# GeV Majorana neutrinos in top-quark decay at the LHC

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We explore the  $\Delta L = 2$  same-sign dilepton signal from top-quark decay via a Majorana neutrino at the LHC in the top anti-top pair production samples. The signature is same-sign dilepton plus multijets with no significant missing energy. The most optimistic region lies where the Majorana neutrino mass is between 15–65 GeV. For 300 fb<sup>-1</sup> integrated luminosity, it is possible to probe  $S_{ij}$ , the effective mixing parameter, to  $\mathcal{O}(10^{-5})$ .

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## I. INTRODUCTION

Evidence for neutrino mass clearly indicates the need for new physics beyond the standard model (SM) [1]. The  $10^{12}$ order hierarchy in  $m_t/m_v$  and the large mixing in the neutrino sector also suggest a possible different mechanism for neutrino mass generation from the SM Yukawa interactions. In addition, its electric neutrality allows for the possibility of neutrinos being Majorana fermions. Consequently,  $\Delta L = 2$  lepton number violation will always occur in those theories [2–4]. Taking an effective theory approach, Majorana neutrino mass generation can be categorized into a SM gauge invariant nonrenormalizable operator [2]

# $\lambda \ell \ell HH/\Lambda_{\not\!L}$

where the  $\ell$  and H are  $SU(2)$  doublets, and  $\Lambda_{\ell}$  is the new<br>physics scale at which lepton number violation occurs. The physics scale at which lepton number violation occurs. The smallness of neutrino masses then suggests a large  $\Lambda_{\ell}$ .<br>Verious poutrino models have employed this so called Various neutrino models have employed this so-called "seesaw" spirit [3,4]. For instance, given  $\lambda \sim \mathcal{O}(1)$ , the lepton number violation scale  $\Lambda_{\nu}$  needs to be  $M_{\text{GUE}}$  to lepton number violation scale  $\Lambda_{\ell}$  needs to be  $M_{\text{GUT}}$  to obtain  $m_{\ell} \approx 0.1$  eV. This can be realized in a type Lecesary obtain  $m_{\nu} \sim 0.1 \text{ eV}$ . This can be realized in a type I seesaw<br>model [3] where a standard model singlet Majorana neumodel [3] where a standard model singlet Majorana neutrino  $N<sup>c</sup>$  is introduced per generation and the interaction is as

$$
\ell N^c H + M_N N^c N^c.
$$

The Large Hadron Collider (LHC) at CERN will provide a great opportunity for exploring physics at TeV scale. There were recently several proposals to test the neutrino mass generation mechanisms at the LHC where the new physics responsible for neutrino mass generation is of  $\mathcal{O}(10-10^3 \text{ GeV})$ . For instance, in some extended type I models, Majorana neutrino  $N$  may be accessible at the LHC [5–7]. Following the same notation in [5], in the

<span id="page-0-0"></span>presence of three Majorana neutrino states, the neutrino gauge eigenstate can be written as

$$
\nu_{iL} = \sum_{m=1}^{3} U_{im} \nu_{mL} + \sum_{m'=4}^{6} V_{im'} N_{m'L'}^{c}, \qquad (1)
$$

where  $i = e$ ,  $\mu$ ,  $\tau$ . Therefore, the interaction between charged lepton and Majorana neutrino mass eigenstates is as

$$
\mathcal{L} = -\frac{g}{2\sqrt{2}} V_{ij} W^+_\mu l_i \gamma^\mu (1 - \gamma_5) N^c_j + \text{H.c.}
$$
 (2)

In the conventional type I seesaw model where  $M_N$  is of order  $10^{14}$  GeV, the mixing  $V_{ii}$  are highly suppressed. However, in some extended type I models, this constraint can be released [5]. Here, we adopt the philosophy in [5] by taking a pure phenomenology approach without assuming any a priori relationship among the mass and mixing parameters.

