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CP-violation parameters from decay rates of $B^{\pm} \to DK^{\pm}$, $D \to$ multibody final states

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We describe a method for measuring CP-violation parameters from which the Cabibbo-Kobayashi-Maskawa angle γ may be extracted. The method makes use of the total decay rates in $B^{\pm} \to DK^{\pm}$ decays, where the neutral D meson decays to multibody final states. We analyze the error of the method using experimental CP-violation analysis variables that enable straightforward sensitivity comparison with other methods for extracting γ, and discuss the use of B-factory and charm-factory data to obtain the relevant charm-decay information needed for this measurement. Measurement sensitivities are estimated for the currently available B-factory data sample, and D decay modes for which use of this method can make a significant contribution toward reducing the total error on γ are identified.

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I. INTRODUCTION

An important part of the program to study CP violation is the measurement of the angle $\gamma = \arg(-V_{ud}V_{ub}^{*}/V_{cd}V_{cb}^{*})$ of the unitarity triangle related to the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix [1]. Measurement of γ performed with tree-level processes defines an experimentally allowed region for the apex of the unitarity triangle. This region should overlap with the region obtained from $B^0 - \bar{B}^0$ and $B_s - \bar{B}_s$ mixing, assuming there are no significant new-physics contributions in the mixing amplitude. With this assumption, current Tevatron measurements [2] of the $B_s - \bar{B}_s$ mixing rate yield an indirect constraint on γ that is much tighter than direct measurements [3]. Therefore, precise direct determination of γ presents an opportunity to conduct an accurate test of the standard model.

The decays $B \to DK$ can be used to measure γ with essentially no hadronic uncertainties, exploiting interference between the $b \to u\bar{c}s$ and $b \to c\bar{u}s$ amplitudes of the decays $B \to \bar{D}^0 K$ and $B \to D^0 K$, respectively [4]. Interference takes place when the D meson[1] is observed in a final state F that is accessible to both D^0 and $\bar{D}^0$ decays. Such measurements can be conducted with quite a few D and B decay modes, including those with excited charm and strange mesons, involving different methods for constructing and optimizing CP-violation observables and measuring parameters related to γ. In fact, there has been a healthy stream of new ideas in this area since the basic method was first proposed in 1991 [4]. The different parameters of the various measurements are then combined statistically, yielding confidence intervals for γ [3]. The statistical sensitivity provided by each mode and method is generally poor, mainly due to the strong CKM suppression (and, for most modes [5], color suppression) in the $b \to u\bar{c}s$ transition. This necessitates the exploitation of as many modes and methods as possible, in order to achieve a small combined error on γ.

The most accurate γ measurement method to date determines γ by analyzing the D-decay event distribution in $B^{\pm} \to DK^{\pm}$ with multibody D decays [6,7]. This method was initially applied to the Cabibbo-favored decay $D \to K_S^0\pi^+\pi^-$ [8,9], and the BABAR Collaboration later used it with $K_S^0K^+K^-$ [10] and the Cabibbo-suppressed decay $D \to \pi^+\pi^-\pi^0$ [11]. A simulation study has also been conducted for the four-body mode $D \to K^+K^-\pi^+\pi^-$ [12].

As originally proposed [6], this method extracts the angle γ from measurements of $d\Gamma_{\pm}^{F}(P)/dP$, the differential decay rates of $B^{\pm} \to DK^{\pm}$ at each phase-space point P of the multibody D-decay final state F. However, measurements done with the final states $F = K_S^0\pi^+\pi^-$ and $F = K_S^0K^+K^-$ have only made use of the phase-space distributions, given by the relative differential rates

$$\frac{d\hat{\Gamma}_{\pm}^{F}(P)}{dP} \equiv \frac{d\Gamma_{\pm}^{F}(P)}{dP}\frac{1}{\Gamma_{\pm}^{F}}, \tag{1}$$

where

$$\Gamma_{\pm}^{F} \equiv \int \frac{d\Gamma_{\pm}^{F}(P)}{dP}dP \tag{2}$$

are the total decay rates. Thus, these measurements were sensitive only to the dependence of the rates on the point P, not to their integrated values $\Gamma_{\pm}F$. By contrast, the BABAR measurement with $F = \pi^+\pi^-\pi^0$ used both $d\hat{\Gamma}_{\pm}^{F}(P)/dP$ and $\Gamma_{\pm}^{F}$. For that mode, the total decay rate $\Gamma_{\pm}^{F}$ gave more precise information about the CP-violation parameters than the phase-space distribution $d\hat{\Gamma}_{\pm}^{F}(P)/dP$. While most measurements and sensitivity estimates have focused on use of $d\hat{\Gamma}_{\pm}^{F}(P)/dP$ for learning about γ, it is

[1]We use the symbol D to indicate any linear combination of a D^0 and a $\bar{D}^0$ meson state.

important to identify and study the decay modes for which the total decay rate has competitive sensitivity to the CP-violation parameters. This will help ensure that all useful modes are utilized for measuring γ, while preventing much effort from being wasted on data analysis of decay modes that are not promising.

The purpose of this paper is to provide the tools for estimating the CP-parameter sensitivities of measurements of the absolute decay rates $\Gamma_\pm^F$ for different D decay modes. We demonstrate that a good estimate of the sensitivities is provided by a single mode-dependent parameter. The impact of each mode on the combined error of γ depends on values of strong phases and decay distributions that in many cases are not well known yet. However, our general analysis of the sensitivities, performed in terms of CP-violation parameters similar to those used in the most accurate experimental analyses to date, provides a good indication as to when using the integrated decay rates is expected to improve the overall precision on γ. Since the combined error on γ depends on many measurements, its full estimation is not within the scope of this paper and is not attempted here. Rather, we compare the sensitivity of the absolute-rate analysis to that of the current-best phase-space-distribution analysis using comparable experimental CP-violation variables.

We present the formalism for the decay rates in $B^- \to DK^-$ with multibody D decays in Sec. II. Methods for measuring important charm-decay quantities are discussed in Sec. III. The sensitivities with which the CP-violation parameters are obtained from the total-rates are calculated in Sec. IV, then estimated for self-conjugate D decay modes in Sec. IV A and for non-self-conjugate modes in Sec. IV B. We provide numerical estimates for several cases, in which enough information is available for carrying out this calculation, indicating the promising and not-so-promising final states for this type of analysis. Actual data analysis of the type discussed here has been performed for only one of the decay modes we study, $D \to \pi^+\pi^-\pi^0$. For all other modes, the estimates we provide are new.

