Pseudoscalar glueball mass from η-η'-G mixing

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We deduce the mass of the pseudoscalar glueball G from an η-η'-G mixing formalism based on the anomalous Ward identity for transition matrix elements. With the inputs from the recent KLOE experiment, we find a solution for the pseudoscalar glueball mass around (1.4 ± 0.1) GeV, which is fairly insensitive to a range of inputs with or without Okubo-Zweig-Iizuka-rule violating effects. This affirms that $\eta(1405)$, having a large production rate in the radiative J/Ψ decay and not seen in $\gamma\gamma$ reactions, is indeed a leading candidate for the pseudoscalar glueball. Other relevant quantities including the anomaly and pseudoscalar density matrix elements are obtained. The decay widths for $G \to \gamma\gamma, \ell^+\ell^-$ are also predicted.

I. INTRODUCTION

The quest for pseudoscalar glueballs has continued for decades. An $E(1420)$ meson with a mass of 1426 MeV was first discovered at CERN in 1963 through $p\bar{p}$ interactions [1]. In 1980, Mark II observed that a J/ψ meson decays via photon emission into a resonance at a mass around 1440 MeV [2]. This new state, named $\iota(1440)$ by Mark II and Crystal Ball Collaborations [3], was also once called $G(1440)$ in [4,5]. Shortly after the Mark II experiment, $E(1420)$ and $\iota(1440)$ were first proposed to be the pseudoscalar glueball candidates in [6] and in [4,5,7], respectively, while an opposite opinion that $E(1420)$ was an 1^+ $s\bar{s}$ quark state was advocated in [8]. As the experimental situation was sorted out, $E(1420)$ turned out to be an 1^+ meson now known as $f_1(1420)$, and $\iota(1440)$ was a pseudoscalar state now known as $\eta(1405)$. For an excellent review of the E and ι mesons, see [9].

$\eta(1405)$ indeed behaves like a glueball in its productions and decays. The $K\bar{K}\pi$ and $\eta\pi\pi$ channels in $\gamma\gamma$ collisions have been investigated [10]. While $\eta(1475)$ in $K\bar{K}\pi$ was observed, $\eta(1405)$ in $\eta\pi\pi$ was not. Since the glueball production is presumably suppressed in $\gamma\gamma$ collisions, the above observations suggest that the latter state has a large glueball content [11]. J/ψ radiative decays through γgg have been considered as the ideal channels of searching for glueballs. The branching ratio $\mathcal{B}(J/\psi \to \gamma\eta(1405))$ of order 10^{-3} is much larger than the decays $J/\psi \to \gamma\eta(1295), \gamma\eta(2225), \ldots$ which are either not seen or are of order 10^{-4}. The decay of a nearby $\eta(1475) \to \gamma\gamma$ has been observed [10], but $\eta(1405) \to \gamma\gamma$ has not. All these features support the proposal that $\eta(1405)$ is a good pseudoscalar glueball candidate [9]. There has also been

theoretical support based on the closed flux-tube model [12] and the model that combines the octet, the singlet, and the glueball into a decuplet [13]. Besides $\eta(1405)$, other states with masses below 2 GeV have also been proposed as the candidates, such as $\eta(1760)$ in [14] and $X(1835)$ in [15].

As for the scalar glueball, two of the authors (H. Y. C. and K. F. L.) and Chua [16] have considered a model for the glueball and $q\bar{q}$ mixing, which involves the neutral scalar mesons $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$, based on two lattice results: (i) a much better SU(3) symmetry in the scalar sector than in the other meson sectors [17] and (ii) an unmixed scalar glueball at about 1.7 GeV in the quenched approximation [18]. It was found that $f_0(1500)$ is a fairly pure octet, having very little mixing with the singlet and the glueball, while $f_0(1370)$ and $f_0(1710)$ are dominated by the glueball and the $q\bar{q}$ singlet, respectively, with about 10% mixing between them. The observed enhancement of $\omega f_0(1710)$ production over $\phi f_0(1710)$ in hadronic J/ψ decays and the copious $f_0(1710)$ production in radiative J/ψ decays lend further support to the prominent glueball nature of $f_0(1710)$.

Contrary to the above case, the pseudoscalar glueball interpretation for $\eta(1405)$ is, however, not favored by quenched lattice gauge calculations, which predicted the mass of the 0^{-+} state to be above 2 GeV in [19] and around 2.6 GeV in [18,20]. It is not favored by the sum-rule analysis with predictions higher than 1.8 GeV [21,22] either. Readers are referred to [23] for a recent review on the results of the glueball masses. Note that the above lattice calculations were performed under the quenched approximation without the fermion determinants. It is believed that dynamical fermions may have a significant effect in the pseudoscalar channel, because they raise the singlet would-be-Goldstone boson mass from that of the pion to η and η'. It has been argued that the pseudoscalar

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glueball mass in full QCD is substantially lower than that in the quenched approximation [22]. In view of the fact that the topological susceptibility is large ($\approx (191 \text{ MeV})^4$) in the quenched approximation [24], and yet is zero for full QCD in the chiral limit, it is conceivable that full QCD has a large effect on the glueball as it does on η and η'.

In this paper, we infer the pseudoscalar glueball mass m_G from the η-η'-G mixing, where G denotes the physical pseudoscalar glueball. Implementing this mixing into the equations of motion for the anomalous Ward identity, that connects the vacuum to η, η', and G transition matrix elements of the divergence of axial-vector currents to those of pseudoscalar densities and the U(1) anomaly, m_G is related to other phenomenological quantities such as the η, η' masses, the decay constants, and the mixing angles. Since the mixing angles have been measured recently from the $\phi \to \gamma \eta$, $\gamma \eta'$ decays by KLOE [25], m_G can be solved. Our numerical study gives a fairly robust result $m_G \approx 1.4$ GeV, which is insensitive to a range of inputs. We also obtain the matrix elements for the pseudoscalar densities and axial U(1) anomaly associated with the η, η', and G states. The values of the pseudoscalar density matrix elements for the η, η' mesons are close to those obtained in the Feldmann-Kroll-Stech (FKS) scheme [26], which does not include the mixing with the pseudoscalar glueball. The results of the anomaly matrix elements for the above states are quite consistent with those estimated from the topological susceptibility [27–29] and the lattice evaluation [18], indicating that the $J/\psi \to \gamma \eta'$ branching ratio could be comparable to that of $J/\psi \to \gamma G$. We then study the pseudoscalar glueball decays into two photons and two leptons $G \to \gamma\gamma$, $\ell^+\ell^-$. The comparison of our analysis with the properties of known mesons suggests that the $\eta(1405)$ meson is a strong pseudoscalar glueball candidate.