This interaction will lead to direct production of Majorana neutrinos. The signal consists of dijet plus same-sign dilepton associated with no significant  $E_T$ ,

$$
q\bar{q}' \to l^{\pm}N \to l^{\pm}l^{\pm}(W^{\mp})^* \to l^{\pm}l^{\pm}jj'.
$$

Currently, the Majorana nature of neutrinos is being tested at neutrinoless double beta decay experiments  $(0\nu\beta\beta)$  [8] and it provides the strongest bound on  $V_{eN}$  as [5]

$$
\sum_{N} \frac{|V_{eN}|^2}{M_N} < 5 \times 10^{-8} \text{ GeV}^{-1}.\tag{3}
$$

The CERN LEP experiment suggests  $|V_{\mu N}|^2$ ,  $|V_{\tau N}|^2 \le$ <br>10<sup>-4</sup> 10<sup>-5</sup> for  $M_{\odot} \approx$  5.80 GeV 15.9.101. The D0 and  $10^{-4}$ – $10^{-5}$  for  $M_N \sim 5$ –80 GeV [5,9,10]. The D0 and<br>CDE detectors at Tevatron have also performed a direct CDF detectors at Tevatron have also performed a direct search the light Majorana neutrino [11].

The LHC is a ''top factory'' with a next-to-leading order production rate of about 800 pb and single top rate of about 400 pb. In this top rich environment, similar to  $W^{\pm} \rightarrow$  $l^{\pm}N \rightarrow l^{\pm}l^{\pm}(W^{\mp})^*$ , we explore top decay into  $N^c$ . The

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<span id="page-1-0"></span>unique signal final state which consists of same-sign dilepton with no significant  $\not\hspace{-.15cm}/\,F_T$  makes the discovery possible. The width for top decaying into  $N<sup>c</sup>$  has also been computed in [7] in a different treatment. In Sec. II, we will discuss the top decay into Majorana neutrino. Finally, Sec. III, we will study this specific decay mode in  $t\bar{t}$  pair production at the LHC.

# II. TOP-QUARK DECAY TO A MAJORANA NEUTRINO

As discussed in the introduction, if a Majorana neutrino occurs as an intermediate state in W decay, we will encounter a same-sign dilepton ( $\Delta L = 2$ ) final state as

$$
W^{\pm} \longrightarrow l^{\pm}N \longrightarrow l^{\pm}l^{\pm}(W^{\mp})^*.
$$

To avoid a combinatorial problem in the lepton final states, we require the  $W^*$  to decay hadronically. Therefore, for the top-quark decay through a Majorana neutrino, we are interested in the cascade as (Fig. 1)

$$
t(p) \to b(p_b) + l_i^+(l_i) + l_j^+(l_j) + q(j_1) + \bar{q}'(j_2), \quad (4)
$$

where  $p, p_b$ , etc. denote the 4-momentum of the corresponding particles. The differential decay width for this channel is given as

$$
d\Gamma_{t\rightarrow b l^{+}l^{+}q\bar{q}'} = \frac{1}{2m_{t}}|\mathcal{M}_{t\rightarrow b l^{+}l^{+}q\bar{q}'}|^{2} d\text{PS}_{5},\tag{5}
$$

where  $dPS_5$  denotes the 5-body phase space, and  $m_t$  represents the top-quark mass.

<span id="page-1-1"></span>The corresponding matrix element squared is given as follows:

$$
|\mathcal{M}_{t \to b l^{+} l^{+} q \bar{q}'}|^{2}
$$
\n
$$
= \frac{g^{8} N_{c} M_{N}^{2} |V_{iN} V_{jN}|^{2} |V_{tb}|^{2} |V_{qq'}|^{2} (1 - \frac{1}{2} \delta_{ij})}{[(p_{w}^{2} - m_{W}^{2})^{2} + \Gamma_{W}^{2} m_{W}^{2}] [(p_{w}^{2} - m_{W}^{2})^{2} + \Gamma_{W}^{2} m_{W}^{2}]}
$$
\n
$$
\times \left\{ F - \frac{G}{D_{11} D_{22}} + [l_{i} \leftrightarrow l_{j}] \right\},
$$
\n(6)

where  $g = e / \sin \theta_W$ ,  $N_c = 3$ ,  $\Gamma_W(m_W)$  is the width (mass)



FIG. 1. Like-sign lepton pair production in top-quark decay:  $t \rightarrow bl^+l^+q\bar{q}'.$ 

of the W boson,  $M_N$  is the heavy neutrino mass,  $V_{tb/qq'}$  is the CKM matrix elements,  $V_{iN}$  is the rotation of neutrino mass eigenstates defined in Eq. [\(1](#page-0-0)), and  $G, F, D_{11}, D_{22}$  are defined in the Appendix.

