoscalar mesons in terms of the measured resonances (as given by PDG [34]) is the following:

$$\begin{pmatrix} \pi^* \\ \eta^*_{nn} \\ \eta^*_{ss} \\ K^* \end{pmatrix} = \begin{pmatrix} \pi(1300) \\ \eta(1295) \\ \eta(1405), \eta(1475) \\ K(1460) \end{pmatrix}. \tag{9}$$

Because of the mass degeneracy of the $\pi(1300)$ with the $\eta(1295)$, the interpretation of the latter state as dominantly composed of $n\bar{n}$ is strengthened and points to a rather small mixing angle with any other higher state. The question whether the $\eta(1405)$ or the $\eta(1475)$ is a quarkonium or an exotic state remains open despite several recent attempts to disentangle this problem, possibly connected to the existence of a glueball (see e.g. Ref. [35], an overview on the search of the pseudoscalar glueball is given in [36]). We will show in the framework of our analysis how to possibly clarify this situation with further measurements.

The $K(1460)$ is not yet accepted by PDG, but it is the only candidate with the right quantum numbers for an excited kaon in this mass range. Its mass, which has been measured to be between 1400 and 1460 MeV, is compatible with a kaon state about 150 MeV higher than the $\pi(1300)$ due to the s quark. For the $K(1460)$ we will study the effect of different masses and finite-width effects. Besides the puzzling situation concerning the supernumerous state, the identification in this sector is rather clear.

2. Decays and fit

In Table I, we give our fit (chiral approach) to the $^3P_0$ model [18,19], indicating latter results and the predictions of other theoretical approaches [3,11] for the decays of the first radial pseudoscalar excitation into pseudoscalar and vector mesons. The coupling constant results are

$$c_{P^*PV} = 4.95\ \mathrm{GeV}^{-1}, \tag{10}$$

where the value does not depend on which of the η_{ss} candidates is used. Note that chiral and $^3P_0$ approaches have an agreement in the results for the $P^* \to PV$ decay widths, which are largely determined by quantum numbers, phase space, and flavor symmetry which are common to both models. In addition, the $^3P_0$ model assumes identical wave functions across flavor multiplets, also in agreement with the chiral model.

TABLE I. Theoretical results for the $P^* \to PV$ decay widths (in MeV).

Decay mode		Gerasimov *et al.* [3]	Volkov *et al.* [11]	Barnes *et al.* [18,19]	Chiral approach
$\pi(1300)$	$\to \pi\rho$	630 ± 160	220	209	527
$\eta_{ss}(1415)$	$\to K\bar{K}^*$	$\cdots$	~ 0	11	10
$\eta_{ss}(1500)$	$\to K\bar{K}^*$	$\cdots$	~ 0	100	82
$K(1460)$	$\to \rho K$	$\cdots$	50	73	65
	$\to \omega K$	$\cdots$	$\cdots$	23	20
	$\to \pi K^*$	$\cdots$	100	101	127
	$\to \eta K^*$	$\cdots$	$\cdots$	3	2

It is important to mention that in Ref. [11] the excited kaon mass was assumed as 1.3 GeV, which leads to a considerable reduction in phase space. Because of interference the approach in [11] predicts a very narrow width for the decay of $\eta_{ss}(1470) \to K\bar{K}^*$). In Ref. [3], the decay width $\Gamma(\pi^* \to \pi\rho)$ was calculated using available data on $\Gamma(\rho^* \to \omega\pi)$ at that time, i.e. assuming a mass of 1.1 to 1.2 GeV.

3. The η_{ss} excitation

Barnes *et al.* assumed in [18,19] that the η_{ss} lies between 1.415 and 1.5 GeV. Presently, the experimental candidates for this meson are the $\eta(1405)$ and $\eta(1475)$. It is still unclear which one is to be considered supernumerous and what values the mixing angle may take. Using the coupling constant of (10), we give the decay widths for these masses including and, in brackets $(\cdots)$, excluding finite-width effects

$$\Gamma(\eta_{ss}(1476) \to K\bar{K}^*) = 67\ \mathrm{MeV}\,(57\ \mathrm{MeV}), \tag{11}$$

$$\Gamma(\eta_{ss}(1410) \to K\bar{K}^*) = 13\ \mathrm{MeV}\,(7\ \mathrm{MeV}). \tag{12}$$

The decay width strongly depends on the mass value of the state and on finite-width effects, especially for the lower state. If we compare this value with the full width, the result seems to point to an $s\bar{s}$ interpretation of the $\eta(1475)$: The full width of the experimental state $\eta(1475)$ is 87 MeV and the decay mode $K\bar{K}\pi$ fed by $K\bar{K}^*$ is dominant. The absence of the $K\bar{K}^*$ decay mode for the experimental candidate $\eta(1405)$ seems to point to the conclusion that its $s\bar{s}$ component is small. However, the expected decay width would be small anyway due to the strong kinematical suppression. It is important to stress the fact that the non-observation of this mode is not necessarily clear evidence for the absence of a large $s\bar{s}$ admixture.