In Sec. II, we set up the formalism for the η-η'-G mixing, assuming that the glueball only mixes with the flavor-singlet η_1, but not with the flavor-octet η_8. Our parametrization for the mixing matrix contains only two angles and differs from that in [25], where it is assumed that η does not mix with the glueball state. The solution for the pseudoscalar gluaball mass m_G is derived in Sec. III with the phenomenological inputs from KLOE [25]. The solutions with the inputs from [30] and from [26] as a limit of vanishing mixture with the glueball state are also pre-

sented for comparison. It will be shown in Sec. IV that the result for m_G is stable against the variations of phenomenological inputs and of corrections violating the Okubo-Zweig-Iizuka (OZI) rule [31]. The $G \to \gamma\gamma$, $\ell^+\ell^-$ decay widths are also estimated. Section V is the conclusion.

II. η-η'-G MIXING

We extend the FKS formalism [26] for the η-η' mixing to include the pseudoscalar glueball G. In the FKS scheme, the conventional singlet-octet basis and the quark-flavor basis have been proposed. For the latter, the $q\bar{q} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$ flavor states, labeled by the η_q and η_s mesons, respectively, are defined. In the extension to the η-η'-G mixing, the physical states η, η', and G are related to the octet, singlet, and unmixed glueball states η_8, η_1, and g, respectively, through the combination of rotations

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \\ |G\rangle \end{pmatrix} = U_3(\theta)U_1(\phi_G) \begin{pmatrix} |\eta_8\rangle \\ |\eta_1\rangle \\ |g\rangle \end{pmatrix}, \qquad (1)$$

with the matrices

$$\begin{aligned} U_3(\theta) &= \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\ U_1(\phi_G) &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_G & \sin\phi_G \\ 0 & -\sin\phi_G & \cos\phi_G \end{pmatrix}. \end{aligned} \qquad (2)$$

The matrix U_1 (U_3) represents a rotation around the axis along the η_8 meson (unmixed glueball g). Equation (1) is based on the assumption that η_8 does not mix with the glueball, under which two mixing angles θ and ϕ_G are sufficient.

The octet and singlet states are related to the flavor states via

$$\begin{pmatrix} |\eta_8\rangle \\ |\eta_1\rangle \\ |g\rangle \end{pmatrix} = U_3(\theta_i) \begin{pmatrix} |\eta_q\rangle \\ |\eta_s\rangle \\ |g\rangle \end{pmatrix}, \qquad (3)$$

where θ_i is the ideal mixing angle with $\cos\theta_i = \sqrt{1/3}$ and $\sin\theta_i = \sqrt{2/3}$, i.e., $\theta_i = 54.7°$. The flavor states are then transformed into the physical states through the mixing matrix

$$U(\phi, \phi_G) = U_3(\theta)U_1(\phi_G)U_3(\theta_i) = \begin{pmatrix} \cos\phi + \sin\theta\sin\theta_i\Delta_G & -\sin\phi + \sin\theta\cos\theta_i\Delta_G & -\sin\theta\sin\phi_G \\ \sin\phi - \cos\theta\sin\theta_i\Delta_G & \cos\phi - \cos\theta\cos\theta_i\Delta_G & \cos\theta\sin\phi_G \\ -\sin\theta_i\sin\phi_G & -\cos\theta_i\sin\phi_G & \cos\phi_G \end{pmatrix}, \qquad (4)$$

with the angle $\phi = \theta + \theta_i$ and the abbreviation $\Delta_G = 1 - \cos\phi_G$. U has been written in the form, which approaches the FKS mixing matrix [26] in the $\phi_G \to 0$ limit. That is, the angle ϕ plays the same role as the mixing angle in the FKS scheme.

Our formalism assumes isospin symmetry, i.e., no mixing with π^0, and neglects other possible admixtures from $c\bar{c}$ states and radial excitations. The widely studied decay constants f_q and f_s are defined by [26]

$$\langle 0|\bar{q}\gamma^\mu\gamma_5 q|\eta_q(P)\rangle = -\frac{i}{\sqrt{2}}f_q P^\mu,$$

$$\langle 0|\bar{s}\gamma^\mu\gamma_5 s|\eta_s(P)\rangle = -if_s P^\mu \tag{5}$$

for the light quark $q = u$ or d. The η_q (η_s) meson decay constant f_q^s (f_s^q) through the s (q) quark current [32], and the unmixed glueball decay constants $f_g^{q,s}$ through the q and s quark currents, can be defined in a similar way:

$$\langle 0|\bar{q}\gamma^\mu\gamma_5 q|\eta_s(P), g(P)\rangle = -\frac{i}{\sqrt{2}}f_{s,g}^q P^\mu,$$

$$\langle 0|\bar{s}\gamma^\mu\gamma_5 s|\eta_q(P), g(P)\rangle = -if_{q,g}^s P^\mu. \tag{6}$$

The decay constants associated with the η meson, η' meson, and the physical glueball defined in

$$\langle 0|\bar{q}\gamma^\mu\gamma_5 q|\eta(P), \eta'(P), G(P)\rangle = -\frac{i}{\sqrt{2}}f_{\eta,\eta',G}^q P^\mu,$$

$$\langle 0|\bar{s}\gamma^\mu\gamma_5 s|\eta(P), \eta'(P), G(P)\rangle = -if_{\eta,\eta',G}^s P^\mu \tag{7}$$

are related to those associated with the η_q, η_s, and g states

via the same mixing matrix

$$\begin{pmatrix} f_\eta^q & f_\eta^s \\ f_{\eta'}^q & f_{\eta'}^s \\ f_G^q & f_G^s \end{pmatrix} = U(\phi, \phi_G) \begin{pmatrix} f_q & f_q^s \\ f_s^q & f_s \\ f_g^q & f_g^s \end{pmatrix}. \tag{8}$$

Sandwiching the equations of motion for the anomalous Ward identity

$$\partial_\mu(\bar{q}\gamma^\mu\gamma_5 q) = 2im_q\bar{q}\gamma_5 q + \frac{\alpha_s}{4\pi}G_{\mu\nu}\tilde{G}^{\mu\nu},$$

$$\partial_\mu(\bar{s}\gamma^\mu\gamma_5 s) = 2im_s\bar{s}\gamma_5 s + \frac{\alpha_s}{4\pi}G_{\mu\nu}\tilde{G}^{\mu\nu} \tag{9}$$

between vacuum and $|\eta\rangle$, $|\eta'\rangle$ and $|G\rangle$, where $G_{\mu\nu}$ is the field-strength tensor and $\tilde{G}^{\mu\nu}$ the dual field-strength tensor, and following the procedure in [32], we derive