Majorana neutrino  $N$  width is

$$
\Gamma_N = \sum_{i=e,\mu,\tau} 18|V_{iN}|^2 \left(\frac{G_F^2 M_N^5}{192\pi^3}\right) \quad (M_N \ll m_W), \quad (7)
$$

and

$$
\Gamma_N = \sum_{i=e,\mu,\tau} |V_{iN}|^2 \left(\frac{G_F M_N^3}{8}\right) \quad (M_N \gg m_Z, m_H). \tag{8}
$$

Since the total width of a Majorana neutrino contains a factor as  $\sum_{i=e,\mu,\tau} |V_{iN}|^2$  and it will appear in the Majorana<br>neutrino proposator, we follow [5] to define an effective neutrino propagator, we follow [5] to define an effective mixing parameter as

$$
S_{ij} = \frac{|V_{iN}V_{jN}|^2}{\sum_{i=e,\mu,\tau} |V_{iN}|^2}.
$$
 (9)

We then can normalize the physics variables by  $S_{ij}$ .

The normalized branching ratio for  $t \rightarrow bl^{+}l^{+}jj'$  vs the ajorana neutrino mass  $M_{\nu}$  is plotted in Fig. 2 Majorana neutrino mass  $M_N$  is plotted in Fig. 2.

For  $M_N$  below  $m_W$ , the on-shell decay of W into Majorana neutrino can be as large as  $0.02S<sub>ij</sub>$ . If the Majorana neutrino is within  $m_W < M_N < m_t$ , the top three body decay  $t \rightarrow bl^{+}N$  with on-shell Majorana neutrino varies between  $10^{-5}S_{ij}$  and  $10^{-10}S_{ij}$ . If  $M_N > m_t$ , the decay branching ratio is less  $10^{-10}S_{ii}$  and irrelevant to our search.



FIG. 2. Normalized decay branching ratio of the process  $t \rightarrow$  $bl^+l^+q\bar{q}'.$ 

## III. DISCOVERY AT THE LHC

The LHC is a top rich environment, which enables us to use the  $t\bar{t}$  events to investigate the Majorana neutrino signals. From Fig. [2,](#page-1-0) if  $N$  is off-shell produced or from top three body decay, the chance to discover this channel will be extremely tiny. We focus on the region where  $N$  can be on-shell produced from W. The most striking signature for the Majorana neutrino production is from a same-sign dilepton  $l^{\pm}l^{\pm}$ . Therefore, the visibility of two isolated same-sign leptons is essential to our search. If  $M_N$  is in close degeneracy with  $m_W$ , the lepton from W will be extremely soft and very hard to detect. At another extreme where the  $N$  is very light, the decay products from the  $N$ will be also soft and the N boost will make the lepton and hadrons collimated and hence difficult to isolate. Therefore, the most optimistic region will be  $M_N$  within the 15–65 GeV range. We choose  $M_N = 15$  GeV for the purpose of illustration.

The total cross section of  $t\bar{t}$  production at hadron colliders is defined as follows:

$$
d\bar{\sigma} = \int dx_1 dx_2 f_{a/A}(x_1) f_{b/B}(x_2) d\hat{\sigma}_{ab \to t\bar{b}} \qquad (10)
$$

where  $f(x)$  denotes the parton distribution function, and  $d\hat{\sigma}$  represents the differential cross section at the parton level. At Tevatron and LHC, there are two dominant partonic processes:

$$
q(p_1) + \bar{q}(p_2) \to t(k_1) + \bar{t}(k_2),
$$
 (11)

$$
g(p_1) + g(p_2) \to t(k_1) + \bar{t}(k_2)
$$
 (12)

at leading order of QCD. Their differential cross sections are given as follows [12]:

$$
d\hat{\sigma}_{ab \to t\bar{t}} = \frac{1}{2\hat{s}} |\mathcal{M}_{ab}|^2 d\text{PS}_2,
$$
 (13)

where  $\hat{s} = (p_1 + p_2)^2$ ,  $dPS_2$  is the two body phase space,<br>and the corresponding matrix elements squared are as and the corresponding matrix elements squared are as follows:

$$
|\mathcal{M}_{q\bar{q}}|^2 = \frac{g_s^2 (N_c^2 - 1)}{4N_c^2} \{2 - \beta^2 (1 - y^2)\},
$$
  
\n
$$
|\mathcal{M}_{gg}|^2 = \frac{g_s^2 [N_c^2 (1 + \beta^2 y^2) - 2]}{2N_c (N_c^2 - 1)(1 - \beta^2 y^2)^2} \{1 + 2\beta^2 (1 - y^2) - \beta^4 [1 + (1 - y^2)^2]\},
$$
\n(14)

with  $y = \vec{p}_1 \cdot \vec{k}_1 / |p_1||k_1|$ , and  $\beta = \sqrt{1 - 4m_t^2 / \hat{s}}$  and  $N_c =$ 3.