II. $B^- \to DK^-$ DECAY RATES

Consider the decay $B^\pm \to DK^\pm$, $D \to F(P)$, where D is a superposition of the D^0 and $\bar{D}^0$ states, F represents the particles comprising a multibody final state accessible through both D^0 and $\bar{D}^0$ decays, and P is a specific point in the phase space of F. We are also interested in events involving the decay $D \to \bar{F}(\bar{P})$, where $\bar{F}(\bar{P})$ is the CP conjugate of $F(P)$. The B-meson decay amplitudes to final states with specific charm flavor are parameterized and denoted in this paper in the following way:

$$A(B^- \to D^0K^-) = A(B^+ \to \bar{D}^0K^+) = A_B,$$
$$A(B^- \to \bar{D}^0K^-) = A_B z_-, \qquad A(B^+ \to D^0K^+) = A_B z_+, \tag{3}$$

where the complex numbers

$$z_\pm \equiv r_B e^{i(\delta_B \pm \gamma)} \tag{4}$$

are the CP-violation parameters of interest, $r_B \sim 0.1$ is the non-negative ratio between the magnitudes of the interfering $b \to u\bar{c}s$ and $b \to c\bar{u}s$ amplitudes, and δ_B is the CP-even phase difference between them. The magnitude $|A_B|$ is measured [13] from the rate of the process $B^- \to D^0K^-$, $D^0 \to K^-\pi^+$, where contamination by the interfering decay chain $B^- \to \bar{D}^0K^-$, $\bar{D}^0 \to K^-\pi^+$ is doubly Cabibbo supressed as well as r_B suppressed.

We define the magnitudes A_F and R_FA_F to be the square roots of the total D^0 decay rates into F and $\bar{F}$,

$$A_F \equiv \sqrt{\Gamma(D^0 \to F)} = \sqrt{\Gamma(\bar{D}^0 \to \bar{F})},$$
$$R_F \equiv \frac{1}{A_F}\sqrt{\Gamma(D^0 \to \bar{F})} = \frac{1}{A_F}\sqrt{\Gamma(\bar{D}^0 \to F)}. \tag{5}$$

The ratio R_F equals 1 for charge self-conjugate final states ($F = \bar{F}$), but can in general have any non-negative value. Equation (5) ignores the possible impact of CP violation in D decays. In addition, our use below of A_F and R_F will also ignore the effect of $D^0 - \bar{D}^0$ mixing. It has been demonstrated [14] that these effects can be neglected for the purpose of measuring γ, as long as this is done consistently for the D mesons produced in the B decay as well as for those used to determine necessary D-decay quantities, discussed in Sec. III. Alternatively, previously measured mixing and CP violation in D decays can be explicitly accounted for in the formalism [15]. For the purpose of the current discussion, it is sufficient to neglect these effects, as we do throughout this paper.

We define the normalized amplitude-distribution functions for the P-dependent charm meson decays

$$f_{D^0}^F(P) \equiv \frac{A(D^0 \to F(P))}{A_F}, \qquad f_{\bar{D}^0}^F(P) \equiv \frac{A(\bar{D}^0 \to F(P))}{A_FR_F},$$
$$f_{D^0}^{\bar{F}}(\bar{P}) \equiv \frac{A(D^0 \to \bar{F}(\bar{P}))}{A_FR_F}, \qquad f_{\bar{D}^0}^{\bar{F}}(\bar{P}) \equiv \frac{A(\bar{D}^0 \to \bar{F}(\bar{P}))}{A_F}. \tag{6}$$

These functions satisfy the relations

$$f_{D^0}^{\bar{F}}(\bar{P}) = f_{\bar{D}^0}^F(P), \qquad f_{\bar{D}^0}^{\bar{F}}(\bar{P}) = f_{D^0}^F(P) \tag{7}$$

as a result of CP conservation in the charm meson decays, and are explicitly normalized, such that

$$\int |f_{D^0}^F(P)|^2 dP = \int |f_{\bar{D}^0}^F(P)|^2 dP = 1. \tag{8}$$

Accounting for the interference between the $b \to u\bar{c}s$ and $b \to c\bar{u}s$ amplitudes in the B meson decays, the amplitudes for the four full decay chains are obtained from Eqs. (3), (5), and (6),

$$\begin{aligned} A(B^- \to F(P)K^-) &= A_0(f^F_{D^0}(P) + R_F f^F_{\bar{D}^0}(P) z_-),\\ A(B^+ \to F(P)K^+) &= A_0(R_F f^F_{\bar{D}^0}(P) + f^F_{D^0}(P) z_+),\\ A(B^- \to \bar{F}(\bar{P})K^-) &= A_0(R_F f^{\bar{F}}_{D^0}(\bar{P}) + f^{\bar{F}}_{\bar{D}^0}(\bar{P}) z_-),\\ A(B^+ \to \bar{F}(\bar{P})K^+) &= A_0(f^{\bar{F}}_{\bar{D}^0}(\bar{P}) + R_F f^{\bar{F}}_{D^0}(\bar{P}) z_+), \end{aligned} \tag{9}$$

where $A_0 \equiv |A_B| A_F$. The observable P-dependent B-decay rates are the squares of these amplitudes,

$$\begin{aligned} \frac{d\Gamma^F_\pm(P)}{dP} &= |A(B^\pm \to F(P)K^\pm)|^2,\\ \frac{d\Gamma^{\bar{F}}_\pm(\bar{P})}{d\bar{P}} &= |A(B^\pm \to \bar{F}(\bar{P})K^\pm)|^2. \end{aligned} \tag{10}$$

In the case $\bar{F} = F$, namely, when the D-decay final state is self-conjugate, only two of the four Eqs. (10) are unique. These are the modes that have been studied experimentally so far [8–11,16]. As mentioned above, measurements of $z_\pm$ using $F = K^0_S \pi^+\pi^-$ and $F = K^0_S K^+ K^-$ have been performed by analyzing only the P dependence of the event distributions $d\hat{\Gamma}_\pm(P)/dP$, disregarding the total decay rates $\Gamma^F_\pm$. Since fitting $d\hat{\Gamma}_\pm(P)/dP$ in terms of r_B, γ, and δ_B leads to an average upward bias in r_B when r_B is of order its experimental error, Refs. [8–10,16] used the CP-violation parameters

$$x_\pm \equiv \Re\{z_\pm\}, \qquad y_\pm \equiv \Im\{z_\pm\}, \tag{11}$$

which are unbiased for this type of analysis. After these parameters are measured in the analysis of $d\hat{\Gamma}^F_\pm(P)/dP$, they are converted into (in general, non-Gaussian) confidence regions in terms of the "physical" parameters r_B, γ, and δ_B.