Further measurements of the decay to $K\bar{K}^*$ would be useful in this case to better identify the η_{ss} state in this region and to sort out the supernumerous state. A measurement of the direct three-body decays $\eta(1405) \rightarrow \eta\pi\pi$, $K\bar{K}\pi$, to be discriminated from resonance-fed decays, could help to estimate the mixing angle in different mixing scenarios in this mass region, since a chiral approach could describe three-body decays as well. Unfortunately, the lack of experimental data makes a serious analysis impossible at the moment.

4. Finite-width effects of $\eta(1295)$ and $\pi(1300)$

The experimental state $\eta(1295)$ has a width of 55 ± 5 MeV. The radially excited pion is even broader, $\Gamma(\pi(1300)) \geq 200$ MeV. Now we include finite-width effects to study the $K\bar{K}^*$ decay channel.

We compute the $K\bar{K}^*$ decay width using expression (6) and find

$$\Gamma(\pi(1300) \rightarrow K\bar{K}^*) = 10.37 \text{ MeV}, \tag{13}$$

$$\Gamma(\eta(1295) \rightarrow K\bar{K}^*) = 0.07 \text{ MeV}. \tag{14}$$

For a $n\bar{n}$ η state at 1.3 GeV the decay width into $K\bar{K}^*$ is negligible. The excited pion however should decay with a small finite width into KK^*. Note that the decay $\pi(1300) \rightarrow \pi\rho$ is absolutely dominant as stated by the theoretical predictions and seems to feed the full width of 200–600 MeV. Indeed, the decay into $K\bar{K}^*$ has not been seen so far. For the η an upper bound of the $\pi\pi K$ width exists which is consistent with the present small estimate for this width.

5. The kaon

The experimental situation regarding the kaon is not very clear, there are two possible mass assignments (1.4 GeV and 1.46 GeV). The total width is large, around 250 MeV in both cases. In Table II we give the decay widths using our approach including finite-width effects for both candidates.

The decay modes ηK^* and ϕK are considerably enhanced by finite-width effects. The observed decay width to ρK of around 34 MeV (no error is given by PDG) is overestimated by a factor of 3 in the model prediction. The decay width to $K^*\pi$ is measured as 109 MeV and lies in the range of our fit. A direct fit of the coupling constant to the data would not allow an improved correspondence, since the ratio

$$\frac{\Gamma(K(1400) \rightarrow \rho K)}{\Gamma(K(1400) \rightarrow \pi K^*)} \sim 6 \tag{15}$$

is fixed in the present model, but experimentally it is currently found to be ~ 0.3. Even though the decay to $K^*(1430)\pi$ is strongly suppressed kinematically, the experimental value of 117 MeV is rather large. Further data on decay modes involving the kaon would be useful to establish this state better, the experimental data used here is extracted from one single experiment and is not confirmed by PDG. Since its mass lies in a reasonable range with respect to the pion-nonet partner, no exotic scenario is evident here.

TABLE II. Decay width including finite-width effects for the kaon (in MeV).

Decay mode		$m = 1.4$ GeV	$m = 1.46$ GeV	Data [34]
K	$\rightarrow \rho K$	73	96	34
	$\rightarrow \omega K$	21	30	...
	$\rightarrow \pi K^*$	118	152	109
	$\rightarrow \eta K^*$	21	34	...
	$\rightarrow \phi K$	8	13	...

B. Second radial pseudoscalar excitation

The mass region of the second radially excited pseudoscalar mesons is interesting since it lies closer to the region in which lattice QCD predicts the pseudoscalar glueball with mass of about 2.3 ± 0.2 GeV [37]. Mixing might lower the mass considerably and affect the pattern in this mass region. So far, no supernumerous state has been observed in this region, but no clear candidate for η_{ss} has been observed so far either.

1. Experimental situation

A possible interpretation of the second radial excitations would be as follows:

$$\begin{pmatrix} \pi^{**} \\ \eta_{nn}^{**} \\ \eta_{ss}^{**} \\ K^{**} \end{pmatrix} = \begin{pmatrix} \pi(1800) \\ \eta(1760) \\ ? \\ K(1830) \end{pmatrix}. \tag{16}$$

There is no clear candidate for η_{ss}. The next resonance with suitable quantum numbers would be the $\eta(2225)$ which lies considerably higher than expected and has

TABLE III. Decay widths of second radial pseudoscalar excitation to pseudoscalars and vectors (in MeV).

Decay mode		Gerasimov *et al.* [3]	Barnes *et al.* [18,19]	Chiral approach (1)	Chiral approach (2)
$\pi(1800)$	$\rightarrow \pi\rho$	32–50	31	54	77
	$\rightarrow K\bar{K}^*$	...	36	13	18
$\eta_{nn}(1800)$	$\rightarrow K\bar{K}^*$	...	36	13	18
$\eta_{ss}(1950)$	$\rightarrow K\bar{K}^*$	...	53	43	61
$K(1830)$	$\rightarrow \rho K$	...	21	15	21
	$\rightarrow \omega K$	...	7	5	7
	$\rightarrow \phi K$	...	18	4	6
	$\rightarrow \pi K^*$	...	16	18	25
	$\rightarrow \eta K^*$	...	27	9	12

TABLE IV. Decay widths of second radial pseudoscalar excitation to two vectors (in MeV).