$$M_{qsg}^2 = U^\dagger(\phi, \phi_G)M^2 U(\phi, \phi_G)\tilde{J}. \tag{10}$$

In the above expression the matrices are written as

$$M_{qsg}^2 = \begin{pmatrix} m_{qq}^2 + (\sqrt{2}/f_q)\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_q\rangle & m_{sq}^2 + (1/f_s)\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_q\rangle & 0 \\ m_{qs}^2 + (\sqrt{2}/f_q)\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_s\rangle & m_{ss}^2 + (1/f_s)\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_s\rangle & 0 \\ m_{qg}^2 + (\sqrt{2}/f_q)\langle 0|\alpha_s G\tilde{G}/(4\pi)|g\rangle & m_{sg}^2 + (1/f_s)\langle 0|\alpha_s G\tilde{G}/(4\pi)|g\rangle & 0 \end{pmatrix},$$

$$M^2 = \begin{pmatrix} m_\eta^2 & 0 & 0 \\ 0 & m_{\eta'}^2 & 0 \\ 0 & 0 & m_G^2 \end{pmatrix}, \qquad \tilde{J} = \begin{pmatrix} 1 & f_q^s/f_s & 0 \\ f_s^q/f_q & 1 & 0 \\ f_g^q/f_q & f_g^s/f_s & 0 \end{pmatrix}, \tag{11}$$

with the abbreviation

$$m_{qq,qs,qg}^2 \equiv \frac{\sqrt{2}}{f_q}\langle 0|m_u\bar{u}i\gamma_5 u + m_d\bar{d}i\gamma_5 d|\eta_q, \eta_s, g\rangle,$$

$$m_{sq,ss,sg}^2 \equiv \frac{2}{f_s}\langle 0|m_s\bar{s}i\gamma_5 s|\eta_q, \eta_s, g\rangle. \tag{12}$$

In the limit of the large color number N_c, the scaling for the decay constants, the pseudoscalar densities, and the anomaly matrix elements is [33]

$$f_{q,s} \sim O(\sqrt{N_c}), \qquad f_g^{q,s} \sim O(1), \qquad f_q^s \sim f_s^q \sim O(1/\sqrt{N_c}),$$

$$m_G \sim O(1), \qquad \phi_G \sim O(1/\sqrt{N_c}), \qquad m_{qq}^2 \sim O(1),$$

$$m_{ss}^2 \sim O(1), \qquad m_{qg}^2 \sim m_{sg}^2 \sim O(1/\sqrt{N_c}),$$

$$m_{qs}^2 \sim m_{sq}^2 \sim O(1/N_c), \qquad \langle 0|\alpha_s G\tilde{G}/(4\pi)|g\rangle \sim O(1),$$

$$\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_q\rangle \sim \langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_s\rangle \sim O(1/\sqrt{N_c}). \tag{13}$$

The pseudoscalar meson and glueball masses scale as $O(1)$ in large N_c. However, it has been pointed out [27–29] that

the subleading $O(1/N_c)$ term in the η_1 mass squared $m_{\eta_1}^2 \sim O(1) + O(1/N_c)$ is numerically large due to the $U(1)$ anomaly, and is related to the topological susceptibility χ in the quenched QCD without fermions. In the chiral limit, the relation $m_{\eta'}^2 = 4N_F\chi/f_\pi^2 = 2\sqrt{N_F}\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta'\rangle/f_\pi$ with N_F being the number of flavors gives $\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta'\rangle \approx 0.035$ GeV^3 for $\chi = (191$ MeV$)^4$ [24]. Although being $O(1/\sqrt{N_c})$ in large N_c, this anomaly matrix element is numerically much larger than the $O(\sqrt{N_c})$ quantities $m_{qq}^2 f_q \approx 0.0026$ GeV^3 for $m_{qq}^2 \approx m_\pi^2$ and comparable to $m_{ss}^2 f_s \approx 0.087$ GeV^3 for $m_{ss}^2 \approx 2m_K^2 - m_\pi^2$. In view of this, we shall keep all the anomaly matrix elements for η_q, η_s, and g in the following analysis. On the other hand, we expect the decay constants and the pseudoscalar density matrix elements to have the normal ordering in terms of N_c. That is, we expect $f_{q,s} > f_g^{q,s} > f_s^q, f_q^s$, $m_{qq}^2 > m_{qg}^2 > m_{qs}^2$, and $m_{ss}^2 > m_{sg}^2 > m_{sq}^2$. The above ordering is consistent with the OZI rule in that double quark annihilation, which is present in f_q^s, f_s^q, m_{qs}^2, and m_{sq}^2 but not in others, is OZI-rule violating and suppressed. We note that the two sides of each of the equations in Eq. (10) have the same N_c scaling, implying consistency of our formalism in terms of N_c.

III. PSEUDOSCALAR GLUEBALL MASS

The explicit expansion of Eq. (10) leads to

$$U_{11}^\dagger[U_{11} + U_{12}R' + U_{13}r']m_\eta^2 + U_{12}^\dagger[U_{21} + U_{22}R' \\ + U_{23}r']m_{\eta'}^2 + U_{13}^\dagger[U_{31} + U_{32}R' + U_{33}r']m_G^2 \\ = m_{qq}^2 + (\sqrt{2}/f_q)\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_q\rangle, \qquad (14)$$

$$U_{11}^\dagger[U_{11}R + U_{12} + U_{13}r]m_\eta^2 + U_{12}^\dagger[U_{21}R + U_{22} \\ + U_{23}r]m_{\eta'}^2 + U_{13}^\dagger[U_{31}R + U_{32} + U_{33}r]m_G^2 \\ = m_{sq}^2 + (1/f_s)\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_q\rangle, \qquad (15)$$

$$= m_{qs}^2 + (\sqrt{2}/f_q)\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_s\rangle, \qquad (16)$$

$$U_{21}^\dagger[U_{11}R + U_{12} + U_{13}r]m_\eta^2 + U_{22}^\dagger[U_{21}R + U_{22} \\ + U_{23}r]m_{\eta'}^2 + U_{23}^\dagger[U_{31}R + U_{32} + U_{33}r]m_G^2 \\ = m_{ss}^2 + (1/f_s)\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_s\rangle, \qquad (17)$$

$$U_{31}^\dagger[U_{11} + U_{12}R' + U_{13}r']m_\eta^2 + U_{32}^\dagger[U_{21} + U_{22}R' \\ + U_{23}r']m_{\eta'}^2 + U_{33}^\dagger[U_{31} + U_{32}R' + U_{33}r']m_G^2 \\ = m_{qg}^2 + (\sqrt{2}/f_q)\langle 0|\alpha_s G\tilde{G}/(4\pi)|g\rangle, \qquad (18)$$

$$U_{31}^\dagger[U_{11}R + U_{12} + U_{13}r]m_\eta^2 + U_{32}^\dagger[U_{21}R + U_{22} \\ + U_{23}r]m_{\eta'}^2 + U_{33}^\dagger[U_{31}R + U_{32} + U_{33}r]m_G^2 \\ = m_{sg}^2 + (1/f_s)\langle 0|\alpha_s G\tilde{G}/(4\pi)|g\rangle, \qquad (19)$$

where the parameters $r \equiv f_g^s/f_s$, $r' \equiv f_g^q/f_q$, $R \equiv f_q^s/f_s$, and $R' \equiv f_s^q/f_q$ are introduced, and U_{ij} denotes the matrix element of U. In developing our mixing formalism, the flavor-independent couplings between the glueball g and the pseudoscalar $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ states are assumed, so that g only mixes with the flavor-singlet η_1. Accordingly, we postulate $f_g^q = \sqrt{2}f_g^s$ and $f_s^q = f_q^s$, and thus the relations

$$r' = \sqrt{2}\frac{f_s}{f_q}r \qquad R' = \frac{f_s}{f_q}R, \qquad (20)$$

which will be adopted in the numerical study in Sec. IV.