Here, however, we wish to focus on and generalize the approach used experimentally in Ref. [11] and first studied theoretically in Ref. [17], by examining the additional information that can be extracted from the total decay rates $\Gamma^F_\pm$ and $\Gamma^{\bar{F}}_\pm$. The expressions for these rates are obtained by taking the squared absolute value of Eqs. (9) and integrating over all phase-space points,

$$\begin{aligned} \Gamma^F_- &= A_0^2(1 + R_F^2|z_-|^2 - 2R_F\Re\{z_F^* z_-\}),\\ \Gamma^F_+ &= A_0^2(R_F^2 + |z_+|^2 - 2R_F\Re\{z_F z_+\}),\\ \Gamma^{\bar{F}}_- &= A_0^2(R_F^2 + |z_-|^2 - 2R_F\Re\{z_F z_-\}),\\ \Gamma^{\bar{F}}_+ &= A_0^2(1 + R_F^2|z_+|^2 - 2R_F\Re\{z_F^* z_+\}), \end{aligned} \tag{12}$$

where

$$z_F \equiv -\int f^F_{D^0}(P)(f^F_{\bar{D}^0}(P))^* dP = -\int f^F_{D^0}(P)(f^{\bar{F}}_{D^0}(\bar{P}))^* dP \tag{13}$$

is a measure of the interference between the D^0 and $\bar{D}^0$ decay amplitudes into the final state F, averaged over the final-state phase space. The absolute value and argument of z_F are, respectively, the coherence parameter and average strong phase of Ref. [17]. For the purpose of this discussion, it will be more useful to graphically think of z_F as a coordinate-system offset parameter for $z_\pm$. Methods to measure z_F are outlined in Sec. III. The important point for now is that z_F can be measured significantly more precisely than $z_\pm$ from high-statistics D decay samples, namely,

$$\sigma_{z_F} \ll \sigma_{z_\pm}. \tag{14}$$

It is useful to represent $z_\pm$ in terms of the parameters

$$\rho_\pm \equiv z_\pm - \frac{1}{R_F} z_F, \qquad \bar{\rho}_\pm \equiv z_\pm - R_F z_F^*. \tag{15}$$

We follow Ref. [11] in referring to $\rho_\pm$ and $\bar{\rho}_\pm$ as the polar-coordinate parameters. This designation is motivated by the fact that measurement of the absolute decay rates is directly related to the radii $|\rho_\pm|$ and $|\bar{\rho}_\pm|$, via the relations

$$\begin{aligned} \Gamma^F_- &= A_0^2(1 + R_F^2|\rho_-|^2 - |z_F|^2),\\ \Gamma^F_+ &= A_0^2(R_F^2 + |\bar{\rho}_+|^2 - R_F^2|z_F|^2),\\ \Gamma^{\bar{F}}_- &= A_0^2(R_F^2 + |\bar{\rho}_-|^2 - R_F^2|z_F|^2),\\ \Gamma^{\bar{F}}_+ &= A_0^2(1 + R_F^2|\rho_+|^2 - |z_F|^2). \end{aligned} \tag{16}$$

Figure 1 demonstrates the relationship between the polar coordinates ρ_-, $\bar{\rho}_-$ and the Cartesian coordinates x_-, y_- for specific values of z_- and z_F. The absolute values $|\rho_\pm|$, $|\bar{\rho}_\pm|$ extracted from the total decay rates of Eq. (16) yield two possible values for z_- and two for z_+, for a solution of γ with a four-fold ambiguity. In that sense, this is identical to measuring γ with two, two-body D modes, as in the

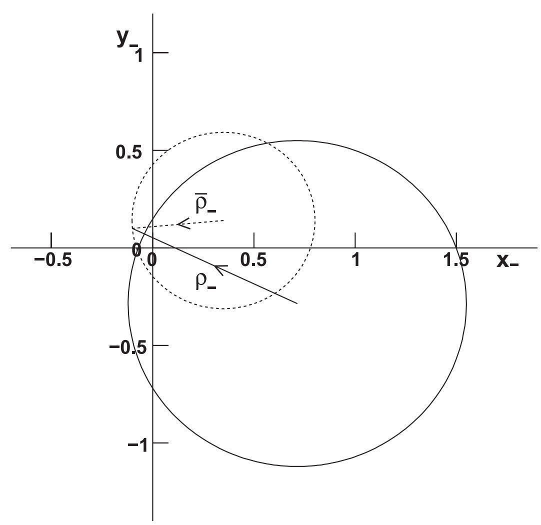

FIG. 1. The relationship between the Cartesian and polar coordinates for $z_F = 0.5 - 0.2i$, $z_- = -0.1 + 0.1i$, $R_F = 0.7$. The solid (dotted) arrow corresponds to the complex number ρ_- ($\bar{\rho}_-$) of Eq. (15). A measurement of $|\rho_-|$ ($|\bar{\rho}_-|$) implies that the true value of z_- may be anywhere on the solid (dotted) circle in the (x_-, y_-) plane. The two crossing points of the circles are the possible solutions of z_-.

method of Ref. [19], whose discrete ambiguities are further discussed in Ref. [20]. In the case of multibody D modes, analysis of the distribution of events throughout the F phase space reduces the ambiguity to two-fold, in addition to improving the total precision [6]. In effect, the event phase-space-distribution analysis measures not only the absolute value but also the phase of $\rho_\pm$ and $\bar{\rho}_\pm$ [11]. Combining the phase-space-distribution analysis with the total-rates analysis yields the most precise measurement of γ for a given D decay mode.

III. MEASURING z_F

A general approach for measuring the components of z_F from decay rates of the $\psi(3770)$ into neutral-D final states has been developed in Ref. [17]. Consider the case in which one of the $\psi(3770)$ daughters decays into $F(P)$ and the other decays into $F'(P')$, where the phase-space points P and P' do not have to be related. Because of the quantum numbers $J^{PC} = 1^{--}$ of the $\psi(3770)$, its two-D decay wave function is antisymmetric under exchange of the daughters, and hence must be $(D^0\bar{D}^0 - \bar{D}^0 D^0)/\sqrt{2}$. The normalized event density in the PP' phase space is

$$\frac{2}{B_{D^0\bar{D}^0}A_F^2 A_{F'}^2}\frac{d\Gamma_{F,F'}(P,P')}{dPdP'} = |f_{D^0}^F(P)R_{F'}f_{\bar{D}^0}^{F'}(P') - R_F f_{\bar{D}^0}^F(P) f_{D^0}^{F'}(P')|^2, \quad (17)$$

where $B_{D^0\bar{D}^0}$ is the $\psi(3770) \to D^0\bar{D}^0$ branching fraction. Integrating this expression over phase space yields the normalized rate