Decay mode		Barnes *et al.* [18,19]	Chiral approach
$\pi(1800)$	$\to \rho\omega$	73	83
$\eta_{nn}(1800)$	$\to \rho\rho$	112	130
	$\to \omega\omega$	36	40
$\eta_{ss}(1950)$	$\to K^*\bar{K}^*$	67	55
$K(1830)$	$\to \rho K^*$	45	36
	$\to \omega K^*$	14	11

been discussed as a glueball candidate. The candidate for the kaon has a rather low mass, being very close to the pion. This phenomenon can be understood by the following argument. The larger strange quark mass implies a smaller excitation energy for kaons. Eventually the kaon-like states come closer to and become even lighter than the nonstrange counterparts, typically around the second or third excitations. The candidate for η_{nn} lies somehow lower than the corresponding pion state, which might allude to a possible mixing scenario which we will discuss.

2. The fit

In the chiral approach, these widths are related to the couplings $c_{P^{**}PV}$ and to $c_{P^{**}VV}$, the best fit to the results of the $^3P_0$ model gives

$$c_{P^{**}PV} = 0.94589\ \text{GeV}^{-1}, \qquad c_{P^{**}VV} = 0.29759\ \text{GeV}^{-1}. \tag{17}$$

Our results in comparison with other theoretical predictions are given in Tables III, IV, and V.

3. General discussion

The authors in Ref. [11] do not give predictions for the decays of the second radial excitation of the pseudoscalar (3^1S_0). In the vector-vector channel, the chiral approach reproduces the decay pattern rather well. The order of magnitude of the decay widths are equivalent, the general pattern is the same. In the vector-pseudoscalar channel strong deviations between the results of the $^3P_0$ model and the present predictions are observed. In Ref. [3], the fact that

$$\Gamma(\pi^{**} \to \pi\rho) \ll \Gamma(\pi^* \to \pi\rho) \tag{18}$$

is well explained. The reason for this naively unexpected behavior can be traced to the node structure of the radial wave functions of the excited pion states. The prediction was indeed confirmed experimentally and solidifies the present interpretation of π^*, π^{**} as radial excitations. This behavior is also present in the basis for our fit (1), that is the predictions of the $^3P_0$ model in [18,19]. Here, the suppression of the $\pi^{**} \to \pi\rho$ mode is reflected by a rather small coupling constant in order to fit the small width $\Gamma(\pi(1800) \to \pi\rho) = 31$ MeV found by Barnes *et al.* [18,19]. At the same time, the additional decay width for $K\bar{K}^*$ is consequently smaller in the chiral approach. Our second fit (2) does not include the decay width to $\pi\rho$ and leads to a larger coupling constant:

$$c_{P^{**}PV} = 1.1295\ \text{GeV}^{-1}. \tag{19}$$

The relation $\Gamma(\eta(1800) \to K\bar{K}^*) = \Gamma(\pi(1800) \to K\bar{K}^*)$ also emerges naturally in the chiral approach, but the authors of Ref. [18,19] find $\Gamma(\pi(1800) \to \rho\pi) \leq \Gamma(\pi(1800) \to K\bar{K}^*)$. In our approach, independent of the choice of the coupling, we have

$$\frac{\Gamma(\pi(1800) \to \rho\pi)}{\Gamma(\pi(1800) \to K\bar{K}^*)} = 2\frac{\lambda(m_{\pi^{**}}^2, m_\pi^2, m_\rho^2)^{3/2}}{\lambda(m_{\pi^{**}}^2, m_K^2, m_{\bar{K}^*}^2)^{3/2}} = 4.35. \tag{20}$$

4. η_{nn} and π

We can use our results to identify the theoretical states with experimental candidates. The experimental states for the second radial excitation of η_{nn} and π are easily found: They are the $\eta(1760)$ with mass 1756 ± 9 MeV and total width 96 ± 70 MeV. The approximately mass-degenerate pion-partner would be the $\pi(1800)$ with mass 1816 ± 14 MeV and width 208 ± 12 MeV.

The predictions for the partial decay widths of $\eta(1800)$, given in [18,19] with a mass of 1.8 GeV, are too high. Because of the lower mass of the experimental candidate (1.76 GeV) they have to be corrected. Since the decays to $\rho\rho$ and $\omega\omega$ are close to threshold, the small change in mass changes the decay pattern considerably. The large width of the initial state has to be considered as well to check whether we can expect a sizable decay to $K^*\bar{K}^*$ which lies kinematically near threshold. For the experimental candidates, we find the decays listed in Table V.

The $\eta(1760) \to K^*\bar{K}^*$ decay mode is essentially suppressed, for the $\pi(1800)$ however, we can indeed expect to see this decay. The enhancement of the $\rho\omega$ channel for the π is very strong, too. For the $\pi(1800)$ the situation is

TABLE V. Prediction of decay widths for experimental candidates (in MeV).