<span id="page-2-0"></span>To minimize the lepton combinatorial problem, we require the second top to decay hadronically. At leading order, the final state consists of six jets (two of them are b-jets) and same-sign dilepton with no significant  $E_T$ ,

$$
pp \to t\bar{t} \to b\bar{b} + l^{\pm}l^{\pm} + j_1j_2j_3j_4. \tag{15}
$$

 $t\bar{t}$  production involves very active QCD radiation and the jets from virtual W decay are as soft as the radiation jets. It is hard to require an inclusive signature of exactly six jets. Therefore, at the trigger level, we do not impose the six jets requirement and we use the two-top reconstruction to categorize jets.

The key feature for this channel is the same-sign dilepton with no missing energy associated with it. However, due to the measurement of jet energy or electromagnetic energy of leptons,  $\not{\!\mathbb{E}}_T$  may appear. To simulate the detector effects on the energy-momentum measurements, we smear the electromagnetic energy and the muon momentum by a Gaussian distribution whose width is parameterized as [13]

$$
\frac{\Delta E}{E} = \frac{a_{\text{cal}}}{\sqrt{E/\text{GeV}}} \oplus b_{\text{cal}}, \qquad a_{\text{cal}} = 5\%, \qquad b_{\text{cal}} = 0.55\%,
$$
\n(16)

$$
\frac{\Delta p_T}{p_T} = \frac{a_{\text{track}} p_T}{\text{TeV}} \oplus \frac{b_{\text{track}}}{\sqrt{\sin \theta}}, \quad a_{\text{track}} = 15\%, \quad b_{\text{track}} = 0.5\%.
$$
\n(17)

The jet energies are also smeared using the same Gaussian formula as in Eq.  $(16)$ , but with  $[13]$ 

$$
a_{\text{cal}} = 100\%, \qquad b_{\text{cal}} = 5\%. \tag{18}
$$

The smearing simulation in Fig. 3 shows that  $\not\!\!E_T$  cannot be neglected.



FIG. 3.  $\mathbb{Z}_T$  distribution of  $pp \to t\bar{t} \to 6j + l^{\pm}l^{\pm}$  after detector smearing effect, normalized by the mixing parameter S. smearing effect, normalized by the mixing parameter  $S_{ij}$ .

We require that there is no significant  $\not\hspace{-.15cm}/\,^T T$  as

$$
\not\!E_T < 25 \text{ GeV.} \tag{19}
$$

We propose the basic cuts as

- (i) same-sign dilepton with  $p_T(l) > 10 \text{ GeV}$  and  $|\eta(l)| < 2.8$ ,
- (ii) at least 3 jets with  $p_T(j) > 50$  GeV and  $|\eta(j)| < 3.0$ ,
- (iii)  $\n *E*<sub>T</sub> < 25 \text{ GeV}, \text{ and}$
- (iv)  $R_{jl}, R_{jj}, R_{ll} > 0.4$ .

We only require three hard jets at the trigger level. However, to identify the signal, the first step is to reconstruct two tops. We demand two b-tagged jets, plus four more jets, along with the two same-sign dilepton. By first taking the three-jet invariant mass which is closest to  $m_t$ , one can group the three jets from hadronic top decay then group everything else together to construct invariant mass. Figure 4 shows the simulated signal event following this jet categorization procedure.

The top reconstruction serves two purposes. One is to identify the event and remove the multijets  $+ W^{\pm} W^{\pm}$  or  $t\bar{t}W^{\pm}$  background. By requiring the second invariant mass

$$
|M_{\text{inv}} - m_t| < 30 \text{ GeV},\tag{20}
$$

one can argue that there is no standard model background and the signal is essentially event-counting.