$$\begin{aligned}\tilde{\Gamma}_{F,F'} &\equiv \frac{2}{B_{D^0\bar{D}^0}A_F^2 A_{F'}^2}\Gamma_{F,F'} \\ &= R_F^2 + R_{F'}^2 - 2R_F R_{F'}\Re\{z_F z_{F'}^*\} \\ &= R_F^2 + R_{F'}^2 - 2R_F R_{F'}(x_F x_{F'} + y_F y_{F'}),\end{aligned} \quad (18)$$

where Eq. (13) was used, and we have separated z_F and $z_{F'}$ into their real and imaginary parts,

$$z_F \equiv x_F + iy_F, \qquad z_{F'} \equiv x_{F'} + iy_{F'}. \quad (19)$$

By measuring the decay rate of Eq. (18) for different final states F', one obtains all the information about z_F. We begin with $F' = F$, for which Eq. (18) becomes

$$\tilde{\Gamma}_{F,F} = 2R_F^2(1 - |z_F|^2) = 2R_F^2(1 - x_F^2 - y_F^2). \quad (20)$$

Next, we take F' to be a CP-even or CP-odd state, namely,

$$F' = D_\pm^0 = \frac{1}{\sqrt{2}}(D^0 \pm \bar{D}^0). \quad (21)$$

The inverse relations of Eq. (21) yield

$$z_{D_\pm^0} = \pm 1, \qquad R_{D_\pm^0} = 1, \quad (22)$$

where the normalization condition Eq. (8) has been taken into account. Then Eq. (18) becomes

$$\tilde{\Gamma}_{F,D_\pm} = 2(1 \mp x_F). \quad (23)$$

Equations (20) and (23) are sufficient for obtaining x_F and y_F, the latter with a sign ambiguity. To resolve this ambiguity, we now take F' to be the two-body state $K^-\pi^+$. Equation (18) then gives

$$\tilde{\Gamma}_{F,K^-\pi^+} = R_F^2 + R_{K^-\pi^+}^2 - 2R_F R_{K^-\pi^+}(x_F x_{K^-\pi^+} + y_F y_{K^-\pi^+}). \quad (24)$$

We have yet to determine $x_{K^-\pi^+}$ and $y_{K^-\pi^+}$. These are obtained from the rates

$$\begin{aligned}\tilde{\Gamma}_{K^-\pi^+,K^-\pi^+} &= 2R_{K^-\pi^+}^2(1 - x_{K^-\pi^+}^2 - y_{K^-\pi^+}^2), \\ \tilde{\Gamma}_{K^-\pi^+,D_\pm} &= 2(1 \mp x_{K^-\pi^+}),\end{aligned} \quad (25)$$

as in Eqs. (20) and (23), respectively. Since $K^-\pi^+$ is a two-body state, $f_{D^0}^{K^-\pi^+}$ and $f_{\bar{D}^0}^{K^-\pi^+}$ are numbers rather than functions. It then follows from Eq. (8), that $z_{K^-\pi^+}$ has unit magnitude, and the constraint

$$x_{K^-\pi^+}^2 + y_{K^-\pi^+}^2 = 1, \quad (26)$$

resolves the ambiguity in $y_{K^-\pi^+}$. Thus, it is possible to measure the real and imaginary parts of z_F with no ambiguities. Further information about y_F may be obtained from e^+e^- to $\bar{D}D\gamma$ events , where the $\bar{D}D$ pair is in a C-even state [32].

Several studies [6,21,22] have shown that when obtaining D-decay parameters from $\psi(3770)$ decays, the expected error on the CP-violation parameters due to the finite $\psi(3770)$ statistics is relatively small, given current CESR-c and B-factory integrated luminosities. A detailed simulation study [22] has shown that in the phase-space-distribution analysis with $F = K_S^0\pi^+\pi^-$, the error on γ due to the finite $\psi(3770)$ statistics is about 4 times smaller than the error due to the finite $B^- \to DK^-$ statistics in the currently available, $\sim 1\ \mathrm{ab}^{-1}$ B-factory data sample. Measurements performed by CLEO-c with 818 pb^{-1} of $e^+e^- \to \psi(3770)$ data have yielded an estimated γ error of 1°–2° due to the measurement of the $D \to K_S\pi^+\pi^-$ decay parameters [23]. CLEO-c has also measured z_F for the modes $K^-\pi^+\pi^0$ and $K^-\pi^+\pi^-\pi^+$, obtaining the preliminary results [24] $|z_{K^-\pi^+\pi^0}| = 0.79 \pm 0.08$, $\arg\{z_{K^-\pi^+\pi^0}\} = (197^{+28}_{-27})°$, $|z_{K^-\pi^+\pi^+\pi^-}| = 0.24^{+0.21}_{-0.17}$, $\arg\{z_{K^-\pi^+\pi^0}\} = (161^{+85}_{-48})°$. We note that while these errors are large, their impact on the errors of $|\bar{\rho}_\pm|$, which are the relevant CP-violation parameters for small-R_F modes (see Sec. IV B) is suppressed by R_F, as seen from Eq. (15).

Furthermore, the newly launched BEPC-II charm factory, with a design luminosity almost 20 times that of CESR-c, will be able to supply the charm data needed to match the large B samples that will be collected at LHCb and possibly at a proposed e^+e^- "super B factory." We conclude that the error on $z_\pm$ in a decay-rate analysis of the type presented in this paper will be dominated by the experimental error on $|\rho_\pm|$ and not by knowledge of z_F.

A. Self-conjugate modes

So far, all the multibody D-decay modes studied experimentally within the context of $B^\pm \to DK^\pm$ have been charge self-conjugate, i.e., $F = \bar{F}$. From Eq. (5), one sees that self-conjugate modes satisfy $R_F = 1$. In addition, Eq. (7), together with the condition $F = \bar{F}$, implies

$$f^F_{D^0}(\bar{P}) = f^F_{\bar{D}^0}(P). \tag{27}$$

As a result, such states satisfy

$$y_F = 0, \tag{28}$$

as we demonstrate by dividing the phase space of F into two equal-volume regions V and $\bar{V}$, such that every point $P \in V$ is related to a point $\bar{P} \in \bar{V}$ by the CP transformation. For example, in a three-body decay of the type $D \to a^+a^-b^0$, the division is along the line $(p_{a^+} + p_{b^0})^2 = (p_{a^-} + p_{b^0})^2$, where p_j is the four momentum of particle j. Such a division can be performed for any multibody final state that is self-conjugate, regardless of its particle multiplicity. Then

$$y_F = \Im\left\{\int_V f^F_{D^0}(P)(f^F_{\bar{D}^0}(P))^* dP\right\} + \Im\left\{\int_{\bar{V}} f^F_{D^0}(\bar{P})(f^F_{\bar{D}^0}(\bar{P}))^* d\bar{P}\right\}. \tag{29}$$

Using Eq. (27), the second integral in Eq. (29) can be written as

$$\Im\left[\int_V f^F_{\bar{D}^0}(P)(f^F_{D^0}(P))^* dP\right]. \tag{30}$$

The integrand in Eq. (30) is the complex conjugate of the integrand of the first term in Eq. (29). Therefore, their imaginary parts cancel in the sum, yielding $y_F = 0$.