Decay mode		Chiral approach (zero width)	Chiral approach (finite width)
$\eta(1760)$	$\to K\bar{K}^*$	10	10
	$\to \rho\rho$	95	94
	$\to \omega\omega$	29	29
	$\to K^*K^*$	0	2
$\pi(1800)$	$\to K^*\bar{K}^*$	2	26
	$\to \rho\omega$	90	217
	$\to KK^*$	13	13
	$\to \pi\rho$	56	55

presently rather unclear, since the mode $\pi\rho$ has not been observed yet. Also, the dominant decay channel $\rho\omega$, which feeds the 5π final state, has not been detected yet.

5. *Identification of* η_{ss}

The η_{ss} meson is assumed to lie around 2000 MeV (the approach in [18,19] uses a mass of 1950 MeV), close to the $\phi\phi$ threshold. Predictions for this decay mode will therefore depend strongly on the mass and width of the $\eta_{ss}(\approx 2000)$.

The identification of η_{ss} is difficult: No pseudoscalar isoscalar state has been observed so far near 2000 MeV. The next candidate would be the resonance $\eta(2225)$, which, however, lies much higher than one would naively expect. It is a broad state (the width is about 150 MeV), hence finite-width effects can be important.

6. *Mixing scenario*

As the η_{nn} candidate lies somewhat lower, one can analyze a mixing scenario of a bare η_{nn} around 1812 MeV and a bare η_{ss} at a mass lower than 2225 MeV. One finds

$$\begin{pmatrix} \eta(1760) \\ \eta(2225) \end{pmatrix} = \begin{pmatrix} 0.95 & 0.32 \\ -0.32 & 0.95 \end{pmatrix} \begin{pmatrix} \eta_{nn}(1812) \\ \eta_{ss}(2183) \end{pmatrix}, \qquad (21)$$

which corresponds to a mixing angle of $\alpha = 0.32$. This scenario may help to explain the large mass of the η_{ss} candidate and the low mass of η_{nn}. However, no strong mixing dynamics is presently known which would induce such mass shifts. If the pseudoscalar glueball is present in this mass region, mixing of this state with η_{nn} and η_{ss} might also result in a similar mass shift.

Let us assume that the $\eta(2225)$ state is indeed mainly η_{ss}. Its decay pattern is listed in Table VI. The total width of the experimental candidate with $\Gamma = 150^{+300}_{-60} \pm 60$ MeV would be largely overestimated in the fit of the coupling constant to the $^3P_0$ data. The decay rates have not been measured yet.

An important question would be to clarify whether this state is the third radial excitation of the η_{nn} configuration. A mass-degenerate pion triplet with the same mass would of course favor this interpretation. For this purpose we analyze the decay ratios for both an $\eta_{ss}(2225)$ and $\eta_{nn}(2225)$, where we assume the width to be 150 MeV (and the threshold set to 1 GeV):

$$\Gamma(\eta_{ss} \to \phi\phi):\Gamma(K^*\bar{K}^*) = 1{:}6, \qquad (22)$$

$$\Gamma(\eta_{nn} \to \omega\omega):\Gamma(K^*\bar{K}^*):\Gamma(\rho\rho) = 1{:}1.20{:}4.62. \qquad (23)$$

While the $K^*\bar{K}^*$ decay mode is the strongest VV mode for the η_{ss} state, for η_{nn} we should rather expect the $\rho\rho$ as dominant decay mode. A determination of the $K^*\bar{K}^*$ channel would help in further clarification of this state.

The situation for the $\eta(2225)$ is not satisfactory. The knowledge of the decay widths might clarify the nature of the $\eta(2225)$ and give evidence for or against its interpretation as mainly η_{ss}.

C. First radial vector excitations

1. *Experimental situation*

The experimental candidates for the first radial vector excitations are the following:

$$\begin{pmatrix} \rho^* \\ \omega^* \\ \phi^* \\ K^* \end{pmatrix} = \begin{pmatrix} \rho(1450) \\ \omega(1420) \\ \phi(1680) \\ K^*(1680), K^*(1410) \end{pmatrix}. \qquad (24)$$

The candidates are rather well established, but the interpretation of the K^* states is still an open issue: Experimentally, one finds $K^*(1680)$ and $K^*(1410)$ which would be possible candidates, even though the mass of $K^*(1410)$ (current PDG average is 1414 ± 15 MeV) would be considerably low and possibly indicates mixing with a hybrid meson state in this mass region. In [18,19], the excited vector kaon state has been considered with mass 1414 MeV and 1580 MeV. We will also discuss which decay patterns are expected for the $K^* = K(1680)$.

2. *Decays and fit*

We have performed fits to the $^3P_0$ data including both candidates for the excited vector kaon separately. The resulting coupling constants do not depend on this choice.

We do, however, not include the very narrow decay width $\Gamma(K^*(1580) \to \eta' K)$ in our fit as this value seems to be a breakout. In Table VII, we give the experimental total decay widths and the sum of the widths of the modes considered here, in the chiral approach and the results by

TABLE VI. Predictions for the partial decay widths of $\eta_{ss}(2220)$ (in MeV).

Decay mode		Chiral approach (zero width)	Chiral approach (finite width)
$\eta_{ss}(2220)$	$\to K\bar{K}^*$	79	78
	$\to \phi\phi$	40	44
	$\to K^*\bar{K}^*$	265	264

TABLE VII. Total decay widths of first vector radial excitation (in MeV).