We first explore the implication of the η-η'-G mixing on the glueball mass m_G. To simplify the matter, the ratios r, r', R, and R' are neglected, which are $O(1/\sqrt{N_c})$ and $O(1/N_c)$, respectively, in large N_c as shown in Eq. (13). We also neglect m_{qg}^2 and m_{sg}^2 relative to the numerically large anomaly term $\langle 0|\alpha_s G\tilde{G}/(4\pi)|g\rangle/f_{q,s}$ as an approximation. It should be safe to drop m_{qg}^2, since it is, like the small $m_{qq}^2 \approx m_\pi^2$, proportional to the light u/d quark mass. On the other hand, it is not clear if it is safe to drop m_{sg}^2. Although it is $O(1/\sqrt{N_c})$ compared to m_{ss}^2, but the latter, being proportional to the strange quark mass, is larger than $\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta'\rangle/f_s$ in the chiral limit as discussed in Sec. II. This subject will be investigated in a more detailed numerical analysis later in Sec. IV. Having made the above assumptions, we take the ratio of Eqs. (18) and (19), and obtain

$$\frac{c\theta(s\phi - c\theta s\theta_i\Delta_G)m_{\eta'}^2 - s\theta(c\phi + s\theta s\theta_i\Delta_G)^2 m_\eta^2 - s\theta_i c\phi_G m_G^2}{c\theta(c\phi - c\theta c\theta_i\Delta_G)m_{\eta'}^2 + s\theta(s\phi - s\theta c\theta_i\Delta_G)^2 m_\eta^2 - c\theta_i c\phi_G m_G^2} = \frac{\sqrt{2}f_s}{f_q}, \qquad (21)$$

where $c\phi$ ($s\phi$) is the shorthand notation for $\cos\phi$ ($\sin\phi$) and similarly for others.

Note that the above simple formula still holds, even after keeping the r'- and r-dependent terms, as long as they obey Eq. (20). In other words, the r'-dependent terms in Eq. (18) and the r-dependent terms in Eq. (19) can be absorbed into the right-hand sides of these equations and are therefore canceled after taking the ratio of Eqs. (18) and (19). The factor $\sin\phi_G$ in the numerator and the denominator of Eq. (21) has been canceled out, so that the ϕ_G dependence appears at order of $\Delta_G \approx \phi_G^2$ for small ϕ_G. As such, we find that the solution for m_G is stable against the most uncertain input ϕ_G, as long as the η, η' mesons do not mix with the glueball g intensively. The solution depends on the ratio f_s/f_q, which is, through Eqs. (15) and (16), related to

$$\frac{\sqrt{2}f_s}{f_q} = \frac{\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_q\rangle}{\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_s\rangle}, \qquad (22)$$

if the pseudoscalar density matrix elements m_{qs}^2 and m_{sq}^2 are neglected. It implies that the SU(3) symmetry breaking in the axial anomaly matrix element echoes the symmetry breaking in the decay constants, and plays a sensitive role in the determination of the pseudoscalar glueball mass. For a given ϕ_G, m_G increases with decreasing f_s/f_q. By the same token, when the anomaly matrix element for the η_s becomes larger relative to that of the η_q meson, the mass of the pseudoscalar glueball gets higher.

Before solving for m_G from Eq. (21), we explain the strategies for data fitting adopted in [25,30], which led to different extractions of the mixing angle ϕ_G. In Ref. [25], the decay constants $f_q = (1 \pm 0.01)f_\pi$ and $f_s = (1.4 \pm 0.014)f_\pi$ [34], and the parameters associated with meson wave function overlaps [35] were fixed as inputs. The angles $\phi = (39.7 \pm 0.7)°$ and $\phi_G = (22 \pm 3)°$ were then determined from the relevant data. A tiny error (1%) was assigned to f_q and f_s, which is one of the reasons why a

high precision was reached for the determination of ϕ_G. In [30] the data of $P \to \gamma V$ and $V \to \gamma P$ were first considered, which do not depend on f_q and f_s, and the fit gave the outcomes $\phi = (41.4 \pm 1.3)°$ and $\phi_G = (12 \pm 13)°$. Without precise inputs of f_q and f_s, and with the hadronic parameters for meson wave function overlaps being free, it is not unexpected to get a wide range for ϕ_G. The value of ϕ_G, being consistent with zero [36], means that the data could be accommodated by the hadronic uncertainty alone. The extracted ϕ and ϕ_G were then used as inputs to determine f_q and f_s from the η, $\eta' \to \gamma\gamma$ data. Since ϕ_G has a wide range, the results $f_q = (1.05 \pm 0.03)f_\pi$ and $f_s = (1.57 \pm 0.28)f_\pi$ also have larger errors. The correlation between ϕ_G and f_s (a smaller ϕ_G corresponding to a larger f_s) is a consequence of the constraint from these data. We also note that a larger mixing angle $\phi_G = (33 \pm 13)°$ has been extracted from the $J/\psi \to VP$ data recently [37]. In summary, both sets of parameters in [25,30] can fit the data, and are consistent with each other within their uncertainties. It is seen that f_q, f_s, and ϕ are more or less certain, but ϕ_G varies in a wider range. Fortunately, the solution for the pseudoscalar glueball mass m_G is not sensitive to ϕ_G as discussed above and will be explored further in the remainder of this paper.

As stated before, KLOE postulated that the glueball does not mix with η [25]. We shall point out that this postulate does not yield a solution for m_G in our formalism. The KLOE parametrization for the η-η'-G mixing matrix is written as

$$U_{\text{KLOE}} = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi\cos\phi_G & \cos\phi\cos\phi_G & \sin\phi_G \\ -\sin\phi\sin\phi_G & -\cos\phi\sin\phi_G & \cos\phi_G \end{pmatrix}. \quad (23)$$

Repeating the above procedure, Eq. (21) is modified to

$$\frac{s\phi c\phi_G s\phi_G(m_{\eta'}^2 - m_G^2)}{c\phi c\phi_G s\phi_G(m_{\eta'}^2 - m_G^2)} = \frac{s\phi}{c\phi} = \frac{\sqrt{2}f_s}{f_q}. \quad (24)$$

For the KLOE parameter set $f_q = f_\pi$, $f_s = 1.4f_\pi$, and $\phi = 39.7°$ [25], Eq. (24) does not hold, and there is no solution for m_G as a result.