The second purpose is to properly group the jets. In this channel, there is no significant missing  $E<sub>T</sub>$  in the final states. This provides us a way of using only invariant mass variables to fully reconstruct the events.





FIG. 5. The minimal jet transverse momentum distribution  $\min\{p_T(j)\}\$  of  $N \rightarrow ljj'$  normalized by the mixing parameter  $S_{ij}$ .

In the case of  $M_N = 15$  GeV, the decay products from  $N \rightarrow ljj'$  will be very soft and W's from  $t \rightarrow bW$  are onshell produced. Then the N boost will enhance the jet  $p_T$ and make the  $N \rightarrow ljj'$  collimated in the N boost direction. Figure 5 shows the min $\{p_T(j)\}\$ in the event and Fig. 6 shows the min $\{\Delta R_{lj}\}$  due to N boost.

We define a cone of all these soft jets and one lepton then construct the invariant mass, which gives us the  $M_N$  as shown in Fig. [7.](#page-4-0)



FIG. 4 (color online). Invariant mass distribution of two fully reconstructed tops normalized by the mixing parameter  $S_{ij}$ . The solid line corresponds to the first-reconstructed hadronic top and the dashed line corresponds to the leptonic top.

FIG. 6. The minimal separation between lepton and jet distribution min $\{\Delta R_{lj}\}$  normalized by the mixing parameter  $S_{ij}$ .

<span id="page-4-0"></span>

FIG. 7. Invariant mass distribution of jets and lepton that reconstruct  $M_N$  normalized by the mixing parameter  $S_{ij}$ .

To illustrate another mass region, we show in Fig. 8 the total cross section of the top-quark decay to a Majorana neutrino versus  $M_N$  at the LHC energy. The solid (dashed) curve represents the cross section without (or with) the basic kinematic cuts as

- (i) same-sign dilepton with  $p_T(l) > 10$  GeV and  $|\eta(l)| < 2.8$ ,
- (ii) six jets with  $p_T(j) > 15$  GeV and  $|\eta(j)| < 3.0$ ,
- (iii)  $E_T < 25$  GeV, and
- (iv)  $R_{il}$ ,  $R_{ij}$ ,  $R_{ll}$  > 0.4.

As we argued earlier, the two-top reconstruction requirement reduces the SM background to a negligible level so the signal is just event-counting. To summarize the reach of different mass of  $N$ , in Fig. 8, we also show the three events' contour of this channel at the LHC. Since  $M_N$  is

fully reconstructable, one can use the event-counting to probe the effective mixing parameter  $S_{ij}$ .

#### IV. SUMMARY

Because of the large event sample size of the top quarks at the LHC, we consider the signal of a Majorana neutrino from top-quark decay. The signature is same-sign dilepton plus multijets with no significant missing energy. The most optimistic region lies where the Majorana neutrino mass is between 15–65 GeV. For 300 fb<sup>-1</sup> integrated luminosity, it is possible to probe the effective mixing parameter  $S_{ii}$  to  $\mathcal{O}(10^{-5})$ . Since the bounds on  $|V_{eN}|^2$  already ruled out the reach at LHC if one can identify  $e^{\pm}e^{\pm}$  final states in the reach at LHC, if one can identify  $e^{\pm}e^{\pm}$  final states in the top decay chain, it will be from  $\tau^{\pm} \tau^{\pm}$  leptonic decay. In order to get a better sensitivity than the LEP experiments on  $|V_{\mu N}|^2$ ,  $|V_{\tau N}|^2$ , it will require total integrated luminosity<br>to be higher than 200 fb<sup>-1</sup> to be higher than 200 fb<sup>-1</sup>.

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FIG. 8. Left: cross section of  $pp \rightarrow t\bar{t} \rightarrow b\bar{b}llj\bar{j}$  at LHC. Solid/dashed line without/with cuts, normalized by the mixing parameter  $S_{\text{L}}$ . Bight: three events' contour of N decay from  $t\bar{t}$  pair production at  $S_{ij}$ . Right: three events' contour of N decay from  $t\bar{t}$  pair production at the LHC.