State	Data [34]	Sum of widths (Barnes *et al.* [18,19])	Sum of widths (chiral approach)
$\rho(1450)$	400 ± 60	275	330
$\omega(1420)$	180–250	376	454
$\phi(1680)$	150 ± 50	378	361

TABLE VIII. Decay widths of first vector radial excitation to two pseudoscalars (in MeV).

Decay mode		Gerasimov *et al.* [3]	Volkov *et al.* [11]	Barnes *et al.* [18,19]	Chiral approach
$\rho(1450)$	$\to \pi\pi$	7	22	74	108
	$\to K\bar{K}$	⋯	⋯	35	23
$\omega(1420)$	$\to K\bar{K}$	⋯	⋯	31	19
$\phi(1680)$	$\to K\bar{K}$	⋯	10	89	91
$K^*(1414)$	$\to \pi K$	⋯	20	55	50
	$\to \eta K$	⋯	⋯	42	23
$K^*(1580)$	$\to \pi K$	⋯	⋯	61	76
	$\to \eta K$	⋯	⋯	60	44
	$\to \eta' K$	⋯	⋯	0.5	6

Barnes *et al.* [18,19]. The results for the partial decay widths of the first vector radial excitation are listed in Tables VIII and IX. The results obtained in [11] lie considerably below both the $3P_0$-model predictions and consequently our fit. Indeed, the experimental total decay width is overestimated by the two latter approaches. The candidates for the kaon are discussed below. One can expect that the decay modes not considered here (such as decays to scalar mesons) are important for the ρ and weaker for the ω and ϕ.

Note that in [3] the ρ^* meson mass was placed at 1220 MeV. In this approach, the decay width $\rho^* \to \pi\pi$ is strongly suppressed as a result of the node structure of the wave function.

For the two pseudoscalar modes the coupling strength results are

$$c_{V^*PP} = 1.65\ \text{GeV}^{-1}. \tag{25}$$

Again, for the pseudoscalar-vector modes we do not fit the very small decay widths. The coupling strength results in this case are

$$c_{V^*VP} = 1.20\ \text{GeV}^{-1}. \tag{26}$$

3. The kaon

The actual experimental candidates for the vector kaon are the $K^*(1680)$ and the $K^*(1410)$ with masses 1717 ± 27 MeV, 1414 ± 15 MeV and total widths of 322 ± 110 MeV and 232 ± 21 MeV, respectively. The $K^*(1414)$ lies somewhat too low with respect to the other better established members of the nonet; $K^*(1680)$ (with a mass of 1717 MeV) has a very high mass, above the $\phi(1680)$.

Table X shows the known decay widths of $K^*(1410)$ and $K^*(1680)$ and our predictions in the chiral approach. Neither candidate is described well by the chiral approach. Although the decay ratio

$$\frac{\Gamma(K^*(1680) \to K\rho)}{\Gamma(K^*(1680) \to K^*\pi)} \approx 1 \tag{27}$$

is reproduced approximately by the model, the total width of $K^*(1680)$ is largely overestimated, as observed before for ω and ρ.

The deviation from the experimental data is still stronger for the case of the $K^*(1414)$, where neither the current pattern of the decay widths can be reproduced nor the decay width to $K\pi$. The decay widths to two pseudoscalars was not overestimated so strongly to explain a discrepancy by a factor 4.

Further measurements of $K\eta$ and $K^*\eta$ would help to confirm or disprove the interpretation of $K^*(1680)$ as the first radially excited vector kaon. The interpretation of the kaon remains an open issue. In this vein, we should stress that SU(3) flavor symmetry breaking effects could be quite important for modes involving strange mesons. This can be

TABLE IX. Decay widths of first vector radial excitation to pseudoscalar and vector mesons (in MeV).

Decay mode		Volkov *et al.* [11]	Barnes *et al.* [18,19]	Chiral approach
$\rho(1450)$	$\to \omega\pi$	75	122	165
	$\to \rho\eta$	⋯	25	19
	$\to KK^*$	⋯	19	15
$\omega(1420)$	$\to K\bar{K}^*$	⋯	5	4
	$\to \rho\pi$	225	328	422
	$\to \omega\eta$	⋯	12	9
$\phi(1680)$	$\to KK^*$	90	245	241
	$\to \eta\phi$	⋯	44	29
$K^*(1414)$	$\to \omega K$	⋯	10	8
	$\to \rho K$	⋯	34	26
	$\to \pi K^*$	⋯	55	63
	$\to \eta K^*$	⋯	0	0
$K^*(1580)$	$\to \omega K$	⋯	29	29
	$\to \rho K$	⋯	90	91
	$\to \pi K^*$	⋯	99	135
	$\to \eta K^*$	⋯	1	28
	$\to \phi K$	⋯	9	6

TABLE X. Decay widths of K^* candidates and predictions (in MeV).