In the γ-related measurements performed so far with $B^\pm \to DK^\pm$ and $D \to F$ decays into a multibody, self-conjugate state, a particular model was assumed for the functional form of $f^F_{D^0}(P)$. The parameters of the model were obtained by fitting the phase-space distribution of $D^0 \to F$ decays, where the flavor of the D^0 was tagged by its production in the decay $D^{*+} \to D^0\pi^+$. In this case, Eq. (27) guarantees that z_F can be fully determined by inserting the model $f^F_{D^0}(P)$ into Eq. (13), as done in Ref. [11]. The same cannot be done for modes that are not self-conjugate, where one must resort to the use of $\psi(3770)$ decays.

IV. EXPERIMENTAL SENSITIVITIES

Because of the linear relationship (16) between the experimentally observable rates and $|\rho_\pm|^2$, $|\bar{\rho}_\pm|^2$, these squared radii are the unbiased CP-violation parameters of choice for the rates analysis, given that decay rates can almost always be obtained from reasonably unbiased estimators. If the errors on $|\rho_\pm|^2$ and $|\bar{\rho}_\pm|^2$ are significantly smaller than the values of these parameters, then their roots $|\rho_\pm|$ and $|\bar{\rho}_\pm|$ are also unbiased parameters.

In terms of the errors on the rates, the errors on $|\rho_\pm|$ and $|\bar{\rho}_\pm|$ are

$$\sigma_{|\rho_-|} = \frac{\sigma_{\Gamma^F_-}}{2|\rho_-|A_0^2R_F^2}, \qquad \sigma_{|\bar{\rho}_+|} = \frac{\sigma_{\Gamma^F_+}}{2|\bar{\rho}_+|A_0^2},$$
$$\sigma_{|\bar{\rho}_-|} = \frac{\sigma_{\Gamma^{\bar{F}}_-}}{2|\bar{\rho}_-|A_0^2}, \qquad \sigma_{|\rho_+|} = \frac{\sigma_{\Gamma^{\bar{F}}_+}}{2|\rho_+|A_0^2R_F^2}, \tag{31}$$

where we have used Eq. (14) to neglect the error on $|z_F|$. As a result of Eqs. (14) and (15), the errors on $|\rho_\pm|$ and $|\bar{\rho}_\pm|$ are similar in magnitude to the errors on $x_\pm$ and $y_\pm$. Therefore, studying Eq. (31) provides a simple means to compare the γ sensitivity of a rates analysis using any final state F to the sensitivity of the current-best measurement, namely, that of $x_\pm$ and $y_\pm$ from the phase-space-distribution analysis of $F = K^0_S\pi^+\pi^-$. In what follows, we make quantitative estimates of the errors on $|\rho_\pm|$ and $|\bar{\rho}_\pm|$.

A. Self-conjugate modes

As a result of Eq. (28), Eq. (15) simplifies to

$$\bar{\rho}_\pm = \rho_\pm = z_\pm - x_F \tag{32}$$

for self-conjugate modes. Therefore, the two circles of Fig. 1 collapse onto each other, and the rates measurement of the radii $|\rho_\pm|$ is no longer sufficient for fully determining $z_\pm$. This is hardly a problem, for two reasons. First, the phases of $\rho_\pm$ may be determined from the event-distribution analysis, as was done in Ref. [11], yielding a measurement of $z_\pm$ whose precision is enhanced due to the use of all available experimental information. Second, as stated in the introduction, precise knowledge of γ can in any case be obtained only by combining many measurements of parameters related to γ. Therefore, measurement of $|\rho_\pm|$ helps reduce the overall error on γ, even if it is not sufficient for extracting γ without information obtained from other γ-related measurements.

Since x_F is well known, it is useful to estimate the errors on $|\rho_\pm|$ for relevant D decay modes, as they will correspond closely to the errors on $z_\pm$. Since we are dealing with the case $R_F = 1$, Eq. (31) becomes

$$\sigma_{|\rho_\pm|} = \frac{\sigma_{\Gamma_\pm}}{2|\rho_\pm|A_0^2} = \frac{1 + |\rho_\pm|^2 - x_F^2}{2|\rho_\pm|} \cdot \frac{\sigma_{\Gamma_\pm}}{\Gamma_\pm}, \tag{33}$$

where we have used $\Gamma_\pm \equiv \Gamma_\pm F = \Gamma^{\bar{F}}_\pm$, and the second equality of Eq. (33), obtained from Eq. (16), conveniently relates the $|\rho_\pm|$ errors to the relative errors on the signal branching fractions. We rely on previous "reference" experimental studies of the relevant decay modes to obtain these relative errors for any hypothetical value of $|\rho_\pm|$.

Suppose that in a reference measurement performed with B-factory data of integrated luminosity $\tilde{L}$, one observed $\tilde{N}_\pm$ signal $B^\pm \to FK^\pm$ events, from which the rates $\tilde{\Gamma}^F_\pm$ were determined and the CP-violation parameter values $|\tilde{\rho}_\pm|^2$ were calculated. Let $\tilde{N} \equiv \tilde{N}_+ + \tilde{N}_-$. Then the numbers of signal events that would be observed in an experimentally identical, hypothetical measurement of luminosity L given hypothetical values $|\rho_\pm|^2$ for the CP-violation parameters, are

$$N_\pm = \tilde{N}\tilde{r}_\pm \frac{L}{\tilde{L}}, \tag{34}$$

where

$$\tilde{r}_\pm \equiv \frac{\Gamma^F_\pm}{\tilde{\Gamma}^F_- + \tilde{\Gamma}^F_+} = \frac{1 + |\rho_\pm|^2 - x_F^2}{2 + |\tilde{\rho}_-|^2 + |\tilde{\rho}_+|^2 - 2x_F^2} \tag{35}$$

is the ratio between the value of the rate $\Gamma^F_\pm$ given the hypothetical parameter values $\rho_\pm$ and the sum of the rates $\tilde{\Gamma}^F_- + \tilde{\Gamma}^F_+$ measured in the reference measurement. The second equality in Eq. (35) arises from Eq. (16).