#### APPENDIX: TOP DECAY TO A MAJORANA NEUTRINO

In this appendix, we give the derivation of the Majorana neutrino decay partial width calculation. As is well known, the top-quark width  $(\Gamma_t)$  is much smaller than its mass  $(m_t)$ . The leading pole approximation (LPA) can then be applied. Under LPA, the cross section of the process ([15](#page-2-0)) can be factorized into two parts:  $t\bar{t}$  pair production and top-quark decays, i.e.,

$$
d\sigma = \frac{1}{\Gamma_i^2} d\bar{\sigma}_{pp/p\bar{p}\to t\bar{t}} \{d\Gamma_{t\to b\bar{t}^+l^+j_1j_2} d\Gamma_{\bar{t}\to b\bar{j}_3j_4} + d\Gamma_{\bar{t}\to b\bar{t}^-l^-j_1j_2} d\Gamma_{t\to b\bar{j}_3j_4} \},
$$
 (A1)

where  $d\bar{\sigma}$  denotes the differential cross section for  $t\bar{t}$ production, and  $d\Gamma$  is the corresponding top-quark decay differential decay width.  $\Gamma_t$  is the total decay width of the top quark

$$
t(p) \to b(p_b) + l_i^+(l_i) + l_j^+(l_j) + q(j_1) + \bar{q}'(j_2), \quad (A2)
$$

where  $p$ ,  $p<sub>b</sub>$ , etc. denotes the 4-momentum of the corresponding particles. Its differential decay width is given as follows:

$$
d\Gamma_{t\to b l^+l^+q\bar{q}'} = \frac{1}{2m_t} |\mathcal{M}_{t\to b l^+l^+q\bar{q}'}|^2 d\text{PS}_5, \tag{A3}
$$

where  $dPS_5$  denotes the 5-body phase space. The quark pair  $q\bar{q}'$  is mainly  $d\bar{u}$  and  $s\bar{c}$ . The corresponding matrix element squared is given as follows:

$$
|\mathcal{M}_{t \to b l^{+} l^{+} q \bar{q}'}|^{2}
$$
\n
$$
= \frac{g^{8} N_{c} M_{N}^{2} |V_{i N} V_{i N}|^{2} |V_{tb}|^{2} |V_{qq'}|^{2} (1 - \frac{1}{2} \delta_{ij})}{[(p_{w}^{2} - m_{W}^{2})^{2} + \Gamma_{W}^{2} m_{W}^{2}] [(p_{w}^{2} - m_{W}^{2})^{2} + \Gamma_{W}^{2} m_{W}^{2}]}
$$
\n
$$
\times \left\{ F - \frac{G}{D_{11} D_{22}} + [l_{i} \leftrightarrow l_{j}] \right\},
$$
\n(A4)

where  $g = e / \sin \theta_W$ , and

$$
F = \frac{2(l_j j_1)}{D_{11}} \Big[ 4(pj_2)(p_b l_i) - \frac{2m_t^2}{m_W^2} [(j_2 p_b)[l_i \cdot (p - 2p_b)] + (pj_2)(p_b l_i) - (j_2 l_i)(p p_b)]
$$
  
+ 
$$
\frac{m_t^2 (p p_b)}{m_W^4} [2(j_2 p_w)(l_i p_w) - (l_i j_2)[m_t^2 - 2(p p_b)]] \Big],
$$
  

$$
D_{11} = D_1^2 + \Gamma_N^2 M_N^2, \qquad D_1 = (l_i - p_w)^2 - m_W^2, \qquad p_w = p - p_b,
$$
  

$$
D_{22} = D_2^2 + \Gamma_N^2 M_N^2, \qquad D_2 = (l_j - p_w)^2 - m_W^2, \qquad p_w' = j_1 + j_2.
$$
 (A5)

We use the notation  $(pp_b) \equiv p \cdot p_b$ , etc. The term G in Eq. [\(6](#page-1-1)) is from the interference between the two diagrams of Fig. [1](#page-1-0):

$$
G = [D_1 D_2 + \Gamma_N^2 M_N^2] G_1 + [D_1 - D_2] \Gamma_N M_N G_2,
$$
\n(A6)