Decay mode	$K^*(1410)$ Data [34]	$K^*(1410)$ Chiral approach	$K^*(1680)$ Data [34]	$K^*(1680)$ Chiral approach
$K\pi$	15	52	125	109
$K\eta$	⋯	26	⋯	50
$K\eta'$	⋯	≈ 0	⋯	0.5
$K\omega$	⋯	14	⋯	58
$K\rho$	<16	43	101	178
πK^*	>93	76	96	221
ηK^*	⋯	13	⋯	103
$\eta' K^*$	⋯	≈ 0	⋯	0.4
$K\phi$	⋯	4	⋯	49

accommodated in the chiral approach by including the terms in the Lagrangian responsible for such symmetry breaking contributions. We plan to make this improvement of the approach in further work, where the presence of sufficient data allows such a detailed analysis.

4. The ϕ and the ω

The dominance of the $\pi\rho$ channel for the ω and the K^*K channel for the ϕ agree well with experiment: these two decays are stated by the PDG as the dominant ones. The $^3P_0$ model as well as our fit overestimate largely the total widths, especially in the vector-pseudoscalar channel. The dominance of the $K\bar{K}^*$ mode, which is stated by PDG for the experimental candidate, is confirmed in the model and the fit. The interpretation of $\phi(1680)$ as a dominant $s\bar{s}$ partner of the $\omega(1420)$ is therefore clear. The width of this meson is estimated by PDG to be 215 ± 35 MeV, the sum of widths in [18,19] and in our approach lie considerably above that.

A more quantitative measurement of the widths of the single decay modes would help to improve the quality of the present interpretation considerably. For example our estimate for $\frac{\Gamma(\phi\to K\bar{K})}{\Gamma(\phi\to K\bar{K}^*)} \approx \frac{1}{3}$ is far from being consistent with the actual (not confirmed) value of 0.07 ± 0.01.

IV. SUMMARY

We have studied the strong decays of radially excited mesons in a chiral approach. Because of the lack of sufficient data points we chose to adjust the phenomenological coupling strengths of the present chiral approach for the decays of radial pseudoscalar and vector excitations to the average values of the $^3P_0$ model of Refs. [18,19]. Although the absolute values of the decay widths are fixed in such a fit, the relative decay rates are a prediction of the chiral approach when staying both in the initial and final states within fixed meson nonets. Our goal was to test the possibility of a phenomenological analysis within such an approach, which would have also the advantage of being able to incorporate three-body decays.

The picture we have drawn is satisfactory. Most of the presently known decay widths are reproduced rather well, especially when taking into account the small number of parameters. A chiral approach analysis might therefore help to resolve the remaining interpretation issues in the future. The possibility to include three-body decays in our analysis might help to better understand the decay pattern, for example, in the context of the $\eta(1405)$, $\eta(1475)$ puzzle. Open interpretation issues were addressed but could not be resolved unambiguously, since the lack of experimental data prohibits a direct fit of our parameters. In the pseudoscalar sector an interesting extension is also given by the possible presence of a glueball state, which can mix with the quarkonia configurations. The phenomenological consequences of this additional glueball configuration on the decay patterns of pseudoscalar mesons are presently studied.

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APPENDIX A: MATRICES $V_{\mu\nu}$, $V^*_{\mu\nu}$, P^* AND P^{**}

$$V_{\mu\nu} = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho_0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}_{\mu\nu}. \tag{A1}$$

$$V^*_{\mu\nu} = \begin{pmatrix} \frac{\rho^0(1450)}{\sqrt{2}} + \frac{\omega(1420)}{\sqrt{2}} & \rho^+(1450) & K^{*+}(1680) \\ \rho^-(1450) & -\frac{\rho^0(1450)}{\sqrt{2}} + \frac{\omega(1420)}{\sqrt{2}} & K^{*0}(1680) \\ K^{*-}(1680) & \bar{K}^{*0}(1680) & \phi(1680) \end{pmatrix}_{\mu\nu}. \tag{A2}$$

$$P^* = \begin{pmatrix} \frac{\pi^0(1300)}{\sqrt{2}} + \eta(1295) & \pi^+(1300) & K^+(1460) \\ \pi^-(1300) & -\frac{\pi^0(1300)}{\sqrt{2}} + \eta(1295) & K^0(1460) \\ K^-(1460) & \bar{K}^0(1460) & \eta(1475) \end{pmatrix}. \tag{A3}$$

$$P^{**} = \begin{pmatrix} \frac{\pi^0(1800)}{\sqrt{2}} + \eta(1760) & \pi^+(1800) & K^+(1830) \\ \pi^-(1800) & -\frac{\pi^0(1800)}{\sqrt{2}} + \eta(1760) & K^0(1830) \\ K^-(1830) & \bar{K}^0(1830) & \eta(2225) \end{pmatrix}. \tag{A4}$$

APPENDIX B: TREE-LEVEL AMPLITUDES FOR THE DECAY OF EXCITED MESONS

The tree-level amplitudes can easily be read off the original Lagrangian. We state them here for future reference.

(1) Decay amplitudes of excited pseudoscalar mesons are given in Table XI.

TABLE XI. Decay amplitudes squared (excluding the decay strength constant).