We assume that the error on the number of events $\tilde{N}$ in the reference measurement can be written as the sum in quadrature of a Poisson signal part and a background part, namely,

$$\sigma^2_{\tilde{N}} = \tilde{N} + \sigma^2_{\tilde{N},bgd}. \tag{36}$$

Using this relation and the published reference-measurement quantities $\tilde{N}$ and $\sigma_{\tilde{N}}$, we obtain the background contribution to the error, which we assume to be CP symmetric. Then the errors on the numbers of events in the hypothetical measurement, in which $N_\pm$ will be observed, are

$$\sigma_{N_\pm} = \sqrt{\frac{1}{2}(\sigma^2_{\tilde{N}} - \tilde{N}) + \tilde{N}\tilde{r}_\pm}\sqrt{\frac{L}{\tilde{L}}}, \tag{37}$$

where the statistical assumption leading to Eq. (36) was again used. From Eqs. (34) and (37), we obtain the relative branching-fraction errors for the hypothetical measurement

$$\frac{\sigma_{\Gamma_\pm}}{\Gamma_\pm} = \frac{\sigma_{N_\pm}}{N_\pm} = \frac{\sqrt{\frac{1}{2}(\sigma^2_{\tilde{N}} - \tilde{N}) + \tilde{N}\tilde{r}_\pm}}{\tilde{N}\tilde{r}_\pm}\sqrt{\frac{\tilde{L}}{L}}. \tag{38}$$

In Table I we report x_F, $|\rho_\pm|$, and $\sigma_{|\rho_\pm|}$ for several three-body D decay modes, assuming a data sample of 10^9 $e^+e^- \to B\bar{B}$ events, similar to the currently available B-factory sample. We obtain the values of x_F from Eq. (13), using the Dalitz-plot distributions $f^F_{D^0}(P)$, whose parameterizations are reported in Refs. [8,10,11,25] for the D-decay final states $\pi^+\pi^-\pi^0$, $K^0_S\pi^+\pi^-$, $K^0_S K^+K^-$, and $K^+K^-\pi^0$, respectively. We also obtain $\tilde{N}$, $\sigma_{\tilde{N}}$, and $\tilde{L}$ from these references, except for $K^+K^-\pi^0$, where we estimate $\tilde{N}$ and $\sigma_{\tilde{N}}$ from their values in $\pi^+\pi^-\pi^0$ [11], taking into account the ratio of branching fractions $\mathcal{B}(D^0 \to K^+K^-\pi^0)/\mathcal{B}(D^0 \to \pi^+\pi^-\pi^0)$ [26] and an assessment that the background yield in $K^+K^-\pi^0$ will be 20% of that in $\pi^+\pi^-\pi^0$. Since extraction of $|\tilde{\rho}_\pm|^2$ from the total rates has been reported only for the $F = \pi^+\pi^-\pi^0$ mode [11], we take $|\tilde{\rho}_\pm| = |\rho_\pm|$ when evaluating $\tilde{r}_\pm$ for all other modes, for lack of a better value. We take the hypothetical CP-violation parameter values $|\rho_\pm|$ from the averages of the values of $x_\pm$ and $y_\pm$ reported in Refs. [10,16],

TABLE I. Values of the inputs to Eq. (33) and the expected errors $\sigma_{|\rho_\pm|}$ for different D-decay modes and a B-factory data sample of 10^9 $e^+e^- \to B\bar{B}$ events, calculated with the CP-violation parameters of Eq. (39).

Mode	x_F	$\frac{\sigma_{\Gamma_-}}{\Gamma_-}$	$\frac{\sigma_{\Gamma_+}}{\Gamma_+}$	$\vert\rho_-\vert$	$\vert\rho_+\vert$	$\sigma_{\vert\rho_-\vert}$	$\sigma_{\vert\rho_+\vert}$
$\pi^+\pi^-\pi^0$	0.85	0.13	0.10	0.75	0.94	0.07	0.06
$K^0_S\pi^+\pi^-$	−0.02	0.05	0.05	0.13	0.08	0.19	0.32
$K^0_S K^+K^-$	−0.31	0.09	0.10	0.41	0.23	0.12	0.20
$K^+K^-\pi^0$	0.20	0.11	0.11	0.12	0.29	0.48	0.19

$$x_- = 0.097 \pm 0.034, \qquad y_- = 0.054 \pm 0.058,$$
$$x_+ = -0.087 \pm 0.031, \qquad y_+ = -0.038 \pm 0.042. \tag{39}$$

The errors of Eq. (39) reflect the sensitivity of a measurement conducted with 1.04×10^9 $e^+e^- \to B\bar{B}$ events, comparable to the value used to produce Table I.

One can see from Eq. (33), that the error $\sigma_{|\rho_\pm|}$ is small when $|\rho_\pm|$ is large. Large $|\rho_\pm|$ requires x_F to be large, by virtue of Eq. (32) and the smallness of $|z_\pm|$, demonstrated in Eq. (39). We note that some insight into the value of x_F for a particular mode can be obtained by studying the distribution of events in the D^0-decay Dalitz plot, since a generally, high level of apparent symmetry under the exchange of the two charged particles leads to a high value of x_F.

It is evident from Table I that of the three-body modes studied here, only $x_{D\to\pi^+\pi^-\pi^0}$ is large enough for Eq. (33) to yield $|\rho_\pm|$ errors that are competitive with the errors of Eq. (39). In particular, the high-statistics, low-background mode $K^0_S\pi^+\pi^-$ ends up having large $|\rho_\pm|$ errors due to the very small value of $x_{K^0_S\pi^+\pi^-}$. On the other hand, we expect that the methods for suppression of the significant background in $\pi^+\pi^-\pi^0$, which were first developed in Ref. [27], will improve in upcoming analyses. That should reduce $\sigma_{|\rho_\pm|}$ for this mode below the simple extrapolation shown in Table I.