Since we have changed the mixing matrix from KLOE's in Eq. (23) to Eq. (4), we need to refit ϕ and ϕ_G in principle. However, comparing Eqs. (4) and (23), it is easy to find that their 2×2 submatrices in the left-upper hand corner have almost equal elements for $\phi \approx 40°$ and $\phi_G \approx 22°$:

$$U = \begin{pmatrix} 0.751 & -0.654 & 0.097 \\ 0.585 & 0.725 & 0.362 \\ -0.306 & -0.216 & 0.927 \end{pmatrix},$$

$$U_{\text{KLOE}} = \begin{pmatrix} 0.766 & -0.643 & 0 \\ 0.596 & 0.710 & 0.375 \\ -0.241 & -0.287 & 0.927 \end{pmatrix}. \quad (25)$$

These four elements, which are responsible for the quark

mixing, are the only ones involved in the data fitting of $\phi \to \gamma\eta$, $\gamma\eta'$, $\eta' \to \gamma\rho$, $\gamma\omega$, and η, $\eta' \to \gamma\gamma$ mentioned above. Therefore, it is expected that the refit of the data using our parametrization will give the mixing angles close to KLOE's. That is, the KLOE parameter set can be employed directly in our numerical analysis within uncertainty. The other off-diagonal elements in Eq. (25), describing the mixing among the η, η' mesons and the glueball, do have different values. It is thus understood why the two parametrizations have similar mixing angles, but the ratios in Eqs. (21) and (24) exhibit different behaviors as far as m_G is concerned.

It is also interesting to consider the parameter set from [26] with $f_q = (1.07 \pm 0.02)f_\pi$, $f_s = (1.34 \pm 0.06)f_\pi$, $\phi = (39.3 \pm 1.0)°$, and $\phi_G = 0$ (no mixing with the pseudoscalar glueball). Note that the lower f_s in [26] arises from combined experimental and phenomenological constraints. If only the experimental constraints were used, mainly those of the η, $\eta' \to \gamma\gamma$ data, its central value would increase and the range is enlarged, giving $f_s = (1.42 \pm 0.16)f_\pi$ close to that extracted in [25]. Using the central values of f_s/f_q and ϕ_G from [25,26,30] as inputs, we derive the pseudoscalar glueball mass from Eq. (21) [see also Eqs. (27)–(29) below]

$$m_G = 1.41, 1.56, 1.30 \text{ GeV}, \quad (26)$$

respectively. The above investigation leads to $m_G = (1.4 \pm 0.1)$ GeV with the currently determined phenomenological parameters. The proximity of the predicted m_G to the mass of $\eta(1405)$ and other properties of $\eta(1405)$ make it a very strong candidate for the pseudoscalar glueball. We shall come back to visit the robustness of our prediction in the next section, when higher order effects in $1/N_c$ are included.

One may question whether other pseudoscalar mesons with masses around 1.4 GeV, such as $\eta(1295)$ and $\eta(1475)$, should be included into our mixing formalism. We note that $\eta(1295)$ and $\eta(1475)$ have been assigned as the 2^1S_0 states, namely, the radial excitations of $q\bar{q}$ and $s\bar{s}$, respectively [38,39]. As stated in the previous section, these radial excitations do not mix with the η and η' mesons by definition, since they are diagonalized under the same Hamiltonian. As for the mixing of the radial excitations with the glueball, we speculate that it is negligible for the following reason. $\eta(1295)$ is practically degenerate with the radially excited pion $\pi(1300)$, and $\eta(1475)$ is about 200 MeV above $\eta(1295)$, a situation similar to the ideal mixing in the vector meson sector with $\phi(1020)$ being $\sim$200 MeV above $\omega(780)$. This suggests that $\eta(1295)$ and $\eta(1475)$ are much like the radially excited isovector pseudoscalar $q\bar{q}$ and pseudoscalar $s\bar{s}$ without annihilation. The difference between the isoscalar η, η' mesons and the isovector pion is that the former have disconnected insertions (annihilation) with the coupling going through the contact term in the topological suscep-

tibility which pushes their masses up. By virtue of the fact that $\eta(1295)$ is degenerate with $\pi(1300)$ and $\eta(1475)$ is ~ 200 MeV above, they do not seem to acquire such an enhancement for their masses. Therefore, we venture to suggest that the annihilation process is not important for these two mesons, and their mixing with the glueball is weak.

A pseudoscalar glueball mass about 1.4 GeV was also determined from the framework of the η-η'-G mixing in [40], but with a strategy quite different from ours: The mixing is assumed to occur through a perturbative potential, so that the mixing angles are parametrized in terms of the transition strength among the states η_q, η_s, and g and their masses m_{η_q}, m_{η_s}, and m_g [40]. These parameters were then fixed from data fitting. Hence, it is the unmixed glueball mass m_g, instead of the physical glueball mass m_G, that was derived in [40]. Moreover, the result of [40] is a consequence of data fitting, while ours comes from the solution to Eq. (21). If the quark-flavor states do not mix strongly with the glueball, m_G is expected to be close to that of m_g. Following this reasoning, three possible 0^{-+} glueball candidates, $\eta(1405)$, $\eta(1475)$, and $X(1835)$ with masses around 1.4 GeV, have been examined in [40], and the latter two were found to be experimentally disfavored.

IV. NUMERICAL ANALYSIS

We now proceed to solve Eqs. (14)–(19) based on the large N_c hierarchy in Eq. (13). As discussed in Sec. II, all the anomaly matrix elements will be kept. Even though they are small parametrically ($O(1)$ and $O(1/\sqrt{N_c})$), they are large numerically. As a follow up to the last section, we first neglect the decay constants f_q^s, f_s^q, and $f_g^{q,s}$, which are $O(1/\sqrt{N_c})$ and $O(1/N_c)$ lower than $f_{q,s}$, respectively. We also neglect the pseudoscalar density matrix elements m_{qg}^2, m_{sg}^2, m_{qs}^2, and m_{sq}^2, which are similarly suppressed as compared to m_{qq}^2 and m_{ss}^2.[1] Under this approximation, our formalism involves six unknowns: three mass related terms m_G, m_{qq}^2, and m_{ss}^2, and three anomaly matrix elements $\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_q\rangle$, $\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_s\rangle$, and $\langle 0|\alpha_s G\tilde{G}/(4\pi)|g\rangle$, provided that the phenomenological quantities m_η^2, $m_{\eta'}^2$, f_q, f_s, ϕ, and ϕ_G are given as inputs. There are six equations from Eq. (10), so the six unknowns can be solved in principle. We note in passing that the four unknowns m_{qq}^2, m_{ss}^2, $\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_q\rangle$, and $\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_s\rangle$ were solved for five given inputs m_η^2, $m_{\eta'}^2$, f_q, f_s, and ϕ in the η-η' mixing case [41].