Decay	Products	Squared amplitude
$\pi^* \to PV$	$\pi\rho$	4
	$K\bar{K}^*$	2
$\pi^* \to VV$	$\rho\phi$	$8\cos^2\phi_V$
	$\rho\omega$	$8\sin^2\phi_V$
	$K^*\bar{K}^*$	4
$\eta^{*\prime} \to PV$	$K\bar{K}^*$	$6\sin^2\theta_P$
$\eta^{*\prime} \to VV$	$\rho^0\rho^0$	$2\sin^2\phi_P$
	$\rho^+\rho^-$	$8\sin^2\phi_P$
	$\phi\phi$	$2(\sin\phi_P^*\cos\phi_V^2 + \sqrt{2}\cos\phi_P^*\sin\phi_V^2)^2$
	$\phi\omega$	$8\sin^2\phi_V\cos^2\phi_V(-\sin\phi_P^* + \sqrt{2}\cos\phi_P^*)^2$
	$\omega\omega$	$2(\sin\phi_P^*\sin\phi_V^2 + \sqrt{2}\cos\phi_P^*\cos\phi_V^2)^2$
	$K^*\bar{K}^*$	$2(\sin\phi_P^* + \sqrt{2}\cos\phi_P^*)^2$
$K^* \to PV$	$K\rho$	$\frac{3}{2}$
	$K\omega$	$\frac{3}{2}\sin^2\theta_V$
	$K\phi$	$\frac{3}{2}\cos^2\theta_V$
	ηK^*	$\frac{3}{2}\cos^2\theta_P$
	$\eta' K^*$	$\frac{3}{2}\sin^2\theta_P$
	πK^*	$\frac{3}{2}$
$K^* \to VV$	$K^*\rho$	6
	$K^*\omega$	$4(\sin\theta_V + \sqrt{2}\cos\theta_V)^2$
	$K^*\phi$	$4(\cos\theta_V - \sqrt{2}\sin\theta_V)^2$

(2) Decay amplitudes of excited vector mesons are given in Table XII.

TABLE XII. Decay amplitudes squared (excluding the decay strength constants).

Decay	Products	Squared amplitude
$\rho^* \to PP$	$K\bar{K}$	4
	$\pi\pi$	8
$\rho^* \to VP$	$\bar{K}^* K$	2
	$\rho\eta$	$2\cos^2\phi_P$
	$\rho\eta'$	$2\sin^2\phi_P$
	$\omega\pi$	$2\sin^2\phi_V$
	$\phi\pi$	$2\cos^2\phi_V$
$\omega^* \to PP$	$K\bar{K}$	$12\sin^2\theta_V^*$
$\omega^* \to VP$	$\phi\eta$	$2(\sqrt{2}\cos\phi_V^*\sin\phi_V\sin\phi_P + \sin\phi_V^*\cos\phi_V\cos\phi_P)^2$
	$\phi\eta'$	$2(\sqrt{2}\cos\phi_V^*\sin\phi_V\cos\phi_P - \sin\phi_V^*\cos\phi_V\sin\phi_P)^2$
	$\omega\eta$	$2(\sqrt{2}\cos\phi_V^*\cos\phi_V\sin\phi_P - \sin\phi_V^*\sin\phi_V\cos\phi_P)^2$
	$\omega\eta'$	$2(\sqrt{2}\cos\phi_V^*\cos\phi_V\cos\phi_P + \sin\phi_V^*\sin\phi_V\sin\phi_P)^2$
	$\rho\pi$	$6\sin^2\phi_V^*$
	$K^*\bar{K}$	$2(\sin\phi_V^* + \sqrt{2}\cos\phi_V^*)^2$
$\phi^* \to PP$	$K\bar{K}$	$12\cos^2\theta_V^*$
$\phi^* \to VP$	$\phi\eta$	$2(-\sqrt{2}\sin\phi_V^*\sin\phi_V\sin\phi_P + \cos\phi_V^*\cos\phi_V\cos\phi_P)^2$
	$\phi\eta'$	$2(\sqrt{2}\sin\phi_V^*\sin\phi_V\cos\phi_P + \cos\phi_V^*\cos\phi_V\sin\phi_P)^2$
	$\omega\eta$	$2(\sqrt{2}\sin\phi_V^*\cos\phi_V\sin\phi_P + \cos\phi_V^*\sin\phi_V\cos\phi_P)^2$
	$\omega\eta^*$	$2(-\sqrt{2}\sin\phi_V^*\cos\phi_V\cos\phi_P + \cos\phi_V^*\sin\phi_V\sin\phi_P)^2$
	$\rho\pi$	$6\cos^2\phi_V^*$
	$K^*\bar{K}$	$2(\cos\phi_V^* - \sqrt{2}\sin\phi_V^*)^2$
$K^* \to PP$	$K\pi$	6
	$K\eta$	$4(\frac{1}{\sqrt{2}}\cos\phi_P + \sin\phi_P)^2$
	$K\eta'$	$4(\frac{1}{\sqrt{2}}\sin\phi_P - \cos\phi_P)^2$
$K^* \to VP$	$K^*\pi$	$\frac{3}{2}$
	$K^*\eta$	$(\frac{\cos\phi_P}{\sqrt{2}} + \sin\phi_P)^2$
	$K^*\eta'$	$(\frac{\sin\phi_P}{\sqrt{2}} - \cos\phi_P)^2$
	ωK	$(\frac{\sin\phi_V}{\sqrt{2}} + \cos\phi_V)^2$
	ϕK	$(\frac{\cos\phi_V}{\sqrt{2}} - \sin\phi_V)^2$
	ρK	3