where

$$
G_{1} = 4(pj_{2})(l_{j1})(p_{b}l_{i}) + (j_{1}l_{i})(p_{b}l_{j}) - (p_{b}j_{1})(l_{i}l_{j})\} + \frac{m_{t}^{2}(pp_{b})}{m_{W}^{4}} \{2(l_{i}j_{1})[2(j_{2}p_{w})(l_{j}p_{w}) - (l_{j}j_{2})[m_{t}^{2} - 2(pp_{b})]\} - (l_{i}l_{j})[2(j_{1}p_{w})(j_{2}p_{w}) - (j_{1}j_{2})[m_{t}^{2} - 2(pp_{b})]]\} + \frac{2m_{t}^{2}}{m_{W}^{2}} \{-2(j_{1}l_{j})[(p_{b}j_{2})[l_{i} \cdot (p - 2p_{b})] + (pj_{2})(p_{b}l_{i}) - (j_{2}l_{i})(pp_{b})]\} - (j_{2}l_{i})(pp_{b})] + (l_{i}l_{j})[(p_{b}j_{1})[j_{2} \cdot (p - 2p_{b})] + (pj_{1})(j_{2}p_{b}) - (j_{1}j_{2})(pp_{b})]\}
$$
  
\n
$$
G_{2} = (l_{i}l_{j})\left\{\omega\epsilon_{j_{1}j_{2}(l_{i}-l_{j})p_{b}} - \frac{2m_{t}^{2}(pp_{b})}{m_{W}^{4}}\epsilon_{j_{1}j_{2}l_{i}l_{j}}\right\} - 2(j_{1}l_{i})\left\{\omega\epsilon_{(j_{1}-l_{i})j_{2}l_{j}p_{b}} + \frac{2m_{t}^{2}(pp_{b})}{m_{W}^{4}}\epsilon_{j_{1}j_{2}l_{i}l_{j}}\right\} + \epsilon_{j_{1}j_{2}l_{i}l_{j}}\right\} + \epsilon_{j_{1}j_{2}l_{i}l_{j}}\left\{2\omega(j_{2}p_{b}) + \left(1 + \frac{m_{t}^{2}}{m_{W}^{2}}\right)(pp_{b})\right\} + 2\left(3 - \frac{m_{t}^{2}}{m_{W}^{2}}\right)(j_{2}l_{i})\epsilon_{j_{1}(l_{i}+j_{2})l_{j}p_{b}} + 4(p_{b}l_{i})\epsilon_{j_{1}j_{2}l_{j}p_{b}} + \left(3 - \frac{m_{t}^{2}}{m_{
$$

$$
\omega = \frac{m_t^2 - m_W^2}{m_W^2}, \qquad \epsilon_{j_1 j_2 l_i l_j} \equiv \epsilon_{\mu \nu \rho \sigma} j_1^{\mu} j_2^{\nu} l_i^{\rho} l_j^{\sigma}.
$$
\n(A8)

- [1] See e.g., V. Barger, D. Marfatia, and K. Whisnant, Int. J. Mod. Phys. E 12, 569 (2003); B. Kayser, Phys. Lett. B 592, 1 (2004); M. C. Gonzalez-Garcia and M. Maltoni, Phys. Rep. 460, 1 (2008); R. N. Mohapatra and A. Y. Smirnov, Annu. Rev. Nucl. Part. Sci. 56, 569 (2006); A. Strumia and F. Vissani, arXiv:hep-ph/0606054.
- [2] S. Weinberg, Phys. Rev. Lett. **43**, 1566 (1979).
- [3] P. Minkowski, Phys. Lett. 67B, 421 (1977); T. Yanagida, KEK Report No. 79-18, Tsukuba (1979), p. 95; M. Gell-Mann, P. Ramond, and R. Slansky, in Supergravity, edited by P. van Nieuwenhuizen et al. (North-Holland, Amsterdam, 1979), p. 315; S.L. Glashow, in Quarks and Leptons, Cargèse, edited by M. Lévy et al. (Plenum, New York, 1980), p. 707; R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. 44, 912 (1980).
- [4] W. Konetschny and W. Kummer, Phys. Lett. 70B, 433 (1977); T. P. Cheng and L. F. Li, Phys. Rev. D 22, 2860 (1980); G. Lazarides, Q. Shafi, and C. Wetterich, Nucl. Phys. B181, 287 (1981); J. Schechter and J. W. F. Valle, Phys. Rev. D 22, 2227 (1980); R. N. Mohapatra and G. Senjanović, Phys. Rev. D 23, 165 (1981).
- [5] T. Han and B. Zhang, Phys. Rev. Lett. **97**, 171804 (2006). For a more comprehensive treatment on  $\Delta L = 2$  processes at hadron colliders and in rare decays, see e.g., A. Atre, T. Han, S. Pascoli, and B. Zhang (unpublished).
- [6] S. F. King and T. Yanagida, Prog. Theor. Phys. 114, 1035 (2005); F. M. L. de Almeida, Y. D. A. Coutinho, J. A. Martins Simoes, A. J. Ramalho, S. Wulck, and M. A. B. do Vale, Phys. Rev. D 75, 075002 (2007); J. Kersten and A. Y. Smirnov, Phys. Rev. D 76, 073005 (2007); A. de Gouvea, arXiv:0706.1732; M. L. Graesser, arXiv:0705.2190; F. del Aguila, J. A. Aguilar-Saavedra, and R. Pittau, J. High Energy Phys. 10 (2007) 047; K. Huitu, S. Khalil, H. Okada, and S. K. Rai, Phys. Rev. Lett. 101, 181802 (2008); S. Bar-Shalom, G. Eilam, T. Han, and A. Soni, Phys. Rev. D 77, 115019 (2008); F. del Aguila and J. A. Aguilar-Saavedra, arXiv:0808.2468; A. Pilaftsis, Z. Phys. C 55, 275 (1992); D. Dicus, D. Karatas, and P. Roy, Phys. Rev. D 44, 2033 (1991); A. Datta, M. Guchait,