Using the central values of the parameter sets from [25,26,30] for f_q, f_s, ϕ, and ϕ_G as inputs, we obtain the following solutions:

$$m_{qq}^2 = -0.073 \text{ GeV}^2,$$
$$\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_q\rangle = 0.069 \text{ GeV}^3,$$
$$m_{ss}^2 = 0.52 \text{ GeV}^2,$$
$$\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_s\rangle = 0.035 \text{ GeV}^3, \quad (27)$$
$$m_G = 1.41 \text{ GeV},$$
$$\langle 0|\alpha_s G\tilde{G}/(4\pi)|g\rangle = -0.033 \text{ GeV}^3,$$

$$m_{qq}^2 = -0.084 \text{ GeV}^2,$$
$$\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_q\rangle = 0.067 \text{ GeV}^3,$$
$$m_{ss}^2 = 0.50 \text{ GeV}^2,$$
$$\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_s\rangle = 0.032 \text{ GeV}^3, \quad (28)$$
$$m_G = 1.30 \text{ GeV},$$
$$\langle 0|\alpha_s G\tilde{G}/(4\pi)|g\rangle = -0.015 \text{ GeV}^3,$$

and

$$m_{qq}^2 = 0.012 \text{ GeV}^2,$$
$$\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_q\rangle = 0.054 \text{ GeV}^3,$$
$$m_{ss}^2 = 0.50 \text{ GeV}^2,$$
$$\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_s\rangle = 0.030 \text{ GeV}^3, \quad (29)$$
$$m_G = 1.56 \text{ GeV},$$
$$\langle 0|\alpha_s G\tilde{G}/(4\pi)|g\rangle = 0 \text{ GeV}^3.$$

The above solutions give us an idea of the range of uncertainties in our predictions. It is observed that the solutions for the anomaly matrix elements associated with the η_q and η_s mesons change little in Eqs. (27)–(29). However, $\langle 0|\alpha_s G\tilde{G}/(4\pi)|g\rangle$ for the pseudoscalar glueball varies with the inputs of ϕ_G and f_s/f_q strongly as can be seen from Eqs. (18) and (19). The solutions of m_{qq}^2 in Eqs. (27) and (28) deviate from the naive expectation $m_{qq}^2 = m_\pi^2$ [26], while that in Eq. (29) is in better agreement with m_π^2 due to a smaller f_s. The solutions of m_{ss}^2, on the other hand, are stable with respect to the various inputs, and are close to the expected leading N_c result $m_{ss}^2 = 2m_K^2 - m_\pi^2$. The values for m_G have been shown in Eq. (26) already. We should comment that m_{qq}^2 is small because of the cancellation of two large terms as pointed out in [41]. It flips sign easily, depending on the inputs of f_s/f_q and OZI-rule violating effects, which have been considered before in the two-angle formalism for the η-η' mixing [42,43]. Our opinion is that introducing the OZI-rule suppressed decay constants f_q^s, f_s^q [32] is more transparent than employing the multiple-angle formalism. It has been observed that the tiny corrections from f_q^s and f_s^q can turn a negative m_{qq}^2 into a positive value easily due to the smallness of m_{qq}^2 [32].

In the following, we investigate the higher $O(1/N_c)$ effects from the decay constants $f_g^{q,s}$, f_q^s, and f_s^q, i.e.,

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[1]The off-diagonal mass terms m_{sq}^2 and m_{qs}^2 have been absorbed into the matrix elements $\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_q\rangle$ and $\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_s\rangle$, respectively, in [26].

TABLE I. Solutions for various inputs of $\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_q\rangle = 0.065$ GeV^3 (the first row), 0.050 GeV^3 (the second row), and 0.035 GeV^3 (the third row) with $m_{qq}^2 = m_\pi^2$, $m_{ss}^2 = 2m_K^2 - m_\pi^2$, $m_{sg}^2 = m_{qg}^2 = m_{qs}^2 = m_{sq}^2 = 0$, and $\phi = 42.4°$. The upper (lower) table is for $\phi_G = 22°$ ($\phi_G = 12°$).

r	R	f_s	m_G (GeV)	$\langle 0|\frac{\alpha_s}{4\pi}G\tilde{G}|\eta_s\rangle$ (GeV^3)	$\langle 0|\frac{\alpha_s}{4\pi}G\tilde{G}|g\rangle$ (GeV^3)
0.004	−0.002	$1.25f_\pi$	1.50	0.037	−0.038
0.22	−0.002	$1.25f_\pi$	1.50	0.028	0.036
0.44	−0.002	$1.25f_\pi$	1.50	0.020	0.111
−0.26	−0.003	$1.28f_\pi$	1.44	0.036	−0.108
0.16	−0.003	$1.28f_\pi$	1.44	0.028	0.035
0.58	−0.003	$1.28f_\pi$	1.44	0.019	0.178

from r and R on our solutions. The pseudoscalar density matrix elements m_{qg}^2, m_{sg}^2, m_{qs}^2, and m_{sq}^2 are still ignored. We take $m_{qq}^2 = m_\pi^2$, $m_{ss}^2 = 2m_K^2 - m_\pi^2$ and $\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_q\rangle = 0.065$, 0.050, and 0.035 GeV^3 [the typical values from Eqs. (27)–(29)], as the inputs in order to solve for the unknowns r, R, and f_s. The relation $m_{ss}^2 = 2m_K^2 - m_\pi^2$ seems to hold well for the earlier solutions in Eqs. (27)–(29). Thus, it is reasonable to fix it to its leading N_c value. Taking $f_q = f_\pi$, $\phi = 42.4°$, and $\phi_G = 22°$ and 12°, the corresponding solutions are listed in Table I. The results of R, f_s, and m_G are independent of the inputs of $\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_q\rangle$, reaffirming that m_G is independent of r as seen from Eq. (21). The magnitude of R is smaller than that of r, which in turn is smaller than unity. This finding is in agreement with the large N_c counting rule. The decay constant f_s turns out to be lower than those in [25,30], following from the observation that a smaller f_s leads to a positive m_{qq}^2 [41]. The values of m_G and $\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_s\rangle$ are consistent with the range derived in Eqs. (27)–(29), implying that these higher $O(1/N_c)$ effects are small. The parameters r and $\langle 0|\alpha_s G\tilde{G}/(4\pi)|g\rangle$ are found to be sensitive to the inputs, and both of them increase with decreasing $\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_q\rangle$.