[1] E. Klempt and A. Zaitsev, Phys. Rep. **454**, 1 (2007).
[2] V. Mathieu, N. Kochelev, and V. Vento, arXiv:0810.4453.
[3] S. B. Gerasimov and A. B. Govorkov, Z. Phys. C **13**, 43 (1982); A. B. Govorkov, Z. Phys. C **32**, 405 (1986); S. B. Gerasimov and A. B. Govorkov, Z. Phys. C **29**, 61 (1985).
[4] S. B. Gerasimov and A. B. Govorkov, Z. Phys. C **36**, 435 (1987); A. B. Govorkov, Yad. Fiz. **48**, 237 (1988) [Sov. J. Nucl. Phys. **48**, 150 (1988)].
[5] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).
[6] Yu. S. Surovtsev and P. Bydzovsky, Nucl. Phys. **A807**, 145 (2008).
[7] N. N. Achasov and A. A. Kozhevnikov, Phys. Rev. D **62**, 117503 (2000).
[8] N. N. Achasov and A. A. Kozhevnikov, Yad. Fiz. **65**, 158 (2002) [Phys. At. Nucl. **65**, 153 (2002)].
[9] Yu. S. Surovtsev, R. Kaminski, D. Krupa, and M. Nagy, Int. J. Mod. Phys. A **22**, 649 (2007).
[10] M. K. Volkov and C. Weiss, Phys. Rev. D **56**, 221 (1997).
[11] M. K. Volkov, D. Ebert, and M. Nagy, Int. J. Mod. Phys. A **13**, 5443 (1998).
[12] M. K. Volkov, Yad. Fiz. **60**, 1094 (1997) [Phys. At. Nucl. **60**, 1920 (1997)].
[13] M. K. Volkov and V. L. Yudichev, Fiz. Elem. Chastits At. Yadra **31**, 576 (2000) [Phys. Part. Nucl. **31**, 282 (2000)].
[14] M. K. Volkov and V. L. Yudichev, Yad. Fiz. **63**, 1924 (2000) [Phys. At. Nucl. **63**, 1835 (2000)].
[15] M. K. Volkov and V. L. Yudichev, Yad. Fiz. **63**, 527 (2000) [Phys. At. Nucl. **63**, 455 (2000)].
[16] A. Holl, A. Krassnigg, and C. D. Roberts, Phys. Rev. C **70**, 042203(R) (2004).

[17] A. Holl, A. Krassnigg, P. Maris, C. D.Roberts, and S. V. Wright, Phys. Rev. C **71**, 065204 (2005).
[18] T. Barnes, F. E. Close, P. R. Page, and E. S. Swanson, Phys. Rev. D **55**, 4157 (1997).
[19] T. Barnes, N. Black, and P. R. Page, Phys. Rev. D **68**, 054014 (2003).
[20] T. Barnes, S. Godfrey, and E. S. Swanson, Phys. Rev. D **72**, 054026 (2005).
[21] G. Busetto and L. Oliver, Z. Phys. C **20**, 247 (1983).
[22] S. Weinberg, Physica A (Amsterdam) **96**, 327 (1979).
[23] J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.) **158**, 142 (1984).
[24] J. Gasser and H. Leutwyler, Nucl. Phys. **B250**, 465 (1985).
[25] G. Ecker, J. Gasser, A. Pich, and E. de Rafael, Nucl. Phys. **B321**, 311 (1989); G. Ecker, J. Gasser, H. Leutwyler, A. Pich, and E. de Rafael, Phys. Lett. B **223**, 425 (1989).
[26] F. Giacosa, T. Gutsche, V. E. Lyubovitskij, and A. Faessler, Phys. Lett. B **622**, 277 (2005).
[27] F. Giacosa, T. Gutsche, V. E. Lyubovitskij, and A. Faessler, Phys. Rev. D **72**, 114021 (2005).
[28] F. Giacosa, T. Gutsche, V. E. Lyubovitskij, and A. Faessler, Phys. Rev. D **72**, 094006 (2005).
[29] V. Cirigliano, G. Ecker, H. Neufeld, and A. Pich, J. High Energy Phys. 06 (2003) 012.
[30] E. P. Venugopal and B. R. Holstein, Phys. Rev. D **57**, 4397 (1998).
[31] K. Kawarabayashi and N. Ohta, Nucl. Phys. **B175**, 477 (1980).
[32] M. Maruyama and T. Ueda, Nucl. Phys. **A364**, 297 (1981).
[33] V. V. Anisovich, Usp. Fiz. Nauk **174**, 49 (2004) [Phys. Usp. **47**, 45 (2004)].
[34] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667**, 1 (2008).
[35] S. B. Gerasimov, M. Majewski, and V. A. Meshcheryakov, arXiv:0708.3762.
[36] A. Masoni, C. Cicalo, and G. L. Usai, J. Phys. G **32**, R293 (2006).
[37] G. S. Bali, K. Schilling, A. Hulsebos, A. C. Irving, C. Michael, and P. W. Stephenson (UKQCD Collaboration), Phys. Lett. B **309**, 378 (1993).