B. Non-self-conjugate modes

We proceed to estimate the errors on $\rho_\pm$ and $\bar{\rho}_\pm$ in D-decay final states that are not self-conjugate, i.e., $F \neq \bar{F}$. As in the procedure leading up to Eq. (33), we replace A_0^2 in Eq. (31) using Eq. (16):

$$\sigma_{|\rho_-|} = \frac{\nu_{\rho_-}}{2|\rho_-|R_F^2} \cdot \frac{\sigma_{\Gamma_-^F}}{\Gamma_-^F}, \qquad \sigma_{|\bar\rho_+|} = \frac{\bar\nu_{\bar\rho_+}}{2|\bar\rho_+|} \cdot \frac{\sigma_{\Gamma_+^F}}{\Gamma_+^F},$$
$$\sigma_{|\bar\rho_-|} = \frac{\bar\nu_{\bar\rho_-}}{2|\bar\rho_-|} \cdot \frac{\sigma_{\Gamma_-^{\bar F}}}{\Gamma_-^{\bar F}}, \qquad \sigma_{|\rho_+|} = \frac{\nu_{\rho_+}}{2|\rho_+|R_F^2} \cdot \frac{\sigma_{\Gamma_+^{\bar F}}}{\Gamma_+^{\bar F}}, \tag{40}$$

where

$$\nu_{\rho_\pm} \equiv 1 + R_F^2|\rho_\pm|^2 - |z_F|^2,$$
$$\bar\nu_{\bar\rho_\pm} \equiv R_F^2 + |\bar\rho_\pm|^2 - R_F^2|z_F|^2. \tag{41}$$

As in Eq. (38), the relative errors in Eq. (40) are obtained from the number of signal events $\tilde N^F$, $\tilde N^{\bar F}$ and their errors, observed in existing reference measurements

$$\frac{\sigma_{\Gamma_\pm^F}}{\Gamma_\pm^F} = \frac{\sqrt{\frac{1}{2}(\sigma^2_{\tilde N^F} - \tilde N^F) + \tilde N^F \tilde r_\pm^F}}{\tilde N^F \tilde r_\pm^F}\sqrt{\frac{\tilde L}{L}}, \tag{42}$$

with an analogous expression for $\bar F$, where by analogy with Eq. (35)

$$\tilde r_-^F \equiv \frac{\nu_{\rho_-}}{\nu_{\tilde\rho_-} + \bar\nu_{\tilde{\bar\rho}_+}}, \qquad \tilde r_+^F \equiv \frac{\bar\nu_{\bar\rho_+}}{\nu_{\tilde\rho_-} + \bar\nu_{\tilde{\bar\rho}_+}},$$
$$\tilde r_-^{\bar F} \equiv \frac{\bar\nu_{\bar\rho_-}}{\nu_{\tilde\rho_+} + \bar\nu_{\tilde{\bar\rho}_-}}, \qquad \tilde r_+^{\bar F} \equiv \frac{\nu_{\rho_+}}{\nu_{\tilde\rho_+} + \bar\nu_{\tilde{\bar\rho}_-}}. \tag{43}$$

As in Sec. IV A, the symbols $\tilde\rho_\pm$, $\tilde{\bar\rho}_\pm$ in Eq. (43) refer to the CP-violation parameters extracted from the reference measurements $\tilde N^F$ and $\tilde N^{\bar F}$. If the total rates were not used to extract CP-violation parameters, one can naively take $\tilde\rho_\pm$ and $\tilde{\bar\rho}_\pm$ from Eq. (39) for the purpose of performing this error estimate.

Let us consider this error estimate in the case of the non-self-conjugate, three-body final state $F = K_S^0 K^- \pi^+$. With as little as 5% of their currently available data sample, the BABAR Collaboration has performed a preliminary analysis of this mode's Dalitz-plot amplitude-distribution functions $f_{D^0}^F(P)$ and $f_{D^0}^{\bar F}(\bar P)$ [28], from which we compute $|z_F| = 0.47$. The ratio $R_{K_S^0K^-\pi^+} = 0.68$ is easily extracted from the results reported in Ref. [28]. With $R_{K_S^0K^-\pi^+}$ being different from 1 yet of order 1, this mode is in a class of Cabibbo-suppressed decays expected to exhibit large interference between the $b \to u\bar c s$ and $b \to c\bar u s$ decays [29]. Unfortunately, as we show below, the combination of a small branching fraction and a medium-sized $|z_F|$ render $K_S^0 K^- \pi^+$ unattractive for extracting γ via the total-rate method.

In addition to $R_{K_S^0K^-\pi^+}$ and $|z_{K_S^0K^-\pi^+}|$, calculation of all four errors of Eq. (40) also requires knowledge of $\arg\{z_{K_S^0K^-\pi^+}\}$, which has not been measured. However, a rough estimate of the CP-parameter errors shows them to be comparable to those of the $K_S^0K^+K^-$ mode, due to the following two observations. First, the combined branching fraction $\mathcal{B}(D^0 \to K_S^0K^-\pi^+) + \mathcal{B}(D^0 \to K_S^0K^+\pi^-)$ is approximately 85% of $\mathcal{B}(K_S^0K^+K^-)$. One therefore expects the relative error on $N^{K_S^0K\pi}$ to be somewhat larger than that on $N^{K_S^0K^+K^-}$. Experimental details, such as kaon vs pion multiplicities and combinatoric background under the larger $K_S^0K\pi$ Dalitz plot, slightly increase our expectation for the ratio between the relative errors on $N^{K_S^0K\pi}$ and $N^{K_S^0K^+K^-}$. The second observation is that $|z_{K_S^0K^-\pi^+}|$ is about 50% larger than $x_{K_S^0K^+K^-}$. Combining these two competing effects, we conclude that the errors on components of the CP-violation parameters obtained from $K_S^0K\pi$ and $K_S^0K^+K^-$ should be of similar magnitudes. As seen in Table I, this implies error values that are too large to be of practical interest.

We note that Eq. (40) also holds for Cabibbo-allowed final states involving a single charged kaon, such as $F = K^-\pi^+\pi^0$, for which $R_{K^-\pi^+\pi^0} \approx 0.05$ [30] (where we have ignored the effect of $D^0 - \bar D^0$ mixing [14]). Equation (16) shows that in this case, the sensitivity of Γ_-^F and $\Gamma_+^{\bar F}$ to the CP-violation parameters is suppressed by $R^2_{K^-\pi^+\pi^0}$, making these rates useful for obtaining A_B, as mentioned in Sec. II for the $D^0 \to K^-\pi^+$ decay. However, the absolute rates Γ_+^F and $\Gamma_-^{\bar F}$ do provide a good measurement of $|\bar\rho_\pm|$. Searching for these decays in a data sample of 226×10^6 $e^+e^- \to B\bar B$ events, BABAR [31] has put an upper limit on the ratio

$$R_{\rm ADS} \equiv \frac{\Gamma_-^{\bar F} + \Gamma_+^F}{\Gamma_-^F + \Gamma_+^{\bar F}} = \frac{\bar\nu_{\bar\rho_-} + \bar\nu_{\bar\rho_+}}{\nu_{\rho_-} + \nu_{\rho_+}}, \tag{44}$$