and A. Pilaftsis, Phys. Rev. D 50, 3195 (1994); F. M. L. Almeida, Y. A. Coutinho, J. A. M. Simoes, and M. A. B. Vale, Phys. Rev. D 62, 075004 (2000); O. Panella, M. Cannoni, C. Carimalo, and Y. N. Srivastava, Phys. Rev. D 65, 035005 (2002).

- [7] S. Bar-Shalom, N. G. Deshpande, G. Eilam, J. Jiang, and A. Soni, Phys. Lett. B 643, 342 (2006).
- [8] For a recent review on neutrinoless double- $\beta$  decay, see e.g., S. Elliott and J. Engel, J. Phys. G 30, R183 (2004); P. Bamert, C. P. Burgess, and R. N. Mohapatra, Nucl. Phys. B438, 3 (1995); E. Nardi, E. Roulet, and D. Tommasini, Phys. Lett. B 344, 225 (1995); G. Bélanger, F. Boudjema, D. London, and H. Nadeau, Phys. Rev. D 53, 6292 (1996); P. Benes, A. Faessler, F. Simkovic, and S. Kovalenko, Phys. Rev. D 71, 077901 (2005); A. S. Barabash, JINST 1, P07002 (2006).
- [9] O. Adriani et al. (L3 Collaboratioin), Phys. Lett. B 295, 371 (1992); P. Achard et al., Phys. Lett. B 517, 67 (2001).
- [10] P. Abreu et al. (DELPHI Collaboration), Z. Phys. C 74, 57 (1997); 75, 580 (1997); M. Z. Akrawy et al., Phys. Lett. B 247, 448 (1990).
- [11] A. Abulencia et al. (CDF Collaboration), Phys. Rev. Lett. 98, 221803 (2007).
- [12] P. Nason, S. Dawson, and R. K. Ellis, Nucl. Phys. **B303**, 607 (1988); W. Beenakker, H. Kuijf, W. L. van Neerven, and J. Smith, Phys. Rev. D 40, 54 (1989); W. Beenakker, W. L. van Neerven, R. Meng, G. A. Schuler, and J. Smith, Nucl. Phys. B351, 507 (1991); W. Bernreuther, A. Brandenburg, and Z. G. Si, Phys. Lett. B 483, 99 (2000); W. Bernreuther, A. Brandenburg, Z. G. Si, and P. Uwer, Phys. Lett. B 509, 53 (2001); Phys. Rev. Lett. 87, 242002 (2001); Nucl. Phys. B690, 81 (2004); A. Brandenburg, Phys. Lett. B 388, 626 (1996); A. Czarnecki, M. Jezabek, and J. H. Kuhn, Nucl. Phys. B351, 70 (1991); A. Brandenburg, Z. G. Si, and P. Uwer, Phys. Lett. B 539, 235 (2002).
- [13] CMS Collaboration, Report No. CERN-LHCC-2006-021; ATLAS Collaboration, Report No.CERN-LHCC-99-15.