Finally, we explore the impact of m_{sg}^2 on our solutions. To do so, we add f_s as an input so that m_{sg}^2 can be introduced as an unknown. m_{qg}^2 is not considered, because its effect should be very minor as explained before. The results for the various inputs of $f_s = (1.24-1.30)f_\pi$, $\phi_G = 22°$ and 12°, and $\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_q\rangle = 0.050(0.035)$ GeV^3

are listed in Table II (III). In the large N_c analysis for the resolution of the $U(1)$ anomaly [27–29], it is the combined contribution from a contact term and the glueball that cancels the η' contribution to give a zero topological susceptibility in full QCD in the chiral limit. This combined contribution is just the topological susceptibility χ_quench in the quenched QCD, which leads to the Witten-Veneziano mass formula $m_{\eta'}^2 = 4N_F\chi_\text{quench}/f_\pi^2$. χ_quench is calculated to be ≈ 0.00133 GeV^4 [24], and the quenched glueball contributes about -11% to χ_quench [18], which makes the contact term to be ≈ 0.00148 GeV^4. It is observed that the anomaly matrix element $\langle 0|\alpha_s G\tilde{G}/(4\pi)|g\rangle$ is 0.105 GeV^3 or larger in Table III. This anomaly matrix element contributes $\langle 0|\alpha_s G\tilde{G}/(4\pi)|g\rangle^2/(-4m_g^2) \approx -0.00141$ GeV^4 to the topological susceptibility for $\langle 0|\alpha_s G\tilde{G}/(4\pi)|g\rangle = 0.105$ GeV^3 and $m_g = 1.4$ GeV. Namely, the glueball contribution is as large as but destructive to the contact term. As the glueball contribution and the contact term already cancel each other to a large extent, there is no room left for the contact term to cancel the sizable η and η' contributions in order to end in a very small topological susceptibility in full QCD, which has a value $\chi(\text{full QCD}) = -\langle \bar{\psi}\psi\rangle/(1/m_u + 1/m_d + 1/m_s) \sim 4 \times 10^{-5}$ GeV^4 [44]. It implies that the anomaly matrix element $\langle 0|\alpha_s G\tilde{G}/(4\pi)|g\rangle = 0.105$ GeV^3 is probably too large.

Based on the above reasoning, we believe that $\langle 0|\alpha_s G\tilde{G}/(4\pi)|g\rangle \geq 0.105$ GeV^3 is not likely to be a viable solution. This criterion would exclude all the results in

TABLE II. Same as Table I except that $\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_q\rangle = 0.050$ GeV^3 and f_s is fixed to trade for m_{sg}^2 as a free parameter.

f_s	r	R	m_{sg}^2 (GeV^2)	m_G (GeV)	$\langle 0|\frac{\alpha_s}{4\pi}G\tilde{G}|\eta_s\rangle$ (GeV^3)	$\langle 0|\frac{\alpha_s}{4\pi}G\tilde{G}|g\rangle$ (GeV^3)
$1.24f_\pi$	0.22	−0.001	−0.009	1.60	0.028	0.036
$1.26f_\pi$	0.22	−0.003	0.004	1.47	0.028	0.036
$1.28f_\pi$	0.23	−0.005	0.016	1.34	0.028	0.038
$1.30f_\pi$	0.24	−0.007	0.029	1.21	0.028	0.040
$1.24f_\pi$	0.12	0.001	−0.054	2.15	0.027	0.030
$1.26f_\pi$	0.13	−0.001	−0.029	1.84	0.027	0.031
$1.28f_\pi$	0.15	−0.003	−0.005	1.52	0.027	0.034
$1.30f_\pi$	0.24	−0.005	0.018	1.16	0.028	0.045

TABLE III.   Same as Table II except $\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_q\rangle = 0.035$ GeV^3.

f_s	r	R	m_{sg}^2 (GeV^2)	m_G (GeV)	$\langle 0|\frac{\alpha_s}{4\pi}G\tilde{G}|\eta_s\rangle$ (GeV^3)	$\langle 0|\frac{\alpha_s}{4\pi}G\tilde{G}|g\rangle$ (GeV^3)
$1.24f_\pi$	0.40	-0.001	-0.009	1.60	0.019	0.105
$1.26f_\pi$	0.45	-0.003	0.004	1.47	0.020	0.113
$1.28f_\pi$	0.54	-0.005	0.016	1.34	0.021	0.126
$1.30f_\pi$	0.69	-0.007	0.029	1.21	0.022	0.148
$1.24f_\pi$	0.27	0.001	-0.054	2.15	0.018	0.136
$1.26f_\pi$	0.34	-0.001	-0.029	1.84	0.018	0.146
$1.28f_\pi$	0.51	-0.003	-0.005	1.52	0.019	0.168
$1.30f_\pi$	1.18	-0.005	0.018	1.16	0.023	0.262

Table III with $\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_q\rangle = 0.035$ GeV^3 as an input. For Table II, it is seen that m_{sg}^2 and m_G do depend on f_s sensitively. In some cases, we have m_G as large as 1.84 GeV and 2.15 GeV, for which m_{sg}^2 are negative and large. We cannot discard these solutions of m_{sg}^2 *a priori*, but they are not favored due to their negative values. This issue can be sorted out, when lattice calculations of m_{sg}^2 with dynamical fermions are available. As $f_s \geq 1.30f_\pi$, m_G becomes smaller than 1.2 GeV, where there are no pseudoscalar glueball candidates. Therefore, if excluding the solutions with large and negative m_{sg}^2, the range (1.4 ± 0.1) GeV of the pseudoscalar glueball mass obtained in Sec. III will be more or less respected.

Having studied the higher $O(1/N_c)$ effects and confirmed that they are small, modulo the uncertainty regarding m_{sg}^2, we shall simply use the typical results in Eq. (27) [Eq. (28)] to obtain the anomaly matrix elements for the physical states η, η', and G

$$\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta\rangle = 0.026(0.028) \text{ GeV}^3,$$

$$\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta'\rangle = 0.054(0.057) \text{ GeV}^3, \qquad (30)$$

$$\langle 0|\alpha_s G\tilde{G}/(4\pi)|G\rangle = -0.059(-0.041) \text{ GeV}^3.$$

It is found that the value of $\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta\rangle$ is close to $\sqrt{3/2}f_\eta m_\eta^2/3 \approx 0.021$ GeV^3 obtained in [45], and $\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta'\rangle$ is within a factor of 2 from its chiral limit estimated from the topological susceptibility, i.e., $\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta'\rangle = 2\sqrt{N_F}\chi/f_\pi = 0.035$ GeV^3. Equation (30) also reveals that $\langle 0|\alpha_s G\tilde{G}/(4\pi)|G\rangle$ is almost the same as that from the quenched lattice QCD calculation, which gives $|\langle 0|\alpha_s G\tilde{G}/(4\pi)|G\rangle| = (0.06 \pm 0.01)$ GeV^3 [18]. The fact that $\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta'\rangle$ is com-

parable to $\langle 0|\alpha_s G\tilde{G}/(4\pi)|G\rangle$, which defies the large N_c scaling in Eq. (13), implies that the η' meson production in the J/ψ radiative decay may have a branching ratio as large as that for the pseudoscalar glueball production.