for which the central value obtained was $\tilde R_{\rm ADS} = 0.013^{+0.010}_{-0.004}$. The rates that appear in the numerator of Eq. (44), to which we refer as the Atwood-Dunietz-Soni (ADS) rates [19], are suppressed by factors of second order in the small parameters r_B, $R_{K^-\pi^+\pi^0}$ relative to the rates in the denominator. The error on $R_{\rm ADS}$ is dominated by the statistical errors on the ADS rates. To properly account for this when calculating the relative errors on the ADS rates, we evaluate Eq. (42) with

$$\tilde r_\pm^{K\pi\pi^0} = \frac{\bar\nu_{\bar\rho_\pm}}{\bar\nu_{\tilde{\bar\rho}_-} + \bar\nu_{\tilde{\bar\rho}_+}} \tag{45}$$

instead of the expressions in Eq. (43), and take $\tilde N^{K\pi\pi^0}$ to be the number of ADS events detected in Ref. [31], namely, 19 ± 10, where the 10-event error is obtained from the naive average of the positive and negative errors on $\tilde R_{\rm ADS}$.

The resulting errors on $|\bar\rho_\pm|$ are shown in Fig. 2, calculated with Eq. (40) for different values of z_F. As in the case of Table I, we have assumed a data sample of 10^9 $e^+e^- \to B\bar B$ events and the CP-violation parameter values of Eq. (39). The errors reach values as low as 0.016 and as high as 0.035 (0.045) for $|\bar\rho_-|$ ($|\bar\rho_+|$). We see that at least one of the errors is smaller than about 0.025 for any value of z_F. For the CLEO-c central values of $z_{K^-\pi^+\pi^0}$ [24], we find $\sigma_{|\bar\rho_\pm|} \approx 0.02$.

These results suggest that one can expect measurement of the CP-violation parameters with $F = K^-\pi^+\pi^0$ to

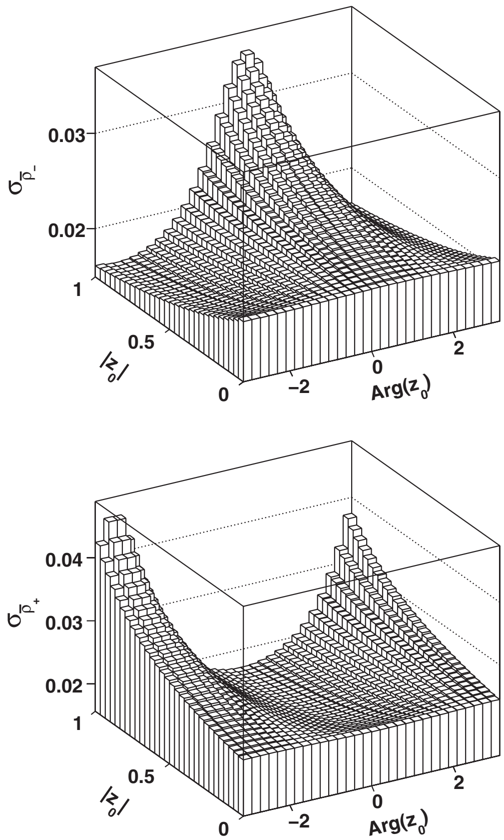

FIG. 2. The errors on $|\bar{\rho}_-|$ (top) and $|\bar{\rho}_+|$ (bottom) as functions of the absolute value and phase of z_F for $F = K^-\pi^+\pi^0$, calculated for 10^9 $e^+e^- \to B\bar{B}$ events with the CP-violation parameters of Eq. (39).

yield errors that are very competitive with the current-best measurement, Eq. (39), once the luminosity is high enough for observation of the ADS decays. Additional information about the use of phase-space distributions and absolute decay rates for measuring γ with $D \to K^-\pi^+\pi^0$ and similar modes can be found in Refs. .

V. DISCUSSION

Of the self-conjugate final states studied quantitatively here, the errors obtained from $\pi^+\pi^-\pi^0$ are the smallest, due to the large value of $x_F = \Re\{z_F\} = 0.85$ in this mode. The errors are expected to decrease beyond the estimate shown in Table I, as background suppression improves in subsequent analyses of this mode. By contrast, the final state $K^0_S\pi^+\pi^-$, which thanks in part to its large branching fraction and high purity has yielded the most precise phase-space-distribution measurements of γ to date, has a very small x_F, rendering its absolute decay rates poor measures of the CP-violation parameters.

Our calculations show that measuring $|\bar{\rho}_\pm|$ with the final state $K^-\pi^+\pi^0$ can yield very small errors, smaller than or of similar magnitude to the errors from the phase-space-distribution analysis of $K^0_S\pi^+\pi^-$. We note that similar precision may be obtained with the two- and four-body final states $K^-\pi^+$ and $K^-\pi^+\pi^-\pi^+$, whose study is outside the scope of this paper.

The results presented here cover the major three-body D-decay final states with known and significant branching fractions. It is possible that the absolute decay rates into some of the higher-multiplicity states will also turn out to yield competitive errors on γ. Among the Cabibbo-suppressed modes, this includes the final state $K^+K^-\pi^+\pi^-$, whose phase-space-distribution analysis has been studied in simulation [12], and $2\pi^+2\pi^-$. The Cabibbo-favored mode $D^0 \to K^0_S\pi^+\pi^-\pi^0$ has a large branching fraction $(5.3 \pm 0.6)\%$ [30], and may therefore be attractive for both phase-space-distribution and absolute-decay-rate analyses. Since almost half the rate is due to the resonant contribution $K^{*-}(892)\rho^+$, the phase-space distribution is highly asymmetric under exchange of the two charged pions. Therefore, it is unlikely that x_F is large for this mode. Nonetheless, given the large branching fraction, even x_F as small as 0.1 could make this mode attractive for studying γ.

VI. CONCLUSIONS

We have studied the use of the absolute $B^\pm \to DK^\pm$ decay rates, where the D decays to a multibody final state, for obtaining information with which to improve the overall knowledge of the CKM unitarity-triangle phase γ. This information is complementary to that obtained from other γ-related measurements, including analysis of the $D^0-\bar{D}^0$ interference pattern seen in the phase-space distributions of the D decay products. We have developed a formalism for estimating the error on the CP-violating parameters $|\rho_\pm|$ and $|\bar{\rho}_\pm|$. The parameter that most strongly affects the errors is z_F of Eq. (13). We have evaluated z_F for three-body D final states for which the necessary input information is available, and have estimated the errors on the CP-violation parameters for these self-conjugate modes and for the modes $K^0_SK^-\pi^+$ and $K^-\pi^+\pi^0$.

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