Given the mixing angles, we can predict the widths of the $G \to \gamma\gamma$, $\ell^+\ell^-$ decays, assuming that they take place through the quark content [11]. The ratio of the $G \to \gamma\gamma$ width over the $\pi^0 \to \gamma\gamma$ one is expressed as

$$\frac{\Gamma(G \to \gamma\gamma)}{\Gamma(\pi^0 \to \gamma\gamma)} = \frac{1}{9}\left(\frac{m_G}{m_{\pi^0}}\right)^3\left(5\frac{f_\pi}{f_q}\sin\theta_i\sin\phi_G\right.$$
$$\left. + \sqrt{2}\frac{f_\pi}{f_s}\cos\theta_i\sin\phi_G\right)^2. \qquad (31)$$

We have confirmed that both the parameter sets in [25,30] give the η, $\eta' \to \gamma\gamma$ widths in agreement with the data $\Gamma(\eta \to \gamma\gamma) \approx 0.51$ keV and $\Gamma(\eta' \to \gamma\gamma) \approx 4.28$ keV [38], by considering the similar ratios for the η, η' mesons. The parameter set in [25] ([30]) leads to a ratio 387 (83.3) in Eq. (31), i.e., the decay width $\Gamma(G \to \gamma\gamma) = 3(0.6)$ keV for $\Gamma(\pi^0 \to \gamma\gamma) = 7.7$ eV [38]. If $\eta(1405)$ is a pseudoscalar glueball, we predict the branching ratio $\mathcal{B}(\eta(1405) \to \gamma\gamma) = 6(1) \times 10^{-5}$, i.e., an order of 10^{-5} for the total decay width $\Gamma(\eta(1405)) = 51$ MeV [38]. The above result can be confronted with future experimental data. The "stickiness" S has been proposed to be a useful quantity for identifying a glueball rich state [46], which is defined as the ratio of $\Gamma(J/\psi \to \gamma G)$ to $\Gamma(G \to \gamma\gamma)$ with the phase space factors taken out. Combining our predictions for the pseudoscalar glueball production and decay, we obtain $S = 18$–80 for G, which is much larger than $S = 1$ as defined for the η meson.

For the $G \to \ell^+\ell^-$ decays, we calibrate their widths using the available $\pi^0 \to e^+e^-$ and $\eta \to \mu^+\mu^-$ data:

$$\frac{\Gamma(G \to e^+e^-)}{\Gamma(\pi^0 \to e^+e^-)} = \frac{1}{9}\left(\frac{m_G}{m_{\pi^0}}\right)^3\left(5\frac{f_\pi}{f_q}\sin\theta_i\sin\phi_G + \sqrt{2}\frac{f_\pi}{f_s}\cos\theta_i\sin\phi_G\right)^2,$$

$$\frac{\Gamma(G \to \mu^+\mu^-)}{\Gamma(\eta \to \mu^+\mu^-)} = \left(\frac{m_G}{m_\eta}\right)^3\left(5\frac{f_\pi}{f_q}\sin\theta_i\sin\phi_G + \sqrt{2}\frac{f_\pi}{f_s}\cos\theta_i\sin\phi_G\right)^2$$
$$\times\left[5\frac{f_\pi}{f_q}(\cos\phi + \sin\theta\sin\theta_i\Delta_G) - \sqrt{2}\frac{f_\pi}{f_s}(\sin\phi + \sin\theta\cos\theta_i\Delta_G)\right]^{-2}. \qquad (32)$$

For $\Gamma(\pi^0 \to e^+e^-) = 4.8 \times 10^{-7}$ eV [38], we obtain $\Gamma(G \to e^+e^-) = 1.9(0.4) \times 10^{-4}$ eV using the parameter set from [25] ([30]). For $\Gamma(\eta \to \mu^+\mu^-) = 7.5 \times 10^{-3}$ eV [38], we have $\Gamma(G \to \mu^+\mu^-) = 4.0(1.0) \times 10^{-2}$ eV. If $\eta(1405)$ is a pseudoscalar glueball, the above predictions correspond to the branching ratios $\mathcal{B}(\eta(1405) \to e^+e^-) = 4(0.8) \times 10^{-12}$ and $\mathcal{B}(\eta(1405) \to \mu^+\mu^-) = 8(2) \times 10^{-10}$, which would be quite a challenge to observe experimentally.

V. CONCLUSION

In this paper, we have formulated the η-η'-G mixing scheme via the vacuum to meson transition matrix elements for the anomalous Ward identity. The extension to include the glueball mixing with the flavor-singlet η_1 is a generalization of the FKS scheme for the η-η' mixing [26]. Therefore, only one extra angle ϕ_G for the mixing of the glueball state g and η_1 is introduced in addition to the angle ϕ in the FKS scheme. We have explained the different parameter extractions from the same set of η, η' meson data in [25,30], which give an idea of the uncertainties contained in the inputs. The obtained pseudoscalar glueball mass m_G around 1.4 GeV is much lower than the results from quenched lattice QCD (> 2.0 GeV). It has been examined that our solution for m_G depends weakly on the ratio of the decay constants f_s/f_q in the favored phenomenological range and is stable against the variation of ϕ_G and the higher $O(1/N_c)$ corrections.

There may not exist a unique feature that tells a glueball apart from a quark-antiquark state. We need to combine information from J/ψ radiative decays, hadronic decays, as well as $\gamma\gamma$ and leptonic decays as advocated in [47]. The comparison of our solutions with the available data suggests that $\eta(1405)$, which is copiously produced in the J/ψ radiative decay but has not been seen in the $\gamma\gamma$ reaction, is a strong pseudoscalar glueball candidate. The anomaly matrix elements $\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta'\rangle$ and $\langle 0|\alpha_s G\tilde{G}/(4\pi)|G\rangle$ in Eq. (30) are in reasonable agreement with those estimated from the topological susceptibility and quenched lattice calculation. According to our analysis, the $\eta(1405) \to \gamma\gamma$ decay width is 0.6–3 keV, and the leptonic decays $\eta(1405) \to \ell^+\ell^-$ are very small. Both predictions can be confronted with future experiments.

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